Superconformal constraints for QCD conformal anomalies

A. V. Belitsky
C. N. Yang Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794-3840

D. Müller
Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany

(Received 4 July 2001; published 20 February 2002)

Anomalous superconformal Ward identities and commutator algebra in $\mathcal{N}=1$ super-Yang-Mills theory give rise to constraints between the QCD special conformal anomalies of conformal composite operators. We evaluate the superconformal anomalies that appear in the product of renormalized conformal operators and the trace anomaly in the supersymmetric spinor current and check the constraints at one-loop order. In this way we prove the universality of QCD conformal anomalies, which define the nondiagonal part of the anomalous dimension matrix responsible for scaling violations of exclusive QCD amplitudes at the next-to-leading order.

DOI: 10.1103/PhysRevD.65.054037 PACS number(s): 12.38.Bx, 11.10.Gh, 11.30.Pb

I. INTRODUCTION

Supersymmetric models [1], having higher space-time symmetry as compared to conventional ones, provide a strong consistency requirement on theoretical predictions. For the purposes of testing massless QCD calculations an especially illuminating example is $\mathcal{N}=1$ supersymmetric Yang-Mills theory, since both models have, up to a difference in color representation of fermion fields, the same Lagrangian. Thus, we can map a QCD result to an $\mathcal{N}=1$ super-Yang-Mills theory one by identifying the color Casimir operators in corresponding representations, i.e., $C_A=C_F=2N_fT_F\equiv N_c$. After this procedure a QCD result has to satisfy constraints arising from the supersymmetry that connects gluonic and quark sectors of the theory. In this way the use of supersymmetry has allowed us to find a set of identities [2,3] between the entries of the forward anomalous dimension matrices of leading twist-2 composite operators. They were valuable to clarify subtleties appearing in two-loop computations of anomalous dimensions. At the tree level both theories are invariant under conformal transformations. Thus, the $\mathcal{N}=1$ supersymmetric Yang-Mills theory is also invariant under superconformal transformations [4,5], which can give rise to a new set of constraints for certain conformal quantities that appear in the special conformal Ward identities for composite operators, the so-called special conformal matrix. In addition to the breaking of conformal symmetry at the quantum level by the trace anomaly in the energy-momentum tensor, we also have to deal with a superconformal anomaly due to the nonvanishing trace in a supersymmetric spinor current. Nevertheless, an explicit calculation of this anomaly will allow us to check the special conformal anomalies calculated in QCD.

Composite operators appear in various QCD applications by means of operator product expansion and consequently their hadronic matrix elements contain a nonperturbative input, which is needed as an initial condition for the solution of the evolution equations. In the case of exclusive processes the off-forwardness of hadronic matrix elements, given in terms of distribution amplitudes and skewed parton distributions, requires operators with total derivatives. To ensure that the twist-2 operators do not mix under renormalization at leading order in the coupling constant, i.e., their anomalous dimension matrix has the diagonal form $\gamma_j\delta_{jk}$, it is necessary to arrange the operators in such a way that they have a covariant behavior under conformal transformations. This can be easily done. However, beyond the one-loop approximation the anomalous dimension matrix γ_{jk} develops nonzero, nondiagonal, $j>k$, elements $\gamma_{jk}^{\rm ND}\propto\mathcal{O}(\alpha_s^2)$.

The ordinary conformal algebra provides severe restrictions [6,7] on the nonforward anomalous dimensions γ of the conformal operators. In Refs. [6,7] we developed a formalism based on the use of the broken conformal Ward identities for evaluation of the nondiagonal part $\gamma^{\rm ND}$ of the complete anomalous dimensions matrix $\gamma=\gamma^{\rm D}+\gamma^{\rm ND}$. This nondiagonal part arises entirely due to the violation of the special conformal symmetry in perturbation theory. The corresponding anomalies have been calculated to one-loop order accuracy in the minimal subtraction scheme using dimensional regularization, which implies the two-loop approximation for $\gamma^{\rm ND}$. To check our results, one can employ $\mathcal{N}=1$ super-Yang-Mills constraints, valid in a renormalization scheme that respects supersymmetry, for the entries of the nonforward anomalous dimension matrix, derived in [3]. Unfortunately, this is not the case for the dimensionally regularized theory. Thus, one has to find finite renormalization constants from the latter to the dimensional reduction scheme, which is expected to preserve the supersymmetry. But there arises a subtlety in the evaluation of this rotation matrix for the gluonic sector[1] which prevents it from being unambiguously fixed [3]. Nevertheless, our result for two-loop nonforward anomalous dimensions is supported by the fact that the constraints can be satisfied by a finite multiplicative renormalization, which proves the existence of a supersymmetric regularization scheme.

Alternatively, we derive in this paper constraints directly for the special conformal anomalies at one-loop level and show that they are indeed satisfied. Our consequent presentation is organized as follows. In Sec. II we define conformal

[1]This complication does not show up in the forward kinematics.

 ©2002 The American Physical Society

operators, their anomalous dimensions, and the relations of the latter to conformal anomalies. Section III is devoted to the study of translational and conformal superanomalies on the Lagrangian level in the dimensional regularization scheme. Then in Sec. IV we present the transformation properties of conformal operators under the relevant superconformal variations required for a derivation of the Ward identities discussed in the same section. In Sec. V we give a derivation of relations between the scale and special conformal anomalies of the conformal operators. Furthermore, we show that the latter acquire anomalous contributions originating from the product of the trace anomaly in the spinor current and the conformal operators. They are explicitly evaluated in Sec. VI, where it is demonstrated that indeed the anomalous constraints are satisfied with special conformal anomalies from [7]. Finally, we conclude. A few Appendixes are devoted to technical details that we found inappropriate to include in the body of the paper.

II. PRELIMINARIES

In this paper we discuss relations between the QCD scale and special conformal anomalies of conformal operators implied by the $\mathcal{N}=1$ supersymmetry. In $\mathcal{N}=1$ super-Yang-Mills theory we introduce the conformal operators (for the chiral even sector discussed throughout)

$$
{}^{Q}\mathcal{O}^{i}_{jl}=\frac{1}{2}\bar{\psi}^{a}_{+}(i\partial_{+})^{l}C^{3/2}_{j}\left(\frac{\overleftrightarrow{\mathcal{D}}_{+}}{\partial_{+}}\right)\Gamma^{i}\psi^{a}_{+}\,,
$$

$$
{}^{G}\mathcal{O}^{i}_{jl}=G^{a\perp}_{+\mu}(i\partial_{+})^{l-1}C^{5/2}_{j-1}\left(\frac{\overleftrightarrow{\mathcal{D}}_{+}}{\partial_{+}}\right)\mathcal{T}^{i}_{\mu\nu}G^{a\perp}_{\nu+}\,, \qquad (1)
$$

where C^{ν}_{j} are the Gegenbauer polynomials and the tensor structures are $\Gamma^{(V;A)}=(\gamma_{+};\gamma_{+}\gamma_{5})$, $\mathcal{T}^{(V;A)}_{\mu\nu}=(g^{\perp}_{\mu\nu}\equiv g_{\mu\nu}-n_{\mu}n^{\star}_{\nu}-n^{\star}_{\mu}n_{\nu};i\epsilon_{\mu\nu\rho\sigma}n^{\star}_{\rho}n_{\sigma})$. We use the convention $\partial\lambda=\overrightarrow{\partial}+\overleftarrow{\partial}$ and $\overleftrightarrow{\mathcal{D}}=\overrightarrow{\mathcal{D}}-\overleftarrow{\mathcal{D}}$ with the adjoint covariant derivative defined by $\mathcal{D}^{ab}_{\mu}=\partial_{\mu}\delta^{ab}+gf^{acb}B^{c}_{\mu}$. The $+$ sign as a subscript stands for contraction with the lightlike vector n_{μ} which specifies a direction along the light cone. For the latter purposes we introduce another vector $n^{\star}_{\mu}$ such as $n^{2}=n^{\star 2}=0$ and $n\cdot n^{\star}=1$. Obviously, the only difference from QCD arises in the gluino, which we loosely call the quark, sector, which now belongs to the adjoint representation of the color group. The factor $\frac{1}{2}$ in Eq. (1) is related to the Majorana nature of the quarks in the model.

The renormalization group equation for these operators looks like

$$
\frac{d}{d\ln\mu}[\boldsymbol{\mathcal{O}}_{jl}]=-\sum_{k=0}^{j}\gamma_{jk}[\boldsymbol{\mathcal{O}}_{kl}], \qquad (2)
$$

where the square brackets will denote the renormalized operators defined by $[\boldsymbol{\mathcal{O}}_{jl}]=\Sigma^{j}_{k=0}\boldsymbol{Z}_{jk}\boldsymbol{\mathcal{O}}_{kl}$, with the renormalization constant matrix $\boldsymbol{Z}_{jk}$, which generate finite Green functions with elementary field operators $\phi=\{\psi,B_{\mu}\}$. We use everywhere the matrix notation and introduce the vector $\boldsymbol{\mathcal{O}}=\binom{{}^{Q}\mathcal{O}}{{}^{G}\mathcal{O}}$ of quark and gluon conformal operators, which mix with each other under renormalization. As mentioned in the Introduction, the only modification of a given QCD result is to identify the color Casimir operators. Since at leading order the conformal anomalies have a unique color structure[2] it presents no difficulty to disentangle the separate components.

We introduce as well the fermionic operator which is related to the bosonic ones (1) by supersymmetry:

$$
{}^{F}\mathcal{O}^{i}_{jl}=G^{a\perp}_{+\mu}(i\partial_{+})^{l}P^{(2,1)}_{j}\left(\frac{\overleftrightarrow{\mathcal{D}}_{+}}{\partial_{+}}\right)\mathcal{F}^{i}_{\mu}\psi^{a}_{+}\,. \qquad (3)
$$

Here $P^{(a,b)}_{j}$ are the Jacobi polynomials and the vertices read $\mathcal{F}^{(V;A)}=(\gamma^{\perp}_{\mu};\gamma^{\perp}_{\mu}\gamma_{5})$. The operators that form a representation of the supersymmetry algebra are defined by linear combinations of Eq. (1)

$$
\left\{\begin{matrix}\mathcal{S}^{a}\\ \mathcal{P}^{a}\end{matrix}\right\}_{jl}={}^{Q}\omega^{a}_{j}{}^{Q}\mathcal{O}^{\Gamma}_{jl}+{}^{G}\omega^{a}_{j}{}^{G}\mathcal{O}^{\Gamma}_{jl}\,,\quad \mathcal{V}_{jl}=\varrho_{j}{}^{F}\mathcal{O}^{V},
$$

$$
\mathcal{U}_{jl}=\varrho_{j}{}^{F}\mathcal{O}^{A}, \qquad (4)
$$

with $\Gamma=V(A)$ standing for the $\mathcal{S}$ ($\mathcal{P}$) operator, and coefficients ${}^{Q}\omega^{1}_{j}=1$, ${}^{G}\omega^{1}_{j}=6/j$, ${}^{Q}\omega^{2}_{j}=-(j+3)/(j+1)$, ${}^{G}\omega^{2}_{j}=6/(j+1)$, and $\varrho_{j}=(j+2)(j+3)/(j+1)$. Obviously, $\mathcal{U}=-\gamma_{5}\mathcal{V}$. Note that the bosonic and fermionic conformal operators form the $\mathcal{N}=1$ chiral superfield

$$
\Phi=\mathcal{A}+2\theta\chi-\theta^{2}\mathcal{F}, \qquad (5)
$$

with the operators $\mathcal{S}^{1}_{jl}$ and $\mathcal{P}^{1}_{jl}$ ($\mathcal{S}^{2}_{jl}$ and $\mathcal{P}^{2}_{jl}$) being the real and imaginary parts of the $\mathcal{A}$ ($\mathcal{F}^{\dagger}$) complex fields, and $\mathcal{V}_{j-1,l}$ identified with the Majorana fermion $\binom{\chi^{\alpha}}{\bar{\chi}\dot{\alpha}}$ constructed from the Weyl spinor χ. Transformation between operators under supersymmetry arises from the conventional equation $[\bar{\zeta}\mathcal{Q},\Phi]_{-}=[\zeta\mathcal{Q}+\bar{\mathcal{Q}}\bar{\zeta},\Phi]_{-}=\{\zeta r+\bar{r}\bar{\zeta}\}\Phi$, with $r=i(\partial/\partial\theta)$ and $\bar{r}=-i(\partial/\partial\bar{\theta})+2\theta\partial\!\!\!/$.

Now let us briefly point out how the nondiagonal part of the anomalous dimension matrix is induced by the special conformal anomaly matrix. In four-dimensional space-time the 15-parameter conformal group SO(4,2) is defined by its algebra containing the Poincaré, dilatation $\mathcal{D}$, and special conformal $\mathcal{K}_{\mu}$ generators. The conformal anomalies are defined by the renormalized Ward identities. The generic form of the latter, however, written in an unrenormalized cast, reads

$$
\langle[\boldsymbol{\mathcal{O}}_{jl}]\delta\mathcal{X}\rangle=-\langle\delta[\boldsymbol{\mathcal{O}}_{jl}]\mathcal{X}\rangle-\langle i[\boldsymbol{\mathcal{O}}_{jl}]\delta S\mathcal{X}\rangle, \qquad (6)
$$

where $\mathcal{X}=\Pi_{l}\phi(x_{l})$ is a product of elementary fields appearing in the classical Lagrangian. Here δ is any of the varia-

[2]More precisely, the anomalous dimensions in the gluon-gluon channel have in addition to the C_{A} term also trivial N_{f} dependent contributions, which arise from the self-energy insertion.

tions from the symmetry algebra in question. When the transformation is a symmetry of the theory on the quantum level then $\delta S = 0$ up to possible BRST exact operators. In the (dimensionally) regularized theory the action no longer vanishes for conformal, i.e., both scaling, $\delta^S \phi(x) = i[\phi(x),\mathcal{D}] = -(d_\phi + x_\nu \partial_\nu)\phi(x)$, and special conformal, $\delta^C_\mu \phi(x) = i[\phi(x),\mathcal{K}_\mu] = -(2d_\phi x_\mu - x^2\partial_\mu + 2x_\mu x_\nu \partial_\nu - 2ix_\nu \Sigma_{\mu\nu})\phi(x)$, variations, where d_ϕ and $\Sigma_{\mu\nu}$ are the canonical dimension and the spin operator of the field ϕ, respectively. Thus, the renormalization of the operator product $i[\mathcal{O}_{jl}]\delta S$ is responsible for the conformal anomalies.

