Effective Lagrangian for $\bar{s}bg$ and $\bar{s}b\gamma$ vertices in the minimal supergravity model

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Complete expressions of the $\bar{s}bg$ and $\bar{s}b\gamma$ vertices are derived in the framework of supersymmetry with minimal flavor violation. As examples, the branching ratios of charmless B decays $[B\rightarrow K+X$ (no charm)] and exclusive processes $B_s\rightarrow\gamma\gamma$ are calculated with the minimal supergravity assumptions.

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I. INTRODUCTION

The rare B decays serve as a good test for new physics beyond the standard model (SM) since they are not seriously affected by the uncertainties due to long distance effects. The forthcoming B factories will make more precise measurements on the rare B-decay processes and those measurements should set more strict constraints on the new physics beyond SM. The main purpose of investigating B decays, especially the rare decay modes, is to search for traces of new physics and determine its parameter space. In all the extensions of the SM, supersymmetry is considered as one of the most plausible candidates. In the general supersymmetric extension of the SM, new sources of flavor violation may appear in those soft breaking terms [1]. Applying the mass insertion method, the influence of those nonuniversal soft breaking terms on various flavor changing neutral current (FCNC) processes is discussed in the literatures [2]. However, too many free parameters which exist in the supersymmetry model with nonuniversal soft breaking terms decrease the model prediction ability. Thus for a practical calculation whose results can be compared with the data, one needs to reduce the number of the free parameters in some way, i.e., by enforcing some physical conditions and assuming reasonable symmetries. A realization of this idea is the minimal supergravity (MSUGRA), which is fully specified by only five parameters [3]. In this work, we perform a strict analysis on the $\bar{s}bg$ $(\bar{s}b\gamma)$ effective Lagrangian in the minimal flavor violation supersymmetry up to the leading order. The next leading order (NLO) supersymmetric SUSY QCD corrections to those processes have been evaluated in our another work [4].

The most general form of the superpotential which does not violate gauge invariance and the conservation laws in SM is

$$\mathcal{W}=\mu\epsilon_{ij}\hat{H}_i^1\hat{H}_j^2+\epsilon_{ij}h_l^I\hat{H}_i^1\hat{L}_j^I\hat{R}^I-h_d^I(\hat{H}_1^1\hat{Q}_2^I-\hat{H}_2^1V^{IJ}\hat{Q}_1^J)\hat{D}^I$$
$$-h_u^I(\hat{H}_1^2V^{*JI}\hat{Q}_2^J-\hat{H}_2^2\hat{Q}_1^I)\hat{U}^I. \quad (1)$$

Here $\hat{H}^1,\hat{H}^2$ are Higgs superfields; $\hat{Q}^I$ and $\hat{L}^I$ are quark and lepton superfields in doublets of the weak SU(2) group, where $I=1,2,3$ are the indices of generations; the rest superfields $\hat{U}^I$, $\hat{D}^I$ and $\hat{R}^I$ are quark superfields of the u- and d-types and charged leptons in singlets of the weak SU(2) respectively. Indices i,j are contracted for the SU(2) group, and $h_l,h_{u,d}$ are the Yukawa couplings. In order to break the supersymmetry, the soft breaking terms are introduced as

$$\mathcal{L}_{soft}=-m_{H^1}^2H_i^{1*}H_i^1-m_{H^2}^2H_i^{2*}H_i^2-m_{L^I}^2\tilde{L}_i^{I*}\tilde{L}_i^I-m_{R^I}^2\tilde{R}^{I*}\tilde{R}^I$$
$$-m_{Q^I}^2\tilde{Q}_i^{I*}\tilde{Q}_i^I-m_{U^I}^2\tilde{U}^{I*}\tilde{U}^I-m_{D^I}^2\tilde{D}^{I*}\tilde{D}^I$$
$$+(m_1\lambda_B\lambda_1+m_2\lambda_A^i\lambda_A^i+m_3\lambda_G^a\lambda_G^a+\text{H.c.})$$
$$+[B\mu\epsilon_{ij}H_i^1H_j^2+\epsilon_{ij}A_l^Ih_l^IH_i^1\tilde{L}_j^I\tilde{R}^I$$
$$-A_d^Ih_d^I(H_1^1\tilde{Q}_2^I-H_2^1V^{IJ}\tilde{Q}_1^J)\tilde{D}^I$$
$$-A_u^Ih_u^I(H_1^2V^{*JI}\tilde{Q}_2^J-H_2^2\tilde{Q}_1^I)\tilde{U}^I+\text{H.c.}], \quad (2)$$

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where $m^2_{H^1}$, $m^2_{H^2}$, $m^2_{L^I}$, $m^2_{R^I}$, $m^2_{Q^I}$, $m^2_{U^I}$ and $m^2_{D^I}$ are the parameters in unit of mass squared, m_3, m_2, m_1 denote the masses of λ^a_G $(a=1,2, \ldots, 8)$, λ^i_A $(i=1,2,3)$ and λ_B, which are the $SU(3)\times SU(2)\times U(1)$ gauginos. B is a free parameter in unit of mass. A^I_l, A^I_u, A^I_d $(I=1,2,3)$ are the soft breaking parameters that result in mass splitting between leptons, quarks and their supersymmetric partners. Taking into account of the soft breaking terms Eq. (2), we can study the phenomenology within the minimal supersymmetric extension of the standard model (MSSM). The resultant mass matrix of the up-type scalar quarks is written as

$$m^2_{\tilde{U}^I}=\begin{pmatrix} m^2_{Q^I}+m^2_{u^I}+\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)\cos 2\beta m^2_Z & -m_{u^I}(A^I_u+\mu\cot\beta) \\ -m_{u^I}(A^I_u+\mu\cot\beta) & m^2_{U^I}+m^2_{u^I}+\frac{2}{3}\sin^2\theta_W\cos 2\beta m^2_Z \end{pmatrix}, \tag{3}$$

and the corresponding mass matrix of the down-type scalar quarks is

$$m^2_{\tilde{D}^I}=\begin{pmatrix} m^2_{Q^I}+m^2_{d^I}+\left(\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)\cos 2\beta m^2_Z & -m_{d^I}(A^I_d+\mu\tan\beta) \\ -m_{d^I}(A^I_d+\mu\tan\beta) & m^2_{D^I}+m^2_{d^I}-\frac{1}{3}\sin^2\theta_W\cos 2\beta m^2_Z \end{pmatrix}, \tag{4}$$

with m_{u^I}, m_{d^I} $(I=1,2,3)$ being the masses of the Ith generation quarks. One difference between the MSSM and SM is the Higgs sector. There are four charged scalars, two of them are physical massive Higgs bosons and other are massless Goldstones bosons in the SUSY extension. The mixing matrix can be written as

$$\mathcal{Z}_H=\begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \tag{5}$$

with $\tan\beta=v_2/v_1$ and v_1, v_2 being the vacuum expectation values of the two Higgs scalars. Another matrix that we will use in the later derivation is the chargino mixing matrix. The SUSY partners of the charged Higgs boson and $W^\pm$ combine to give four Dirac fermions: $\chi^\pm_1, \chi^\pm_2$. The two mixing matrices $\mathcal{Z}^\pm$ appearing in the Lagrangian are defined as

$$(\mathcal{Z}^-)^T\mathcal{M}_c\mathcal{Z}^+=\mathrm{diag}(m_{\chi_1}, m_{\chi_2}), \tag{6}$$

where $\mathcal{M}_c$ is the mass matrix of charginos. In a similar way, $Z_{U,D}$ diagonalize the mass matrices of the up- and down-type squarks respectively:

$$\mathcal{Z}^\dagger_{\tilde{U}^I}m^2_{\tilde{U}^I}\mathcal{Z}_{\tilde{U}^I}=\mathrm{diag}(m^2_{\tilde{U}^I_1}, m^2_{\tilde{U}^I_2}),$$

$$\mathcal{Z}^\dagger_{\tilde{D}^I}m^2_{\tilde{D}^I}\mathcal{Z}_{\tilde{D}^I}=\mathrm{diag}(m^2_{\tilde{D}^I_1}, m^2_{\tilde{D}^I_2}). \tag{7}$$

In the framework of minimal supergravity (MSUGRA), the unification assumptions at the ground unified theory (GUT) scale are expressed as [3]

$$A^I_l=A^I_d=A^I_u=A_0, \tag{8}$$

$$B=A_0-1,$$

$$m^2_{H^1}=m^2_{H^2}=m^2_{L^I}=m^2_{R^I}=m^2_{Q^I}=m^2_{U^I}$$
$$=m^2_{D^I}=m^2_0,$$

$$m_1=m_2=m_3=m_{1/2}.$$

Under these assumptions, the MSUGRA is specified by five parameters:

$$A_0, m_0, m_{1/2}, \tan\beta, \mathrm{sgn}(\mu),$$

and the flavor structure of the model is similar to SM, i.e., flavors change only via the CKM matrix.

The supersymmetric contributions will modify the Wilson coefficients of the effective $\overline{s}bg$ and $\overline{s}b\gamma$ vertices. For the W-boson propagator, we adopt the nonlinear R_ξ gauge whose gauge fixing term is [5]

$$\mathcal{L}_{gauge\text{-}fixing}=-\frac{1}{\xi}f^\dagger f \tag{9}$$

with $f=(\partial_\mu W^{+\mu}-ieA_\mu W^{+\mu}-i\xi m_W\phi^+)$ in our calculations. A thorough discussion about the gauge invariance was given by Deshpande *et al.* [6,7].

