

## Information on $B \rightarrow \pi\pi$ provided by the semileptonic process $B \rightarrow \pi l \nu$

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Analysis of the present data on the semileptonic process  $B \rightarrow \pi l \nu$  indicates that they have not yet reached the precision to provide adequate information on the  $B \rightarrow \pi$  form factor  $F_+(q^2)$ , which for  $q^2 = m_\pi^2$  is known to be related to the factorized color-favored (“ $T$ ,” or “tree”) contribution to  $B^0 \rightarrow \pi^+ \pi^-$ . It is shown here that with around 500  $B \rightarrow \pi l \nu$  events in which rate and spectrum are measured one can improve the accuracy of  $T$  by a significant amount. A recent CLEO determination of the  $D^* D \pi$  coupling constant is compared with an earlier prediction, and its role in the description of the  $B \rightarrow \pi$  form factors is noted. When combined with an estimate of the penguin amplitude (“ $P$ ”) obtained using flavor SU(3) symmetry from  $B \rightarrow K \pi$  decays, information on  $T$  allows one to gauge the effects of the penguin amplitude on extraction of the weak phase  $\alpha = \phi_2$  from the time-dependent  $CP$ -violating rate asymmetry in  $B^0 \rightarrow \pi^+ \pi^-$ . The constraint on  $\alpha$  implied by a recent experimental result on this asymmetry is described.

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### I. INTRODUCTION

The semileptonic process  $B \rightarrow \pi l \nu$  is known to provide information on the  $B \rightarrow \pi$  form factor  $F_+(q^2)$ , which for  $q^2 = m_\pi^2$  is related to the factorized color-favored (“ $T$ ,” or “tree”) contribution to  $B^0 \rightarrow \pi^+ \pi^-$ . In the present paper we show that while present semileptonic data have not yet reached adequate precision, with around 500  $B \rightarrow \pi l \nu$  events in which rate and spectrum are measured one can improve the accuracy of  $T$  by a significant amount. We then discuss the benefits of such a determination.

A connection between the decays  $B^0 \rightarrow \pi^- l^+ \nu_l$  and  $B^0 \rightarrow \pi^+ \pi^-$  was noted some time ago by Voloshin [1], who derived the relation

$$\frac{\Gamma(B^0 \rightarrow \pi^- e^+ \nu_e)}{\Gamma(B^0 \rightarrow \pi^+ \pi^-)} = \frac{M_B^2}{12\pi^2 f_\pi^2} \approx 13.7 \quad (f_\pi = 131 \text{ MeV}), \quad (1)$$

using a pole model for the  $B \rightarrow \pi$  form factor  $F_+(q^2)$ . This relation assumes the dominance of a tree ( $T$ ) contribution to  $B^0 \rightarrow \pi^+ \pi^-$  in the notation of Ref. [2]. The CLEO [3] and Belle [4] Collaborations have measured the branching ratio for the semileptonic process. Averaging their results yields

$$\mathcal{B}(B^0 \rightarrow \pi^- e^+ \nu_e) = (1.4 \pm 0.3) \times 10^{-4}, \quad (2)$$

while an average of CLEO [5], Belle [6], and BaBar [7] ( $B^0$ - and  $\bar{B}^0$ -averaged) branching ratios [8] gives

$$\mathcal{B}(B^0 \rightarrow \pi^+ \pi^-) = (4.4 \pm 0.9) \times 10^{-6}. \quad (3)$$

The experimental ratio of these two branching ratios is  $\Gamma(B^0 \rightarrow \pi^- e^+ \nu_e)/\Gamma(B^0 \rightarrow \pi^+ \pi^-) = 32 \pm 9$ , a factor of 2.3 above Eq. (1), which indicates either that the tree contribution is substantially overestimated in Eq. (1), or that some

other process is interfering destructively with the tree amplitude to reduce the  $B^0 \rightarrow \pi^+ \pi^-$  decay rate. A prime candidate for this amplitude is the “penguin,” or  $P$  amplitude in the notation of [2]. If this amplitude were sufficiently important to reduce the expected  $B^0 \rightarrow \pi^+ \pi^-$  rate by roughly a factor of 2.3, it could have important effects on the extraction of the weak phase  $\alpha = \phi_2$  entering the Cabibbo-Kobayashi-Maskawa (CKM) matrix [9]. This question has now acquired particular urgency as a result of the first report of results on  $CP$ -violating parameters in  $B^0 \rightarrow \pi^+ \pi^-$  [10].