Moreover, the commutator $[\mathcal{D},\mathcal{K}_-]_- = i\mathcal{K}_-$, where $\mathcal{K}_-$ is the $n^\star_\mu$-light-cone projection of $\mathcal{K}_\mu$, provides a connection between the conformal anomalies. In Ref. [7] the nondiagonal elements of the next-to-leading anomalous dimensions

$$\boldsymbol{\gamma}^{\mathrm{ND}(1)} = [\boldsymbol{\gamma}^{\mathrm{D}(0)}, \boldsymbol{d}(\beta_0 - \boldsymbol{\gamma}^{\mathrm{D}(0)}) + \boldsymbol{g}]_- \tag{7}$$

of the QCD quark and gluon conformal operators were found in terms of the one-loop special conformal anomaly matrix

$$a^{-1}_{jk}(B)\boldsymbol{\gamma}^{c(0)}_{jk} \equiv -\boldsymbol{d}_{jk}(\boldsymbol{\gamma}^{\mathrm{D}(0)}_k - \beta_0 \boldsymbol{P}^G) + \boldsymbol{g}_{jk}. \tag{8}$$

It is constructed out of the leading order anomalous dimensions of conformal operators $\boldsymbol{\gamma}^{\mathrm{D}(0)}$ and the first expansion coefficient of the QCD β function $\beta_0 = \frac{4}{3}T_F N_f - \frac{11}{3}C_A$ times $a_{jk}(B)d_{jk}(F)|_{j>k} = -2(2k+3)$ with the a_{jk} matrix from the conformal transformation of $\boldsymbol{\mathcal{O}}_{jk}$ [see Eq. (29) below]. The projector $\boldsymbol{P}^G = \left(\begin{smallmatrix}0&0\\0&1\end{smallmatrix}\right)$ in Eq. (8) singles out the gluonic component. Finally, the $\boldsymbol{g}$ matrix has appeared from the renormalization of the product of the conformal operator $[\boldsymbol{\mathcal{O}}_{jk}]$ and the integrated trace anomaly $\delta^C_- S \propto -\int d^d x 2x_- \Theta_{\mu\mu}(x)$ in the energy-momentum tensor

$$[\boldsymbol{\mathcal{O}}_{jl}]\delta^C_- S = i\sum_{k=0}^{j} \boldsymbol{\gamma}^c_{jk}[\boldsymbol{\mathcal{O}}_{kl-1}] + \cdots. \tag{9}$$

In the dimensionally regularized theory, i.e., $d = 4 - 2\varepsilon$, the conformal variations of the QCD action can be calculated in a straightforward manner. Choosing the scaling dimensions of the physical fields equal to their canonical values in four dimensions[3] ($d_\psi = \frac{3}{2}$, $d_B = 1$) and setting the scaling dimensions of the ghost fields as $d_\omega = 0$ and $d_{\bar\omega} = d - 2$, the result

$$\delta^B S = \int d^d x\, w^B(x)\bigg\{ -\frac{d-4}{2}(\mathcal{O}_A(x) + \mathcal{O}_B(x) + \Omega_{\bar\omega}(x) - \Omega_{\bar\psi\psi}(x) - \Omega_D(x)) + (d-2)\partial_\mu \mathcal{O}_{B\mu}(x)\bigg\}, \quad \text{with} \quad B = \{D, C\}, \tag{10}$$

is decomposed into operators that can be easily classified according to their renormalization properties. Here the weight function reads $w^D = 1$ and $w^C = 2x_-$ for scale and special conformal transformations, respectively. We introduced the following set of type A and B operators:

$$\mathcal{O}_A(x) = \frac{1}{2}(G^a_{\mu\nu})^2, \quad \mathcal{O}_B(x) = \delta^{\mathrm{BRST}}(\bar\omega^a \partial_\mu B^a_\mu),$$
$$\mathcal{O}_{B\mu}(x) = \delta^{\mathrm{BRST}}(\bar\omega^a B^a_\mu), \tag{11}$$

as well as class C equation-of-motion operators

$$\Omega_G(x) = B^a_\mu \frac{\delta S}{\delta B^a_\mu}, \quad \Omega_{\bar\psi\psi}(x) = \frac{\delta S}{\delta \psi}\psi + \bar\psi \frac{\delta S}{\delta \bar\psi},$$
$$\Omega_{\bar\omega}(x) = \bar\omega^a \frac{\delta S}{\delta \bar\omega^a}, \quad \Omega_D(x) = D^a \frac{\delta S}{\delta D^a}. \tag{12}$$

The renormalization of Eq. (10) is straightforward and the renormalization of the operator products and the resulting renormalized conformal Ward identities are given in Ref. [7].

As a side remark let us note that, in spite of the fact that the conformal field transformation laws for the dimensionally reduced [from $d = 4 - 2\varepsilon$ (and $\varepsilon < 0$) to 4 dimensions] theory differ from the ones in dimensional regularization by the presence of ε-scalar contributions, e.g.,[4] $\tilde\delta^D B^a_\mu = x_{\tilde\nu} G^a_{\mu\nu} - B^a_{\bar\mu}$ and $\tilde\delta^C_- B^a_\mu = (2x_- x_{\tilde\nu} - x^2 n^\star_{\tilde\nu}) G^a_{\mu\nu} - 2x_- B^a_{\bar\mu}$ for the gauge covariant variations of four-dimensional fields, nevertheless, the final result for the variation of the action takes the same form as in Eq. (10) but with boson fields being four dimensional instead.

III. SUPERCONFORMAL ANOMALIES

In four-dimensional space-time the classical action of the $\mathcal{N} = 1$ SU(N_c) super-Yang-Mills theory in the Wess-Zumino gauge, i.e.,

$$S_{\mathrm{cl}} \equiv \int d^4 x\, \mathcal{L}_{\mathrm{cl}}(x) = \int d^4 x \bigg\{ -\frac{1}{4}(G^a_{\mu\nu})^2 + \frac{i}{2}\bar\psi^a \mathcal{D}^{ab}\psi^b + \frac{1}{2}(D^a)^2 \bigg\}, \tag{13}$$

contains the Yang-Mills field strength $G^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g f^{abc} B^b_\mu B^c_\nu$, the Majorana field ψ^a satisfying the conventional condition $\psi^T C^{(+)} = \bar\psi$, and an auxiliary field D^a. It is invariant under transformation of the superconformal group, which consists besides the conformal group also of the trans-

[3]This choice is legitimate since the infinitesimal conformal variation is linear in d_ϕ and thus does not affect the Ward identities, since the anomalous part will show up as a renormalization counterterm of the product of conformal and equation-of-motion operators.

[4]The indices μ, $\tilde\mu$, and $\bar\mu$ refer to the 4-, d-, and 2ε-dimensional spaces, respectively.

lational Q and conformal S supertransformations. The latter two are defined infinitesimally by their action on field operators as

$$\delta^F \psi^a = \frac{i}{2} G^a_{\mu\nu} \sigma_{\mu\nu} \zeta - i D^a \gamma_5 \zeta, \quad \delta^F B^a_\mu = -i \bar{\zeta} \gamma_\mu \psi^a,$$

$$\delta^F D^a = \bar{\zeta} \not{\mathcal{D}}^{ab} \gamma_5 \psi^b, \tag{14}$$

where $\zeta \equiv \zeta_0 - i \not{x} \zeta_1$ is the Grassmann parameter. For $\zeta_1 = 0$ $(F=Q)$ we have restricted supertransformations, while for $\zeta_0 = 0$ $(F=S)$ these equations define the superconformal variations.

The superconformal group is defined by its algebra from which we will be interested in one particular commutator

$$[\mathcal{Q}, \mathcal{K}_-]_- = \gamma_- \mathcal{S}, \tag{15}$$

with $\mathcal{Q}$ ($\mathcal{S}$) being super (conformal) generators. Note, however, that for the short supermultiplet (B^a_μ, ψ^a, D^a) this commutation relation is modified for action on fermions. To restore it one has to use Jackiw's gauge covariant conformal transformation $\tilde{\delta}^C_\mu \equiv \delta^C_\mu + \delta^{\rm gauge}_\mu$ [8], where the gauge transformation is defined with the field-dependent parameter $\epsilon^a_\mu \equiv (2x_\mu x_\nu - x^2 g_{\mu\nu}) B^a_\nu$, instead of the conventional δ^C_μ variation defined above. For the action on a space spanned by gauge invariant operators this modification is irrelevant. The commutator (15), when applied on a Green function with conformal operator insertion, will provide in the supersymmetric limit (identifying color factors) nontrivial relations between the aforementioned QCD special conformal anomalies.

To quantize the theory described by the action (13) we have to add a gauge fixing and a ghost term. We do it via the covariant gauge fixing

$$S_{\rm gf} = \int d^4 x \left\{ -\frac{1}{2\xi} (\partial_\mu B^a_\mu)^2 + \partial_\mu \bar{\omega}^a \mathcal{D}^{ab}_\mu \omega^b \right\}. \tag{16}$$

Although it explicitly breaks the supersymmetry on the Lagrangian level, it will not affect gauge invariant quantities since the supersymmetry variations (14) commute with BRST transformations on the gauge fixing function,[5] i.e., $[\delta^F, \delta^{\rm BRST}]_- (\partial_\mu B^a_\mu) = 0$.

Translational and conformal supervariation of the action $S = S_{\rm cl} + S_{\rm gf}$ regularized by means of the dimensional regularization $4 \to d = 4 - 2\varepsilon$ leads to

$$i\delta^Q S = -(\bar{\zeta}_0 \mathcal{O}_{3\psi}) - \mathcal{O}^{\rm BRST}_Q, \tag{17}$$

$$i\delta^S S = \frac{d-4}{2} (\bar{\zeta}_1 \mathcal{A}) + (\bar{\zeta}_1 \mathcal{O}^-_{3\psi}) - \mathcal{O}^{\rm BRST}_S, \tag{18}$$

where $\mathcal{O} \equiv \int d^d x \mathcal{O}(x)$ and the operator insertions read

$$\mathcal{A}(x) = \sigma_{\mu\nu} G^a_{\mu\nu} \psi^a, \tag{19}$$

$$\mathcal{O}^{\rm BRST}_Q(x) = i\delta^{\rm BRST} \delta^Q (\bar{\omega}^a \partial_\mu B^a_\mu),$$

$$\mathcal{O}^{\rm BRST}_S(x) = i\delta^{\rm BRST} \delta^S (\bar{\omega}^a \partial_\mu B^a_\mu), \tag{20}$$

$$\mathcal{O}_{3\psi}(x) = i\frac{g}{2} f^{abc} (\bar{\psi}^a \gamma_\mu \psi^b)(\gamma_\mu \psi^c),$$

$$\mathcal{O}^-_{3\psi}(x) = \frac{g}{2} f^{abc} (\bar{\psi}^a \gamma_\mu \psi^b)(\not{x} \gamma_\mu \psi^c), \tag{21}$$

with the operator $\mathcal{A}$ being the superconformal anomaly [10,11] in the trace of the supersymmetric current, i.e., $\mathcal{Q}_\rho = \frac{1}{2} G^a_{\mu\nu} \sigma_{\mu\nu} \gamma_\rho \psi^a$. We used in Eq. (20) the identity

$$\delta^F \delta^{\rm BRST} (\bar{\omega}^a \partial_\mu B^a_\mu) = 2\delta^{\rm BRST} \delta^F (\bar{\omega}^a \partial_\mu B^a_\mu) + \delta^F \Omega_{\bar{\omega}}^-.$$

Note that the three-fermion operators $\mathcal{O}_{3\psi}$ and $\mathcal{O}^-_{3\psi}$ vanish in four dimensions by means of Fierz rearrangement. Moreover, $\mathcal{O}^-_{3\psi}$ can be generated by a special conformal variation of the operator $\mathcal{O}_{3\psi}$, namely, $\delta^C_\mu \mathcal{O}_{3\psi} = i\gamma_\mu \mathcal{O}^-_{3\psi} + \mathcal{O}(\varepsilon^2)$, where at one-loop level we can safely neglect the remainder.

Later on we will concentrate on the use of the $\xi = -3$ gauge in the derivation of the constraints, which ensures renormalized supersymmetry at one-loop order [12]. This means that the quark and gluon anomalous dimensions are equal, $\gamma_\phi \equiv \gamma_G = \gamma_\psi$. Furthermore, at one-loop order it was found that the anomaly $\mathcal{A}$ does not acquire gauge variant counterterms [11], provided one uses this particular value of the gauge fixing parameter. Therefore, we can write to this accuracy $\varepsilon\mathcal{A}$ in terms of the renormalized operator

$$\frac{d-4}{2} \mathcal{A} = \frac{\beta_\varepsilon}{g} [\mathcal{A}] + \mathcal{O}(\alpha_s^2), \tag{22}$$

with d-dimensional β function $\beta_\varepsilon = -\varepsilon g + \beta$. Finally, we write the superconformal variation of the action to one-loop accuracy as

$$i\delta^S S = \frac{\beta_\varepsilon}{g} \bar{\zeta}_1 [\mathcal{A}] + \bar{\zeta}_1 \mathcal{O}^-_{3\psi} - \mathcal{O}^{\rm BRST}_S + \mathcal{O}(\alpha_s^2). \tag{23}$$

Let us point out a further consequence of the $\xi = -3$ gauge, which also leads to the equality of the anomalous dimensions γ_ϕ and the β function, namely, $\beta/g = \gamma_\phi = -\alpha_s/(4\pi) 3N_c$. Consequently, the renormalization of the conformal variation of the action will be simplified and its integrand in Eq. (10) reads in one-loop approximation (see, e.g., Ref. [7])

---

[5]We write for brevity $\delta^{\rm BRST}$ instead of $\delta^{\rm BRST}/\delta\lambda$, i.e., after transformation the infinitesimal Grassmann variable is canceled from the right. We recall that the BRST transformations are given by the set of equations $\delta^{\rm BRST} B^a_\mu = \mathcal{D}^{ab}_\mu \omega^b$, $\delta^{\rm BRST} \psi^a = g f^{abc} \omega^b \psi^c$, $\delta^{\rm BRST} \omega^a = (g/2) f^{abc} \omega^b \omega^c$, and $\delta^{\rm BRST} \bar{\omega}^a = (1/\xi) \partial_\mu B^a_\mu$.

$$-\frac{d-4}{2}[\mathcal{O}_A(x)+\mathcal{O}_B(x)+\Omega_{\bar\omega}(x)-\Omega_{\bar\psi\psi}(x)-\Omega_D(x)]$$

$$=-\frac{\beta_\varepsilon}{g}\{[\mathcal{O}_A(x)]-\Omega_{\bar\psi\psi}(x)\}-\frac{d-4}{2}\{[\mathcal{O}_B(x)]+\Omega_{\bar\omega}(x)-\Omega_D(x)\}+2\gamma_{\bar\omega}\Omega_{\bar\omega}(x)+(d-2)\partial_\mu[\mathcal{O}_{B\mu}(x)], \quad (24)$$

with ghost anomalous dimension $\gamma_{\bar\omega}$.