As in the case of SM [12], the operator basis for $b\to sg$ in the supersymmetry consists of

$$\mathcal{O}_1=\frac{1}{(4\pi)^2}\overline{s}(i\not{D})^3\omega_-b, \tag{10}$$

$$\mathcal{O}_2=\frac{1}{(4\pi)^2}\overline{s}\{i\not{D}, g_sG\cdot\sigma\}\omega_-b,$$

$$\mathcal{O}_3=\frac{1}{(4\pi)^2}\overline{s}iD_\mu(ig_sG^{\mu\nu})\gamma_\nu\omega_-b,$$

$$\mathcal{O}_4=\frac{1}{(4\pi)^2}\overline{s}(i\not{D})^2(m_s\omega_-+m_b\omega_+)b,$$

$$\mathcal{O}_5=\frac{1}{(4\pi)^2}\overline{s}g_sG\cdot\sigma(m_s\omega_-+m_b\omega_+)b.$$

In these operators, $D_\mu\equiv\partial_\mu-ig_sG_\mu$ and $G_{\mu\nu}\equiv G^a_{\mu\nu}T^a$ denotes the gluon field strength tensor with $G^a_{\mu\nu}=\partial_\mu G^a_\nu-\partial_\nu G^a_\mu+g_sf^{abc}G^b_\mu G^c_\nu$, and $G\cdot\sigma\equiv G_{\mu\nu}\sigma^{\mu\nu}$.

For transition $b\to s\gamma$, the operator basis is somewhat different from those in Eq. (10) and the changes are reflected in the following replacements:

$$\mathcal{O}_2\to\mathcal{O}_6=\frac{1}{(4\pi)^2}\overline{s}\{i\not{D},eQ_dF\cdot\sigma\}\omega_-b, \tag{11}$$

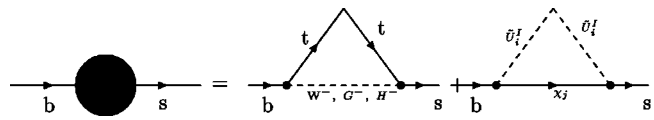

FIG. 1. The one-loop self-energy diagrams for $b\to s$ in the SUSY model with minimal flavor violation.

$$\mathcal{O}_3\to\mathcal{O}_7=\frac{1}{(4\pi)^2}\overline{s}iD_\mu(ieQ_dF^{\mu\nu})\gamma_\nu\omega_-b,$$

$$\mathcal{O}_5\to\mathcal{O}_8=\frac{1}{(4\pi)^2}\overline{s}eQ_dF\cdot\sigma(m_s\omega_-+m_b\omega_+)b$$

with $F_{\mu\nu}$ being the electromagnetic field strength tensor and $F\cdot\sigma\equiv F_{\mu\nu}\sigma^{\mu\nu}$.

II. THE EFFECTIVE LAGRANGIAN FOR $\overline{s}bg$ ($\overline{s}b\gamma$)

At first, we present the analysis of $\overline{s}b$ mixing. The self-energy diagrams are drawn in Fig. 1. The unrenormalized $\overline{s}b$ self-energy is given as

$$\Sigma=\frac{ig_2^2}{32\pi^2}\sum_{i=u,c,t}V_{ib}V^*_{is}\Bigg\{\Bigg(A_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})+\frac{p^2}{m_W^2}A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Bigg)\not{p}\omega_-+\Bigg(B_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})$$
$$+\frac{p^2}{m_W^2}B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Bigg)(m_s\omega_-+m_b\omega_+)+C_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\frac{m_bm_s}{m_W^2}\not{p}\omega_+\Bigg\} \tag{12}$$

with the symbolic definitions $x_i=m_i^2/m_W^2$, $x_H=m^2_{H^+}m_W^2$, $x_{\tilde{U}^i_\alpha}=m^2_{\tilde{U}^i_\alpha}/m_W^2$, $x_{\chi_\beta}=m^2_{\chi_\beta}/m_W^2$ with $i=u,c,t$. Those form factors A_0, A_1, B_0, B_1 and C_0 are complicated functions of the parameters and their explicit expressions are collected in Appendix A.

We renormalize the $\overline{s}b$ self-energy according to the well-known prescription, namely by demanding that the renormalized self-energy $\hat{\Sigma}$ vanish when one of the external legs is on its mass shell [8–10]. Obviously, this is a necessary physical condition which must be satisfied. This is realized as

$$\hat{\Sigma}=\frac{ig_2^2}{32\pi^2}\sum_{i=u,c,t}V_{ib}V^*_{is}\Bigg\{\Bigg(A^*+A_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})+\frac{p^2}{m_W^2}A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Bigg)\not{p}\omega_-+\Bigg(B^*_s+B_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})$$
$$+\frac{p^2}{m_W^2}B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Bigg)m_s\omega_-+\Bigg(B^*_b+B_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})+\frac{p^2}{m_W^2}B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\Bigg)m_b\omega_+$$
$$+(C^*+C_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}))\frac{m_bm_s}{m_W^2}\not{p}\omega_+\Bigg\}, \tag{13}$$

where

$$A^*=-A_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})-\frac{m_b^2+m_s^2}{m_W^2}(A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})+B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})),$$

$$B^*_b=-B_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})-\frac{m_s^2}{m_W^2}(A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})+B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})), \tag{14}$$

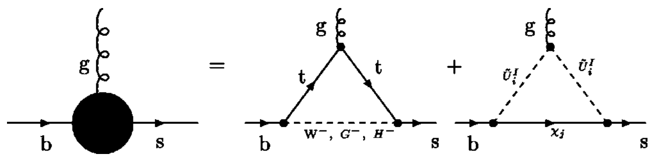

FIG. 2. The one-loop diagrams for $b\to sg$ in the SUSY model with minimal flavor violation.

$$B_s^*=-B_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})-\frac{m_b^2}{m_{\rm W}^2}(A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})+B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})),$$

$$C^*=-\frac{m_bm_s}{m_{\rm W}^2}(A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})+2B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})+C_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})).$$

After carrying out the renormalization procedure described above, the self-energy is written as

$$\begin{aligned}\hat{\Sigma}=\frac{ig_2^2}{32\pi^2}\sum_{i=u,c,t}V_{ib}V_{is}^*\Bigg\{&\left[\frac{p^2-m_b^2-m_s^2}{m_{\rm W}^2}A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})-\frac{m_b^2+m_s^2}{m_{\rm W}^2}B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\right]\not{p}\,\omega_-+\left[\frac{p^2}{m_{\rm W}^2}B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\right.\\&\left.+\frac{m_b^2}{m_{\rm W}^2}(A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})+B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}))\right]m_s\omega_-+\left[\frac{p^2}{m_{\rm W}^2}B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})+\frac{m_s^2}{m_{\rm W}^2}(A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\right.\\&\left.+B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}))\right]m_b\omega_+-\frac{m_bm_s}{m_{\rm W}^2}(A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})+2B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta}))\not{p}\,\omega_+\Bigg\}.\end{aligned}\tag{15}$$

This procedure is exactly the same as that adopted in the SM case [17].

Next, let us calculate the unrenormalized $\bar{s}bg$ vertex $\Gamma_\rho(p,q)$ corresponding to Fig. 2. Keeping terms up to order $p^2,q^2/m_{\rm W}^2$ [11,12,14–16], we have

$$\begin{aligned}\Gamma_\rho^{b\to sg}=g_sT^a\frac{ig_2^2}{32\pi^2}\sum_{i=u,c,t}V_{ib}V_{is}^*\Bigg\{&A_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\gamma_\rho\omega_-+A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\frac{p^2\gamma_\rho+(p+q)^2\gamma_\rho+\not{p}\gamma_\rho\not{p}}{m_{\rm W}^2}\omega_-\\&+F_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\frac{q^2}{m_{\rm W}^2}\gamma_\rho\omega_-+F_2(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\frac{\not{p}\gamma_\rho\not{q}}{m_{\rm W}^2}\omega_-+F_3(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\frac{\not{q}\gamma_\rho\not{p}}{m_{\rm W}^2}\omega_-\\&+F_4(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\frac{\not{q}\gamma_\rho\not{q}}{m_{\rm W}^2}\omega_-+B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\frac{1}{m_{\rm W}^2}((\not{p}+\not{q})\gamma_\rho+\gamma_\rho\not{p})(m_s\omega_-+m_b\omega_+)\\&+F_5(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\frac{1}{m_{\rm W}^2}[\not{q},\gamma_\rho](m_s\omega_-+m_b\omega_+)+C_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})\frac{m_bm_s}{m_{\rm W}^2}\gamma_\rho\omega_+\Bigg\},\end{aligned}\tag{16}$$

where $F_i(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})$ $(i=1,\ldots,5)$ are collected in Appendix A. From Eq. (12) and Eq. (16), it is easy to show that $\Gamma_\rho^{b\to sg}$ obeys the Ward-Takahashi identity

$$q^\rho\Gamma_\rho^{b\to sg}(p,q)=g_sT^a[\Sigma(p+q)-\Sigma(p)].\tag{17}$$

According to the general principle of renormalization, $\bar{s}bg$ vertex does not exist in the fundamental Lagrangian, thus it does not need to be renormalized. In other words, the divergence would be canceled as the physical conditions are taken into account. In the nonlinear R_ξ gauge, as well as in the unitary gauge, the one-loop penguin diagram results in a divergence. On other side, all the one-loop diagrams which contribute to the $b\to sg$ or $b\to s\gamma$ processes must constitute a convergent subgroup. Thus obviously, the renormalizations of the penguin and flavor-changing self-energies are associated. In fact, the Ward-Takahashi identity holds for the unrenormalized penguin, to renormalize the $\bar{s}bg$ vertex, we demand that the Ward-Takahashi identity be preserved for the renormalized vertex $\hat{\Gamma}_\rho^{b\to sg}$ [17],

$$q^\rho \hat{\Gamma}_\rho^{b\to sg}(p,q) = g_s T^a[\hat{\Sigma}(p+q) - \hat{\Sigma}(p)]. \tag{18}$$

It is noted that with this requirement, just as in the SM case [17], the renormalization of the $\bar{s}bg$ vertex is realized when we renormalize the self-energy by enforcing the physical condition $\hat{\Sigma}=0$ as one of the external legs being on its mass shell. Moreover, indeed, the renormalization scheme of the $\bar{s}bg$ vertex pledges the current conservation for an on-shell transition, since the renormalized self-energies $\hat{\Sigma}(p+q)$ and $\hat{\Sigma}(p)$ are zero as both b and s are on mass shell [17].

