PHYSICAL REVIEW D, VOLUME 65, 055001

Search for the fourth standard model family fermions and E_6 quarks at $\mu^+\mu^-$ colliders

A. K. Çiftçi
Physics Department, Faculty of Sciences, Ankara University, 06100 Tandogan, Ankara, Turkey

R. Çiftçi
Physics Department, Faculty of Sciences and Arts, Gazi University, 06500 Teknikokullar, Ankara, Turkey

S. Sultansoy
Physics Department, Faculty of Sciences and Arts, Gazi University, 06500 Teknikokullar, Ankara, Turkey
and Institute of Physics, Academy of Sciences, H. Cavid Avenue 33, Baku, Azerbaijan

(Received 22 June 2001; published 22 January 2002)

The potential of $\mu^+\mu^-$ colliders to investigate the fourth standard model family fermions predicted by flavor democracy is analyzed. It is shown that muon colliders are advantageous for both pair production of fourth family fermions and resonance production of fourth family quarkonia. Also, isosinglet quark production at $\mu^+\mu^-$ colliders is investigated.

DOI: 10.1103/PhysRevD.65.055001 PACS number(s): 12.60.−i, 13.90.+i, 14.60.−z, 14.65.−q

I. INTRODUCTION

The mass spectrum and the mixing of fundamental fermions are among the most important unsolved problems of particle physics. According to the standard model (SM), these masses and mixings arise from the interaction with the Higgs doublet via spontaneous symmetry breaking.

In the framework of the SM, fermions with the same quantum number (electric charge, weak isospin, etc.) are indistinguishable before the symmetry breaking. Therefore, in the fermion-Higgs interaction, the Lagrangian terms corresponding to fermions with the same quantum numbers should come with equal strength. As a result, one deals with singular mass matrices after the spontaneous symmetry breaking.

According to the democratic mass matrix (DMM) approach [1–5] in the case of n SM families, $(n-1)$ families are massless and nth family fermions have masses $na\eta$ (here a is the common strength of Higgs-fermion interactions). Taking the real mass spectrum of the third family fermions into account necessarily leads to the assumption that at least a fourth SM family must exist [6–8] (for a recent situation, see Ref. [9]).

The existence of the fourth SM family and masses of the fourth SM family quarks will be determined as a result of experiments done at the CERN Large Hadron Collider (LHC) [10,11]. In our opinion, muon colliders will be advantageous for investigation of the fourth SM family leptons and quarkonia.

II. THE PRODUCTION OF THE FOURTH SM FAMILY FERMIONS AT $\mu^+\mu^-$ COLLIDERS

It is clear that direct pair production of the fourth family fermions will be possible at future high-energy colliders only, since their predicted masses lie between 300 and 700 GeV [7]. Therefore, lepton colliders with $\sqrt{s}\geqslant 1.5$ TeV and sufficiently high luminosity will give us the opportunity to search for all fermions from the fourth SM family.

Linear e^+e^- colliders with high energy are some of the necessary devices to explore the fundamental ingredients of matter and their interactions. But the advantage of $\mu^+\mu^-$ colliders over e^+e^- colliders is that the former have more monochromatic particle beams (this is especially important since narrow width quarkonia are involved in the following sections). For example, while the energy spread of e^+e^- colliders is more than 1%, that of $\mu^+\mu^-$ colliders is between 0.1% and 0.014% [12]. In addition, since the mass of a muon

TABLE I. The production cross-section values for the fourth SM family fermions with $m_4=500$ GeV at different $\sqrt{s}$ and $L=5\times10^{33}$ cm^{-2} s^{-1}.