IV. SUPERCONFORMAL WARD IDENTITIES

In order to derive Ward identities we have to know the change of the conformal operators under the superconformal symmetry. Using the rules in Eq. (14) one finds that the translational supersymmetry transformation laws are given by {here and everywhere $\sigma_j=\frac{1}{2}[1-(-1)^j]$}

$$\delta^Q\mathcal{S}^1_{jl}=\sigma_j\overline{\zeta_0}\mathcal{V}_{j-1l}, \quad \delta^Q\mathcal{S}^2_{jl}=\sigma_j\overline{\zeta_0}\mathcal{V}_{jl},$$

$$\delta^Q\mathcal{P}^1_{jl}=\sigma_{j+1}\overline{\zeta_0}\mathcal{U}_{j-1l}, \quad \delta^Q\mathcal{P}^2_{jl}=\sigma_{j+1}\overline{\zeta_0}\mathcal{U}_{jl}, \quad (25)$$

$$\delta^Q\mathcal{V}_{j-1l-1}=-\gamma_-\zeta_0\{\mathcal{S}^1_{jl}+\mathcal{S}^2_{j-1l}\}-\gamma_-\gamma_5\zeta_0\{\mathcal{P}^1_{jl}+\mathcal{P}^2_{j-1l}\}. \quad (26)$$

These equations clarify our comment about the formation of the conformal operators into the chiral superfield: Eqs. (25),(25) are in one-to-one correspondence with the supersymmetric rules for the Wess-Zumino multiplet [4]. Under superconformal variations conformal operators behave as follows:

$$\delta^S\mathcal{S}^1_{jl}=-\sigma_j(j+l+3)\overline{\zeta_1}\gamma_+\mathcal{V}_{j-1l-1},$$

$$\delta^S\mathcal{S}^2_{jl}=-\sigma_j(l-j)\overline{\zeta_1}\gamma_+\mathcal{V}_{jl-1},$$

$$\delta^S\mathcal{P}^1_{jl}=-\sigma_{j+1}(j+l+3)\overline{\zeta_1}\gamma_+\mathcal{U}_{j-1l-1},$$

$$\delta^S\mathcal{P}^2_{jl}=-\sigma_{j+1}(l-j)\overline{\zeta_1}\gamma_+\mathcal{U}_{jl-1}, \quad (27)$$

$$\delta^S\mathcal{V}_{j-1l-1}=-\gamma_-\gamma_+\zeta_1\{(l-j)\mathcal{S}^1_{jl-1}+(j+l+2)\mathcal{S}^2_{j-1l-1}\}-\gamma_-\gamma_5\gamma_+\zeta_1\{(l-j)\times\mathcal{P}^1_{jl-1}+(j+l+2)\mathcal{P}^2_{j-1l-1}\}. \quad (28)$$

Note that Eqs. (25),(27) do not require Fierz rearrangement and, therefore, they do not change their form when the theory is regularized via dimensional regularization. Finally, let us recall that the transformation laws of conformal operators under scaling and special conformal variations are given by

$$\delta^{D\Omega}\mathcal{O}_{jl}=-[l+d(\Omega)]^\Omega\mathcal{O}_{jl}, \quad \delta^{C\,\Omega}_-\mathcal{O}_{jl}=ia_{jl}(\Omega)^\Omega\mathcal{O}_{jl-1}. \quad (29)$$

Here $d(B)\equiv d(G)=d(Q)=3$ and $d(F)=\frac{7}{2}$ as well as $a_{jl}(B)\equiv a_{jl}(Q)=a_{jl}(G)=a(j,l,1,1)$ and $a_{jl}(F)=a(j,l,2,1)$ with $a(j,l,\nu_1,\nu_2)=2(j-l)(j+l+\nu_1+\nu_2+1)$, where $\nu_\phi=d_\phi+s_\phi-1$ and s_ϕ is the spin of the field ϕ. Again the scale dimension d_ϕ is chosen to coincide with its canonical value in four dimensions.

Because of difficulties in preserving the supersymmetry of the theory with quantization and regularization procedures our modest goal will be, therefore, a derivation of the constraints for the special conformal anomalies of the QCD conformal operators stemming from the commutator equation (15) at one-loop level only. We will choose the covariant gauge with $\xi=-3$, which gives us the advantages mentioned above.

The dilatation and special conformal Ward identities for a conformal operator ${}^\Omega\mathcal{O}$, which is either bosonic ($\Omega=B$) or fermionic ($\Omega=F$), look now very simple (cf. Ref. [7]):

$$\langle[{}^\Omega\mathcal{O}_{jl}]\delta^D\mathcal{X}\rangle=\sum_{k=0}^{j}\{[l+d(\Omega)]\mathbf{1}+\boldsymbol{\gamma}(\Omega)\}_{jk}\langle[{}^\Omega\mathcal{O}_{kl}]\mathcal{X}\rangle+\frac{\beta}{g}\langle i[{}^\Omega\mathcal{O}_{jl}(\mathcal{O}_A-\Omega_{\bar\psi\psi})]\mathcal{X}\rangle, \quad (30)$$

$$\langle[{}^\Omega\mathcal{O}_{jl}]\delta^C_-\mathcal{X}\rangle=-i\sum_{k=0}^{j}\{a_{jl}(\Omega)\mathbf{1}+\boldsymbol{\gamma}^c(\Omega)\}_{jk}\langle[{}^\Omega\mathcal{O}_{kl-1}]\mathcal{X}\rangle+\frac{\beta}{g}\langle i[{}^\Omega\mathcal{O}_{jl}(\mathcal{O}^-_A-\Omega^-_{\bar\psi\psi})]\mathcal{X}\rangle-2\langle i[{}^\Omega\mathcal{O}_{jl}\Delta^-_{\rm ext}]\mathcal{X}\rangle, \quad (31)$$

with a 2×2 unit matrix $\mathbf{1}\equiv1_{[2]\times[2]}$. Here $\boldsymbol{\gamma}$ and $\boldsymbol{\gamma}^c$ are the scale and special conformal anomalies. It is well known that in the scale Ward identity coincides with the Callan-Symanzik equation. Thus, $\boldsymbol{\gamma}$ is the conventional anomalous dimension matrix and the combination $(l+d)\mathbf{1}+\boldsymbol{\gamma}$ is the scale dimension matrix of conformal operators. Obviously $\mathcal{O}_A(x)-\Omega_{\bar\psi\psi}(x)=-2\mathcal{L}_{\rm cl}(x)$ is the classical Lagrangian (13) without auxiliary fields. In Eqs. (30),(31) we have also introduced a new convention for the operator insertions weighted with different functions, i.e., $\mathcal{O}=\int d^dx\mathcal{O}(x)$, $\mathcal{O}^-=\int d^dx2x_-\mathcal{O}(x)$ (analogously for equation-of-motion operators), and $\Delta^-_{\rm ext}=\int d^dx2x_-\partial_\mu\mathcal{O}_{B\mu}(x)$. The precise definition of the renormalized operator products is given in the next section.

Let us turn to the renormalized supersymmetric Ward identities. For definiteness let us consider parity even $\mathcal{S}$ operators. From the unrenormalized Ward identity (6), the superconformal variations of the action (17),(23) and the operators (25),(27) we can immediately derive the renormalized Q and S supersymmetric Ward identities in one-loop approximation:

$$\langle[\mathcal{S}^a_{jl}]\delta^Q\mathcal{X}\rangle=-\langle[\delta^Q\mathcal{S}^a_{jl}]\mathcal{X}\rangle+\langle(\overline{\zeta_0}\mathcal{O}_{3\psi})[\mathcal{S}^a_{jl}]\mathcal{X}\rangle + \langle[\mathcal{S}^a_{jl}]\mathcal{O}^{\rm BRST}_Q\mathcal{X}\rangle, \quad (32)$$

$$\langle[\mathcal{S}^a_{jl}]\delta^S\mathcal{X}\rangle=-\langle[\delta^S\mathcal{S}^a_{jl}]\mathcal{X}\rangle+\sigma_j\overline{\zeta_1}\gamma_+\sum_{k=0}^{j} r^{a;V[1]}_{jk}\langle[\mathcal{V}_{kl-1}]\mathcal{X}\rangle - \frac{\beta}{g}\langle(\overline{\zeta_1}\mathcal{A})[\mathcal{S}^a_{jl}]\mathcal{X}\rangle-\langle(\overline{\zeta_1}\mathcal{O}^-_{3\psi})[\mathcal{S}^a_{jl}]\mathcal{X}\rangle + \langle[\mathcal{S}^a_{jl}]\mathcal{O}^{\rm BRST}_S\mathcal{X}\rangle. \quad (33)$$

Here the superconformal anomaly $r^{[1]}_{jk}$ is the residue of the renormalization constant

$$r_{jk}=r^{[0]}_{jk}+\frac{1}{\varepsilon}r^{[1]}_{jk}+\cdots, \quad (34)$$

arising from the renormalization of the operator product

$$(\overline{\zeta_1}[\mathcal{A}])[\mathcal{S}^a_{jl}]=[(\overline{\zeta_1}\mathcal{A})\mathcal{S}^a_{jl}]+\sigma_j\overline{\zeta_1}\gamma_+\sum_{k=0}^{j} {}^a r^V_{jk}[\mathcal{V}_{k,l-1}] \quad (35)$$

and induced by the ε term of β_ε. Note that, since the three-fermion operators vanish in four dimensions, their product with the bosonic operators will give a finite contribution at one-loop order. Similar equations hold for $\mathcal{P}_{jl}$ with the replacement of the index V by A and σ_j by σ_{j+1}.

We have neglected infinite terms in the above Ward identities, since they have to cancel each other. It is instructive to discuss this issue in more detail for the supersymmetric Ward identity of the two-vector $\boldsymbol{\mathcal{S}}=\binom{\mathcal{S}^1}{\mathcal{S}^2}$

$$\langle[\boldsymbol{\mathcal{S}}_{jl}]\delta^Q\mathcal{X}\rangle=-\langle\delta^Q[\boldsymbol{\mathcal{S}}_{jl}]\mathcal{X}\rangle-\langle i[\boldsymbol{\mathcal{S}}_{jl}](\delta^Q S)\mathcal{X}\rangle. \quad (36)$$

The variation of the "good" component of the fermion field, $\psi_+\equiv\frac{1}{2}\gamma_-\gamma_+\psi$, entering in $\mathcal{X}$ may cause a divergency on the left-hand side (LHS) since it contains a composite field strength $G_{\mu\nu}$. Fortunately, in the light-cone gauge the latter can be expressed in terms of the elementary vector potential B_μ and, therefore, the LHS is finite by definition. This gauge, together with the use of dimensional reduction which implies $\delta^Q S=0$, leads to

$$\langle\delta^Q[\boldsymbol{\mathcal{S}}_{jl}]\mathcal{X}\rangle=\text{finite}. \quad (37)$$

Since the renormalization of the composite operators is both gauge and scheme independent at leading order, Eq. (37) holds true also for our choice of scheme. Furthermore, in Eq. (32) the renormalization of the product $[\boldsymbol{\mathcal{S}}_{jl}]\mathcal{O}_{3\psi}$ is finite at leading order because $\mathcal{O}_{3\psi}\sim\mathcal{O}(\varepsilon)$ and cancels a pole in one-loop diagrams. Consequently, the product $[\boldsymbol{\mathcal{S}}_{jl}]\mathcal{O}^{\rm BRST}_Q=$finite, or if divergent it cancels the singularities in $[\boldsymbol{\mathcal{S}}_{jl}]\delta^Q\psi$ mentioned above.

Now we discuss the consequences of Eq. (37). The components of the renormalized operators $[\boldsymbol{\mathcal{S}}_{jk}]$ are defined in terms of unrenormalized ones, constructed though from the renormalized fields ϕ by

$$[\mathcal{S}^a_{jl}]=\sum_{b=1}^{2}\sum_{k=0}^{j}\{{}^{ab}Z_{\mathcal{S}}\}_{jk}\mathcal{S}^b_{kl}, \quad \mathcal{S}^a_{kl}=Z^{-1}_\phi\mathcal{S}^{a(0)}_{kl}, \quad (38)$$

where $\boldsymbol{\mathcal{S}}^{(0)}_{jl}$ is expressed in terms of the bare fields $\phi^{(0)}=\sqrt{Z_\phi}\phi$ and coupling $g^{(0)}=\mu^\varepsilon g/\sqrt{Z_\phi}$. The anomalous dimension matrix of the vector $[\boldsymbol{\mathcal{S}}]$ is defined as usual:

$$\frac{d}{d\ln\mu}[\boldsymbol{\mathcal{S}}]_{jl}=\sum_{k=0}^{j}\boldsymbol{\gamma}^{\mathcal{S}}_{jk}[\boldsymbol{\mathcal{S}}]_{kl} \quad \text{with}$$

$$\boldsymbol{\gamma}^{\mathcal{S}}=\boldsymbol{\gamma}^{\mathcal{S}}_Z+2\gamma_\phi\mathbf{1}=\begin{pmatrix} {}^{11}\gamma^{\mathcal{S}} & {}^{12}\gamma^{\mathcal{S}} \\ {}^{21}\gamma^{\mathcal{S}} & {}^{22}\gamma^{\mathcal{S}}\end{pmatrix}. \quad (39)$$

In our scheme the anomalous dimensions

$$\gamma_\phi=\frac{1}{2}\frac{d}{d\ln\mu}\ln Z_\phi=-\frac{1}{2}\frac{\partial}{\partial\ln g}Z^{[1]}_\phi \quad \text{and}$$

$$\boldsymbol{\gamma}^{\mathcal{S}}_Z=-\left(\frac{d}{d\ln\mu}\boldsymbol{Z}_{\mathcal{S}}\right)\boldsymbol{Z}^{-1}_{\mathcal{S}}=\frac{\partial}{\partial\ln g}\boldsymbol{Z}^{[1]}_{\mathcal{S}} \quad (40)$$

are expressed by the residues of the Laurent expansion of the Z factors $Z=1+Z^{[1]}/\varepsilon+\mathcal{O}(\varepsilon^{-2})$. The renormalized fermionic operator is defined by the same equation (38); however, with the 2×2 matrix $\boldsymbol{Z}_{\mathcal{S}}$ replaced by the numbers $Z_{\mathcal{V}}$.