This renormalization scheme can be understood from another angle. The requirement that the Ward-Takahashi identity holds and condition $\hat{\Sigma}(\text{on-shell})=0$ realize the renormalization of the $\bar{s}bg$ vertex and the scheme is equivalent to summing up the contributions of penguin and flavor-changing self-energies to the transition $\bar{s}bg$ at one-loop level. This procedure can be generalized to two-loop calculations.

Applying Eq. (18), we have

$$\begin{aligned}\hat{\Gamma}_\rho^{b\to sg} = {}& g_s T^a \frac{i g_2^2}{32\pi^2} \sum_{i=u,c,t} V_{ib}V_{is}^* \Bigg\{ -\frac{m_b^2+m_s^2}{m_W^2}\big(A_1(x_i,x_H,x_{\tilde{U}_\alpha^i},x_{\chi_\beta}) + B_1(x_i,x_H,x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\big)\gamma_\rho\omega_- \\ & + A_1(x_i,x_H,x_{\tilde{U}_\alpha^i},x_{\chi_\beta}) \frac{p^2\gamma_\rho + (p+q)^2\gamma_\rho + \not{p}\gamma_\rho\not{p}}{m_W^2}\omega_- + F_1(x_i,x_H,x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\frac{q^2}{m_W^2}\gamma_\rho\omega_- + F_2(x_i,x_H,x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\frac{\not{p}\gamma_\rho\not{q}}{m_W^2}\omega_- \\ & + F_3(x_i,x_H,x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\frac{\not{q}\gamma_\rho\not{p}}{m_W^2}\omega_- + F_4(x_i,x_H,x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\frac{\not{q}\gamma_\rho\not{q}}{m_W^2}\omega_- + B_1(x_i,x_H,x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\frac{1}{m_W^2}((\not{p}+\not{q})\gamma_\rho + \gamma_\rho\not{p}) \\ & \times(m_s\omega_- + m_b\omega_+) + F_5(x_i,x_H,x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\frac{1}{m_W^2}[\not{q},\gamma_\rho](m_s\omega_- + m_b\omega_+) - \big(A_1(x_i,x_H,x_{\tilde{U}_\alpha^i},x_{\chi_\beta}) \\ & + 2B_1(x_i,x_H,x_{\tilde{U}_\alpha^i},x_{\chi_\beta})\big)\frac{m_b m_s}{m_W^2}\gamma_\rho\omega_+ \Bigg\}. \end{aligned} \tag{19}$$

The terms of dimension-four which are related to the $\bar{s}\gamma_\rho\omega_\pm b$ vertex cancel each other as long as we let b and s quarks be on their mass shells [12], so that we do not need to consider them at all. We ignore all terms which vanish as $m_{u,c}^2/m_W^2 \to 0$, whereas in the coefficients keep the part which are proportional to $\ln(m_{u,c}^2/m_W^2)$ in the final effective vertex for $b\to sg$, we can recast Eq. (19) to a form with the operator basis given in Eq. (10):

$$\hat{\Gamma}_\rho^{b\to sg} = \frac{4G_F}{\sqrt{2}}\Bigg\{V_{tb}V_{ts}^*\sum_{i=1}^{5} C_i(\mu_W)\mathcal{O}_i + \bigg(\frac{4}{3}V_{cb}V_{cs}^*\ln x_c + \frac{4}{3}V_{ub}V_{us}^*\ln x_u\bigg)\mathcal{O}_3\Bigg\}. \tag{20}$$

After matching between the effective theory and the full theory [13], we have the effective Lagrangian for $b\to sg$ at the weak scale in the minimal flavor violating supersymmetry as

$$\mathcal{L}_{b\to sg} = \frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\sum_{i=1}^{5} C_i(\mu_W)\mathcal{O}_i, \tag{21}$$

where the Wilson coefficients $C_i(\mu_W)$ $(i=1,\ldots,5)$ can be found in Appendix B.

For the vertex $\bar{s}b\gamma$, the Feynman diagrams are drawn in Fig. 3.

With all unrenormalized quantities the Ward-Takahashi identity for the $\bar{s}b\gamma$ vertex is in form

$$q^\rho \Gamma_\rho^{b\to sg}(p,q) = -\frac{1}{3}e[\Sigma(p+q) - \Sigma(p)]. \tag{22}$$

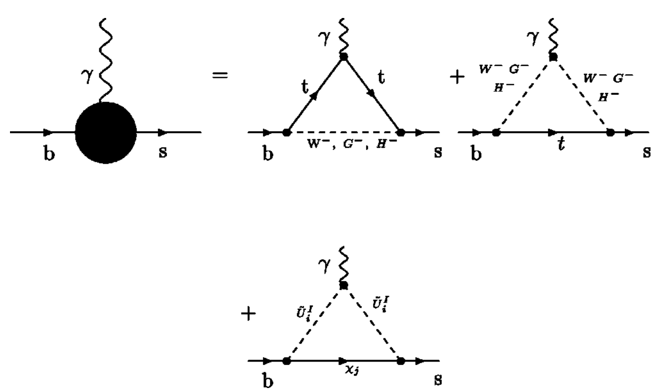

FIG. 3. The one-loop diagrams for $b\to s\gamma$ in the SUSY model with minimal flavor violation.

To renormalize the $\overline{s}b\gamma$ vertex, we demand that the Ward-Takahashi identity be preserved for the renormalized vertex $\hat{\Gamma}_{\rho}^{b\to s\gamma}$ [17],

$$q^{\rho}\hat{\Gamma}_{\rho}^{b\to sg}(p,q)=-\frac{1}{3}e[\hat{\Sigma}(p+q)-\hat{\Sigma}(p)]. \tag{23}$$

The other steps are similar to those applied in the calculation for the $\overline{s}bg$ vertex. The result is written as

$$\mathcal{L}_{b\to s\gamma}=\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^{*}\{C_1(\mu_{\rm W})\mathcal{O}_1+C_4(\mu_{\rm W})\mathcal{O}_4+C_6(\mu_{\rm W})\mathcal{O}_6 +C_7(\mu_{\rm W})\mathcal{O}_7+C_8(\mu_{\rm W})\mathcal{O}_8\} \tag{24}$$

and those coefficient are also collected in Appendix B.

III. THE APPLICATION OF EFFECTIVE LAGRANGIAN

In this section, we apply the effective Lagrangian Eq. (21) and Eq. (24) to calculating the rates of the rare B decays up to the leading order (LO). When the effective Lagrangian is applying at the hadronic scale, we should evolve those Wilson coefficients from the weak scale down to the hadronic scale. The running depends on the anomalous dimension matrix of concerned operators [21]. The coefficients $C_i(m_{\rm W})$ obtained at the weak scale M_W are regarded as the initial conditions for the differential renormalization group equations (RGEs). At present, the most strong constraint on supersymmetry parameter space originates from the rare B processes: $B\to X_s\gamma$. The experimental measurements of the decay $B\to X_s\gamma$ is $\mathrm{BR}(B\to X_s\gamma)=2.32\pm0.57\pm0.35\times10^{-4}$ [22]. The theoretical prediction of the branching ratio for inclusive $B\to X_s\gamma$ is given as

$$\mathrm{Br}(B\to X_s\gamma)=\frac{\Gamma(B\to X_s\gamma)}{\Gamma(B\to X_c e\overline{\nu}_e)} =\frac{|V_{tb}V_{ts}^{*}|^2}{|V_{cb}|^2}\frac{6\alpha_{em}|C_6(m_b)+C_8(m_b)|^2}{\pi\rho\left(\frac{m_c}{m_b}\right)} \times\left(1+\frac{2\alpha_s(m_b)}{3\pi}f\left(\frac{m_c^2}{m_b^2}\right)\right),$$

where α_{em} is the QED fine structure constant and the phase-space factor $\rho(m_c/m_b)=1-8(m_c/m_b)^2+8(m_c/m_b)^6+(m_c/m_b)^8-24(m_c/m_b)^4\ln(m_c/m_b)$. The last term in the bracket is the one-loop corrections to the semileptonic decay, with $f(m_c/m_b))\simeq2.4$ [24]. When we calculate the branching ratios of other rare processes, the $B\to X_s\gamma$ constraint must be taken into account. In the two examples, we will discuss the branching ratios of the QCD induced charmless B decay [$B\to K+X$ (no charm)] and rare process $B_s\to\gamma\gamma$ in the supersymmetric model.