$\sqrt{s}$ (TeV)	$\mu^+\mu^-\to u_4\overline{u}_4$ σ (fb)	Events/year	$\mu^+\mu^-\to d_4\overline{d}_4$ σ (fb)	Events/year	$\mu^+\mu^-\to l_4^+l_4^-$ σ (fb)	Events/year	$\mu^+\mu^-\to \nu_4\overline{\nu}_4$ σ (fb)	Events/year
1.10	62.7	3135	25.5	1275	43.2	2160	6.0	300
1.25	67.8	3390	29.1	1455	45.9	2295	7.1	355
1.50	56.6	2830	25.6	1280	37.5	1875	6.5	325
2.00	35.7	1785	17.1	855	23.1	1155	4.4	220
2.50	23.8	1190	11.7	585	15.2	760	3.1	155
3.00	16.8	840	8.4	420	10.7	535	2.2	110
4.00	9.6	480	4.9	245	6.1	305	1.3	65

TABLE II. The production cross-section values and event numbers per year for the fourth SM family $(\mu_4\overline{u}_4)$ quarkonia at muon colliders with $\sqrt{s}=M$ and $L=5\times10^{33}$ cm^{-2} s^{-1}.

M_{ψ_4} (GeV)	$\sigma^{\rm res}$ (pb)	$\Gamma_{\rm tot}(\psi_4)$ (MeV)	$\Delta E_{\rm coll}$ (GeV)	$\sigma^{\rm av}$ (pb)	Events/year
600	68.2	8.3	0.60	0.94	47000
750	19.5	21.1	0.75	0.55	27500
900	6.8	46.9	0.90	0.35	17500
1050	2.8	93.3	1.05	0.25	12500
1200	1.3	170.5	1.20	0.18	9000
1350	0.6	291.6	1.35	0.13	6500
1500	0.3	472.6	1.50	0.09	4500

is 207 times greater than the mass of an electron, the energy uncertainty from the effect of the opposite beam can be ignored. Design values for the luminosity of $\mu^+\mu^-$ colliders are $L=3\times10^{33}$ cm^{-2} s^{-1} at $\sqrt{s}=0.4$ TeV, $L=5\times10^{33}$ cm^{-2} s^{-1} at $\sqrt{s}=4$ TeV, and $L=3\times10^{35}$ cm^{-2} s^{-1} at $\sqrt{s}=30$ TeV [12].

The cross section for the process $\mu^+\mu^-\to f\overline{f}$ has the form

$$\sigma=\frac{2\pi\alpha^2}{3s}\xi\beta\{Q_f(Q_f-2\chi_1 v v_f)(3-\beta^2)+\chi_2(1+v^2)\times[v_f^2(3-\beta^2)+2\beta^2a_f^2]\}, \qquad (1)$$

where

$$\chi_1=\frac{1}{16\sin^2\theta_W\cos^2\theta_W}\,\frac{s(s-M_Z^2)}{(s-M_Z^2)^2+\Gamma_Z^2M_Z^2},$$

$$\chi_2=\frac{1}{256\sin^4\theta_W\cos^4\theta_W}\,\frac{s^2}{(s-M_Z^2)^2+\Gamma_Z^2M_Z^2},$$

$$v=-1+4\sin^2\theta_W,$$

$$a_f=2T_{3f},$$

$$v_f=2T_{3f}-4Q_f\sin^2\theta_W,$$

$$\beta=\sqrt{1-4m_Q^2/s},$$

$$T_3=\tfrac{1}{2}\quad\text{for } \nu_4 \text{ and } u_4,\quad T_3=-\tfrac{1}{2}\quad\text{for } l_4 \text{ and } d_4,$$

$$\xi=1\quad\text{for leptons},\qquad \xi=3\quad\text{for quarks}.$$

The production cross-section values and corresponding event numbers for $m_4=0.5$ TeV at different $\sqrt{s}$ and $L^{\rm int}=50$ fb^{-1} are given in Table I.