From Eq. (37) we conclude that

$$\sum_{k=0}^{j}\sum_{k'=0}^{k}\begin{pmatrix}\{{}^{11}Z_{\mathcal{S}}\}_{jk} & \{{}^{12}Z_{\mathcal{S}}\}_{jk}\\ \{{}^{21}Z_{\mathcal{S}}\}_{jk} & \{{}^{22}Z_{\mathcal{S}}\}_{jk}\end{pmatrix}\sigma_k\begin{pmatrix}\{Z^{-1}_{\mathcal{V}}\}_{k-1,k'}\\ \{Z^{-1}_{\mathcal{V}}\}_{kk'}\end{pmatrix}[\mathcal{V}_{k'l}] = \text{finite}, \quad (41)$$

where we implied that $Z_{jk}=0$ for $k>j$. Substituting the Laurent series into this result, the $1/\varepsilon$ poles have to cancel. This is ensured by the relations

$$\sum_{k=0}^{j}\{{}^{11}Z^{[1]}_{\mathcal{S}}\}_{jk}\sigma_k[\mathcal{V}_{k-1,l}]+\sum_{k=0}^{j}\{{}^{12}Z^{[1]}_{\mathcal{S}}\}_{jk}\sigma_k[\mathcal{V}_{kl}] = \sigma_j\sum_{k=0}^{j}\{Z^{[1]}_{\mathcal{V}}\}_{j-1,k}[\mathcal{V}_{kl}], \quad (42)$$

$$\sum_{k=0}^{j}\{{}^{21}Z^{[1]}_{\mathcal{S}}\}_{jk}\sigma_k[\mathcal{V}_{k-1,l}]+\sum_{k=0}^{j}\{{}^{22}Z^{[1]}_{\mathcal{S}}\}_{jk}\sigma_k[\mathcal{V}_{kl}] = \sigma_j\sum_{k=0}^{j}\{Z^{[1]}_{\mathcal{V}}\}_{jk}[\mathcal{V}_{kl}],$$

where $[\mathcal{V}_{kl}]$ are independent operators. These when combined together with the analogous results for parity odd operators (replace $\mathcal{S}\to\mathcal{P},\mathcal{V}\to\mathcal{U}$, and $\sigma_j\to\sigma_{j-1}$) and $Z_{\mathcal{V}}=Z_{\mathcal{U}}$ provide constraints for anomalous dimensions (when differentiated with respect to $\ln g$) [2,3]:

$$\{{}^{11}Z_{\mathcal{S}}^{[1]}\}_{2n+1,2m+1}=\{{}^{22}Z_{\mathcal{P}}^{[1]}\}_{2n,2m}=\{Z_{\mathcal{V}}^{[1]}\}_{2n,2m},$$
$$\{{}^{11}Z_{\mathcal{P}}^{[1]}\}_{2n,2m}=\{{}^{22}Z_{\mathcal{S}}^{[1]}\}_{2n-1,2m-1}=\{Z_{\mathcal{V}}^{[1]}\}_{2n-1,2m-1},$$
$$\{{}^{12}Z_{\mathcal{S}}^{[1]}\}_{2n+1,2m+1}=\{{}^{21}Z_{\mathcal{P}}^{[1]}\}_{2n,2m+2}=\{Z_{\mathcal{V}}^{[1]}\}_{2n,2m+1},$$
$$\{{}^{21}Z_{\mathcal{S}}^{[1]}\}_{2n+1,2m+1}=\{{}^{12}Z_{\mathcal{P}}^{[1]}\}_{2n+2,2m}=\{Z_{\mathcal{V}}^{[1]}\}_{2n+1,2m}. \tag{43}$$

Now let us turn to the renormalization of the superconformal Ward identities. An important consequence of these constraints is that the operators are multiplicatively renormalizable in the one-loop approximation, e.g., $\{{}^{12}Z_{\mathcal{S}}^{[1]}\}_{jj}=\{{}^{21}Z_{\mathcal{S}}^{[1]}\}_{jj}=0$ and $\{{}^{11}Z_{\mathcal{S}}^{[1]}\}_{j+1,j+1}=\{{}^{22}Z_{\mathcal{P}}^{[1]}\}_{jj}=\{Z_{\mathcal{V}}^{[1]}\}_{jj}$. Thus, in this approximation the classical transformation laws (27) for superconformal variations remain true also for the renormalized operators. Consequently, the superconformal variation of the renormalized operator provide finite Green functions

$$\langle\delta^S[\boldsymbol{\mathcal{S}}_{jl}]\mathcal{X}\rangle=\langle[\delta^S\boldsymbol{\mathcal{S}}_{jl}]\mathcal{X}\rangle=\text{finite}. \tag{44}$$

Since $\mathcal{O}^-_{3\psi}$ vanishes in four-dimensional space-time, $\langle(\bar{\zeta}_1\mathcal{O}^-_{3\psi})[\mathcal{S}^a_{jl}]\mathcal{X}\rangle$ is finite. Thus also the superconformal Ward identity (33) is renormalized up to possible divergencies on the LHS that are canceled by the renormalization with BRST-exact operators on the RHS of this Ward identity. Note that in our leading order approximation the anomalous term proportional to the β function is given by a tree approximation.

V. CONSTRAINT EQUALITIES FOR CONFORMAL ANOMALIES

Having derived Ward identities we are now able to discuss consequences of the superconformal algebra. To demonstrate the method, we derive first the set of relations (43) between the anomalous dimensions arising from the commutator of super and scaling variations

$$[\delta^Q,\delta^D]_-=\frac{1}{2}\delta^Q, \tag{45}$$

which is deduced from the commutator algebra $[\mathcal{Q},\mathcal{D}]_-=(i/2)\mathcal{Q}$. Next we deal in the same conceptual manner with the commutator (15) of super and special conformal transformations, written in a symbolical form as

$$[\delta^Q,\delta^C_-]_-=-i\gamma_-\delta^S. \tag{46}$$

This provides us the desired constraints for the special conformal anomalies of the conformal operators. We mostly concentrate on the even parity sector and just state the results for the odd one.

Before we start, let us argue that the three-fermion operators and BRST-exact operators will not contribute to the constraints. As already mentioned, the product of the three-fermion operators with composite operators provides in one-loop approximation a finite part, which could possibly contribute to the constraints. However, its evaluation gives a result that depends on the γ_5 and $\mathrm{tr}\,\mathbf{1}$ prescriptions. Let us explain this point in more detail. In calculating the contributions of this operator product one deals with a quark loop that due to different Wick pairings has three terms; two of them contain a string of Dirac matrices while the last one is a trace. Obviously, in four dimensions the sum gives zero result (recall that $\mathcal{O}^{3\psi}$ and $\mathcal{O}^{3\psi}_-$ vanish by means of Fierz rearrangement). However, because of the ε^{-1} pole in the loop momentum integration, we have to keep ε contributions from the spinor algebra that cancel this pole. Obviously, the ε part is ambiguous for the axial channel: the result depends on the handling of γ_5 in the string of Dirac matrices as well as in the trace. Next, since one of the three contributions is given by a trace of Dirac matrices, the result depends on the prescription for the trace of the unit matrix $\mathrm{tr}\,\mathbf{1}$. One of the choices made in most calculations is to adopt a fiction for d-dimensional gamma matrices that still $\mathrm{tr}\,\mathbf{1}=4$. However, in those computations the trace appears as a single overall factor and the above choice is permissible. It results in scheme dependence for the finite part of, e.g., one-loop diagrams. In our case, since we have an additive trace contribution, we have to continue the Clifford algebra in d-dimensional space as well; this results in the rule $\mathrm{tr}\,\mathbf{1}=2^{[d/2]}$. This convention produces a term involving $\ln 2$, reflecting scheme dependence. On the other hand, we certainly know that in leading order the conformal anomalies do not depend on these ambiguities. So we conclude that the contribution of the operator products in questions cannot affect the constraints.

Now we come to the operator products containing unphysical BRST operators. Of course, one expects that these operators do not contribute to the physical sector; however, they may be responsible for the cancellation of unphysical pieces appearing in the renormalization of products containing only gauge invariant operators. Finally, we are interested in relations for physical quantities and as we already know from the constraints on anomalous dimensions these operator products have to be canceled out in Eq. (45). For the commutator (46), we have the superconformal anomaly and the only difficulty could be a gauge dependent term that is canceled by the operator products in questions. From our previous experience in Refs. [6,7], we expect that such a contribution is absent, and this will be shown by an explicit calculation in Sec. VI. So it is justified to neglect the whole unphysical sector from the very beginning.

A. Commutator constraints for anomalous dimensions

First let us demonstrate the derivation of the relations (43) for anomalous dimensions at leading order from the commutator of scale and supersymmetric variations, given in Eq. (45), applied to the Green functions of composite operators

$$\langle[\boldsymbol{\mathcal{S}}_{jl}]([\delta^Q,\delta^D]\mathcal{X})\rangle=\frac{1}{2}\langle[\boldsymbol{\mathcal{S}}_{jl}]\delta^Q\mathcal{X}\rangle. \tag{47}$$

The RHS of this equality is obviously given by the supersymmetric Ward identity (32) times $\frac{1}{2}$. To calculate the LHS of the commutator we employ the Ward identities (30) and (32), with BRST-exact operators being omitted, and find the following contributions:

$$\begin{aligned}\langle[\boldsymbol{\mathcal{S}}_{jl}]\delta^Q\delta^D\mathcal{X}\rangle=&-\frac{\beta}{g}\langle i[(\delta^Q\boldsymbol{\mathcal{S}}_{jl})(\mathcal{O}_A-\Omega_{\bar\psi\psi})]\mathcal{X}\rangle+\left(l+\frac{7}{2}\right)\\&\times\langle(\overline{\zeta}_0\mathcal{O}_{3\psi})[\boldsymbol{\mathcal{S}}_{jl}]\mathcal{X}\rangle-\sigma_j\overline{\zeta}_0\sum_{k=0}^{j}\left\{\left(l+\frac{7}{2}\right)\right.\\&\times\begin{pmatrix}\delta_{j-1,k}&0\\0&\delta_{jk}\end{pmatrix}+\left.\begin{pmatrix}\gamma^{\mathcal{V}}_{j-1,k}&0\\0&\gamma^{\mathcal{V}}_{jk}\end{pmatrix}\right\}\\&\times\left\langle\begin{pmatrix}[\mathcal{V}_{kl}]\\[\mathcal{V}_{kl}]\end{pmatrix}\mathcal{X}\right\rangle,\end{aligned} \tag{48}$$

$$\begin{aligned}\langle[\boldsymbol{\mathcal{S}}_{jl}]\delta^D\delta^Q\mathcal{X}\rangle=&-\frac{\beta}{g}\langle i\,\delta^Q[\boldsymbol{\mathcal{S}}_{jl}(\mathcal{O}_A-\Omega_{\bar\psi\psi})]\mathcal{X}\rangle+(l+3)\\&\times\langle(\overline{\zeta}_0\mathcal{O}_{3\psi})[\boldsymbol{\mathcal{S}}_{jl}]\mathcal{X}\rangle-\overline{\zeta}_0\sum_{k=0}^{j}\sigma_k\left\{(l+3)\right.\\&\times\begin{pmatrix}\delta_{jk}&0\\0&\delta_{jk}\end{pmatrix}+\left.\begin{pmatrix}{}^{11}\gamma^{\mathcal{S}}_{jk}&{}^{12}\gamma^{\mathcal{S}}_{jk}\\{}^{21}\gamma^{\mathcal{S}}_{jk}&{}^{22}\gamma^{\mathcal{S}}_{jk}\end{pmatrix}\right\}\\&\times\left\langle\begin{pmatrix}[\mathcal{V}_{k-1,l}]\\[\mathcal{V}_{kl}]\end{pmatrix}\mathcal{X}\right\rangle.\end{aligned} \tag{49}$$

To derive the RHS of Eq. (48) we have used for $\langle(\overline{\zeta}_0\mathcal{O}_{3\psi})[\boldsymbol{\mathcal{S}}_{jl}]\delta^D\mathcal{X}\rangle$ the scaling Ward identities with $\delta^D\mathcal{O}_{3\psi}=-\frac{1}{2}\mathcal{O}_{3\psi}$ and the variation (29) of the conformal operators. The variation of the action proportional to ε is neglected, since this Green function is already finite at one-loop order. As was already expected from our previous result (43) and discussion about scheme dependence, the three-fermion operator insertion will not affect the constraints at one-loop level.

Now we come to the terms in the above equations proportional to the β function. To derive the RHS of Eqs. (48),(49) we have used the equation (modulo infinite constants which again do not affect the constraints, since the latter are basically relations between finite contributions)

$$\begin{aligned}\langle i[\boldsymbol{\mathcal{S}}_{jl}(\mathcal{O}_A-\Omega_{\bar\psi\psi})]\mathcal{X}\rangle=&\langle i[\boldsymbol{\mathcal{S}}_{jl}]([\mathcal{O}_A]-\Omega_{\bar\psi\psi})\mathcal{X}\rangle\\&-2\langle i[\boldsymbol{\mathcal{S}}_{jl}]\mathcal{X}\rangle\end{aligned} \tag{50}$$

to get rid of the variation sign in the field monomial $\delta^Q\mathcal{X}$. It results from the study of the differential vertex operator insertions in the Green function $\langle[\boldsymbol{\mathcal{O}}_{jl}]\mathcal{X}\rangle$ with bosonic conformal operator $\boldsymbol{\mathcal{O}}=\binom{{}^Q\mathcal{O}}{{}^G\mathcal{O}}$, and one finds [7] that the renormalization constant of the operator product $[\boldsymbol{\mathcal{O}}_{jl}][\mathcal{O}_A]$,

$$\begin{aligned}i[\boldsymbol{\mathcal{O}}_A(x)][\mathcal{O}_{jl}]=&\,i[\boldsymbol{\mathcal{O}}_A(x)\mathcal{O}_{jl}]-\delta^{(d)}(x)\sum_{k=0}^{j}\{\boldsymbol{Z}_A\}_{jk}[\boldsymbol{\mathcal{O}}_{kl}]\\&-\frac{i}{2}\partial_+\delta^{(d)}(x)\sum_{k=0}^{j}\{\boldsymbol{Z}_A^-\}_{jk}[\boldsymbol{\mathcal{O}}_{kl-1}]-\cdots\\&-\left(g\frac{\partial\ln X}{\partial g}-2\xi\frac{\partial\ln X}{\partial\xi}\right)B^a_\mu(x)\\&\times\frac{\delta}{\delta B^a_\mu(x)}[\boldsymbol{\mathcal{O}}_{jl}],\end{aligned} \tag{51}$$

contains a finite contribution (second term on the RHS)

$$\begin{aligned}\boldsymbol{Z}_A=&\left(g\frac{\partial\boldsymbol{Z}}{\partial g}-2\xi\frac{\partial\boldsymbol{Z}}{\partial\xi}\right)\boldsymbol{Z}^{-1}-2\boldsymbol{Z}\boldsymbol{P}_G\boldsymbol{Z}^{-1}\\&-2\left(g\frac{\partial\ln X}{\partial g}-2\xi\frac{\partial\ln X}{\partial\xi}\right)\boldsymbol{Z}\boldsymbol{P}_G\boldsymbol{Z}^{-1}.\end{aligned} \tag{52}$$

The constant X is related to the charge and gluon wave function renormalization constants by the relation $X=Z_g\sqrt{Z_G}$. For the $[\boldsymbol{\mathcal{S}}_{jl}]\Omega_{\bar\psi\psi}$ we have to use the identity

$$i[\mathcal{O}_{jl}]\Omega_\phi(x)=i[\mathcal{O}_{jl}\Omega_\phi(x)]-\phi(x)\frac{\delta}{\delta\phi(x)}[\mathcal{O}_{jl}], \tag{53}$$

where it is obvious that

$$\langle[\mathcal{O}_{jl}\Omega_\phi(x)]\mathcal{X}\rangle=i\left\langle[\mathcal{O}_{jl}]\phi(x)\frac{\delta}{\delta\phi(x)}\mathcal{X}\right\rangle. \tag{54}$$

