# **Photoproduction of**  $J/\psi$  **mesons at high energies in the parton model** and  $K_T$ -factorization approach

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We consider  $J/\psi$  meson photoproduction on protons at high energies at the leading order in  $\alpha_s$  using the conventional parton model and the  $k<sub>T</sub>$ -factorization approach of QCD. It is shown that in both cases the color singlet mechanism gives the correct description for experimental data from DESY HERA for the total cross section and for the  $J/\psi$  meson *z* spectrum at realistic values of a *c*-quark mass and meson wave function at the origin  $\Psi(0)$ . At the same time our predictions for the *p<sub>T</sub>* spectrum of the *J*/ $\psi$  meson and for the *p<sub>T</sub>* dependence of the spin parameter  $\alpha$  obtained in the  $k_T$ -factorization approach are very different from the results obtained in the conventional parton model. Such an experimental study of a polarized  $J/\psi$  meson production at large  $p_T$ should be a direct test of BFKL gluons.

DOI: 10.1103/PhysRevD.65.054041 PACS number(s): 13.85.Ni, 13.87.Fh

## **I. INTRODUCTION**

It is well known that in the processes of  $J/\psi$  meson photoproduction on protons at high energies the photon-gluon fusion partonic subprocess dominates  $[1]$ . In the framework of the general factorization approach of QCD the  $J/\psi$  photoproduction cross section depends on the gluon distribution function in a proton, the hard amplitude of  $c\bar{c}$ -pair production as well as the mechanism of a creation colorless final state with quantum numbers of the  $J/\psi$  meson. In such a way, we suppose that the soft interactions in the initial state are described by introducing a gluon distribution function, the hard partonic amplitude is calculated using perturbative theory of QCD at order in  $\alpha_s(\mu^2)$ , where  $\mu \sim m_c$ , and the soft process of the  $c\bar{c}$ -pair transition into the *J*/ $\psi$  meson is described in nonrelativistic approximation using a series in the small parameters  $\alpha_s$  and *v* (relative velocity of the quarks in the  $J/\psi$  meson). As is said in nonrelativistic QCD  $(NRQCD)$  [2], there are color singlet mechanisms, in which the  $c\bar{c}$  pair is hardly produced in the color singlet state, and color octet mechanisms, in which the  $c\bar{c}$  pair is produced in the color octet state and at a long distance it transforms into a final color singlet state in the soft process. However, as shown in paper  $\lceil 3 \rceil$ , the data from the DESY *ep* collider HERA [4] in the wide region of  $p_T$  and *z* may be described well in the framework of the color singlet model and the color octet contribution is not needed. Based on the abovementioned result we will take into account in our analysis only the color singlet model contribution in the  $J/\psi$  meson photoproduction  $[1]$ . We consider the role of a proton gluon distribution function in the  $J/\psi$  photoproduction in the framework of the conventional parton model as well as in the framework of the  $k_T$ -factorization approach [5]. The last one is based on Balitskii-Fadin-Kuraev-Lipatov (BFKL) evolution equation  $[6]$ , which takes into account large terms proportional to  $\log 1/x$  and  $\log \mu^2/\Lambda_{QCD}^2$  opposite the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation [7], where only large logarithmic terms  $\sim$ log  $\mu^2/\Lambda_{QCD}^2$  are taken into account. In the process of the *J*/ $\psi$  meson photoproduction one has  $x \sim \mu^2/s$  and  $\mu^2 \sim M^2$ , where *M* is the  $J/\psi$  meson mass and *s* is the square of a total energy of colliding particles in the center of mass frame.