Many attempts have been made to use data to estimate the “penguin pollution” of the  $B^0 \rightarrow \pi^+ \pi^-$  amplitude, including an isospin analysis requiring the measurement of  $B^0 \rightarrow \pi^0 \pi^0$  and  $B^+ \rightarrow \pi^+ \pi^0$  decays [11] (we assume charge-conjugate processes are measured when required), methods which use only a partial subset of the above information [12], and numerous methods based on flavor SU(3) [2,13]. Earlier data hinted that the penguin amplitude was interfering destructively with the tree in  $B^0 \rightarrow \pi^+ \pi^-$  [14,15].

In the present paper we describe measurements of  $B^0 \rightarrow \pi^- e^+ \nu_e$  decays which can significantly improve information on the magnitude of the tree ( $T$ ) contribution to  $B^0 \rightarrow \pi^+ \pi^-$ . Such an improvement is needed to tell whether tree and penguin amplitudes are really interfering destructively in  $B^0 \rightarrow \pi^+ \pi^-$ . We discuss the role of the  $B^*$  pole in this process, whose contribution is related through heavy quark symmetry to a recent CLEO measurement of the  $D^* D \pi$  coupling constant [16]. We then show how information on  $T$  helps to determine the weak phase  $\alpha$  using limits on  $CP$  violation in  $B^0 \rightarrow \pi^+ \pi^-$ .

Our approach differs from that advocated in Refs. [14,17,18], in which the tree amplitude is estimated from the rate for  $B^+ \rightarrow \pi^+ \pi^0$ . In that process, there is an additional color-suppressed amplitude (called  $C$  in the language of Ref. [2]), whose magnitude and phase with respect to  $T$  cannot be independently estimated using present data but must be calculated. One then has  $A(B^+ \rightarrow \pi^+ \pi^0) = -(T+C)/\sqrt{2}$ , and with  $C=0.1T$ , one arrives at estimates rather similar to those in the present paper. (The  $C$  amplitude was neglected alto-

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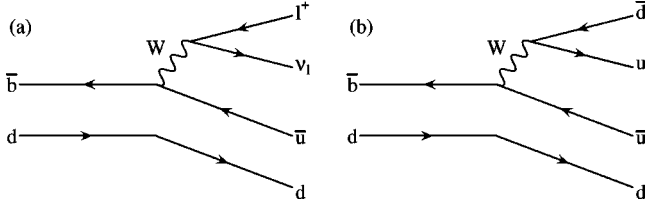


FIG. 1. Feynman diagrams for semileptonic and nonleptonic tree decays of a  $B^0$  meson.

gether in Ref. [14].) The semileptonic process avoids dependence on the theoretical calculation of  $C/T$ .

In Sec. II we give some basic expressions for the  $B^0 \rightarrow \pi^- e^+ \nu_e$  and  $B^0 \rightarrow \pi^+ \pi^-$  decays. Information on the  $B \rightarrow \pi$  form factors is reviewed in Sec. III. The  $D^* D \pi$  measurement and its implications for the  $B^* B \pi$  coupling and the  $B^*$  pole in the  $B \rightarrow \pi$  form factor are described in Sec. IV. We then bracket the possible magnitudes of the tree amplitude  $T$  depending on measurements of the spectrum in  $B^0 \rightarrow \pi^- e^+ \nu_e$  (Sec. V). The extraction of the penguin amplitude from  $B \rightarrow K \pi$  decays with the help of flavor SU(3) allows us to determine the extent to which  $P$  and  $T$  are interfering destructively in  $B^0 \rightarrow \pi^+ \pi^-$ , and hence to determine the correction to the weak phase  $\alpha$  which is needed when extracting it from  $CP$ -violating asymmetries in that process (Sec. VI). We summarize in Sec. VII.

## II. SEMILEPTONIC AND NONLEPTONIC TREE DECAYS

For a generic heavy-to-light decay  $H \rightarrow \pi$ , the non-perturbative matrix element is parametrized by two independent form factors:

$$\langle \pi(p) | \bar{u} \gamma_\mu Q | H(p+q) \rangle = \left( 2p + q - q \frac{m_H^2 - m_\pi^2}{q^2} \right)_\mu F_+(q^2) + q_\mu \frac{m_H^2 - m_\pi^2}{q^2} F_0(q^2), \quad (4)$$

where  $Q = b$  or  $c$  and  $H$  is a  $B$  or  $D$  pseudoscalar meson. The subscript  $H$  has been suppressed in the two form factors. In the case of massless leptons (which is an excellent approximation for  $l = e, \mu$ ), only  $F_+(q^2)$  contributes to the differential decay rate

$$\frac{d\Gamma}{dq^2}(H^0 \rightarrow \pi^- l^+ \nu_l) = \frac{G_F^2 |V_{qQ}|^2}{24\pi^3} |\vec{p}_\pi|^3 |F_+(q^2)|^2, \quad (5)$$

where  $V_{qQ}$  is the relevant CKM matrix element. We will take  $|V_{cd}| = 0.224 \pm 0.016$  and  $|V_{ub}| = 0.0036 \pm 0.0010$  from Ref. [19]. To obtain the total width, one should integrate Eq. (5) over the entire physical region,  $0 \leq q^2 \leq (m_H - m_\pi)^2$ , which requires the precise knowledge of the normalization [i.e.,  $F_+(0)$ ] and  $q^2$  dependence of the form factor.