Thus in Eq. (50) the finite piece appears as a consequence of two contributions: the ${}^G\mathcal{O}$ part from the $[\boldsymbol{\mathcal{S}}_{jl}][\mathcal{O}_A]$ product due to the finite part in Eqs. (51),(52) and the ${}^Q\mathcal{O}$ part from the $[\boldsymbol{\mathcal{S}}_{jl}]\Omega_{\bar\psi\psi}$ product by means of Eq. (53). Similarly, we have for the fermion operators a finite contribution

$$i[\mathcal{V}_{jl}][\mathcal{O}_A]=i[\mathcal{V}_{jl}\mathcal{O}_A]+[\mathcal{V}_{jl}]+\mathcal{O}(\varepsilon^{-r}) \tag{55}$$

for the trace anomaly, and

$$i[\mathcal{V}_{jl}]\Omega_{\bar\psi\psi}=i[\mathcal{V}_{jl}\Omega_{\bar\psi\psi}]-[\mathcal{V}_{jl}]+\mathcal{O}(\varepsilon^{-r}) \tag{56}$$

for the equation-of-motion insertion. In both cases one should note the factor of 1 in front of the second term on the RHS not 2 as for the quark and gluon operators. Although $\Omega_\psi=\Omega_{\bar\psi}$ for Majorana fermions (recall that $\overline{\psi^a}\gamma_\mu\psi^a=0$ due to the Majorana flip properties), ψ and $\overline{\psi}$ are treated as independent variables in the functional integral. Finally, we have

$$\langle i\,\delta^Q[\boldsymbol{\mathcal{S}}_{jl}(\mathcal{O}_A-\Omega_{\bar\psi\psi})]\mathcal{X}\rangle=\langle i[(\delta^Q\boldsymbol{\mathcal{S}}_{jl})(\mathcal{O}_A-\Omega_{\bar\psi\psi})]\mathcal{X}\rangle, \tag{57}$$

which is almost a trivial result. To derive it one uses Eq. (50) and observes that the variation of the last term $\langle[\boldsymbol{\mathcal{S}}_{jl}]\mathcal{X}\rangle$ in it cancels with the second terms in Eqs. (55),(56) so that we are left with the RHS of Eq. (57). All other terms in the commutator of Ward identities are relatively straightforward to handle. Obviously, with the equality (57) the contributions proportional to the β function in Eqs. (48) and (49) cancel each other in the commutator relation.

Subtracting Eq. (48) from Eq. (49) and comparing the difference to the supersymmetric Ward identity (32) we get, after extraction of independent combinations and identifying both parity sectors for fermionic operators, the known supersymmetric relations [3] (see also [2]):

$$
\begin{aligned}
&{}^{11}\gamma^{\mathcal{S}}_{2n+1,2m+1} = {}^{22}\gamma^{\mathcal{P}}_{2n,2m} = \gamma^{\mathcal{V}}_{2n,2m}, \quad m \leqslant n,\\
&{}^{12}\gamma^{\mathcal{S}}_{2n+1,2m+1} = {}^{21}\gamma^{\mathcal{P}}_{2n,2m+2} = \gamma^{\mathcal{V}}_{2n,2m+1}, \quad m \leqslant n-1,\\
&{}^{21}\gamma^{\mathcal{S}}_{2n+1,2m+1} = {}^{12}\gamma^{\mathcal{P}}_{2n+2,2m} = \gamma^{\mathcal{V}}_{2n+1,2m}, \quad m \leqslant n,\\
&{}^{22}\gamma^{\mathcal{S}}_{2n+1,2m+1} = {}^{11}\gamma^{\mathcal{P}}_{2n+2,2m+2} = \gamma^{\mathcal{V}}_{2n+1,2m+1}, \quad m \leqslant n,\\
&{}^{12}\gamma^{\mathcal{S}}_{2n+1,2n+1} = 0, \quad {}^{12}\gamma^{\mathcal{P}}_{2n,2n} = 0.
\end{aligned} \tag{58}
$$

Obviously, these are the same relations as given in Eq. (43). If one relies on a supersymmetry preserving scheme, these equations can be derived in any order of perturbation theory and they have been checked at two-loop order[6] [3].

B. Commutator constraints for special conformal anomalies

The constraints for the special conformal anomalies of the conformal operators result from the commutator (46) of supersymmetric and special conformal variations applied to the Green function:

$$\langle [\boldsymbol{\mathcal{S}}_{jl}]([\delta^{Q},\delta^{D}]\mathcal{X})\rangle = -i\gamma_{-}\langle[\boldsymbol{\mathcal{S}}_{jl}]\delta^{S}\mathcal{X}\rangle. \tag{59}$$

The derivation runs along the same lines as above up to the appearance of the superconformal anomaly on the RHS, given by the Ward identity (33), and the absence of finite contributions to the renormalization of the product $[\boldsymbol{\mathcal{O}}_{jl}][\mathcal{O}^{-}_{A}]$. Again omitting the BRST-exact operator insertions, the commutator of the LHS is given by the two equations

$$
\begin{aligned}
\langle[\boldsymbol{\mathcal{S}}_{jl}]\delta^{Q}\delta^{C}_{-}\mathcal{X}\rangle = &-\frac{\beta}{g}\langle i[(\delta^{Q}\boldsymbol{\mathcal{S}}_{jl})(\mathcal{O}^{-}_{A}-\Omega^{-}_{\bar\psi\psi})]\mathcal{X}\rangle\\
&+\langle[(\bar\zeta_0\mathcal{O}_{3\psi})\boldsymbol{\mathcal{S}}_{jl}]\delta^{C}_{-}\mathcal{X}\rangle - i\sigma_j\bar\zeta_0\sum_{k=0}^{j}\left\{a_{jl}(F)\right.\\
&\times\begin{pmatrix}\delta_{j-1,k} & 0\\ 0 & \delta_{jk}\end{pmatrix} + \left.\begin{pmatrix}\gamma^{c,\mathcal{V}}_{j-1,k} & 0\\ 0 & \gamma^{c,\mathcal{V}}_{jk}\end{pmatrix}\right\}\\
&\times\left\langle\begin{pmatrix}[\mathcal{V}_{k,l-1}]\\ [\mathcal{V}_{k,l-1}]\end{pmatrix}\mathcal{X}\right\rangle,
\end{aligned} \tag{60}
$$

$$
\begin{aligned}
\langle[\boldsymbol{\mathcal{S}}_{jl}]\delta^{C}_{-}\delta^{Q}\mathcal{X}\rangle = &-\frac{\beta}{g}\langle i\delta^{Q}[\boldsymbol{\mathcal{S}}_{jl}(\mathcal{O}^{-}_{A}-\Omega^{-}_{\bar\psi\psi})]\mathcal{X}\rangle - ia_{jl}(B)\\
&\times\langle[(\bar\zeta_0\mathcal{O}_{3\psi})\boldsymbol{\mathcal{S}}_{j,l-1}]\mathcal{X}\rangle\\
&-i\bar\zeta_0\sum_{k=0}^{j}\sigma_k\left\{a_{jl}(B)\begin{pmatrix}\delta_{jk} & 0\\ 0 & \delta_{jk}\end{pmatrix}\right.\\
&+\left.\begin{pmatrix}{}^{11}\gamma^{c,\mathcal{S}}_{jk} & {}^{12}\gamma^{c,\mathcal{S}}_{jk}\\ {}^{21}\gamma^{c,\mathcal{S}}_{jk} & {}^{22}\gamma^{c,\mathcal{S}}_{jk}\end{pmatrix}\right\}\left\langle\begin{pmatrix}[\mathcal{V}_{k-1,l-1}]\\ [\mathcal{V}_{k,l-1}]\end{pmatrix}\mathcal{X}\right\rangle.
\end{aligned} \tag{61}
$$

The conformal variation of the Green function with three-fermion operator appearing in Eq. (60) can be calculated at the tree level:

$$
\begin{aligned}
\langle[(\bar\zeta_0\mathcal{O}_{3\psi})\boldsymbol{\mathcal{S}}_{jl}]\delta^{C}_{-}\mathcal{X}\rangle = &-ia_{jl}(B)\langle[(\bar\zeta_0\mathcal{O}_{3\psi})\boldsymbol{\mathcal{S}}_{j,l-1}]\mathcal{X}\rangle\\
&-\langle[(\bar\zeta_0\delta^{C}_{-}\mathcal{O}_{3\psi})\boldsymbol{\mathcal{S}}_{jl}]\mathcal{X}\rangle.
\end{aligned}
$$

Here we again neglected the BRST-exact operator that arises from the conformal variation of the action. Taking into account the Ward identity (33) and the relation $\delta^{C}_{-}\mathcal{O}_{3\psi} = i\gamma_{-}\mathcal{O}^{-}_{3\psi}$, we observe again that the three-fermion operator contributions cancel each other in the commutator constraint.

Let us now consider the operator product proportional to the β function, which is defined by [7]

$$
\begin{aligned}
&\langle i[\boldsymbol{\mathcal{O}}_{jl}(\mathcal{O}^{-}_{A}-\Omega^{-}_{\bar\psi\psi})]\mathcal{X}\rangle\\
&\quad=\langle i[\boldsymbol{\mathcal{O}}_{jl}][(\mathcal{O}^{-}_{A}-\Omega^{-}_{\bar\psi\psi})]\mathcal{X}\rangle\\
&\qquad-\left\langle\left[\int d^dx\, 2x_{-}\left(\psi\frac{\delta}{\delta\psi}+\bar\psi\frac{\delta}{\delta\bar\psi}\right)[\boldsymbol{\mathcal{O}}_{jl}]\right]\mathcal{X}\right\rangle.
\end{aligned} \tag{62}
$$

The supersymmetric variation of this product gives

$$
\begin{aligned}
&\langle i\delta^{Q}[\boldsymbol{\mathcal{S}}_{jl}(\mathcal{O}^{-}_{A}-\Omega^{-}_{\bar\psi\psi})]\mathcal{X}\rangle\\
&\quad=\langle i[(\delta^{Q}\boldsymbol{\mathcal{S}}_{jl})(\mathcal{O}^{-}_{A}-\Omega^{-}_{\bar\psi\psi})]\mathcal{X}\rangle + i\langle[(\bar\zeta_0\gamma_{-}\mathcal{A})\boldsymbol{\mathcal{S}}_{jl}]\mathcal{X}\rangle\\
&\qquad+\left\langle\left(\int d^dx\, 2x_{-}\psi\frac{\delta}{\delta\psi}\delta^{Q}[\boldsymbol{\mathcal{S}}_{jl}]\right)\mathcal{X}\right\rangle\\
&\qquad-\left\langle\delta^{Q}\left[\int d^dx\, 2x_{-}\left(\psi\frac{\delta}{\delta\psi}+\bar\psi\frac{\delta}{\delta\bar\psi}\right)[\boldsymbol{\mathcal{S}}_{jl}]\right]\mathcal{X}\right\rangle.
\end{aligned} \tag{63}
$$

Here we have used $\delta^{Q}(\mathcal{O}^{-}_{A}-\Omega^{-}_{\bar\psi\psi}) = \bar\zeta_0\gamma_{-}\mathcal{A}+\cdots$, with the ellipsis standing for the $\mathcal{O}_{3\psi}$ operator, which again is irrelevant in one-loop approximation, since the whole contribution (63) is multiplied by the β function and thus starts from α_s. The first two terms on the RHS of this equation ensure the cancellation with the same terms appearing in Eqs. (33),(60); however, the remaining equation-of-motion operators on the RHS will contribute to the constraints for the special conformal anomaly. We can absorb these additional pieces by a redefinition of the special conformal anomaly matrix. The action of the equation-of-motion operators is

[6] Here we evaluated the rotation matrices from the conventional minimal subtraction scheme, in which all next-to-leading anomalous dimensions are available of the dimensional reduction scheme. As mentioned in the Introduction this procedure does not completely fix the nondiagonal part of the rotation matrices.

$$\int d^dx 2x_- \left(\psi \frac{\delta}{\delta\psi} + \overline{\psi}\frac{\delta}{\delta\overline{\psi}} \right) {}^{Q}\mathcal{O}_{jl} = 2i \sum_{k=0}^{j} b_{jk}(B) {}^{Q}\mathcal{O}_{k,l-1},$$
$$\int d^dx 2x_- \psi \frac{\delta}{\delta\psi} \mathcal{V}_{jl} = 2i \sum_{k=0}^{j} b_{jk}(F) \mathcal{V}_{k,l-1}, \qquad (64)$$

where the $b_{jk}(B)$ matrix can be found in [6,7] and the fermionic one is evaluated in Appendix A. Now we shift the QQ entry by ${}^{QQ}\gamma^c_{jk} \to {}^{QQ}\Gamma^c_{jk} \equiv {}^{QQ}\gamma^c_{jk} + 2(\beta/g) b_{jk}(B)$ and set for the remaining channels ${}^{AB}\Gamma^c_{jk} \equiv {}^{AB}\gamma^c_{jk}$. This redefinition treats the β term equivalently for quarks and gluons and instead of Eq. (9) we have at leading order

$$a^{-1}_{jk}(B) \boldsymbol{\Gamma}^{c(0)}_{jk}(B) \equiv -\boldsymbol{d}_{jk}(\boldsymbol{\gamma}^{\mathrm{D}(0)}_k - \beta_0 \mathbf{1}) + \boldsymbol{g}_{jk}. \qquad (65)$$

The new special conformal anomaly matrix

$$\boldsymbol{\Gamma}^c = \begin{pmatrix} {}^{11}\Gamma^c & {}^{12}\Gamma^c \\ {}^{21}\Gamma^c & {}^{22}\Gamma^c \end{pmatrix} \qquad (66)$$

for the $\boldsymbol{\mathcal{S}}_{jl}$ operators can be found from the conventional quark and gluon ones by the transformation

$$\begin{pmatrix} \frac{1}{k}{}^{11}\Gamma^c_{jk} \\ \frac{1}{k+1}{}^{12}\Gamma^c_{jk} \\ \frac{1}{k}{}^{21}\Gamma^c_{jk} \\ \frac{1}{k+1}{}^{22}\Gamma^c_{jk} \end{pmatrix} = \frac{1}{2k+3} \begin{pmatrix} 1 & \frac{k+3}{6} & \frac{6}{j} & \frac{k+3}{j} \\ -1 & \frac{k}{6} & -\frac{6}{j} & \frac{k}{j} \\ -\frac{j+3}{j+1} & -\frac{(k+3)(j+3)}{6(j+1)} & \frac{6}{j+1} & \frac{k+3}{j+1} \\ \frac{j+3}{j+1} & -\frac{k(j+3)}{6(j+1)} & -\frac{6}{j+1} & \frac{k}{j+1} \end{pmatrix} \begin{pmatrix} {}^{QQ}\Gamma^c_{jk} \\ {}^{QG}\Gamma^c_{jk} \\ {}^{GQ}\Gamma^c_{jk} \\ {}^{GG}\Gamma^c_{jk} \end{pmatrix}. \qquad (67)$$