# **II. THE CROSS SECTION FOR**  $\gamma p \rightarrow J/\psi X$  **IN THE** *kT***-FACTORIZATION APPROACH**

Nowadays, there are two approaches calculating the  $J/\psi$ meson or heavy quark production cross sections at high energies. In the conventional parton model  $\lceil 8 \rceil$  it is suggested that the hadronic cross section  $\sigma(\gamma p \rightarrow J/\psi X, s)$  and the relevant partonic cross section  $\hat{\sigma}(\gamma g \rightarrow J/\psi g, \hat{s})$  are connected as follows:

$$
\sigma^{PM}(\gamma p \to J/\psi X, s) = \int dx \hat{\sigma}(\gamma g \to J/\psi g, \hat{s}) G(x, \mu^2),
$$
\n(1)

where  $\hat{s} = xs$ ,  $G(x, \mu^2)$  is the collinear gluon distribution function in a proton, *x* is the fraction of a proton momentum,  $\mu^2$  is the typical scale of a hard process. The  $\mu^2$  dependence of the gluon distribution  $G(x, \mu^2)$  is described by the DGLAP evolution equation [7]. In the region of very high *s* one has  $x \leq 1$ . This fact leads to the BFKL evolution equation [6] for the unintegrated gluon distribution function  $\Phi(x, \mathbf{q}_T^2, \mu^2)$ , where  $\mathbf{q}_T^2$  is the gluon virtuality. The unintegrated gluon distribution function can be related to the conventional gluon distribution by

$$
xG(x,\mu^2) = \int_0^{\mu^2} \Phi(x,\mathbf{q}_T^2,\mu^2) d\mathbf{q}_T^2.
$$
 (2)

The gluon 4-momentum is presented as follows:

 $q = xp_N + q_T$ 

where  $q_T=(0,\mathbf{q}_T,0), p_N=(E_N,0,0,|\mathbf{p}_N|)$  and  $q^2=q_T^2=-\mathbf{q}_T^2$ . \*Electronic address: saleev@ssu.samara.ru The so-called BFKL gluon is off mass shell and it has a

polarization vector along its transverse momentum such as  $\varepsilon^{\mu} = q_T^{\mu}/|\mathbf{q}_T|$ . In the *k<sub>T</sub>*-factorization approach the hadronic and partonic cross sections are related by the following condition:

$$
\sigma(\gamma p \to J/\psi X) = \int \frac{dx}{x} \int_0^{\mu^2} d\mathbf{q}_T^2 \int \frac{d\phi}{2\pi} \Phi(x, \mathbf{q}_T^2, \mu^2)
$$

$$
\times \hat{\sigma}(\gamma g^* \to J/\psi g, \hat{s}, \mathbf{q}_T^2), \tag{3}
$$

where  $\hat{\sigma}(\gamma g^* \rightarrow J/\psi g)$  is the *J*/ $\psi$  meson photoproduction on the BFKL gluon, and  $\phi$  is the azimuthal angle in the transverse *XY* plane between vector  $q_T$  and the fixed *OX* axis.

# **III. UNINTEGRATED GLUON DISTRIBUTION FUNCTION**

At the present time an exact form of the unintegrated gluon distribution  $\Phi(x, \mathbf{q}_T^2, \mu^2)$  is unknown yet because the relevant experimental analysis of the experimental data has never been carried out. There are several theoretical approximations for  $\Phi(x, \mathbf{q}_T^2, \mu^2)$ , which are based on solving BFKL evolution equation  $[9-11,14]$ . In the region of very small *x*  $\leq 0.01$  and moderate  $q_T^2$  ( $\sim 10 \text{ GeV}^2$ ), which is relevant for  $J/\Psi$  photoproduction at HERA, all parametrizations  $[9,10,14]$  approximately coincide (see, for example, discussions in  $[12]$  and  $[13]$ . For our purposes, we will use the unintegrated gluon distribution, which was obtained in Ref. [14]. The proposed method lies upon a straightforward perturbative solution of the BFKL equation where the collinear gluon density  $G(x,\mu^2)$  is used as the boundary condition in the integral form  $(2)$ . Technically, the unintegrated gluon distribution is calculated as a convolution of collinear gluon distribution  $G(x, \mu^2)$  with universal weight factors:

$$
\Phi(x, \mathbf{q}_T^2, \mu^2) = \int_x^1 \mathcal{G}(\eta, \mathbf{q}_T^2, \mu^2) \frac{x}{\eta} G\left(\frac{x}{\eta}, \mu^2\right) d\eta, \qquad (4)
$$

where

$$
\mathcal{G}(\eta, \mathbf{q}_T^2, \mu^2) = \frac{\bar{\alpha}_s}{\eta \mathbf{q}_T^2} J_0 \left[ 2 \sqrt{\bar{\alpha}_s \ln\left(\frac{1}{\eta}\right) \ln\left(\frac{\mu^2}{\mathbf{q}_T^2}\right)} \right]
$$
  
at  $\mathbf{q}_T^2 \leq \mu^2$ , (5)

$$
\mathcal{G}(\eta, \mathbf{q}_T^2, \mu^2) = \frac{\bar{\alpha}_s}{\eta \mathbf{q}_T^2} I_0 \left[ 2 \sqrt{\bar{\alpha}_s \ln \left( \frac{1}{\eta} \right) \ln \left( \frac{\mathbf{q}_T^2}{\mu^2} \right)} \right]
$$
  
at  $\mathbf{q}_T^2 > \mu^2$ , (6)

 $J_0$  and  $I_0$  stand for Bessel functions (of real and imaginary arguments, respectively), and  $\bar{\alpha}_s = \alpha_s/3\pi$ . As an input function  $G(x,\mu^2)$  in Eqs. (5) and (6) we use the standard Gluck-Reya-Vogt (GRV) parametrization [15]. To test the method of calculation for  $\Phi(x, \mathbf{q}_T^2, \mu^2)$  we compare the input collinear gluon distribution  $G(x, \mu^2)$  and the collinear gluon distribution  $\tilde{G}(x,\mu^2)$ , which is obtained using formula (2) from the unintegrated distribution  $\Phi(x, \mathbf{q}_T^2, \mu^2)$  after integration



FIG. 1. Diagrams used for the description of the partonic process  $\gamma$ <sup>+</sup>g→*J*/ $\psi$ <sup>+</sup>g.

over  $\mathbf{q}_T^2$ . In Fig. 1 the result of our calculation for the ratio  $R_1(x,\mu^2) = \tilde{G}(x,\mu^2)/G(x,\mu^2)$  is shown. It is visible that at  $x \le 0.1$  the ratio  $R_1$  does not differ from 1 more than  $1 - 2$  %. Note that in a process of the  $J/\psi$  photoproduction on protons  $x \sim M/\sqrt{s}$  (*M* is the mass of the *J/* $\psi$  meson), and one has  $x \sim 0.03$  at  $\sqrt{s} = 100$  GeV.

# **IV.** AMPLITUDE FOR THE  $\gamma g \rightarrow J/\psi g$  PROCESS

There are six Feynman diagrams  $(Fig. 2)$  which describe the partonic process  $\gamma g \rightarrow J/\psi g$  at the leading order in  $\alpha_s$ and  $\alpha$ . In the framework of the color singlet model and the nonrelativistic approximation the production of the  $J/\psi$  meson is considered as the production of a quark-antiquark system in the color singlet state with orbital momentum  $L=0$ and spin momentum  $S=1$ . The binding energy and relative momentum of quarks in the  $J/\psi$  are neglected. In such a way



FIG. 2. Ratio  $R_1(x,\mu^2)$  at  $\mu^2 = 10 \text{ GeV}^2$  (curve 1) and  $\mu^2$  $= 100 \text{ GeV}^2$  (curve 2).