III. THE PRODUCTION OF THE FOURTH SM FAMILY $\psi_4(^3S_1)$ QUARKONIA AT $\mu^+\mu^-$ COLLIDERS

The condition for forming $(Q\overline{Q})$ quarkonia states with new heavy quarks is [13]

$$m_Q\leqslant(125\ \text{GeV})|V_{Qq}|^{-2/3}, \qquad (2)$$

where Q and q denote new heavy quarks and known quarks, respectively. Therefore, differing from t quarks, fourth family quarks can form the quarkonia if u_4 and d_4 are almost degenerate and their decays are suppressed by small Cabibbo-Kobayashi-Maskawa (CKM) mixings [7,8]. It should be noted that in this case both $(u_4\overline{u}_4)$ and $(d_4\overline{d}_4)$ quarkonia are formed. If u_4 and d_4 are not degenerate but mixings between the fourth and first three families are still small, quarkonia will be formed by the lighter of u_4 and d_4. In the case of parametrization presented in Ref. [6], which predicts rather large mixings, fourth family quarkonia will not be formed at all. The degeneracy of the fourth SM family quarks in parametrization given in [7,8] is guaranteed as a consequence of the form of the perturbed mass matrix, which includes in the third and fourth family the 2×2 submatrix

$$\begin{pmatrix}1+\alpha & 1-\alpha\\ 1-\alpha & 1+\alpha\end{pmatrix}. \qquad (3)$$

TABLE III. Decay widths for main decay modes of $\psi_4(u_4\overline{u}_4)$ for $m_H=150$ GeV.

M_{ψ_4} (GeV)	600	750	900	1050	1200	1350	1500
$\Gamma(\psi_4\to l^+l^-)$, 10^{-2} MeV	1.4	1.6	1.8	2.0	2.1	2.3	2.5
$\Gamma(\psi_4\to u\overline{u})$, 10^{-2} MeV	2.3	2.6	3.0	3.3	3.5	3.8	4.1
$\Gamma(\psi_4\to d\overline{d})$, 10^{-2} MeV	1.0	1.2	1.3	1.5	1.6	1.7	1.8
$\Gamma(\psi_4\to Z\gamma)$, MeV	0.4	0.7	1.1	1.7	2.4	3.3	4.4
$\Gamma(\psi_4\to ZZ)$, 10^{-2} MeV	4.0	7.6	12.6	19.2	27.6	38.0	50.1
$\Gamma(\psi_4\to ZH)$, 10^{-2} MeV	4.3	7.9	13.0	19.6	28.1	38.5	51.1
$\Gamma(\psi_4\to\gamma H)$, 10^{-2} MeV	36.1	66.3	108.5	164.2	234.8	321.9	426.8
$\Gamma(\psi_4\to W^+W^-)$, MeV	6.8	18.9	43.6	88.7	164.4	283.6	462.4

TABLE IV. The production cross-section values and event numbers per year for the $\psi(D_1\overline{D}_1)$ quarkonia of E_6 isosinglet quarks at muon colliders with $\sqrt{s}=M$ and $L=5\times10^{33}$ cm^{-2} s^{-1}.

$M_{\psi_{D_1}}$ (GeV)	σ^{res} (pb)	$\Gamma_{\mathrm{tot}}(\psi_4)$ (MeV)	ΔE_{coll} (GeV)	σ^{av} (pb)	Events/year
300	26.6	13.8	0.3	1.22	61000
600	19.2	7.1	0.6	0.23	11500
900	1.76	44.6	0.9	0.087	4350
1200	0.31	169.8	1.2	0.044	2200
1500	0.082	478.0	1.5	0.026	1300
1800	0.028	1111.7	1.8	0.017	850
2100	0.011	2270.1	2.1	0.012	600

Another form such as

$$\begin{pmatrix} 1 & 1-\alpha \\ 1-\alpha & 1 \end{pmatrix} \tag{4}$$

would not leave them degenerate. However, this parametrization leads to mass splitting of order of 170 GeV. Since a ρ constraint would allow mass splitting of u_4 and d_4 less than 100 GeV [14], we have not considered such a parametrization.