The lepton pair can be replaced with a pion, as shown in Fig. 1 for the decay of a  $B^0$  meson. The resulting diagram is the tree contribution to the nonleptonic decay  $B^0 \rightarrow \pi^+ \pi^-$ .

In the limit of small  $m_\pi$ , the two diagrams in Fig. 1 are related by the Bjorken relation [20]

$$\Gamma_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-) = 6 \pi^2 f_\pi^2 |V_{ud}|^2 |a_1|^2 \left. \frac{d\Gamma(B^0 \rightarrow \pi^- l^+ \nu_l)}{dq^2} \right|_{q^2=m_\pi^2}, \quad (6)$$

where  $|a_1|$  is the QCD correction. We shall take  $|a_1| = 1.0$ , which is a sufficiently good approximation for our present purpose.

## III. $H \rightarrow \pi$ FORM FACTORS

In the absence of a spectrum measurement, one cannot directly employ Eq. (6) to calculate  $T$ . Present extraction of  $T$  using this relation relies on assumptions of particular form factor shapes. One can test such assumptions using data on the  $B^* B \pi$  coupling extracted using heavy quark symmetry from the corresponding  $D^* D \pi$  coupling, and using present information from lattice gauge theories. Form factors parametrized in a manner consistent with such constraints can then be used to anticipate the number of events necessary to extract  $T$  from Eq. (6) in a model-independent way.

Lacking experimental measurements of the form factors  $F_+(q^2)$  and  $F_0(q^2)$ , people have proposed [21] several models to describe their behavior, among which is the single-pole model

$$F_+(q^2) = \frac{f_{H^*}}{2m_{H^*}} \frac{g_{H^* H \pi}}{1 - q^2/m_{H^*}^2}, \quad (7)$$

where we adopt the following convention:

$$\langle 0 | V_\mu | H^*(p, \epsilon) \rangle = f_{H^*} m_{H^*} \epsilon_\mu, \quad (8)$$

$$\langle H^-(p) \pi^+(q) | H^{*0}(p+q, \epsilon) \rangle = g_{H^* H \pi}(q \cdot \epsilon). \quad (9)$$

However, this form factor gives total widths of  $D^0 \rightarrow \pi^- l^+ \nu_l$  and  $B^0 \rightarrow \pi^- l^+ \nu_l$  which are both larger than the experimental values, as will be shown in Sec. IV. So the monopole form factors are not enough to describe the physics involved in the  $H \rightarrow \pi$  decays.

Multipole form factors naturally become our next choice. On the basis of lattice gauge theory calculations, Becirevic and Kaidalov [22] proposed a simple parametrization which is essentially a dipole for  $F_+(q^2)$ ,

$$F_+(q^2) = \frac{c_H(1 - \alpha_H)}{(1 - q^2/m_{H^*}^2)(1 - \alpha_H q^2/m_{H^*}^2)}, \quad (10)$$

$$F_0(q^2) = \frac{c_H(1 - \alpha_H)}{1 - q^2/(\beta_H m_{H^*}^2)}. \quad (11)$$

TABLE I. Vector meson decay constants (MeV) from different calculations.

	$f_{D^*}$	$f_{B^*}$
Becirevic <i>et al.</i> [25]	$245 \pm 20_{-2}^{+3}$	$196 \pm 24_{-2}^{+39}$
UKQCD [26]	$268_{-40}^{+32}$	$236_{-39}^{+45}$
Hwang and Kim [27]	$327 \pm 13$	$252 \pm 10$
Wang and Wu [28]	$354 \pm 90$	$206 \pm 39$
Huang and Luo [29]		$190 \pm 30$

In the infinite quark mass limit, the quantities  $[c_H \sqrt{m_H}, (1 - \alpha_H)m_H, (\beta_H - 1)m_H]$  should scale as constants.  $c_H$  is related to the coupling constant  $g_{H^*H\pi}$  as

$$c_H = \frac{f_{H^*} g_{H^*H\pi}}{2m_{H^*}}. \quad (12)$$

This parametrization has enough freedom to describe lattice results, which typically are obtained for values of  $q^2$  above about  $13 \text{ GeV}^2$  [22–24]. We shall employ it to judge the statistical accuracy needed in extrapolating the  $B \rightarrow \pi l \nu$  spectrum to  $q^2 = m_\pi^2$ , where the Bjorken factorization relation (6) provides an estimate of  $T$ . A similar problem arises when one wishes to extrapolate to the zero-recoil limit in estimating the CKM matrix element  $|V_{cb}|$  from the exclusive process  $B \rightarrow D^{(*)} l \nu$ , since both the normalization and shape of the spectrum have to be determined.