 $M = 2m_c$  and  $p_c = p_c = p/2$ , where *p* is the 4-momentum of the  $J/\psi$ , and  $p_c$  and  $p_{\overline{c}}$  are the 4-momenta of a quark and antiquark. Taking into account the formalism of the projection operator [16] the amplitude of the process  $\gamma g \rightarrow J/\psi g$ may be obtained from the amplitude of the process  $\gamma g$  $\rightarrow$  $\overline{c}cg$  after replacement:

$$
V^{i}(p_{c})\overline{U}^{j}(p_{c}) \rightarrow \frac{\Psi(0)}{2\sqrt{M}}\hat{\varepsilon}(p)(\hat{p}+M)\frac{\delta^{ij}}{\sqrt{3}},
$$
 (7)

where  $\hat{\epsilon}(p) = \epsilon_{\mu}(p)\gamma^{\mu}$ ,  $\epsilon_{\mu}(p)$  is a 4-vector of the *J*/ $\psi$  polarization,  $\delta^{ij}/\sqrt{3}$  is the color factor, and  $\Psi(0)$  is the nonrelativistic meson wave function at the origin. The matrix elements of the process  $\gamma g^* \rightarrow J/\psi g$  may be presented as follows:

$$
M_i = KC^{ab} \varepsilon_\alpha(k_1) \varepsilon_\mu^a(q) \varepsilon_\beta^b(k_2) \varepsilon_\nu(p) M_i^{\alpha \beta \mu \nu}, \qquad (8)
$$

$$
M_1^{\alpha\beta\mu\nu} = \text{Tr}\left[\gamma^{\nu}(\hat{p}+M)\gamma^{\alpha}\frac{\hat{p}_c - \hat{k}_1 + m_c}{(p_c - k_1)^2 - m_c^2}\right]
$$

$$
\times \gamma^{\mu}\frac{-\hat{p}_c - \hat{k}_2 + m_c}{(p_c - k_2)^2 - m_c^2}\gamma^{\beta}\right]
$$
(9)

$$
M_2^{\alpha\beta\mu\nu} = \text{Tr}\left[\gamma^{\nu}(\hat{p}+M)\gamma^{\beta}\frac{\hat{p}_c + \hat{k}_2 + m_c}{(p_c + k_2)^2 - m_c^2}\right]
$$

$$
\times \gamma^{\alpha}\frac{\hat{q} - \hat{p}_c + m_c}{(q - p_c)^2 - m_c^2}\gamma^{\mu}\right]
$$
(10)

$$
M_3^{\alpha\beta\mu\nu} = \text{Tr}\left[\gamma^{\nu}(\hat{p}+M)\gamma^{\alpha}\frac{\hat{p}_c - \hat{k}_1 + m_c}{(p_c - k_1)^2 - m_c^2}\right]
$$

$$
\times \gamma^{\beta}\frac{\hat{q} - \hat{p}_c + m_c}{(q - p_c)^2 - m_c^2}\gamma^{\mu}\right]
$$
(11)

$$
M_4^{\alpha\beta\mu\nu} = \text{Tr}\left[\gamma^{\nu}(\hat{p}+M)\gamma^{\mu}\frac{\hat{p}_c - \hat{q} + m_c}{(p_c - q)^2 - m_c^2}\right]
$$

$$
\times \gamma^{\alpha}\frac{-\hat{p}_c - \hat{k}_2 + m_c}{(k_2 + p_c)^2 - m_c^2}\gamma^{\beta}\right]
$$
(12)

$$
M_S^{\alpha\beta\mu\nu} = \text{Tr}\left[\gamma^{\nu}(\hat{p}+M)\gamma^{\beta}\frac{\hat{p}_c + \hat{k}_2 + m_c}{(p_c + k_2)^2 - m_c^2}\right]
$$

$$
\times \gamma^{\mu}\frac{\hat{k}_1 - \hat{p}_c + m_c}{(k_1 - p_c)^2 - m_c^2}\gamma^{\alpha}\right]
$$
(13)

$$
M_{6}^{\alpha\beta\mu\nu} = \text{Tr}\left[\gamma^{\nu}(\hat{p}+M)\gamma^{\mu}\frac{\hat{p}_{c}-\hat{q}+m_{c}}{(p_{c}-q)^{2}-m_{c}^{2}}\right]
$$