An analogous convention is introduced for the special conformal anomaly matrix of the fermionic operators[7]

$$\Gamma^c_{jk}(F) \equiv \gamma^c_{jk}(F) + 2\frac{\beta}{g} b_{jk}(F). \qquad (68)$$

If we now compare both sides of Eq. (59) in terms of these new conventions we find that in addition to the special conformal anomaly Γ^c only the superconformal anomaly

$${}^{a}\Delta^{i}_{jk} \equiv -2\,{}^{a}r^{i[1]}_{jk}, \qquad (69)$$

arising from the renormalization of $[\mathcal{A}]\boldsymbol{\mathcal{S}}_{jl}$ in Eq. (35), contributes to the desired constraints:

$$\sum_{k=0}^{j} {}^{11}\Gamma^{c,V}_{jk}(B) \sigma_k [\mathcal{V}_{k-1,l-1}] + \sum_{k=0}^{j} {}^{12}\Gamma^{c,V}_{jk}(B) \sigma_k [\mathcal{V}_{kl-1}]$$
$$- \sigma_j \sum_{k=0}^{j} \{\Gamma^c_{j-1,k}(F) - {}^{1}\Delta^V_{jk}\} [\mathcal{V}_{kl-1}] = 0, \qquad (70)$$

$$\sum_{k=0}^{j} {}^{22}\Gamma^{c,V}_{jk}(B) \sigma_k [\mathcal{V}_{kl-1}] + \sum_{k=0}^{j} {}^{21}\Gamma^{c,V}_{jk}(B) \sigma_k [\mathcal{V}_{k-1,l-1}]$$
$$- \sigma_j \sum_{k=0}^{j} \{\Gamma^c_{jk}(F) - {}^{2}\Delta^V_{jk}\} [\mathcal{V}_{kl-1}] = 0, \qquad (71)$$

and for $\mathcal{P}_{jl}$ they read

$$\sum_{k=0}^{j} {}^{11}\Gamma^{c,A}_{jk}(B) \sigma_{k+1} [\mathcal{U}_{k-1,l-1}] + \sum_{k=0}^{j} {}^{12}\Gamma^{c,A}_{jk}(B) \sigma_{k+1} [\mathcal{U}_{kl-1}]$$
$$- \sigma_{j+1} \sum_{k=0}^{j} \{\Gamma^c_{j-1,k}(F) - {}^{1}\Delta^A_{jk}\} [\mathcal{U}_{kl-1}] = 0, \qquad (72)$$

$$\sum_{k=0}^{j} {}^{22}\Gamma^{c,A}_{jk}(B) \sigma_{k+1} [\mathcal{U}_{kl-1}]$$
$$+ \sum_{k=0}^{j} {}^{21}\Gamma^{c,A}_{jk}(B) \sigma_{k+1} [\mathcal{U}_{k-1,l-1}]$$
$$- \sigma_{j+1} \sum_{k=0}^{j} \{\Gamma^c_{jk}(F) - {}^{2}\Delta^A_{jk}\} [\mathcal{U}_{kl-1}] = 0, \qquad (73)$$

where we have implied $\Gamma^c_{jk} = 0$ for $k > j$. Extracting the independent components from Eqs. (70)–(73) we finally obtain four equalities for the nondiagonal elements:

[7]Here the fermionic conformal anomaly $\gamma^c(F)$ has the same structure as the quark one, namely, $\gamma^c(F) = -b(F)\gamma(F) + w(F)$, with the anomalous dimension $\gamma(F)$ of the quark-gluon operator $\mathcal{V}$ ($\mathcal{U}$) and a part $w(F)$ deduced from the renormalization of $\mathcal{V}$ ($\mathcal{U}$) with the trace of the energy-momentum tensor.

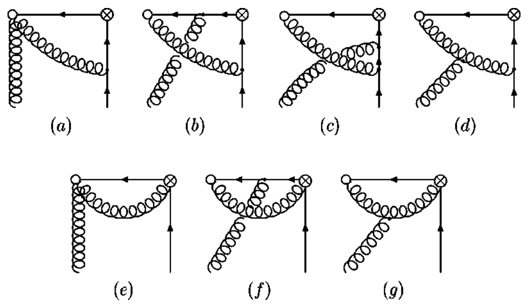

FIG. 1. One-loop diagrams which give rise to divergences in the product of the renormalized operator insertions $i[\mathcal{A}][{}^{Q}\mathcal{O}_{jl}]$.

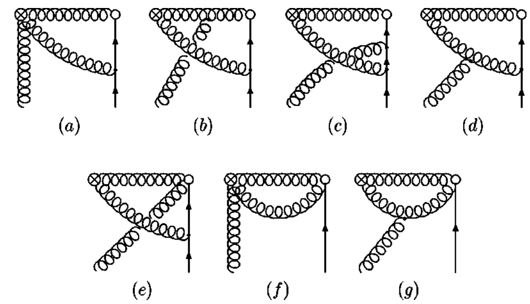

FIG. 2. Same as in Fig. 1 but for the operator product $i[\mathcal{A}][{}^{G}\mathcal{O}_{jl}]$.

$$ {}^{11}\Gamma^{c,V}_{2n+1,2m+1}(B)+{}^{1}\Delta^{V}_{2n+1,2m} = {}^{22}\Gamma^{c,A}_{2n,2m}(B)+{}^{2}\Delta^{A}_{2n,2m}=\Gamma^{c}_{2n,2m}(F), \tag{74}$$

$$ {}^{22}\Gamma^{c,V}_{2n+1,2m+1}(B)+{}^{2}\Delta^{V}_{2n+1,2m+1} = {}^{11}\Gamma^{c,A}_{2n+2,2m+2}(B)+{}^{1}\Delta^{A}_{2n+2,2m+1}=\Gamma^{c}_{2n+1,2m+1}(F), \tag{75}$$

$$ {}^{12}\Gamma^{c,V}_{2n+1,2m+1}(B)+{}^{1}\Delta^{V}_{2n+1,2m+1} = {}^{21}\Gamma^{c,A}_{2n,2m+2}(B)+{}^{2}\Delta^{A}_{2n,2m+1}=\Gamma^{c}_{2n,2m+1}(F), \tag{76}$$

$$ {}^{21}\Gamma^{c,V}_{2n+1,2m+1}(B)+{}^{2}\Delta^{V}_{2n+1,2m} = {}^{12}\Gamma^{c,A}_{2n+2,2m}(B)+{}^{1}\Delta^{A}_{2n+2,2m}=\Gamma^{c}_{2n+1,2m}(F), \tag{77}$$

with $n>m$, and six equations for the diagonal elements, which are of no relevance in prediction (7) for the anomalous dimension matrix. Equations (74)–(77) are the main results of this section. The rest of the paper is devoted to a consistency check of our results for the special conformal anomalies by evaluating the Δ anomalies at one-loop order.

VI. EVALUATION OF SUPERCONFORMAL ANOMALY

To simplify the calculation of the renormalization constants Δ, it is advantageous to use the light-cone position formalism and to this end we introduce the nonlocal light-ray operators

$$ {}^{Q}\mathcal{O}^{i}(\kappa_1,\kappa_2)=\frac{1}{2}\bar{\psi}^{a}_{+}(\kappa_2 n)\Gamma^{i}\Phi^{ab}[\kappa_2,\kappa_1]\psi^{b}_{+}(\kappa_1 n),$$

$$ {}^{G}\mathcal{O}^{i}(\kappa_1,\kappa_2)=G^{a}_{+\mu}(\kappa_2 n)\mathcal{T}^{i}_{\mu\nu}\Phi^{ab}[\kappa_2,\kappa_1]G^{b}_{\nu+}(\kappa_1 n),$$

$$ {}^{F}\mathcal{O}^{i}(\kappa_1,\kappa_2)=G^{a}_{+\mu}(\kappa_2 n)\mathcal{F}^{i}_{\mu}\Phi^{ab}[\kappa_2,\kappa_1]\psi^{b}_{+}(\kappa_1 n). \tag{78}$$

The calculations of the mixing kernels for the product $[\mathcal{A}][\mathcal{O}]$ with fermionic operators are straightforward and go along the lines of Ref. [13] (see also Appendix B of Ref. [7] for a recent review of this formalism in the case when the nonforwardness is essential). The diagrams are represented in Figs. 1 and 2 for quark and gluon operators, respectively. Since these renormalization constants must be gauge independent, we have performed the computation with a general ξ and indeed observed its cancellation in the final result. This shows that the BRST-exact operators cannot contribute to the constraints. The generic form of the result is

$$ [{}^{\Omega}\mathcal{O}^{i}](\kappa_1,\kappa_2)(\overline{\zeta_1}[\mathcal{A}])=\frac{1}{\varepsilon}\frac{\alpha_s}{2\pi}N_c\int_0^1 dx\int_0^1 dy\,\theta(1-x-y)^{\Omega}\times\mathcal{R}^{i}(x,y)(\overline{\zeta_1}\gamma^{F}_{+}\mathcal{O}^{i})(\bar{x}\kappa_1+x\kappa_2,\ y\kappa_1+\bar{y}\kappa_2)\mp(\kappa_1\leftrightarrow\kappa_2), \tag{79}$$

where the $-$ $(+)$ sign in the second term corresponds to the parity even quarks and parity odd gluons (parity even gluons and parity odd quarks) and we use throughout the convention $\bar{x}\equiv 1-x$. Skipping details (see Appendix B) we give the result:

$$ -k_{2+}\,{}^{Q}\mathcal{R}^{V}(x,y)=-\left[\frac{1}{y}\right]_{+}\delta(x),\quad {}^{Q}\mathcal{R}^{A}(x,y)={}^{Q}\mathcal{R}^{V}(x,y), \tag{80}$$

$$ {}^{G}\mathcal{R}^{V}(x,y)=-\left[\frac{1}{x}\right]_{+}\delta(y)-2+\delta(y)+(1-y)\delta(x),$$

$$ {}^{G}\mathcal{R}^{A}(x,y)={}^{G}\mathcal{R}^{V}(x,y)+4y, \tag{81}$$

with the $+$ prescription defined conventionally by

$$ \left[\frac{1}{x}\right]_{+}=\frac{1}{x}-\delta(x)\int_0^1\frac{dy}{y}.$$

We have kept the gluon momentum k_{2+} unintegrated, in the quark sector which stems from the exponential in the Feynman rules for the operator insertion vertex $\times\exp(-i\kappa_1 k_{1+}-i\kappa_2 k_{2+})$. We merely substitute it by the corresponding parton momentum fraction when passing to exclusive type kernels (see Appendix C). Note that the result for the polarized quark kernel is the same as for the parity even case. This could be anticipated, since for divergent parts we can use the anticommutativity of γ_5. However, there is a sign change in the contribution of the diagram in Fig. 1(c) but it gives the contribution $\pm x\,\delta(1-x-y)$ with a $+$ $(-)$ sign for even (odd) parity and together with $\bar{x}\,\delta(1-x-y)$ from Fig. 1(d) gives a vanishing result when the symmetry property of the corresponding quark string operators is used. For gluons the difference between the parity even and odd sectors comes entirely from the contributions of the graphs in Figs. 2(b), 2(c), and 2(e). The contributions from 2(d) and 2(g) annihilate each other, while the remaining diagrams give identical[8] results for the V and A channels.

Evaluation of the conformal moments according to the method spelled out in Appendix C gives for gluons

$$\left(\frac{\alpha_s}{2\pi}N_c\right)^{-1G}\mathcal{R}^{[1]}_{jk}=\frac{1}{3}\theta_{j-1,k}\left\{\frac{(k+1)(k+3)\mp 2}{k+1}+(k+2)\times(k+3)\Psi_{jk}+(-1)^{j+k}(k+2)\times(k+3)\Xi_{jk}\right\}, \tag{82}$$

with the $-$ $(+)$ sign for the vector (axial) channel; and for quarks

$$\left(\frac{\alpha_s}{2\pi}N_c\right)^{-1}{}^{Q}\mathcal{R}^{[1]}_{jk}=2\frac{\theta_{j-1,k}}{(k+1)}\{(-1)^{j+k}(k+2)-1-(-1)^{j+k}(j+1)(j+2)\Psi_{jk}-(j+1)(j+2)\Sigma_{jk}\}. \tag{83}$$

To simplify the presentation we have introduced the matrices, depending only on the logarithmic derivative of the Euler integral $\psi(x)=(d/dx)\ln\Gamma(x)$, via

$$\Psi_{jk}=\psi\left(\frac{j+k+4}{2}\right)-\psi(j+k+4)-\psi\left(\frac{j-k}{2}\right)+\psi(j-k),$$
$$\Xi_{jk}=\psi(j+k+4)+\psi(j-k)-\psi(k+4)-\psi(k+1),$$
$$\Sigma_{jk}=\psi(j+k+4)+\psi(j-k)-2\psi(j+2). \tag{84}$$

The conformal moments $\mathcal{R}_{jk}$ of the kernels $\mathcal{R}(x,y)$ are related to those in Eq. (69) by a normalization factor and, therefore, the superconformal anomaly is

$${}^{a}\Delta_{jk}=-\frac{2}{\varrho_k}({}^{Q}\omega^{aQ}_{j}\mathcal{R}_{jk}+{}^{G}\omega^{aG}_{j}\mathcal{R}_{jk}) \quad \text{with} \quad a=1,2. \tag{85}$$

We should note that for even and odd $j-k$ the ψ functions with argument depending on both j and k enter only in the particular combination

$$A_{jk}=\psi\left(\frac{j+k+4}{2}\right)-\psi\left(\frac{j-k}{2}\right)+2\psi(j-k)-\psi(j+2)-\psi(1),$$

which also arises in all special conformal anomalies γ^c [7].

The final step is to insert our findings for the superconformal anomalies given in Eqs. (82)–(85) and our results for the eight special conformal anomaly matrices from Ref. [7], rotated to the ${}^{ab}\Gamma^{c}$ basis by means of Eq. (67), into the four conformal constraints (74)–(77). Indeed, we find that all of them are identically satisfied. Thus, the universality of the special conformal anomaly matrix for even and odd parity sectors arises from the form of the superconformal anomaly in $\mathcal{N}=1$ Yang-Mills theory.

[8]Discarding the fact that contributions with $\kappa_1\leftrightarrow\kappa_2$ enter with opposite signs.