It should be pointed out that  $f_{D^*}$ ,  $f_{B^*}$  and  $g_{B^*B\pi}$  are far from being determined, though  $g_{D^*D\pi}$  has been measured [16]. Very different values of  $f_{D^*}$  and  $f_{B^*}$  have been obtained on the lattice and in various models (see Table I [25–29]). We will discuss  $g_{B^*B\pi}$  in Sec. IV.

#### IV. IMPLICATIONS OF $g_{D^*D\pi}$ MEASUREMENT

We now describe the CLEO measurement of the  $D^*D\pi$  coupling constant [16] and review its significance in the light of earlier predictions [30–32]. The observed value of the total  $D^{*+}$  width is  $\Gamma(D^{*+}) = (96 \pm 4 \pm 22) \text{ keV}$ , in satisfactory agreement with a prediction of 84 keV made some time ago by comparison with  $K^* \rightarrow K\pi$  and  $K^* \rightarrow K\gamma$  decays

 TABLE II. Predictions for decays  $D^* \rightarrow D\pi$  and  $D^* \rightarrow D\gamma$  based on comparison with  $K^* \rightarrow K\pi$  and  $K^* \rightarrow K\gamma$  decays.

Decay	Predicted		Experimental Branching ratio (%)
	Partial width (keV)	Branching ratio (%)	
$D^{*+} \rightarrow D^+ \pi^0$	25.9	30.9	$30.7 \pm 0.5$
$\rightarrow D^0 \pi^+$	56.9	67.8	$67.7 \pm 0.5$
$\rightarrow D^+ \gamma$	<u>1.1</u>	1.3	$1.6 \pm 0.4$
	83.9		
$D^{*0} \rightarrow D^0 \pi^0$	39.7	70.6	$61.9 \pm 2.9$
$\rightarrow D^0 \gamma$	<u>16.5</u>	29.4	$38.1 \pm 2.9$
	56.2		

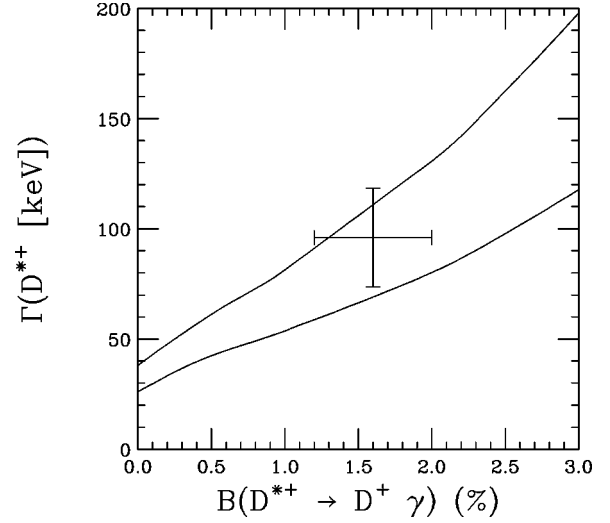


FIG. 2. Prediction of Ref. [31] for  $\Gamma(D^{*+})$  as a function of the branching ratio for  $D^{*+} \rightarrow D^+ \gamma$ , including leading SU(3)-breaking effects. Lines show predicted bounds. The plotted point shows current data [16,19].

[30]. Other predictions of [30] are compared with the current experimental situation [19] in Table II. The agreement is not bad, and can be improved by assuming about a 30% increase in the absolute square of the matrix element for the magnetic dipole transitions  $D^* \rightarrow D\gamma$  with respect to the value in Refs. [30]. The experimental branching ratios at the time of these predictions differed from them much more significantly.

A more detailed set of calculations was performed on the basis of chiral and heavy quark symmetry [31], taking into account SU(3) violating contributions of order  $m_q^{1/2}$ . The experimental values are consistent with the predicted correlation between  $\mathcal{B}(D^{*+} \rightarrow D^+ \gamma)$  and  $\Gamma(D^{*+})$ , as shown in Fig. 2.