$$
\times \gamma^{\beta}\frac{\hat{k}_{1}-\hat{p}_{c}+m_{c}}{(k_{1}-p_{c})^{2}-m_{c}^{2}}\gamma^{\alpha}\right]
$$
(14)

where  $k_1$  is the 4-momentum of the photon,  $q$  is the 4-momentum of the initial gluon, and  $k_2$  is the 4-momentum of the final gluon,

$$
K = e_c e g_s^2 \frac{\Psi(0)}{2\sqrt{M}}, \quad C^{ab} = \frac{1}{\sqrt{3}} \text{Tr} [T^a T^b],
$$
  

$$
e_c = \frac{2}{3}, \quad e = \sqrt{4\pi\alpha}, \quad g_s = \sqrt{4\pi\alpha_s}.
$$

The summation on the photon, the  $J/\psi$  meson and final gluon polarizations is carried out by covariant formulas:

$$
\sum_{\text{spin}} \varepsilon_{\alpha}(k_1) \varepsilon_{\beta}(k_1) = -g_{\alpha\beta},\tag{15}
$$

$$
\sum_{\text{spin}} \varepsilon_{\alpha}(k_2) \varepsilon_{\beta}(k_2) = -g_{\alpha\beta},\tag{16}
$$

$$
\sum_{spin} \varepsilon_{\mu}(p) \varepsilon_{\nu}(p) = -g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{M^2}.
$$
 (17)

In the case of the initial BFKL gluon we use the following prescription:

$$
\sum_{\text{spin}} \varepsilon_{\mu}(q) \varepsilon_{\nu}(q) = \frac{q_{T\mu} q_{T\nu}}{\mathbf{q}_T^2}.
$$
 (18)

For studing  $J/\psi$  polarized photoproduction we introduce the 4-vector of the longitudinal polarization as follows:

$$
\varepsilon_L^{\mu}(p) = \frac{p^{\mu}}{M} - \frac{Mp_N^{\mu}}{(pp_N)}.
$$
 (19)

In the limit of  $s \ge M^2$  the polarization 4-vector satisfies usual conditions  $(\varepsilon_L \varepsilon_L) = -1$ ,  $(\varepsilon_L p) = 0$ .

Traditionally for a description of quarkonium photoproduction processes the invariant variable  $z = (p p_N)/(k_1 p_N)$  is used. In the rest frame of the proton one has  $z = E_{ab}/E_{\gamma}$ . In the  $k_T$ -factorization approach the differential on  $p_T$  and the *z* cross section of the  $J/\psi$  photoproduction may be written as follows:

$$
\frac{d\sigma}{dp_T^2 dz} = \frac{1}{z(1-z)} \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^{\mu^2} d\mathbf{q}_T^2 \Phi(x, \mathbf{q}_T^2, \mu^2)
$$

$$
\times \frac{|\overline{M}|^2}{16\pi (xs)^2}.
$$
(20)

The numerical calculation is performed in the photon and proton center of mass frame where

$$
p_N = \frac{\sqrt{s}}{2} (1,0,0,1), \quad k_1 = \frac{\sqrt{s}}{2} (1,0,0,-1),
$$

$$
p = (E, \mathbf{p}_T, p_{\parallel}), \quad q = \left(\frac{\sqrt{s}}{2} x, \mathbf{q}_T, \frac{\sqrt{s}}{2} x\right).
$$