A. Charmless *B* decay

As an example, we first apply the effective Lagrangian Eq. (21) and Eq. (24) to discuss the branching ratio of charmless B decay: $B\to K+X$ (no charm), which is due to the loop-induced bsg effective coupling. At the quark level, the process $B\to K+X$ (no charm) involve subprocesses $b\to sg$, $b\to sq\overline{q}$, $b\to sgg$, and $b\overline{q}\to s\overline{q}$ with $q=u,d,s$. In principal, the decay mode $b\to su\overline{u}$ ($b\overline{u}\to s\overline{u}$) can be induced by weak charged current at tree level. The contributions from tree level charged current are highly Cabibbo suppressed, that has been first stated in Ref. [25]. In contrast, the penguin diagram contributions, although originating from the one-loop level, are not Cabibbo suppressed as compared to the main b decay modes. With the effective Lagrangian Eq. (21), we obtain the width of the inclusive charmless b decay at the quark level [26]:

$$\begin{aligned}\Gamma(b\to s+X\ (\text{no charm})) &=\Gamma(b\to su\overline{u})+\Gamma(b\to sd\overline{d}) +\Gamma(b\to ss\overline{s})+\Gamma(b\to sg)+\Gamma(b\to sgg) +\Gamma(b\overline{q}\,(q=u,\,d,\,s)\to s\overline{q})\\ &=\frac{G_F^2m_b^5}{144\pi^3}|V_{tb}V_{ts}^{*}|^2\Bigg\{\frac{6\alpha_s}{\pi}|C_2(\mu_b)+C_5(\mu_b)|^2 +\frac{8}{3}\alpha_s^2\left(\frac{f_B}{m_b}\right)^2|C_3(\mu_b)|^2+\frac{\alpha_s^2}{16\pi^2}\\ &\quad\times\left[\frac{35}{6}|C_3(\mu_b)|^2+20\,\mathrm{Re}(C_3(\mu_b)^{*} \times[C_2(\mu_b)+C_5(\mu_b)])\right]\Bigg\}.\end{aligned} \tag{25}$$

Assuming that the partons (quarks and gluons) fragment into hadrons (with an odd number of strangeness) with unit probability and supposing the production rate of Λ or other strange baryons $\Sigma^{\pm},\Sigma^{0}$ is much smaller than that for mesons, one can have [27]

$$\Gamma(b\to s+X\ (\text{no charm}))=\Gamma(B\to K+X\ (\text{no charm})).$$

Note that in above equation, we ignore the interference effects that arise as the partons evolve into the same final state hadrons via different modes. This assumption should not lead to big distinction for the inclusive rate [28]. In order to get branching ratios, following the standard procedure we employ the well-known measured or evaluated decay rates to reduce the uncertainties in numerical analysis

$$\mathrm{Br}(B\to K+X\ (\text{no charm}))$$
$$=\frac{\Gamma(b\to s+X\ (\text{no charm}))}{\Gamma(b\to ce\bar{\nu}_e)}\mathrm{Br}(B\to X_c+e\bar{\nu}_e)$$
$$=\frac{\frac{4}{3}|V_{tb}V_{ts}^*|^2\{\cdots\}}{|V_{cb}|^2\rho\left(\frac{m_c}{m_b}\right)\times 0.88}\times 0.12, \tag{26}$$

where the symbol $\{\cdots\}$ denotes the contents in the parenthesis of Eq. (25), $\rho(m_c/m_b)$ is the standard phase factor, the numerical factor 0.88 is due to the QCD correction to the B semileptonic decays [24] and 0.12 is the measured branching ratio of the B semileptonic decay [23]. Note that the m_b^5 dependence, which exists in the rate of a fermion transiting into three-fermion final states, disappears.

B. The branching ratio of $B_s\to\gamma\gamma$

Now, let us turn to the calculation of the rare process $B_s\to\gamma\gamma$ in the supersymmetry theory. The investigation of $B_s\to\gamma\gamma$ decay is interesting for the following reasons.

It is well known that the QCD corrections to the rare decay $b\to s\gamma$ are relatively large [29–32]. Therefore, we can expect that the QCD corrections to $b\to s\gamma\gamma$ are also large. The leading order (LO) QCD corrections to this decay rate have been given in Refs. [33–36] and found to be large as expected.

In the $B_s\to\gamma\gamma$ decay, the final photons can be in a CP-odd or a CP-even state [37–39]. Therefore we can study CP violation effects in the process.

From the experimental point of view, $B_s\to\gamma\gamma$ decay can be easily identified by putting a cut for the energy of the final photons, e.g., the energy of each photon is larger than 100 MeV. In this case, two hard photons will be easily detected in the experiment [40].

The same as other rare processes, this decay rate is also sensitive to the physics beyond SM.

The effective Lagrangian relevant to $B_s\to\gamma\gamma$ is

$$\mathcal{L}_{B_s\to\gamma\gamma}=\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\sum_{i=1}^{6}C_i^{c}(\mu_W)\mathcal{Q}_i^{c}+\text{Eq. (24)}, \tag{27}$$

where the current operators are defined as

$$\mathcal{Q}_1^c=(\bar{s}_\alpha\gamma_\mu\omega_- b_\alpha)(\bar{c}_\beta\gamma^\mu\omega_- c_\beta),$$
$$\mathcal{Q}_2^c=(\bar{s}_\alpha\gamma_\mu\omega_- b_\beta)(\bar{c}_\beta\gamma^\mu\omega_- c_\alpha),$$
$$\mathcal{Q}_3^c=(\bar{s}_\alpha\gamma_\mu\omega_- b_\alpha)\sum_{q=u,d,s,c,b}(\bar{q}_\beta\gamma^\mu\omega_- q_\beta),$$
$$\mathcal{Q}_4^c=(\bar{s}_\alpha\gamma_\mu\omega_- b_\beta)\sum_{q=u,d,s,c,b}(\bar{q}_\beta\gamma^\mu\omega_- q_\alpha),$$
$$\mathcal{Q}_5^c=(\bar{s}_\alpha\gamma_\mu\omega_- b_\alpha)\sum_{q=u,d,s,c,b}(\bar{q}_\beta\gamma^\mu\omega_+ q_\beta),$$
$$\mathcal{Q}_6^c=(\bar{s}_\alpha\gamma_\mu\omega_- b_\beta)\sum_{q=u,d,s,c,b}(\bar{q}_\beta\gamma^\mu\omega_+ q_\alpha). \tag{28}$$

Here, α,β denote the SU(3) color indices. At the weak scale, we have the initial values for those Wilson coefficients of current operators:

$$C^c_{1,3,\ldots,6}(\mu_W)=0,$$
$$C^c_2(\mu_W)=1. \tag{29}$$

With the initial values of current and penguin operators at the weak scale, we can calculate their contributions at any scale as in the SM case [41].

Using the effective Lagrangian Eq. (27), the amplitude for the decay $B_s\to\gamma\gamma$ can be written as [33]

$$\mathcal{A}_{B_s\to\gamma\gamma}=A^+\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}+iA^-\mathcal{F}_{\mu\nu}\tilde{\mathcal{F}}^{\mu\nu}, \tag{30}$$

where $\tilde{\mathcal{F}}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\mathcal{F}^{\alpha\beta}$. Here, A^+ (A^-) is CP-even (CP-odd) part in a HQET inspired approach:

$$A^+=\frac{\alpha_{em}G_F}{\sqrt{2}\pi}\frac{f_{B_s}}{m_{B_s}}V_{tb}V_{ts}^*\left\{\frac{m_{B_s}^4(m_b^{eff}-m_s^{eff})}{3\bar{\Lambda}_s(m_{B_s}-\bar{\Lambda}_s)(m_b^{eff}+m_s^{eff})}C_8^{eff}(\mu_b)-\frac{4}{9}\frac{m_{B_s}^2}{m_b^{eff}+m_s^{eff}}(-m_bJ(m_b)+m_sJ(m_s))D(\mu_b)\right\},$$

$$A^-=-\frac{\alpha_{em}G_F}{\sqrt{2}\pi}f_{B_s}V_{tb}V_{ts}^*\left\{\frac{m_{B_s}(m_b^{eff}+m_s^{eff})^2+\bar{\Lambda}(m_{B_s}^2-(m_b^{eff}+m_s^{eff})^2)}{3m_{B_s}\bar{\Lambda}_s(m_{B_s}-\bar{\Lambda}_s)}C_8^{eff}(\mu_b)-\sum_{q=u,d,s,c,b}Q_q^2I(m_q)C_q(\mu_b)\right.$$
$$\left.+\frac{(m_b\Delta(m_b)+m_s\Delta(m_s))D(\mu_b)}{9(m_b^{eff}+m_s^{eff})}\right\}, \tag{31}$$

where $Q_q=\frac{2}{3}$ for $q=u,c$ and $Q_q=-1/3$ for $q=d,s,b$. The unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix $\Sigma_{i=u,c,t}V_{ib}V_{is}^{*}=0$ is used when we obtain the above equation, and we have neglected the contribution of $V_{ub}V_{us}^{*}$ due to $V_{ub}V_{us}^{*}\ll V_{tb}V_{ts}^{*}$. The parameter $\overline{\Lambda}_s$ enters Eq. (31) through the bound state kinematics [33]. m_b^{eff},m_s^{eff} denote the effective masses of the quarks in the B_s meson,