The observed  $D^{*+}$  width can be related to a dimensionless  $D^*D\pi$  coupling constant  $\hat{g}$  by the expression [31,33]

$$\Gamma(D^{*+} \rightarrow D^0 \pi^+) = \frac{\hat{g}^2}{6\pi f_\pi^2} |\vec{p}_\pi|^3, \quad (13)$$

where  $f_\pi = 131$  MeV and  $|\vec{p}_\pi| = 39$  MeV/c. Using the branching ratio in Table II we find  $\Gamma(D^{*+} \rightarrow D^0 \pi^+) = 65 \pm 15$  keV and  $\hat{g} = 0.59 \pm 0.07$ . Therefore

$$g_{D^* D \pi} = \frac{2m_D^s \hat{g}}{f_\pi} = 17.8 \pm 2.1, \quad (14)$$

where  $m_D^s = 1973$  MeV is the spin-averaged mass of the  $D^{(*)}$  meson. Taking this value of  $g_{D^* D \pi}$  and  $f_{D^*} = 200$  MeV (which is more than  $1\sigma$  smaller than any determination in Table I), we get  $\mathcal{B}(D^0 \rightarrow \pi^- e^+ \nu_e) = (4.9 \pm 1.2) \times 10^{-3}$ , still larger than the experimental value  $(3.7 \pm 0.6) \times 10^{-3}$  [19]. Higher values of  $f_{D^*}$  yield even larger branching ratios.

Heavy quark symmetry (HQS) predicts

$$g_{B^* B \pi} = \frac{2m_B^s \hat{g}}{f_\pi} = 47.9 \pm 5.7. \quad (15)$$

Again, even if we take a comparatively small value of  $f_{B^*}$  ( $= 160$  MeV) and assume a large (e.g., 40%) violation of HQS (so that  $g_{B^* B \pi}$  can be as small as 29.0), we will get a branching ratio  $\mathcal{B}(B^0 \rightarrow \pi^- e^+ \nu_e) = (2.6 \pm 1.4) \times 10^{-4}$  which is still larger than Eq. (2). Thus we are justified to suspect the single pole form factor (7).

## V. INFORMATION ON $T$ FROM SEMILEPTONIC DECAYS

The Bjorken relation (6) establishes a useful connection between the semileptonic decays and the nonleptonic tree decays. Ideally,  $d\Gamma(B^0 \rightarrow \pi^- l^+ \nu_l)/dq^2$  at  $q^2 = m_\pi^2$  provides the tree contribution to the branching ratio for  $B^0 \rightarrow \pi^+ \pi^-$  (aside from QCD corrections, which have been found to be a few percent in related processes). However, in practice one must measure the semileptonic decay spectrum over a range of  $q^2$  in order to accumulate a sufficient number of events, and therefore must model the spectrum shape, as in extracting  $|V_{cb}|$  from the spectrum for  $B \rightarrow D^{(*)} l \nu$ .

The dipole form factor has enough parameters to allow modeling both a normalization and a spectrum shape. We use it to gain an idea of the statistical requirements for a useful spectrum measurement. The experimental branching ratio (2) for the semileptonic decay  $B^0 \rightarrow \pi^- e^+ \nu_e$  puts a strong constraint on the dipole parameters  $c_B$  and  $\alpha_B$ , as shown in Fig. 3. Accordingly, the tree branching ratio for  $B^0 \rightarrow \pi^+ \pi^-$  is constrained to lie in a certain range (Fig. 4). It should be noted that Fig. 4 does not depend on  $|V_{ub}|$ , though Fig. 3 can be altered by any change in  $|V_{ub}|$ . We can always combine  $|V_{ub}|$  with  $c_B$  and view  $|V_{ub}|c_B$  as a single parameter. This observation plays an important role in estimating  $T$  from Fig. 4.

To determine  $\alpha_B$  and hence  $c_B$  and  $\mathcal{B}_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-)$ , one can measure the normalized spectrum  $[(1/\Gamma)(d\Gamma/dq^2)]$  for  $B^0 \rightarrow \pi^- l^+ \nu_l$ . Note that  $(1/\Gamma)(d\Gamma/dq^2)$  is independent of  $c_B$  and  $|V_{ub}|$ . Thus measuring its dependence on  $q^2$  will give us very clean information about  $\alpha_B$ . Figure 5 shows us that a comparison of the spectrum in the interval  $0 \leq q^2 \leq 11$  GeV<sup>2</sup> with that for  $11 \leq q^2$