Here we take into account that the  $J/\psi$  momentum **p** lies in the *XZ* plane and  $(\mathbf{q}_T \mathbf{p}_T) = |\mathbf{p}_T||\mathbf{q}_T|\cos(\phi)$ . The analytical calculation of the  $\overline{|M|^2}$  is performed with help of the REDUCE package and the results are saved in the FORTRAN codes as a function of  $\hat{s} = (k_1 + q)^2$ ,  $\hat{t} = (p - k_1)^2$ ,  $\hat{u} = (p - k_1)^2$  $(-q)^2$ ,  $\mathbf{p}_T^2$ ,  $\mathbf{q}_T^2$  and cos( $\phi$ ). We directly have tested that

$$
\lim_{\mathbf{q}_T^2 \to 0} \int_0^{2\pi} \frac{d\phi}{2\pi} |M|^2 = |M_{PM}|^2,
$$
\n(21)

where  $\mathbf{p}_T^2 = \hat{t}\hat{u}/\hat{s}$  in the  $|M|^2$  and  $|M_{PM}|^2$  is the square of the amplitude in the conventional parton model  $[3]$ . In the limit of  $q_T^2$ =0 from formula (20) it is easy to find the differential cross section in the parton model, too:

$$
\frac{d\sigma^{PM}}{dp_T^2 dz} = \frac{\overline{|M_{PM}|^2} xG(x,\mu^2)}{16\pi (xs)^2 z(1-z)}.
$$
 (22)

However, making calculations in the parton model we use formula (20), where integration over  $q_T^2$  and  $\phi$  is performed numerically, instead of (22). This method fixes the common normalization factor for both approaches and gives a direct opportunity to study effects connected with virtuality of the initial BFKL gluon in the partonic amplitude.

#### **V. RESULTS AND DISCUSSION**

After we fixed the selection of the gluon distribution function  $G(x,\mu^2)$  there are two parameters only, which values determine the common normalization factor of the cross section under consideration:  $\Psi(0)$  and  $m_c$ . The value of the  $J/\psi$  meson wave function at the origin may be calculated in a potential model or obtained from the experimental well known decay width  $\Gamma(J/\psi \rightarrow \mu^+\mu^-)$ . In our calculation we used the following choice  $|\Psi(0)|^2 = 0.0876 \text{ GeV}^3$  as the same as in Ref. [3]. Concerning a charmed quark mass, the situation is not clear up to the end. From one hand, in the nonrelativistic approximation one has  $m_c = M/2$ , but there are many examples of taking smaller values of a *c*-quark mass in the amplitude of a hard process, for example  $m<sub>c</sub>$  $=1.4$  GeV. Taking into consideration the above-mentioned example we perform calculations as at  $m_c$ =1.4 GeV as well as at  $m_c$ =1.55 GeV. The cinematic region under consideration is determined by the following conditions:  $0.4 < z$  $<$ 0.9 and  $p_T^2$ >1 GeV<sup>2</sup>, which correspond to the H1 and ZEUS Collaborations data  $[4]$ . We assume that the contribution of the color octet mechanism is large at the  $z > 0.9$  only. In the region of the small values of  $z < 0.2$  the contribution of



FIG. 3. Ratio  $R_2(\hat{s}, \mathbf{q}_T^2)$  at  $\hat{s} = 15,50,100 \text{ GeV}^2$ —curves 1, 2, and 3, respectively.

the resolved photon processes  $[17]$  as well as the charm excitation processes  $[18]$  may be large, too. All of these contributions are not in our consideration.

Figure 3 shows the ratio

$$
R_2(\hat{s}, \mathbf{q}_T^2) = \frac{\hat{\sigma}(\hat{s}, \mathbf{q}_T^2)}{\hat{\sigma}(\hat{s}, 0)},
$$
\n(23)

as a function of  $q_T^2$  at the different  $\hat{s}$ . It was necessary to expect ratio (23) to decrease with the growth of  $q_T^2$  and to decrease faster the less the value of *s ˆ*.

Figures 4–6 show our results which were obtained as in the conventional parton model as well as in the  $k_T$ -factorization approach at two values of a charmed quark mass. The dependence of the results on selection of a hard scale parameter  $\mu$  is much less than the dependence on selection of a *c*-quark mass. We put  $\mu^2 = M^2 + \mathbf{p}_T^2$  in a gluon distribution function and in a running constant  $\alpha_s(\mu^2)$ .