VII. CONCLUSIONS

The use of $\mathcal{N}=1$ supersymmetry allows us to find nontrivial relations for both scale and special conformal anomalies of QCD conformal operators. In the former case, the constraints, at leading order in the dimensional regularization scheme as well as to all orders in a supersymmetry preserving scheme, involve only anomalous dimensions. There are six constraints for the diagonal and four for the nondiagonal elements of the four entries in the anomalous dimension matrices in the parity even and odd cases. Although in the supersymmetric limit the color factors have to be identified, the constraints also have predictive power beyond the leading order approximation. As observed in Ref. [3] there is a subtlety in finding the supersymmetry preserving regularization scheme in the nonforward case. Standard dimensional reduction [9] does not serve this purpose. However, the existence of a supersymmetry preserving scheme was proven by the ability to satisfy the constraints by a multiplicative renormalization of the anomalous dimensions.

There exist four constraints for the special conformal anomalies. They contain new ingredients Δ, due to the superconformal symmetry breaking by the trace anomaly $\mathcal{A}$ in the spinor current. The four symmetry violating entries arise as a counterterm in the product of $\mathcal{A}$ and the conformal operators. Thus, there is no predictive power in these constraints; however, they serve for a consistency check of the special conformal anomalies.

Let us mention for completeness the situation with the two further leading twist-2 conformal operators appearing in the definition of the so-called transversity distributions. It is well known that these quark and gluon operators do not mix with each other under renormalization due to different spin representations with respect to the Lorentz group. Their forward anomalous dimensions are related in the supersymmetric limit by one constraint. However, unfortunately, there exist no constraints for the nondiagonal elements of conformal anomalies that do not involve anomalous contributions of fermionic operators [14].

We have evaluated the superconformal anomalies in one-loop approximation and found that our relations are indeed satisfied with the known results for the special conformal anomalies from Refs. [6,7]. In other words, we can now reconstruct the special conformal anomalies of the gluon-gluon and gluon-quark sectors from those in the remaining two channels, provided the former two are not already known. Furthermore, we know that the latter give us the rotation matrix to the conformal scheme in which the conformal operator product expansion of two electromagnetic currents is valid. Since the Wilson coefficients of this expansion in both of these remaining channels are fixed by the ones of the forward kinematics and coincide with the explicit calculation in the minimal subtraction scheme [15], rotated to the conformal scheme [16], we have a complete consistency check of all special conformal anomaly matrices evaluated at one-loop level. Since these conformal anomalies induce, together with terms proportional to the β function, the nondiagonal elements of the anomalous dimension matrix at two-loop level, we have also a complete but indirect check on their correctness from the superconstraints (58). Of course, in the flavor nonsinglet sector the anomalous dimensions arising from our prediction coincide with the conformal moments of the evolution kernel calculated at the two-loop level [17].

Altogether, we have a complete list of consistency checks for the field theoretical treatment of conformal anomalies, their evaluation at one-loop order, and next-to-leading predictions arising from their use. This also supports our results for the transversity sector [18], which has been treated in the same manner.

ACKNOWLEDGMENTS

A.B. would like to thank A. Schäfer for the hospitality extended to him at the Institut für Theoretische Physik, Universität Regensburg, where the final part of this work was done. This work was supported by the Alexander von Humboldt Foundation, in part by the National Science Foundation under grant PHY9722101, by the Graduiertenkolleg Erlangen-Regensburg (A.B.), and by DFG and BMBF (D.M.).

APPENDIX A: EVALUATION OF THE FERMIONIC *b* MATRIX

The fermionic b matrix is defined by Eq. (64). In order to evaluate the matrix it proves convenient to make the Fourier transform on the fields, i.e., $\phi(x)=\int d^d k e^{-k\cdot x}\tilde{\phi}(k)$, and introduce a set of new variables of the integrand, $Y\equiv k_{2+}+k_{1+}$ and $X=(k_{2+}-k_{1+})/(k_{2+}+k_{1+})$, so we get

$$\int d^d x 2x_- \psi\frac{\delta}{\delta\psi}\mathcal{V}_{jl}=2i\varrho_j\int d^dk_1 d^dk_2 Y^{l-1}\{\hat{\mathcal{L}}P_j^{(2,1)}(X)\}\times\tilde{G}_{+\mu}(k_1)\gamma_\mu^\perp\tilde{\psi}(k_2)=2i\sum_{k=0}^{j}\frac{\varrho_j}{\varrho_k}B_{jk}(2,1)\mathcal{V}_{kl-1}, \tag{A1}$$

with the differential operator $\hat{\mathcal{L}}\equiv l+(1-X)d/dX$ and B_{jk} defined by the integral

$$B_{jk}(\alpha,\beta)=\int_{-1}^{1}dX\frac{w(X|\alpha,\beta)}{n_k(\alpha,\beta)}P_k^{(\alpha,\beta)}(X)\hat{\mathcal{L}}P_j^{(\alpha,\beta)}(X). \tag{A2}$$

The weight and normalization factors are given by the standard equations:

$$w(X|\alpha,\beta)=(1-X)^\alpha(1+X)^\beta,$$

$$n_k(\alpha,\beta)=2^{\alpha+\beta+1}\times\frac{\Gamma(k+\alpha+1)\Gamma(k+\beta+1)}{(2k+\alpha+\beta+1)\Gamma(k+1)\Gamma(k+\alpha+\beta+1)}.$$

Using Rodriga's formula for Jacobi polynomials and integrating k (and $k+1$) times by parts we come to integrals that can be easily evaluated by means of the result

$$\int_{-1}^{1}dXw(X|\alpha,\gamma)P_j^{(\alpha,\beta)}(X)=(-1)^j2^{\alpha+\gamma+1}\frac{\Gamma(\alpha+j+1)\Gamma(\beta-\gamma+j)\Gamma(\gamma+1)}{\Gamma(j+1)\Gamma(\beta-\gamma)\Gamma(\alpha+\gamma+j+2)}. \tag{A3}$$

So we finally obtain

$$B_{jk}(\alpha,\beta)=(l-k)\delta_{jk}-\theta_{j,k+1}(-1)^{j-k}(2k+\alpha+\beta+1)\times\frac{\Gamma(j+\alpha+1)\Gamma(k+\alpha+\beta+1)}{\Gamma(k+\alpha+1)\Gamma(j+\alpha+\beta+1)}. \tag{A4}$$

Identifying $\alpha=2$ and $\beta=1$ we get the result

$$b_{jk}(F)\equiv\frac{\varrho_j}{\varrho_k}\left\{(l-k)\delta_{jk}-2(-1)^{j-k}\frac{(k+2)(k+3)}{(j+3)}\theta_{j-1,k}\right\}, \tag{A5}$$

where $\theta_{jk}=\{1 \text{ if } j\geqslant k;\ 0 \text{ if } j<k\}$.

APPENDIX B: RENORMALIZATION OF THE OPERATOR PRODUCT $[\mathcal{A}][\mathcal{O}]$

For the calculation of Z matrices one uses in the light-ray formalism the usual momentum space Feynman rules and vertex$\times\exp(-i\kappa_1k_{1+}-i\kappa_2k_{2+})$ for the nonlocal operator where "vertex" stands for the Dirac or Lorentz tensor. Introducing the Feynman parameters x,y for the propagators we reduce the Feynman integrals to the form

$$\mathcal{J}*\mathcal{R}(x,y)\times\text{vertex with}$$

$$\mathcal{J}=\int_0^1dx\int_0^1dy\,\theta(1-x-y)\times e^{-ik_{1+}((1-x)\kappa_1+x\kappa_2)-ik_{2+}(y\kappa_1+(1-y)\kappa_2)}, \tag{B1}$$

where the exponential corresponds to the Fourier transform of the coordinate dependence of the string operator "after evolution," $\mathcal{O}((1-x)\kappa_1+x\kappa_2,y\kappa_1+(1-y)\kappa_2)$. To calculate the divergent part of the operator product $[\mathcal{A}][\mathcal{O}]$, we have to take in addition the Feynman rule for the anomaly $[\mathcal{A}]$ defined in Eq. (19).

Our calculation has been performed with an arbitrary gauge fixing parameter ξ, which canceled in the sum of diagrams. Therefore, we present the results corresponding to the definition (79) for the separate contributions in $\xi=1$ gauge.

In the quark parity even and odd sectors we get on a diagram-by-diagram basis from Fig. 1

$$
\begin{aligned}
&-k_{2+}\,{}^{Q}\mathcal{R}_{(a)}=\frac{1}{2},\\
&-k_{2+}\,{}^{Q}\mathcal{R}_{(b)}=-\frac{1}{2}(1-y)\,\delta(x),\\
&-k_{2+}\,{}^{Q}\mathcal{R}_{(c)}=\pm\frac{x}{2}\,\delta(1-x-y),\\
&-k_{2+}\,{}^{Q}\mathcal{R}_{(d)}=\frac{y}{2}\,\delta(1-x-y)-\frac{1}{2}\left(1+\frac{y}{2}\right)\delta(x)-\frac{1}{2},\\
&-k_{2+}\,{}^{Q}\mathcal{R}_{(e)}=\left[\frac{1}{y}\right]_{+}\delta(x)-\delta(x),\\
&-k_{2+}\,{}^{Q}\mathcal{R}_{(f)}=-\left[\frac{1}{y}\right]_{+}\delta(x)+\frac{1}{2}(3-y)\,\delta(x),\\
&-k_{2+}\,{}^{Q}\mathcal{R}_{(g)}=-\left[\frac{1}{y}\right]_{+}\delta(x)+\frac{1}{2}\left(1+\frac{y}{2}\right)\delta(x),
\end{aligned}
\tag{B2}
$$

where the $+$ $(-)$ sign in the contribution of diagram (c) stands for the even (odd) case. In these results we dropped contributions of the type const$\cdot\,\delta(x)\,\delta(y)$, since they do not enter into the physical part of the constraints (74)–(77), namely, for $k<j$. In the sum of Eqs. (B2) the term $\frac{1}{2}\cdot\delta(1-x-y)$ for the vector case and $\frac{1}{2}(1-2x)\cdot\delta(1-x-y)$ for the axial one cancels with the $\kappa_1\leftrightarrow\kappa_2$ contribution ($+$ and $-$ sign, respectively) in Eq. (79) and we get the result in Eq. (80).

The gluon case is calculated from the graphs in Fig. 2 and reads for the vector channel

$$
\begin{aligned}
&-k_{2+}\,{}^{G}\mathcal{R}^{V}_{(a|\Phi)}=k_{2+}\,\delta(y)\left\{\left[\frac{2}{x}\right]_{+}-2+2\,\delta(x)\right\},\\
&-k_{2+}\,{}^{G}\mathcal{R}^{V}_{(a|NA)}=\delta(y)\left\{\frac{3}{2}(1-x^2)k_{1+}+\frac{i}{2}\kappa x(1-x)^2k_{1+}^2\right.\\
&\qquad\left.-\frac{3}{2}\,\delta(x)k_{2+}\right\},\\
&-k_{2+}\,{}^{G}\mathcal{R}^{V}_{(b)}=\left(2-\frac{3}{2}y\right)k_{2+}-\frac{3}{2}(1-x)k_{1+}+\frac{i}{2}\kappa\\
&\qquad\times[(1-y)k_{2+}+xk_{1+}][yk_{2+}+(1-x)k_{1+}]\\
&\qquad+\frac{1}{2}(1-x)\,\delta(y)\{(1-3x)k_{1+}-2k_{2+}\\
&\qquad-i\kappa(1-x)k_{1+}(xk_{1+}+k_{2+})\},\\
&-k_{2+}\,{}^{G}\mathcal{R}^{V}_{(c)}=yk_{2+}-(1+x)k_{1+},\\
&-k_{2+}\,{}^{G}\mathcal{R}^{V}_{(d)}=\left(2-\frac{3}{2}y\right)k_{2+}-\frac{3}{2}(1-x)k_{1+}+\frac{i}{2}\kappa\\
&\qquad\times[(1-y)k_{2+}+xk_{1+}]\\
&\qquad\times[yk_{2+}+(1-x)k_{1+}],\\
&-k_{2+}\,{}^{G}\mathcal{R}^{V}_{(e)}=\frac{1}{2}(3-x)k_{1+}-\left(1-\frac{y}{2}\right)k_{2+}-\frac{i}{2}\kappa\\
&\qquad\times[(1-y)k_{2+}+xk_{1+}]\\
&\qquad\times[yk_{2+}+(1-x)k_{1+}],\\
&-k_{2+}\,{}^{G}\mathcal{R}^{V}_{(f|\Phi)}=k_{2+}\,\delta(y)\left\{-\left[\frac{1}{x}\right]_{+}+1-\delta(x)\right\},\\
&-k_{2+}\,{}^{G}\mathcal{R}^{V}_{(f|NA)}=\delta(y)\{-(1-x)k_{1+}+\delta(x)k_{2+}\},\\
&-k_{2+}\,{}^{G}\mathcal{R}^{V}_{(g)}=k^{G}_{2+}\mathcal{R}^{V}_{(d)},
\end{aligned}
\tag{B3}
$$

with $\kappa\equiv\kappa_2-\kappa_1$. The subscript Φ (NA) stands for the contributions originating from the expansion of the path ordered exponential (non-Abelian part of the field strength tensor). The axial case differs from the previous one only in the contributions of diagrams (b), (c), and (e), which are

$$
\begin{aligned}
&-k_{2+}\,{}^{G}\mathcal{R}^{A}_{(b)}=\left(2-\frac{7}{2}y\right)k_{2+}-\left(\frac{3}{2}-\frac{7}{2}y\right)k_{1+}+\frac{i}{2}\kappa[(1-y)\\
&\qquad\times k_{2+}+xk_{1+}][yk_{2+}+(1-x)k_{1+}]\\
&\qquad+\frac{1-x}{2}\,\delta(y)\{(1-3x)k_{1+}-2k_{2+}\\
&\qquad-i\kappa x(1-x)k_{1+}^2-i\kappa(1-x)k_{1+}k_{2+}\},\\
&-k_{2+}\,{}^{G}\mathcal{R}^{A}_{(c)}=-yk_{2+}-(1-x)k_{1+},\\
&-k_{2+}\,{}^{G}\mathcal{R}^{A}_{(e)}=\left(\frac{3}{2}-\frac{5}{2}y\right)k_{1+}-\left(1-\frac{5}{2}y\right)k_{2+}-\frac{i}{2}\kappa\\
&\qquad\times[(1-y)k_{2+}+xk_{1+}][yk_{2+}+(1-x)k_{1+}].
\end{aligned}
\tag{B4}
$$

Summing the separate terms, we have to use the formula (see Ref. [7] for a general result)

$$
\mathcal{J}^{*}\{k_{1+}[1-(1-x)\,\delta(y)]+k_{2+}[1-(1-y)\,\delta(x)]\}=0
\tag{B5}
$$

to reduce the result to its final form (81). Then we use the equation $k_{2+}\widetilde{B}^{a}_{\mu}(k_2)=i\widetilde{G}^{a}_{+\mu}(k_2)$, valid to leading order in the coupling, to reconstruct the field strength from the potential.