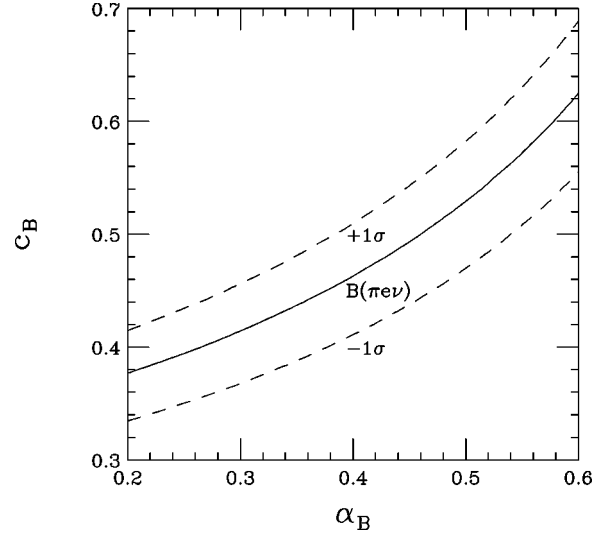


FIG. 3. The dependence of  $c_B$  on  $\alpha_B$  for given values of  $\mathcal{B}(B^0 \rightarrow \pi^- e^+ \nu_e)$ . Solid line:  $\mathcal{B}(B^0 \rightarrow \pi^- e^+ \nu_e) = 1.4 \times 10^{-4}$ ; upper dashed line:  $\mathcal{B}(B^0 \rightarrow \pi^- e^+ \nu_e) = 1.7 \times 10^{-4}$ ; lower dashed line:  $\mathcal{B}(B^0 \rightarrow \pi^- e^+ \nu_e) = 1.1 \times 10^{-4}$ .

$\leq 26$  GeV<sup>2</sup> should be useful in determining  $\alpha_B$ .

A recent lattice calculation [23] obtains values of  $\alpha_B$  ranging from about 0.2 to 0.6,  $c_B$  from about 0.3 to 0.6, and  $F_+(0)$  around 0.27 with a 25% error. A more recent analysis [34] from QCD sum rules on the light-cone obtains  $F_+(0) = 0.26 \pm 0.08$ , in good agreement with the lattice result. This implies that parameters are within the ranges quoted in Figs. 3–5, and leads to values of  $\mathcal{B}_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-)$  ranging between about  $4.5 \times 10^{-6}$  and  $11 \times 10^{-6}$ , as in Fig. 4.

Given the central value of  $\mathcal{B}(B \rightarrow \pi l \nu)$ , Fig. 4 implies that an error  $\Delta\alpha_B = 0.1$  will correspond to an error in  $\Delta\mathcal{B}_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-)$  of about 10%, or an error in  $T$  of about 5%. An additional error will be associated with the statistical error associated with  $\mathcal{B}(B \rightarrow \pi l \nu)$  itself. We shall determine the

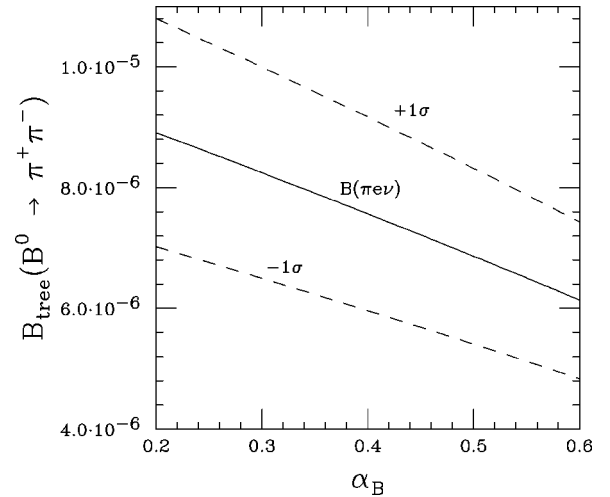


FIG. 4. The dependence of  $\mathcal{B}_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-)$  on  $\alpha_B$  for given values of  $\mathcal{B}(B^0 \rightarrow \pi^- e^+ \nu_e)$ . Solid line:  $\mathcal{B}(B^0 \rightarrow \pi^- e^+ \nu_e) = 1.4 \times 10^{-4}$ ; upper dashed line:  $\mathcal{B}(B^0 \rightarrow \pi^- e^+ \nu_e) = 1.7 \times 10^{-4}$ ; lower dashed line:  $\mathcal{B}(B^0 \rightarrow \pi^- e^+ \nu_e) = 1.1 \times 10^{-4}$ .

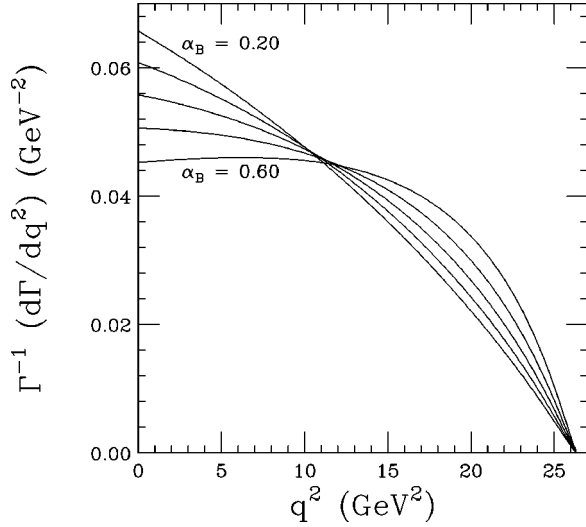


FIG. 5. Normalized spectrum of  $B^0 \rightarrow \pi^- l^+ \nu_l$  for various values of  $\alpha_B$ . At low  $q^2$ , the curves correspond to  $\alpha_B = 0.20, 0.30, 0.40, 0.50, 0.60$ , from top to bottom.

number of events required to achieve an error of  $\Delta\alpha_B = 0.1$ , and estimate the corresponding total error in  $T$ .