Figure 4 shows the dependence of the total  $J/\psi$  photoproduction cross section on  $\sqrt{s}$ . It is visible that the difference between the parton model prediction and the result of the  $k_T$ -factorization approach is much less than between the results obtained at different values of the *c*-quark mass in the both models. At the  $m_c$ =1.55 GeV the obtained cross sections in  $1.5-2$  times are less than experimental data [4], but at the  $m_c$ =1.4 GeV the theoretical curves lie even a shade higher than the experimental points.

There are no contradictions between the theoretical predictions and the data for the *z* spectrum of the  $J/\psi$  mesons. Figure 5 shows that the experimental points lie inside the theoretical corridor as in the parton model and as in the  $k<sub>T</sub>$ -factorization approach.

The count of a transverse momentum of the BFKL gluons in the  $k_T$ -factorization approach results in a flattening of the  $p_T$  spectrum of the *J*/ $\psi$  as contrasted by predictions of the



FIG. 4. The  $J/\psi$  photoproduction cross section as a function of  $\sqrt{s}$  at 0.4<*z*<0.9 and  $p_T^2 > 1$  GeV<sup>2</sup>. The data from [4], the curves 1 and 2, are obtained in the  $k<sub>T</sub>$ -factorization approach at  $m<sub>c</sub>=1.4$  and  $m_c$ =1.55 GeV; the curves 3 and 4 are obtained in the parton model at  $m_c = 1.4$  GeV and  $m_c = 1.55$  GeV.

parton model. For the first time this effect was indicated in Ref.  $[19]$ , and later in Refs.  $[13,20]$ . Figure 6 shows the result of our calculation for the  $p_T$  spectrum of the  $J/\psi$  mesons. Using the  $k_T$ -factorization approach we have obtained the harder  $p_T$  spectrum of the *J*/ $\psi$  than has been predicted in the LO parton model. It is visible that at large values of  $p_T$ only the  $k_T$ -factorization approach gives the correct description of the data  $|4|$ . However, it is impossible to consider this visible effect as a direct indication on nontrivial developments of the small-x physics. In the article  $\lceil 3 \rceil$  was shown that the calculation in the NLO approximation gives a harder

 $d\sigma/dz$ , nb



FIG. 5. The  $J/\psi$  spectrum on *z* at  $\sqrt{s}$  = 100 GeV and  $p_T^2$  $>1$  GeV<sup>2</sup>. The data from [4], the curves, are the same as in Fig. 4.



FIG. 6. The  $J/\psi$  spectrum on  $p_T^2$  at  $\sqrt{s} = 100$  GeV and  $0.4 < z$  $<$  0.9. The data from [4], the curves, are the same as in Fig. 4.

 $p_T$  spectrum of the *J/* $\psi$  meson, too, which will agree with the data at the large  $p_T$ .

As it was mentioned above, the main difference between the  $k_T$ -factorization approach and the conventional parton model is nontrivial polarization of the BFKL gluon. It is obvious that such a spin condition of the initial gluon should result in observed spin effects during the birth of the polarized  $J/\psi$  meson. We have performed calculations for the spin parameter  $\alpha$  as a function *z* or  $p_T$  in the conventional parton model and in the  $k_T$ -factorization approach:

$$
\alpha(z) = \frac{\frac{d\sigma_{tot}}{dz} - 3\frac{d\sigma_L}{dz}}{\frac{d\sigma_{tot}}{dz} + \frac{d\sigma_L}{dz}},
$$
(24)

$$
\alpha(p_T) = \frac{\frac{d\sigma_{tot}}{dp_T} - 3\frac{d\sigma_L}{dp_T}}{\frac{d\sigma_{tot}}{dp_T} + \frac{d\sigma_L}{dp_T}}
$$
(25)