APPENDIX C: CONFORMAL MOMENTS OF REGULARIZED KERNELS

The conformal moments of the transition kernels derived in the body of the text are defined according to

$$\mathcal{R}^{\,i}_{jk} \equiv \int_{-1}^{1} dt\, \frac{w(2,1|t)}{n_k(2,1)} P_k^{(2,1)}(t) \int_0^1 dx \int_0^{\bar{x}} \mathcal{R}(x,y) \times C_j^i([1-x-y]t-x+y), \tag{C1}$$

with the weight $w(2,1|t) \equiv (1-t)^2(1+t)$ and normalization $n_k(2,1) \equiv 8(k+1)/(k+2)(k+3)$. We have for quarks $C_j^Q = C_j^{3/2}$ and $C_j^G = C_{j-1}^{5/2}$ for gluons. From this equation it is straightforward to evaluate the moments of all parts of the kernels (along the lines of Ref. [7]) except for the ones with the + prescription since in this case we obtain the result in terms of derivatives of a hypergeometric function with respect to its indices, which is not easy to handle. In these cases we have to modify our modus operandi and develop a more efficient machinery which leads to more tractable expressions. It can be achieved according to the reexpansion of the integrand, making use of the orthogonality for Gegenbauer polynomials. To be more specific let us consider the gluon kernel $[1/x]_+ \delta(y)$ which in the momentum fraction formalism translates into $[\theta(t-t')/(t-t')]_+$. We use the following regularization of singular distributions:

$$\int_0^1 \frac{dx}{[1-x]_+} \phi(x) \equiv \int_0^1 \frac{dx}{(1-x)^{1-\varepsilon}} [\phi(x)-\phi(1)]. \tag{C2}$$

Then using the representation of the Gegenbauer polynomial in terms of the hypergeometric function ${}_2F_1$ and using Rodriga's formula for Jacobi polynomials we integrate k times by parts to get

$$M_{jk}^G \equiv \left\{ \left[\frac{\theta(t-t')}{(t-t')} \right]_+ \right\}_{jk} = (-1)^{j+k} \frac{(k+2)(k+3)}{6\varepsilon} - (-1)^j \frac{(k+2)(k+3)\Gamma(j+4)}{12\Gamma(j)\Gamma(k+2)} \int_0^1 dx\, x^k (1-x)^{\varepsilon-1} \times \int_0^1 dy\, y^{k+1}(1-y)^{k+2} \frac{d^k}{d(xy)^k} {}_2F_1\left(\begin{matrix} -j+1, j+4 \\ 3 \end{matrix} \middle| xy \right). \tag{C3}$$

The first term on the RHS originates from the $\phi(1)$ contribution in Eq. (C2). The simplicity of the consequent analysis depends on the handling of the derivatives acting on ${}_2F_1$. If we merely differentiate it k times as it appears and perform a y integration this gives ${}_3F_2$. Finally, after x integration Eq. (C3) will be proportional to the derivative of ${}_4F_3$ with respect to a low index [see later, Eq. (C12)]. Fortunately, it is possible to avoid this if one notices that $k-1\varepsilon$ differentiations and y integration lead to a ${}_2F_1$ function with shifted indices. After the last integration is done one ends up with an expression with the ε derivative acting on ${}_3F_2$. In order to achieve this let us reexpand the action of the k^{th} derivative $d/d(xy)$ on ${}_2F_1$ with respect to the complete set of polynomials $C_l^{k+3/2}(2x-1)$:

$$\frac{d}{dx} {}_2F_1\left(\begin{matrix} -j+k, j+k+3 \\ k+2 \end{matrix} \middle| x \right) = \sum_{l=0}^{\infty} c_{j-k,l}\, {}_2F_1\left(\begin{matrix} -l, l+2k+3 \\ k+2 \end{matrix} \middle| x \right), \tag{C4}$$

with expansion coefficients

$$c_{j-k,l} = -[1-(-1)^{j-k-l}]\theta_{j-k,l+1}(2l+2k+3) \times \frac{\Gamma(l+2k+3)}{\Gamma(l+1)} \frac{\Gamma(j-k+1)}{\Gamma(j+k+3)} \tag{C5}$$

easily obtained from the orthogonality of polynomials. Then we get

$$M_{jk}^G = \frac{(-1)^{j+k}}{12} \frac{(k+2)^2(k+3)\Gamma(j+k+3)}{\Gamma(2k+5)\Gamma(j-k+1)} \times \left\{ \psi(1) - \psi(k+1) + \frac{\partial}{\partial\varepsilon} \right\}_{\varepsilon=0} \sum_{l=0}^{\infty} c_{j-k,l} \times {}_3F_2\left(\begin{matrix} -l, l+2k+3, k+1 \\ 2k+5, k+1+\varepsilon \end{matrix} \middle| 1 \right). \tag{C6}$$

Now the way to handle the derivative of ${}_3F_2$ is rather straightforward. First we use the fundamental identity for ${}_3F_2$ ($l \in \mathrm{IN}$)

$${}_3F_2\left(\begin{matrix} -l, l+\alpha, \beta \\ \gamma, \beta+\varepsilon \end{matrix} \middle| 1 \right) = \frac{\Gamma(\gamma)\Gamma(\gamma-\alpha)}{\Gamma(\gamma+l)\Gamma(\gamma-\alpha-l)} {}_3F_2\left(\begin{matrix} -l, l+\alpha, \varepsilon \\ 1+\alpha-\gamma, \beta+\varepsilon \end{matrix} \middle| 1 \right), \tag{C7}$$

where we substitute $\gamma = \alpha + 2 - \rho$ with $\rho \to 0$. Then the expansion with respect to ε is easy to construct:

$${}_3F_2\left(\begin{matrix} -l, l+\alpha, \varepsilon \\ \rho-1, \beta+\varepsilon \end{matrix} \middle| 1 \right) = 1 + \varepsilon \left\{ \frac{l(l+\alpha)}{\beta} + \Gamma(\rho-1)\left(1 + (-1)^l [(l-1) \times (l+\alpha+1)+\beta] \frac{\Gamma(1+l+\alpha-\beta)\Gamma(\beta)}{\Gamma(2+\alpha-\beta)\Gamma(\beta+l)} \right) \right\} + \mathcal{O}(\varepsilon^2), \tag{C8}$$

and together with the identity

$$\frac{\Gamma(\rho-1)}{\Gamma(2-\rho-l)} = (-1)^l \frac{\Gamma(l-1+\rho)}{\Gamma(2-\rho)} \tag{C9}$$

we find

$$
{}_3F_2\left(\begin{matrix}-l,l+\alpha,\beta\\ \alpha+2,\beta+\varepsilon\end{matrix}\middle|1\right)
=\frac{\Gamma(\alpha+2)}{\Gamma(l+\alpha+2)}\Bigg\{\left(1+\varepsilon\frac{l(l+\alpha)}{\beta}\right)(\delta_{l,0}+\delta_{l,1})
+\varepsilon\Gamma(l-1)\Bigg((-1)^l+[(l-1)(l+\alpha+1)+\beta]
\times\frac{\Gamma(1+l+\alpha-\beta)\Gamma(\beta)}{\Gamma(2+\alpha-\beta)\Gamma(\beta+l)}\Bigg)\theta_{l,2}+\mathcal{O}(\varepsilon^2)\Bigg\}. \quad (C10)
$$

Using the results we have just derived, we perform in the last step the summation in Eq. (C6) according to the formula

$$
\frac{\partial}{\partial\varepsilon}\bigg|_{\varepsilon=0}\sum_{l=0}^{\infty}c_{j-k,l}\,{}_3F_2\left(\begin{matrix}-l,l+2k+3,k+1\\ 2k+5,k+1+\varepsilon\end{matrix}\middle|1\right)
=\frac{1}{(k+1)(k+2)}-\frac{(j-k)(j+k+3)}{(k+1)(k+2)(k+3)}
-\frac{1}{k+2}\Bigg\{(-1)^{j+k}\psi\left(\frac{j+k+4}{2}\right)+[1-(-1)^{j+k}]
\times\psi(j+k+4)-(-1)^{j+k}\psi\left(\frac{j-k}{2}\right)+[1+(-1)^{j+k}]
\times\psi(j-k)-\psi(k+4)-\psi(1)\Bigg\}. \quad (C11)
$$

As a by-product we verify the following formula for the derivative of ${}_4F_3$, which is difficult to derive by other means:

$$
\frac{\Gamma(j+k+4)}{\Gamma(j-k)\Gamma(2k+5)}\frac{\partial}{\partial\varepsilon}\bigg|_{\varepsilon=0}{}_4F_3\left(\begin{matrix}-j+k+1,j+k+4,k+2,k+1\\ 2k+5,k+3,k+1+\varepsilon\end{matrix}\middle|1\right)
=\frac{(j-k)(j+k+3)}{(k+1)(k+3)}-\frac{1}{k+1}+(-1)^{j+k}\psi\left(\frac{j+k+4}{2}\right)+[1-(-1)^{j+k}]\psi(j+k+4)-(-1)^{j+k}\psi\left(\frac{j-k}{2}\right)
+[1+(-1)^{j+k}]\psi(j-k)-\psi(k+4)-\psi(1). \quad (C12)
$$

A slightly different procedure holds for quarks. In this case due to the presence of the momentum fraction $k_{2+}=[(1-t)/2](k_1+k_2)_+$ we reexpand the integrand, modified by adding a constant,

$$
M^{Q}_{jk}\equiv\left\{\left[\frac{2}{1-t}\,\frac{\theta(t'-t)}{(t'-t)}\right]_+\right\}_{jk}
=(-1)^k\frac{(k+2)(k+3)(j+1)(j+2)}{k+1}\int_0^1 y^2(1-y)
\times P_k^{(1,2)}(2y-1)\int_0^1 dx\left[\frac{1}{x}\right]_+\frac{1}{y}\left\{{}_2F_1\left(\begin{matrix}-j,j+3\\ 2\end{matrix}\middle|xy\right)
-{}_2F_1\left(\begin{matrix}-j,j+3\\ 2\end{matrix}\middle|0\right)\right\}, \quad (C13)
$$

in the following series:

$$
\frac{1}{x}\left\{{}_2F_1\left(\begin{matrix}-j,j+3\\ 2\end{matrix}\middle|x\right)-{}_2F_1\left(\begin{matrix}-j,j+3\\ 2\end{matrix}\middle|0\right)\right\}
=\sum_{k=0}^{\infty}d_{jk}\,{}_2F_1\left(\begin{matrix}-k,k+3\\ 2\end{matrix}\middle|x\right) \quad (C14)
$$

with the expansion coefficients

$$
d_{jk}=-\theta_{j-1,k}\frac{3+2k}{(j+1)(j+2)}(j-k)(j+k+3). \quad (C15)
$$

Consequent integration and expansion in ε requires Eq. (C10) as well as the following result:

$$
{}_3F_2\left(\begin{matrix}-l,l+\alpha,\beta\\ 1+\alpha,\beta+\varepsilon\end{matrix}\middle|1\right)
=\frac{\Gamma(1+\alpha)}{\Gamma(1+\alpha+l)}\Bigg\{\delta_{l,0}+\varepsilon\Gamma(l)\left(\frac{\Gamma(1+\alpha-\beta+l)\Gamma(\beta)}{\Gamma(1+\alpha-\beta)\Gamma(\beta+l)}
-(-1)^l\right)\theta_{l,1}\Bigg\}+\mathcal{O}(\varepsilon^2), \quad (C16)
$$

which can be deduced using the same recipe as presented above. Final summation gives us the result in Eq. (83).

[1] M. F. Sohnius, Phys. Rep. **128**, 39 (1985).
[2] A. P. Bukhvostov, G. V. Frolov, L. N. Lipatov, and E. A. Kuraev, Nucl. Phys. **B258**, 601 (1985).
[3] A. V. Belitsky, D. Müller, and A. Schäfer, Phys. Lett. B **450**, 126 (1999).
[4] J. Wess and B. Zumino, Nucl. Phys. **B70**, 39 (1974).
[5] S. Ferrara, Nucl. Phys. **B77**, 73 (1974); P. H. Dondi and M. F. Sohnius, *ibid.* **B81**, 317 (1974).
[6] D. Müller, Phys. Rev. D **49**, 2525 (1994); A. V. Belitsky and D. Müller, Nucl. Phys. **B527**, 207 (1998).
[7] A. V. Belitsky and D. Müller, Nucl. Phys. **B537**, 397 (1999).
[8] R. Jackiw, Phys. Rev. Lett. **41**, 1635 (1978); in *Groups and Topology II*, Proceedings of Les Houches Summer School on Theoretical Physics Relativity, edited by B. S. DeWitt and R. Stora (North-Holland, Amsterdam, 1984), p. 221.
[9] W. Siegel, Phys. Lett. **84B**, 193 (1979); **94B**, 37 (1980); D. M. Capper, D. R. T. Jones, and P. van Nieuwenhuizen, Nucl. Phys. **B167**, 479 (1980).
[10] L. F. Abbott, M. T. Grisaru, and H. J. Schnitzer, Phys. Rev. D **16**, 2295 (1977); T. Curtright, Phys. Lett. **71B**, 185 (1977); H. Inagaki, Lett. Nuovo Cimento Soc. Ital. Fis. **20**, 217 (1977); W. Lang, Nucl. Phys. **B150**, 201 (1979); T. Hagiwara, S. Y. Pi, and H. S. Tsao, Ann. Phys. (N.Y.) **130**, 282 (1980); H. Nicolai and P. K. Townsend, Phys. Lett. **93B**, 111 (1980); N. K. Nilsen, Nucl. Phys. **B252**, 401 (1985).
[11] D. Espriu, Z. Phys. C **27**, 603 (1985).
[12] P. Majumdar, E. C. Poggio, and H. J. Schnitzer, Phys. Rev. D **21**, 2203 (1980).
[13] Th. Braunschweig, J. Hořejši, and D. Robaschik, Z. Phys. C **23**, 19 (1984); Th. Braunschweig, B. Geyer, and D. Robaschik, Ann. Phys. (Leipzig) **44**, 403 (1987).
[14] A. V. Belitsky and D. Müller, Nucl. Phys. B (Proc. Suppl.) **79**, 576 (1999).
[15] X. Ji and J. Osborne, Phys. Rev. D **58**, 094018 (1998); L. Mankiewicz, G. Piller, E. Stein, M. Vänttinen, and T. Weigl, Phys. Lett. B **425**, 186 (1998).
[16] D. Müller, Phys. Rev. D **58**, 054005 (1998); A. V. Belitsky and D. Müller, Phys. Lett. B **417**, 129 (1998).
[17] M. H. Sarmadi, Phys. Lett. **143B**, 471 (1984); F.-M. Dittes and A. V. Radyushkin, *ibid.* **134B**, 359 (1984); S. V. Mikhailov and A. V. Radyushkin, Nucl. Phys. **B254**, 89 (1985).
[18] A. V. Belitsky, A. Freund, and D. Müller, Phys. Lett. B **493**, 341 (2000).