$$(m_b^{eff})^2=m_b^2-3\lambda_2,$$
$$(m_s^{eff})^2=(m_b^{eff})^2-m_{B_s}^2+2m_{B_s}\overline{\Lambda}_s, \tag{32}$$

where λ_2 originates from the matrix element of the heavy quark expansion [42]. The LO QCD corrected Wilson coefficients C_i^c $(i=1,\ldots,6)$ show up in combinations [33–35]:

$$C_u(\mu_b)=C_d(\mu_b)=3(C_3^c(\mu_b)-C_5^c(\mu_b))+C_4^c(\mu_b)-C_6^c(\mu_b), \tag{33}$$

$$C_c(\mu_b)=3(C_1^c(\mu_b)+C_3^c(\mu_b)-C_5^c(\mu_b))+C_2^c(\mu_b)+C_4^c(\mu_b)-C_6^c(\mu_b),$$
$$C_s(\mu_b)=C_b(\mu_b)=4(C_3^c(\mu_b)+C_4^c(\mu_b))-3C_5^c(\mu_b)-C_6^c(\mu_b),$$
$$D(\mu_b)=C_5^c(\mu_b)+3C_6^c(\mu_b),$$

where $C_i(\mu)$ $(i=1,\ldots,6)$ are the coefficients of the current operators $\mathcal{Q}_i^c$ at scale μ. The "effective" coefficient of dipole operator $C_8^{eff}(\mu)$ contains renormalization scheme dependent contributions from current operators $\mathcal{Q}_{1,\ldots,6}$. Here, we adopt the naive dimensional reduction (NDR) scheme: $C_8^{eff}(\mu)=C_7(\mu)+C_8(\mu)-\frac{1}{3}C_5(\mu)-C_6(\mu)$ [41]. The functions $I(m_q)$, $J(m_q)$ and $\Delta(m_q)$ originate from the irreducible diagrams with one internal light quark propagating, their expressions are written as

$$I(m_q)=1+\frac{m_q^2}{m_{B_s}^2}\Delta(m_q),$$
$$J(m_q)=1-\frac{m_{B_s}^2-4m_q^2}{4m_{B_s}^2}\Delta(m_q),$$
$$\Delta(m_q)=\begin{cases}\left[\ln\left(\dfrac{m_{B_s}+\sqrt{m_{B_s}^2-4m_q^2}}{m_{B_s}+\sqrt{m_{B_s}^2-4m_q^2}}\right)-i\pi\right]^2 & \text{for } \dfrac{m_{B_s}^2}{4m_q^2}\geqslant 1,\\ -\left[2\tan^{-1}\left(\dfrac{\sqrt{m_{B_s}^2-4m_q^2}}{m_{B_s}}\right)-\pi\right]^2 & \text{for } \dfrac{m_{B_s}^2}{4m_q^2}<1.\end{cases} \tag{34}$$

Using the above expressions, the partial decay width is then given as

$$\Gamma_{B_s\to\gamma\gamma}=\frac{m_{B_s}^3}{16\pi}(|A^+|^2+|A^-|^2). \tag{35}$$

IV. NUMERICAL RESULTS

In this section, we present our numerical analysis about the branching ratios of inclusive charmless B decays [$B\to K+X$ (no charm)] and exclusive process $B_s\to\gamma\gamma$ in the MSUGRA model. As aforementioned, the model is fully specified by five parameters

$$m_0, m_{1/2}, A_0, \tan\beta, \mathrm{sgn}(\mu).$$

Here m_0, $m_{1/2}$, and A_0 are the universal scalar quark mass, gaugino mass and trilinear scalar coupling. They are assumed to arise through supersymmetry breaking in a hidden-sector at the GUT scale $\mu_{GUT}\simeq 2\times 10^{16}$ GeV. In our numerical calculation, to maintain consistency of the theory and the up-to-date experimental observation, when we obtain the numerical value of the Higgs boson mass in the MSUGRA model with the five parameters, we include all one-loop effects in the Higgs potential [18]. Moreover we also employ the two-loop RGEs [19] with one-loop threshold corrections [18,20] as the energy scale runs down from the MSUGRA scale to the lower weak scale.

For the SM parameters, we have $m_c=1.4$ GeV, $m_b=4.8$ GeV, $m_t=174$ GeV, $m_{\rm W}=80.23$ GeV, $m_{\rm Z}=91.12$ GeV, $m_{B_s}=5.369$ GeV, $\alpha_e(m_{\rm W})=\frac{1}{128}$, $\alpha_s(m_{\rm W})=0.12$ at the weak scale [23], together with the experimentally measured lifetime of B_s, $\tau_{B_s}=1.61\times 10^{-12}$ s. The other parameters relat-

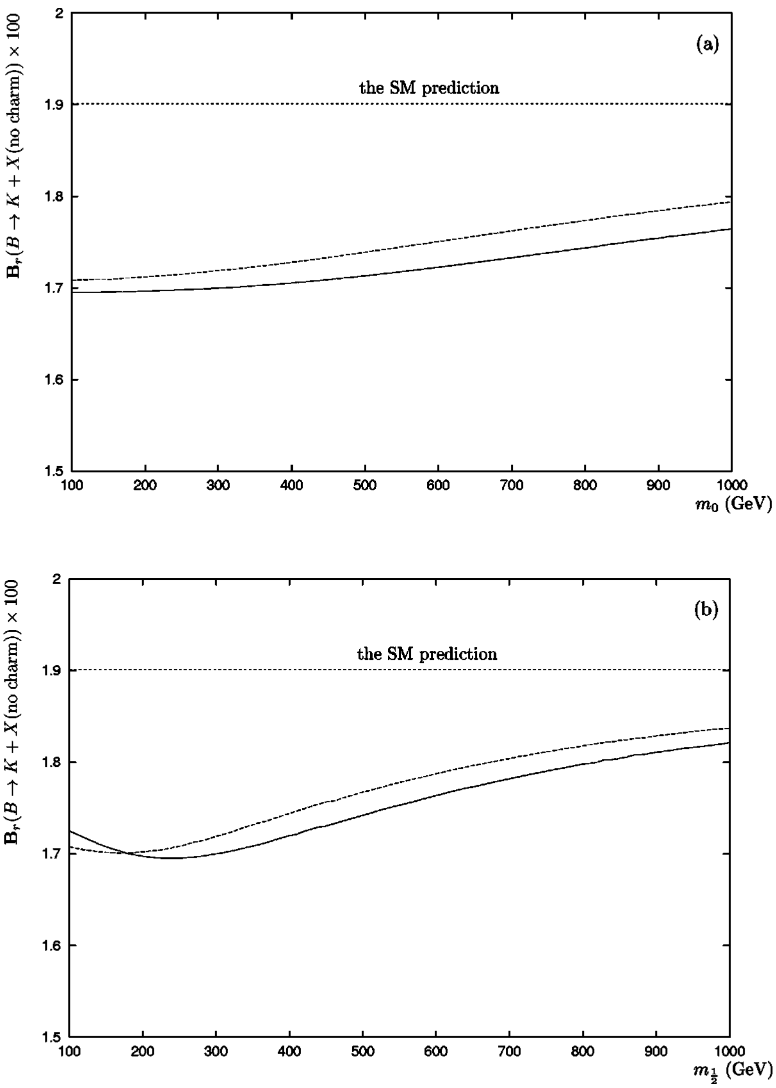

FIG. 4. The branching ratio of $B \to K+X$ (no charm) in MSUGRA versus (a) $m_0(m_{1/2} = 300$ GeV) and (b) $m_{1/2}$ ($m_0 = 300$ GeV). The other parameters are taken as $A_0 = 0$, sin (μ) $= +$ and $\tan\beta = 30$ (solid lines) or $\tan\beta = 10$ (dash lines).

ing to the non-perturbative QCD are taken as $f_B = 0.2$ GeV, $\lambda_2 = 0.12$ GeV^2, and $\overline{\Lambda}_s = 0.57$ GeV [36]. In our later calculations, we always set $A_0 = 0$, sgn(μ) $= +$. Using the measured branching ratios

$$B_{r_{max}}(B \to X_s\gamma) = 3.24 \times 10^{-4},$$

$$B_{r_{min}}(B \to X_s\gamma) = 1.40 \times 10^{-4},$$

together with theoretical uncertainties and experimental errors, we get a possible range for $|C_6(\mu_b) + C_8(\mu_b)|$ as

$$0.1901 \leqslant |C_6(\mu_b) + C_8(\mu_b)| \leqslant 0.4155.$$

In the following analysis we restrict the coefficient $|C_6(\mu_b) + C_8(\mu_b)|$ within this region and study the branching ratios of $B \to K+X$ (no charm) and $B_s \to \gamma\gamma$. We find that the standard model prediction for the branching ratios are $B_r(B \to K+X \text{ (no charm)}) = 1.901 \times 10^{-2}$, $B_r(B_s \to \gamma\gamma) = 3.56 \times 10^{-7}$. Then with the aforementioned inputs of the five SUSY parameters we evaluate the supersymmetric contributions to the branching ratios of those processes.