In Table III we show the fraction  $f$  of  $B^0 \rightarrow \pi^- l^+ \nu_l$  events below  $q^2 = 11 \text{ GeV}^2$  as a function of  $\alpha_B$ . In order to obtain an error of  $\Delta\alpha_B = 0.1$ , one has to determine  $f$  to a precision of  $\Delta f = 0.023$ . For a total of  $N$  events in the spectrum, the error in  $f$  is  $\Delta f = \sqrt{f(1-f)/N}$ , which is about  $0.5/\sqrt{N}$  for  $f$  near 0.5. Thus, one needs about  $(0.5/0.023)^2 \approx 470$   $B \rightarrow \pi l \nu$  events to achieve this accuracy. Such a sample will be associated with an error in the overall  $B \rightarrow \pi l \nu$  rate of  $1/\sqrt{470} \approx 4.6\%$ . When added in quadrature with the 10% error in  $\mathcal{B}_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-)$  associated with the spectrum shape, this leads to an overall error of 11% in  $|T|^2$  or 5.5% in  $T$ . One will need considerably more than  $470/\mathcal{B}(B \rightarrow \pi l \nu) \approx 3.4 \times 10^6$   $B$  decays to obtain a sample of this size, since the efficiency of reconstructing the semileptonic decay is small (e.g., slightly under 2% at Belle [4]). The Belle Collaboration has reported a signal of 107 events on the basis of  $21.2 \text{ fb}^{-1}$ , but the background (148 events) is larger than the signal, and the branching ratio is dominated by systematic error. Thus a sample of about 4.4 times the present size would be the minimum needed to achieve the stated goal, with a larger sample required if background levels are to be reduced.

## VI. INFORMATION ON $P$ AND ITS INTERFERENCE WITH $T$

We shall use present and anticipated information on  $T$  based on the methods described in the preceding section, and

TABLE III. Dependence of the fraction  $f$  of  $B^0 \rightarrow \pi^- l^+ \nu_l$  events below  $q^2 = 11 \text{ GeV}^2$  on the parameter  $\alpha_B$ .

$\alpha_B$	0.2	0.3	0.4	0.5	0.6
$f$	0.618	0.595	0.568	0.538	0.503

flavor SU(3) [2] to obtain information on  $P$  from the mainly penguin process  $B^+ \rightarrow K^0 \pi^+$ . In this manner we shall end up with an estimate  $|P/T| = 0.26 \pm 0.08$ , to be compared to the value of  $0.259 \pm 0.043 \pm 0.052$  quoted by Beneke *et al.* [35] on the basis of a theoretical calculation. (The inclusion of weak annihilation contributions in [35] raises this value to  $0.285 \pm 0.051 \pm 0.057$ .) Improved input data will potentially reduce the error on this ratio considerably, allowing for an estimate of direct  $CP$ -violating effects in  $B^0 \rightarrow \pi^+ \pi^-$  with less recourse to theory. Furthermore, if  $|T|$  turns out to be incompatible with the experimental magnitude of the amplitude  $A(B^0 \rightarrow \pi^+ \pi^-) = -(T+P)$ , we shall obtain a constraint on the product  $\cos \alpha \cos \delta$ , where  $\alpha$  is the CKM phase discussed previously and  $\delta$  is the relative strong phase between tree and penguin amplitudes. Our discussion will be an updated version of that presented in [17].

We shall quote all rates in units of ( $B^0$  branching ratio  $\times 10^6$ ). Thus, the average (3) of  $B^0 \rightarrow \pi^+ \pi^-$  branching ratios implies

$$|T|^2 + |P|^2 - 2|TP| \cos \alpha \cos \delta = 4.4 \pm 0.9 \quad (16)$$

in these units. With  $\mathcal{B}_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-)$  ranging from  $(4.5 \text{ to } 11) \times 10^{-6}$  we then estimate  $|T| = 2.7 \pm 0.6$ . This is identical to the value obtained [36] from  $B^+ \rightarrow \pi^+ \pi^0$  with additional assumptions about the color-suppressed amplitude.