Here  $\sigma_{tot} = \sigma_L + \sigma_T$  is the total *J*/ $\psi$  production cross section,  $\sigma_L$  is the production cross section for the longitudinal polarized  $J/\psi$  mesons, and  $\sigma_T$  is the production cross section for the transverse polarized  $J/\psi$  mesons. The parameter  $\alpha$  controls the angle distribution for leptons in the decay  $J/\psi$  $\rightarrow l^{+}l^{-}$  in the *J/* $\psi$  meson rest frame:

$$
\frac{d\Gamma}{d\cos(\theta)} \sim 1 + \alpha \cos(\theta). \tag{26}
$$

Figure 7 shows the parameter  $\alpha(z)$ , which is calculated in the parton model (curve 2) and in the  $k<sub>T</sub>$ -factorization approach (curve 1). We see that both curves lie near zero at  $z$  $<$ 0.8 and increase at  $z$  $>$ 0.8. The large difference between



FIG. 7. Parameter  $\alpha$  as a function of *z* at  $\sqrt{s} = 100 \text{ GeV}, p_T^2$  $>1$  GeV<sup>2</sup> and  $m_c$ =1.55 GeV. Curve 1 is the result obtained in the  $k<sub>T</sub>$ -factorization approach; curve 2 is the result obtained in the parton model.

predictions is visible only at  $z > 0.9$  where our consideration is not adequate. Let us remark that whether the parameter is gentle depends on the mass of a charmed quark and we demonstrate here only outcomes obtained at the  $m_c = 1.55$ .

For the parameter  $\alpha(p_T)$  we have found strongly opposite predictions in the parton model and in the  $k_T$ -factorization approach, as is visible in Fig. 8. The parton model predicts that  $J/\psi$  mesons should have transverse polarizations at the large  $p_T$  ( $\alpha(p_T) = 0.6$  at  $p_T^2 = 20 \text{ GeV}^2$ ), but the  $k_T$ -factorization approach predicts that  $J/\psi$  mesons should be longitudinally polarized  $(\alpha(p_T) = -0.4$  at  $p_T^2$  $=$  20 GeV<sup>2</sup>). Nowadays, a result of the NLO parton model calculation in the case of the polarized  $J/\psi$  meson photoproduction is unknown. It should be an interesting subject of future investigations. If the count of the NLO corrections will not change predictions of the LO parton model for  $\alpha(p_T)$ , the experimental measurement of this spin effect will be a direct signal about BFKL gluon dynamics.

Nowadays, the experimental data on  $J/\psi$  polarization in photoproduction at large  $p<sub>T</sub>$  are absent. However, there are similar data from the CDF Collaboration [21], where  $J/\psi$ and  $\psi'$   $p_T$  spectra and polarizations have been measured.



FIG. 8. Parameter  $\alpha$  as a function of  $p_T^2$  at  $\sqrt{s} = 100$  GeV, 0.4  $\langle z \rangle$ , and  $m_c$  = 1.55 GeV. The curves are the same as in Fig. 7.

Opposite the case of  $J/\psi$  photoproduction, the hadroproduction data needs to take into account the large color-octet contribution in order to explain  $J/\psi$  and  $\psi'$  production at the Fermilab Tevatron in the conventional collinear parton model. The relative weight of color-octet contribution may be smaller if we use the  $k<sub>T</sub>$ -factorization approach, as was shown recently in  $[22-25]$ . The predicted weight using the collinear parton model transverse polarization of  $J/\psi$  at large  $p_T$  is not supported by the Collider Detector at Fermilab ~CDF! data, which can be roughly explained by the  $k_T$ -factorization approach [23]. In conclusion, the number of theoretical uncertainties in the case of  $J/\psi$  meson hadroproduction is much more than in the case of photoproduction and they need a more complicated investigation, which is why the future experimental analysis of  $J/\psi$  photoproduction at THERA will be a clean check of the collinear parton model and the  $k_T$ -factorization approach.

# **ACKNOWLEDGMENTS**

The author would like to thank S. Baranov and V. Kiselev for discussions on the  $k<sub>T</sub>$ -factorization approach of QCD. This work has been supported in part by the program ''Universities of Russia—Basic Research'' under Grant 02.01.003.

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