In Fig. 4, we plot the branching ratios of charmless B decay $B_r(B \to K+X \text{ (no charm)})$ versus parameter m_0 ($m_{1/2}$), where other parameters are set as $m_{1/2} = 300$ GeV ($m_0 = 300$ GeV), $\tan\beta = 10$ (dash lines) or $\tan\beta = 30$ (solid lines). From Fig. 4, we find that the supersymmetric contributions make the $B_r(B \to K+X \text{ (no charm)})$ deviate from the SM predictions about 10% when those supersymmetry particles have the weak scale masses; when the masses of those supersymmetry particles increases further, the new physics contributions mildly become immaterial.

The branching ratios of $B_s \to \gamma\gamma$ versus parameters m_0 ($m_{1/2}$) are plotted in Fig. 5, with other parameters being set as $m_{1/2} = 300$ GeV ($m_0 = 300$ GeV), $\tan\beta = 10$ (dash lines) and $\tan\beta = 30$ (solid lines). When those supesymmetry particles have the weak scale masses, the supersymmetry corrections enhance the $B_r(B_s \to \gamma\gamma)$ by about 60% compared to the SM prediction. Together with the increase of the masses of the supersymmetric particles, the prediction of $B_r(B_s \to \gamma\gamma)$ turns back to the value determined by SM.

V. DISCUSSIONS

In this work, we discuss the contributions of the SUSY sector to the effective Lagrangian for $b \to sg$ and $b \to s\gamma$ in

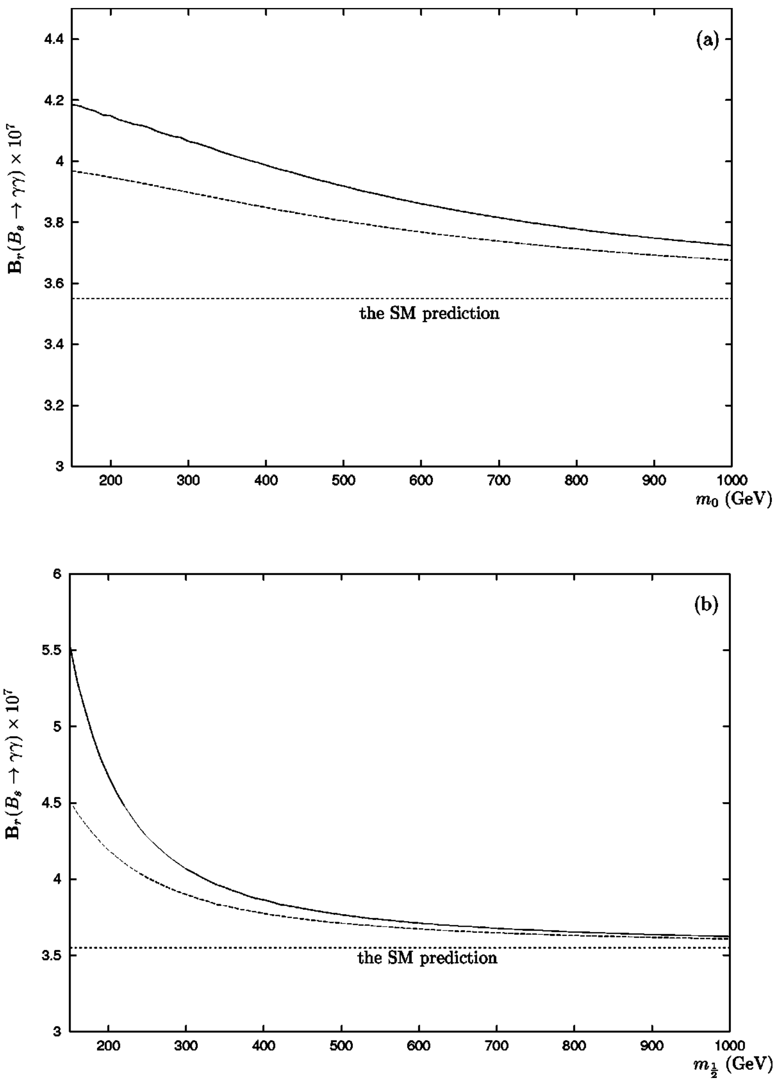

FIG. 5. The branching ratio of $B_s \to \gamma\gamma$ in MSUGRA versus (a) m_0 ($m_{1/2}=300$ GeV) and (b) $m_{1/2}$ ($m_0=300$ GeV). The other parameters are taken as $A_0=0$, sin $(\mu)=+$ and tan $\beta=30$ (solid lines) or tan $\beta=10$ (dash lines).

the MSUGRA model. As many authors suggested, if the masses of the lightest SUSY particles are close to the electroweak energy scale, the contribution from the SUSY sector to the Wilson coefficients of the induced operators is comparable with that from SM.

The strongest constraint on the SUSY model comes from the Higgs boson mass. The recent experimental data have already excluded the range of $M_H<108$ GeV, where H is the lightest Higgs boson in the SUSY model. Anotehr constraint which is closely related to our discussion is the measurement on the branching ratio of the inclusive process $B \to X_s+\gamma$.

Our numerical results indicate that within a reasonable MSUGRA parameter range, the SUSY contributions to branching ratios of $BR(B \to K+X \text{ (no charm)})$ can enlarge the SM prediction by about 10%. When applying the effective $bs\gamma$ vertex, we employ the heavy quark effective theory (HQET) to calculate the branching ratio of $B \to X_s+\gamma$ where the leading order QCD corrections are included. Numerically, for the rare process $B_s \to \gamma\gamma$, the SUSY contributions enhance the SM prediction by about 60%.

If the masses of the SUSY particles are larger, the SUSY contributions to those processes would become weaker. Then as the SUSY particles are heavier and heavier, the contribution of the standard model to the rare processes becomes more and more important and finally the main contribution uniquely is due to SM. This is indeed nothing new, but the decoupling theorem of the SUSY sector demanded by the unitarity of the S-matrix.

In this work, we adopt the nonlinear R_ξ gauge. The advantage is that the Ward-Takahashi identity holds at the one-loop level no matter for the unrenormalized or renormalized quantities. This advantage would be more obvious as we go on doing the two-loop calculations.

Our numerical results also show that as all SUSY particles become very heavy, the values of all coefficients tend to that determined by the SM sector which is consistent with the results obtained before [17].

ACKNOWLEDGMENTS

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APPENDIX A: THE EXPRESSIONS OF THE FORM FACTORS

The form factors in self-energy are given as

$$A_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})=\left(1+\frac{x_i}{2}\right)\left[\Delta+\frac{1}{2}+\ln x_\mu+\frac{x_i}{x_i-1}-\frac{x_i^2\ln x_i}{(x_i-1)^2}\right]+\frac{x_i}{2\tan^2\beta}\left[\Delta+\frac{1}{2}+\ln x_\mu+\frac{x_i}{x_i-x_H}-\frac{x_i^2\ln x_i}{(x_i-x_H)^2}\right.$$
$$\left.+\frac{(2x_Hx_i-x_H^2)\ln x_H}{(x_i-x_H)^2}\right]+\sum_{\alpha,\beta}(\mathcal{A}_i^{\alpha,\beta})^2\left[\Delta+\frac{3}{2}+\ln x_\mu-\frac{x_{\tilde{U}^i_\alpha}}{x_{\tilde{U}^i_\alpha}-x_{\chi_\beta}}\right.$$
$$\left.+\frac{x_{\tilde{U}^i_\alpha}(2x_{\chi_\beta}-x_{\tilde{U}^i_\alpha})\ln x_{\tilde{U}^i_\alpha}}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^2}-\frac{x_{\chi_\beta}^2\ln x_{\chi_\beta}}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^2}\right],$$

$$A_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})=\left(1+\frac{x_i}{2}\right)\left[\frac{2x_i^2+5x_i-1}{3(x_i-1)^3}-\frac{2x_i^2\ln x_i}{(x_i-1)^4}\right]+\frac{x_i}{2\tan^2\beta}\left[\frac{2x_i^2+5x_ix_H-x_H^2}{3(x_i-x_H)^3}-\frac{2x_i^2x_H(\ln x_i-\ln x_H)}{(x_i-x_H)^4}\right]$$
$$+\sum_{\alpha,\beta}(\mathcal{A}_i^{\alpha,\beta})^2\left[\frac{x_{\tilde{U}^i_\alpha}^2-5x_{\tilde{U}^i_\alpha}x_{\chi_\beta}-2x_{\chi_\beta}^2}{3(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^3}+\frac{2x_{\tilde{U}^i_\alpha}x_{\chi_\beta}(\ln x_{\tilde{U}^i_\alpha}-\ln x_{\chi_\beta})}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^4}\right],$$

$$B_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})=\frac{x_i^2\ln x_i}{x_i-1}-\frac{x_i^2\ln x_i-x_ix_H\ln x_H}{x_i-x_H}+2\sum_{\alpha,\beta}\frac{m_{\chi_\beta}}{\sqrt{2}m_{\mathrm{W}}\cos\beta}(\mathcal{A}_3^{\alpha,\beta}\mathcal{B}_3^{\alpha,\beta})$$
$$\times\left[\Delta+1+\ln x_\mu-\frac{x_{\tilde{U}^i_\alpha}\ln x_{\tilde{U}^i_\alpha}-x_{\chi_\beta}\ln x_{\chi_\beta}}{x_{\tilde{U}^i_\alpha}-x_{\chi_\beta}}\right],$$