The cross section for the formation of the fourth family quarkonium and its decay into any X state is given with the relativistic Breit-Wigner equation,

$$\sigma[\mu^+\mu^-\rightarrow(Q\overline{Q})\rightarrow X]=\frac{12\pi(s/M^2)\Gamma_{\mu\mu}\Gamma_X}{(s-M^2)^2+M^2\Gamma^2}, \tag{5}$$

where X corresponds to final-state particles; M is the mass of the fourth family quarkonium; and $\Gamma_{\mu\mu}$, Γ_X, and Γ correspond to the partial decay width to $\mu^+\mu^-$, X state particles, and the total decay width of the fourth family quarkonium, respectively.

Since the $\mu^+\mu^-$ collider has a certain energy spread, the average cross section can be estimated from

$$\sigma^{\mathrm{av}}=\frac{\Gamma_{\mathrm{tot}}}{\Delta E_{\mathrm{coll}}}\sigma^{\mathrm{res}}[\mu^+\mu^-\rightarrow(Q\overline{Q})], \tag{6}$$

where σ^{res} is the resonance value of the cross section [15].

The energy spread is $\Delta E_{\mathrm{coll}}\approx10^{-3}\sqrt{s}$ for the $\mu^+\mu^-$ collider with $\sqrt{s}=O$ (TeV). The estimated cross-section values for $\psi_4(u_4\overline{u}_4)$ are presented in Table II. Corresponding values for $\psi_4(d_4\overline{d}_4)$ are approximately the same. As a result, we obtain a number of events per year, which are given in the last column of Table II.

In this study, we consider only the $\psi_4(^3S_1)$ quarkonia state. Using corresponding formulas from [15] and choosing the Coulomb potential model, we obtain decay widths for the main decay modes of $\psi_4(u_4\overline{u}_4)$, which are given in Table III. One can see that dominant decay modes for ψ_4 quarkonia are $\psi_4\rightarrow W^+W^-$, $\psi_4\rightarrow Z^0\gamma$, and $\psi_4\rightarrow\gamma H$.

IV. THE PRODUCTION OF THE E_6 QUARKS AT $\mu^+\mu^-$ COLLIDERS

Another way to explain the relation $m_{o,\tau}\ll m_t$ is the introduction of exotic fermions. Let us consider as an example the extension of the SM fermion sector, which is inspired by the E_6 grand unified theory (GUT) model initially suggested by Gursey and collaborators [16,17]. It is known that this model is strongly favored in the framework of SUGRA (see Ref. [18] and references therein). For illustration, let us restrict ourselves by quark sector:

$$\begin{pmatrix} u_L^0 \\ d_L^0 \end{pmatrix}, u_R^0, d_R^0; \quad \begin{pmatrix} c_L^0 \\ s_L^0 \end{pmatrix}, c_R^0, s_R^0; \quad \begin{pmatrix} t_L^0 \\ b_L^0 \end{pmatrix}, t_R^0, b_R^0;$$

$$D_{1L}^0, D_{1R}^0; \quad D_{2L}^0, D_{2R}^0; \quad D_{3L}^0, D_{3R}^0.$$

According to flavor democracy, the down quarks' mass matrix has the form

TABLE V. Decay widths for main decay modes of $\psi(D_1\overline{D}_1)$ for $m_H=150$ GeV.

$M_{\psi_{D_1}}$ (GeV)	300	600	900	1200	1500	1800	2100
$\Gamma(\psi_4\rightarrow l^+l^-)$ (keV)	2.3	3.4	4.3	5.2	6.0	6.8	7.5
$\Gamma(\psi_4\rightarrow u\overline{u})$ (keV)	2.6	3.8	4.9	5.9	6.8	7.7	8.5
$\Gamma(\psi_4\rightarrow d\overline{d})$ (keV)	0.9	1.2	1.5	1.8	2.0	2.3	2.5
$\Gamma(\psi_4\rightarrow ZH)$ (keV)	4	27	82	177	321	524	792
$\Gamma(\psi_4\rightarrow\gamma H)$ (keV)	12	90	271	587	1067	1739	2627
$\Gamma(\psi_4\rightarrow W^+W^-)$ (MeV)	0.3	7	44	166	465	1082	2217