$$B_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})=-x_i\left[\frac{x_i+1}{2(x_i-1)^2}-\frac{x_i\ln x_i}{(x_i-1)^3}\right]+x_i\left[\frac{x_i+x_H}{2(x_i-x_H)^2}-\frac{x_ix_H(\ln x_i-\ln x_H)}{(x_i-x_H)^3}\right]$$
$$+\sum_{\alpha,\beta}\frac{m_{\chi_\beta}}{\sqrt{2}m_{\mathrm{W}}\cos\beta}(\mathcal{A}_3^{\alpha,\beta}\mathcal{B}_3^{\alpha,\beta})\left[\frac{x_{\tilde{U}^i_\alpha}+x_{\chi_\beta}}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^2}-\frac{2x_{\tilde{U}^i_\alpha}x_{\chi_\beta}(\ln x_{\tilde{U}^i_\alpha}-\ln x_{\chi_\beta})}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^3}\right],$$

$$C_0(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})=\frac{1}{2}\left[\Delta+\frac{1}{2}+\ln x_\mu+\frac{x_i}{x_i-1}-\frac{x_i^2\ln x_i}{(x_i-1)^2}\right]+\frac{\tan^2\beta}{2}\left[\Delta+\frac{1}{2}+\ln x_\mu+\frac{x_i}{x_i-x_H}\right.$$
$$\left.-\frac{x_i^2\ln x_i}{(x_i-x_H)^2}+\frac{(2x_Hx_i-x_H^2)\ln x_H}{(x_i-x_H)^2}\right]+\sum_{\alpha,\beta}\frac{(\mathcal{B}_3^{\alpha,\beta})^2}{2\cos^2\beta}\left[\Delta+\frac{3}{2}+\ln x_\mu-\frac{x_{\tilde{U}^i_\alpha}}{x_{\tilde{U}^i_\alpha}-x_{\chi_\beta}}\right.$$
$$\left.+\frac{x_{\tilde{U}^i_\alpha}(2x_{\chi_\beta}-x_{\tilde{U}^i_\alpha})\ln x_{\tilde{U}^i_\alpha}}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^2}-\frac{x_{\chi_\beta}^2\ln x_{\chi_\beta}}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^2}\right].\qquad\text{(A1)}$$

The expressions of $F_i(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})$ $(i=1,\ \ldots,5)$ are written as

$$F_1(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})=\left(1+\frac{x_i}{2}\right)\left[\frac{5x_i^2-22x_i+5}{18(x_i-1)^3}+\frac{(3x_i-1)\ln x_i}{3(x_i-1)^4}\right]+\frac{1}{\tan^2\beta}\left[\frac{x_i(5x_i^2-22x_ix_H+5x_H^2)}{36(x_i-x_H)^3}\right.$$
$$\left.-\frac{(x_H^3x_i-3x_H^2x_i^2)(\ln x_i-\ln x_H)}{6(x_i-x_H)^4}\right]+\sum_{\alpha,\beta}(\mathcal{A}_i^{\alpha,\beta})^2\left[\frac{x^2_{\tilde{U}^i_\alpha}-8x_{\tilde{U}^i_\alpha}x_{\chi_\beta}-17x^2_{\chi_\beta}}{36(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^3}\right.$$
$$\left.+\frac{(3x_{\tilde{U}^i_\alpha}x^2_{\chi_\beta}+x^3_{\chi_\beta})(\ln x_{\tilde{U}^i_\alpha}-\ln x_{\chi_\beta})}{6(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^4}\right],$$

$$F_2(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})=\left[-\frac{x_i^3-15x_i^2-12x_i+8}{12(x_i-1)^3}-\frac{(5x_i^2-2x_i)\ln x_i}{2(x_i-1)^4}\right]+\frac{1}{\tan^2\beta}\left[-\frac{x_i(x_i^2-5x_ix_H-2x_H^2)}{12(x_i-x_H)^3}\right.$$
$$\left.-\frac{x_i^2x_H^2(\ln x_i-\ln x_H)}{2(x_i-x_H)^4}\right]+\sum_{\alpha,\beta}(\mathcal{A}_i^{\alpha,\beta})^2\left[-\frac{x^2_{\tilde{U}^i_\alpha}-x_{\tilde{U}^i_\alpha}x_{\chi_\beta}-2x^2_{\chi_\beta}}{6(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^3}-\frac{x_{\tilde{U}^i_\alpha}x^2_{\chi_\beta}(\ln x_{\tilde{U}^i_\alpha}-\ln x_{\chi_\beta})}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^4}\right],$$

$$F_3(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})=\left[\frac{5x_i^3+3x_i^2+6x_i+4}{12(x_i-1)^3}+\frac{x_i(2x_i^2-x_i-2)\ln x_i}{2(x_i-1)^4}\right]+\frac{1}{\tan^2\beta}\left[\frac{x_i(5x_i^2+5x_ix_H-4x_H^2)}{12(x_i-x_H)^3}\right.$$
$$\left.+\frac{(2x_i^3x_H-x_i^2x_H^2)(\ln x_i-\ln x_H)}{2(x_i-x_H)^4}\right]+\sum_{\alpha,\beta}(\mathcal{A}_i^{\alpha,\beta})^2\left[-\frac{x^2_{\tilde{U}^i_\alpha}-x_{\tilde{U}^i_\alpha}x_{\chi_\beta}-2x^2_{\chi_\beta}}{6(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^3}\right.$$
$$\left.-\frac{x_{\tilde{U}^i_\alpha}x^2_{\chi_\beta}(\ln x_{\tilde{U}^i_\alpha}-\ln x_{\chi_\beta})}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^4}\right],$$

$$F_4(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})=-\left(1+\frac{x_i}{2}\right)\left[\frac{5x_i^2-22x_i+5}{18(x_i-1)^3}+\frac{(3x_i-1)\ln x_i}{3(x_i-1)^4}\right]+\frac{1}{\tan^2\beta}\left[-\frac{x_i(5x_i^2-22x_ix_H+5x_H^2)}{36(x_i-x_H)^3}\right.$$
$$\left.+\frac{(x_H^3x_i-3x_H^2x_i^2)(\ln x_i-\ln x_H)}{6(x_i-x_H)^4}\right]+\sum_{\alpha,\beta}(\mathcal{A}_i^{\alpha,\beta})^2\left[\frac{x^2_{\tilde{U}^i_\alpha}-8x_{\tilde{U}^i_\alpha}x_{\chi_\beta}-17x^2_{\chi_\beta}}{36(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^3}\right.$$
$$\left.+\frac{(3x_{\tilde{U}^i_\alpha}x^2_{\chi_\beta}+x^3_{\chi_\beta})(\ln x_{\tilde{U}^i_\alpha}-\ln x_{\chi_\beta})}{6(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^4}\right],$$

$$F_5(x_i,x_H,x_{\tilde{U}^i_\alpha},x_{\chi_\beta})=\left[-\frac{x_i(x_i-3)}{4(x_i-1)^2}-\frac{x_i\ln x_i}{2(x_i-1)^3}\right]+\left[\frac{x_i(x_i-3x_H)}{4(x_i-x_H)^2}+\frac{x_ix_H^2(\ln x_i-\ln x_H)}{2(x_i-x_H)^3}\right]$$
$$+\sum_{\alpha,\beta}\frac{m_{\chi_\beta}}{\sqrt{2}m_W\cos\beta}(\mathcal{A}_3^{\alpha,\beta}\mathcal{B}_3^{\alpha,\beta})\left[\frac{x_{\tilde{U}^i_\alpha}+x_{\chi_\beta}}{2(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^2}+\frac{x_{\tilde{U}^i_\alpha}x_{\chi_\beta}(-\ln x_{\tilde{U}^i_\alpha}+\ln x_{\chi_\beta})}{(x_{\tilde{U}^i_\alpha}-x_{\chi_\beta})^3}\right]. \tag{A2}$$

APPENDIX B: THE WILSON COEFFICIENTS OF THE PENGUIN-INDUCED OPERATORS

The Wilson coefficients for effective $b\overline{s}g(g)$ vertices:

$$C_1(\mu_W)=-\left[\frac{5x_t+1}{2(1-x_t)^3}+\frac{x_t^3+2x_t^2}{(1-x_t)^4}\ln x_t\right]+\frac{1}{\tan^2\beta}\left[\frac{x_t^3x_H(\ln x_t-\ln x_H)}{(x_H-x_t)^4}+\frac{2x_t^3+5x_t^2x_H-x_tx_H^2}{6(x_H-x_t)^3}\right]$$
$$+2\sum_{\alpha,\beta}(\mathcal{A}_3^{\alpha,\beta})^2\left[-\frac{x_{\chi_\beta}^2x_{\tilde{U}_\alpha^3}(\ln x_{\tilde{U}_\alpha^3}-\ln x_{\chi_\beta})}{(x_{\tilde{U}_\alpha^3}-x_{\chi_\beta})^4}-\frac{2x_{\chi_\beta}^2+5x_{\chi_\beta}x_{\tilde{U}_\alpha^3}-x_{\tilde{U}_\alpha^3}^2}{6(x_{\chi_\beta}-x_{\tilde{U}_\alpha^3})^3}\right],$$