$$M^0 = \begin{pmatrix} a\eta & a\eta & a\eta & a\eta & a\eta & a\eta \\ a\eta & a\eta & a\eta & a\eta & a\eta & a\eta \\ a\eta & a\eta & a\eta & a\eta & a\eta & a\eta \\ M & M & M & M & M & M \\ M & M & M & M & M & M \\ M & M & M & M & M & M \end{pmatrix},$$

where M is the scale of ‘‘new’’ physics which determines the masses of the isosinglet quarks. As a result, we obtain five massless quarks, and the sixth quark has the mass $3M + m_t$.

E_6 quarks can be produced at $\mu^+\mu^-$ colliders. To estimate the production cross sections, one can use Eq. (1) with the following minor changes. For the isosinglet D_1 quark, $a_f = 0$ and $v_f = -4Q \sin^2\theta_W$. The result of our estimations shows that the production cross section at $\sqrt{s} = 4$ TeV changes between 1.91 and 1.86 fb (which corresponds to event rates between 95 and 93 per year) for the isosinglet quark mass between 0.1 and 1 TeV.

Additionally, we estimated production cross sections and event numbers per year for isosinglet ψ quarkonia. They are given in Table IV. Decay widths for the main decay modes of ψ_{D_1} are given in Table V, where we use $|V_{D_1 t}| \simeq 0.01$.

V. CONCLUSION

We have shown that $\mu^+\mu^-$ colliders with $\sqrt{s} = O$ (TeV) are a good place to investigate fourth family fermions, quarkonia, and E_6 isosinglet quarkonia. In this study, we have concentrated on the $\psi_4(^3S_1)$ state; other quarkonium states will be considered in a future study.

[1] H. Harari, H. Haut, and J. Weyers, Phys. Lett. **78B**, 459 (1978).

[2] H. Fritzsch, Nucl. Phys. **B155**, 189 (1970).

[3] H. Fritzsch, Phys. Lett. B **184**, 391 (1987).

[4] H. Fritzsch and J. Plankl, Phys. Lett. B **237**, 451 (1990).

[5] H. Fritzsch, CERN-TH.7236/94 (1994).

[6] A. Datta and S. Raychaudhuri, Phys. Rev. D **49**, 4762 (1994).

[7] A. Çelikel, A. K. Çiftçi, and S. Sultansoy, Phys. Lett. B **342**, 257 (1995).

[8] S. Ataǧ *et al.*, Phys. Rev. D **54**, 5745 (1996).

[9] S. Sultansoy, hep-ph/0004271.

[10] ATLAS Detector and Physics Performance Technical Design Report, CERN/LHCC/99-15 (1999), p. 663.

[11] E. Arik *et al.*, ATLAS Internal Note ATL-PHYS-98-125 (1999).

[12] Z. Parsa, in *Muon Collider at the High Energy Frontier—Proceedings of the 6-Month Feasibility Study, Oct’00-Apr’01*, edited by Allen Caldwell and Bruce King (Rinton, Princeton, 2001).

[13] I. Bigi *et al.*, Phys. Lett. B **181**, 157 (1986).

[14] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 104 (2000).

[15] V. Barger *et al.*, Phys. Rev. D **35**, 3366 (1987).

[16] F. Gursey, P. Ramond, and P. Sikivie, Phys. Lett. **60B**, 177 (1976).

[17] F. Gursey and M. Serdaroglu, Lett. Nuovo Cimento Soc. Ital. Fis. **21**, 28 (1978).

[18] J. L. Hewett and T. G. Rizzo, Phys. Rep. **193**, 193 (1989).