$$C_2(\mu_W)=x_t\left[\frac{5x_t-2}{4(1-x_t)^4}\ln x_t+\frac{-4+13x_t-3x_t^2}{8(1-x_t)^3}\right]+\frac{1}{\tan^2\beta}\left[-\frac{x_t^2x_H^2(\ln x_t-\ln x_H)}{4(x_H-x_t)^4}-\frac{2x_tx_H^2+5x_t^2x_H-x_t^3}{24(x_H-x_t)^3}\right]$$
$$+\sum_{\alpha,\beta}(\mathcal{A}_3^{\alpha,\beta})^2\left[\frac{x_{\chi_\beta}^2x_{\tilde{U}_\alpha^3}(\ln x_{\tilde{U}_\alpha^3}-\ln x_{\chi_\beta})}{2(x_{\chi_\beta}-x_{\tilde{U}_\alpha^3})^4}+\frac{2x_{\chi_\beta}^2+5x_{\chi_\beta}x_{\tilde{U}_\alpha^3}-x_{\tilde{U}_\alpha^3}^2}{12(x_{\chi_\beta}-x_{\tilde{U}_\alpha^3})^3}\right],$$

$$C_3(\mu_W)=\left[\frac{-9x_t^2+16x_t-4}{6(1-x_t)^4}\ln x_t+\frac{-x_t^3-11x_t^2+18x_t}{12(1-x_t)^3}\right]+\frac{1}{\tan^2\beta}\left[\frac{(2x_tx_H^3-3x_t^2x_H^2)(\ln x_t-\ln x_H)}{6(x_H-x_t)^4}\right.$$
$$\left.+\frac{16x_tx_H^2-29x_t^2x_H+7x_t^3}{36(x_H-x_t)^3}\right]+\sum_{\alpha,\beta}(\mathcal{A}_3^{\alpha,\beta})^2\left[\frac{x_{\chi_\beta}^3(\ln x_{\tilde{U}_\alpha^3}-\ln x_{\chi_\beta})}{3(x_{\chi_\beta}-x_{\tilde{U}_\alpha^3})^4}+\frac{11x_{\chi_\beta}^2-7x_{\chi_\beta}x_{\tilde{U}_\alpha^3}+2x_{\tilde{U}_\alpha^3}^2}{18(x_{\chi_\beta}-x_{\tilde{U}_\alpha^3})^3}\right],$$

$$C_4(\mu_W)=x_t\left[\frac{x_t^2-x_t}{(1-x_t)^4}\ln x_t+\frac{x_t^2-1}{2(1-x_t)^3}\right]-\left[\frac{x_t^2x_H(\ln x_t-\ln x_H)}{(x_H-x_t)^3}+\frac{x_Hx_t+x_t^2}{2(x_H-x_t)^2}\right]-\sum_{\alpha,\beta}\frac{m_{\chi_\beta}}{\sqrt{2}m_W\cos\beta}(\mathcal{A}_3^{\alpha,\beta}\mathcal{B}_3^{\alpha,\beta})$$
$$\times\left[\frac{2x_{\chi_\beta}x_{\tilde{U}_\alpha^3}(\ln x_{\tilde{U}_\alpha^3}-\ln x_{\chi_\beta})}{(x_{\chi_\beta}-x_{\tilde{U}_\alpha^3})^3}+\frac{x_{\chi_\beta}+x_{\tilde{U}_\alpha^3}}{(x_{\chi_\beta}-x_{\tilde{U}_\alpha^3})^2}\right],$$

$$C_5(\mu_W)=x_t\left[\frac{\ln x_t}{2(1-x_t)^3}+\frac{3-x_t}{4(1-x_t)^2}\right]+\left[\frac{x_tx_H^2(\ln x_t-\ln x_H)}{2(x_H-x_t)^3}+\frac{3x_Hx_t-x_t^2}{4(x_H-x_t)^2}\right]-\sum_{\alpha,\beta}\frac{m_{\chi_\beta}}{\sqrt{2}m_W\cos\beta}(\mathcal{A}_3^{\alpha,\beta}\mathcal{B}_3^{\alpha,\beta})$$
$$\times\left[\frac{x_{\chi_\beta}x_{\tilde{U}_\alpha^3}(\ln x_{\tilde{U}_\alpha^3}-\ln x_{\chi_\beta})}{(x_{\chi_\beta}-x_{\tilde{U}_\alpha^3})^3}+\frac{x_{\chi_\beta}+x_{\tilde{U}_\alpha^3}}{2(x_{\chi_\beta}-x_{\tilde{U}_\alpha^3})^2}\right], \quad \text{(B1)}$$

with

$$\mathcal{A}_i^{\alpha,\beta}=-\mathcal{Z}_{\tilde{U}^i}^{1,\alpha}\mathcal{Z}_{1,\beta}^{+}+\frac{m_{u_i}}{\sqrt{2}m_W\sin\beta}\mathcal{Z}_{\tilde{U}^i}^{2,\alpha}\mathcal{Z}_{2,\beta}^{+},$$

$$\mathcal{B}_i^{\alpha,\beta}=-\mathcal{Z}_{\tilde{U}^i}^{1,\alpha}\mathcal{Z}_{2,\beta}^{-}, \quad \text{(B2)}$$

and the mixing matrices $\mathcal{Z}_{\tilde{U}^i}\mathcal{Z}_{2,\beta}^{\pm}$ are given in Eqs. (6),(7). The first term in each of the above expressions is the SM contributions [12] and the second terms is the charged Higgs contributions. The supersymmetric correction exists in the third term.

The Wilson coefficients for effective $b\overline{s}\gamma$ vertex are

$$C_6(\mu_{\rm W})=x_t\left[\frac{18x_t^2-11x_t-1}{8(1-x_t)^3}+\frac{15x_t^2-16x_t+4}{4(1-x_t)^4}\ln x_t\right]+\frac{x_t}{\tan^2\beta}\left[\frac{4x_t^2+x_tx_H+25x_H^2}{72(x_t-x_H)^3}-\frac{(3x_t^2x_H+2x_tx_H^2)(\ln x_t-\ln x_H)}{12(x_t-x_H)^4}\right]$$
$$+\sum_{\alpha,\beta}(\mathcal{A}_3^{\alpha,\beta})^2\left[-\frac{8x_{\tilde{U}_\alpha^3}^2+5x_{\tilde{U}_\alpha^3}x_{\chi_\beta}-7x_{\chi_\beta}^2}{36(x_{\tilde{U}_\alpha^3}-x_{\chi_\beta})^2}+\frac{(3x_{\tilde{U}_\alpha^3}^2x_{\chi_\beta}-2x_{\tilde{U}_\alpha^3}x_{\chi_\beta}^2)(\ln x_{\tilde{U}_\alpha^3}-\ln x_{\chi_\beta})}{6(x_{\tilde{U}_\alpha^3}-x_{\chi_\beta})^4}\right],$$

$$C_7(\mu_{\rm W})=\left[\frac{-19x_t^3+25x_t^2}{12(1-x_t)^3}+\frac{3x_t^4-30x_t^3+54x_t^2-32x_t+8}{6(1-x_t)^4}\ln x_t\right]+\frac{x_t}{\tan^2\beta}\left[-\frac{19x_t^2+109x_tx_H-98x_H^2}{108(x_t-x_H)^3}\right.$$
$$\left.+\frac{(3x_t^3-9x_t^2x_H-4x_H^3)(\ln x_t-\ln x_H)}{36(x_t-x_H)^4}\right]+\sum_{\alpha,\beta}(\mathcal{A}_3^{\alpha,\beta})^2\left[\frac{52x_{\tilde{U}_\alpha^3}^2-101x_{\tilde{U}_\alpha^3}x_{\chi_\beta}+43x_{\chi_\beta}^2}{54(x_{\tilde{U}_\alpha^3}-x_{\chi_\beta})^3}\right.$$
$$\left.-\frac{(6x_{\tilde{U}_\alpha^3}^3-27x_{\tilde{U}_\alpha^3}^2x_{\chi_\beta}+12x_{\tilde{U}_\alpha^3}x_{\chi_\beta}^2+2x_{\chi_\beta}^3)(\ln x_{\tilde{U}_\alpha^3}-\ln x_{\chi_\beta})}{9(x_{\tilde{U}_\alpha^3}-x_{\chi_\beta})^4}\right],$$

$$C_8(\mu_{\rm W})=x_t\left[\frac{-5x_t^2+8x_t-3}{4(1-x_t)^3}+\frac{3x_t-2}{2(1-x_t)^3}\ln x_t\right]-x_t\left[\frac{1}{2(x_t-x_H)}+\frac{(x_H^2+x_tx_H)(\ln x_t-\ln x_H)}{2(x_t-x_H)^3}\right]$$
$$+\sum_{\alpha,\beta}\frac{m_{\chi_\beta}}{\sqrt{2}m_{\rm W}\cos\beta}(\mathcal{A}_3^{\alpha,\beta}\mathcal{B}_3^{\alpha,\beta})\left[-\frac{7x_{\tilde{U}_\alpha^3}-5x_{\chi_\beta}}{6(x_{\tilde{U}_\alpha^3}-x_{\chi_\beta})^2}+\frac{(3x_{\tilde{U}_\alpha^3}^2+2x_{\tilde{U}_\alpha^3}x_{\chi_\beta})(\ln x_{\tilde{U}_\alpha^3}-\ln x_{\chi_\beta})}{3(x_{\tilde{U}_\alpha^3}-x_{\chi_\beta})^3}\right]. \qquad \text{(B3)}$$

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