The penguin amplitude can be estimated from  $B^+ \rightarrow K^0 \pi^+$ . The average of CLEO [5], Belle [6], and BaBar [7] branching ratios [8] gives

$$\mathcal{B}(B^+ \rightarrow K^0 \pi^+) = (17.2 \pm 2.4) \times 10^{-6}, \quad (17)$$

leading to  $|P'|^2 = (17.2 \pm 2.4)(\tau^0/\tau^+)$ ,  $|P'| = 4.02 \pm 0.28$ , where we use the lifetime ratio  $\tau_{B^+}/\tau_{B^0} = 1.068 \pm 0.016$  [37]. Here  $P'$  refers to the strangeness-changing  $\bar{b} \rightarrow \bar{s}$  penguin amplitude, which is dominated by the CKM combination  $V_{ts} V_{tb}^*$ .

We now estimate the strangeness-preserving  $\bar{b} \rightarrow \bar{d}$  amplitude by assuming it to be dominated by the CKM combination  $V_{td} V_{tb}^*$ . This may induce some uncertainty if the lighter intermediate quarks also play a role [38]. (A slightly different definition of  $P$  is used by [35,39] and avoids this problem.) We find

$$|P/P'| \approx \left| \frac{V_{td}}{V_{ts}} \right| = \lambda |1 - \rho - i\eta| \approx 0.22(0.80 \pm 0.15),$$

$$|P| = 0.71 \pm 0.14, \quad (18)$$

where  $\lambda$ ,  $\rho$ , and  $\eta$  are parameters [40] describing the hierarchy of CKM matrix elements. Combining these results, we find only that  $-0.1 \leq \cos \alpha \cos \delta \leq 1$ , so that destructive interference is possible but not established. Reduced errors on  $|T|$  and  $|P|$  will be needed for a more definitive conclusion. In particular, given the present central values, reduction of the error on  $|T|^2$  to 11%, as achievable with 470  $B \rightarrow \pi l \nu$  events, would allow one to infer the presence of destructive interference at about the  $2.8\sigma$  level.

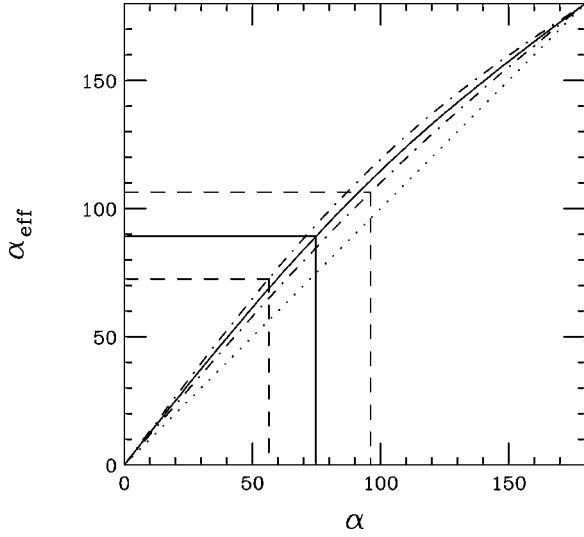


FIG. 6. Relation between  $\alpha_{\text{eff}}$  as measured using  $S_{\pi\pi} = \sin(2\alpha_{\text{eff}})$  and the weak phase  $\alpha$  for  $|P/T|=0.26$  and  $\delta=0$  (solid curve). Dot-dashed curves correspond to  $\pm 1\sigma$  errors on  $|P/T|$ . The dotted line corresponds to  $P=0$ . The solid and dashed lines correspond to the central and  $\pm 1\sigma$  values of  $S_{\pi\pi}$  recently reported by the BaBar Collaboration (allowing also for error in  $|P/T|$ ). We show only the range associated with the region of CKM parameters consistent with other measurements.

With our present estimates of  $|P|$  and  $|T|$ , we then find  $|P/T|=0.26\pm 0.08$ . Errors on this quantity can be decreased by improving the measurements of the branching ratio for  $B\rightarrow\pi l\nu$ , by measuring its spectrum, and by reducing the error on  $|1-\rho-i\eta|$ , which we have taken to be greater than in some other determinations [41].

The presence of the  $P$  amplitude can affect the determination of the weak phase  $\alpha$  using  $CP$ -violating asymmetries in  $B^0\rightarrow\pi^+\pi^-$  decays. The BaBar Collaboration [10] has recently reported the first results for this process. The decay distributions  $f_+$  ( $f_-$ ) in an asymmetric  $e^+e^-$  collider at the  $Y(4S)$  when the tagging particle (opposite to the one produced) is a  $B^0$  ( $\bar{B}^0$ ) are given by [11]

$$f_{\pm}(\Delta t) \simeq e^{-\Delta t/\tau} [1 \pm S_{\pi\pi} \sin(\Delta m_d \Delta t) \mp C_{\pi\pi} \cos(\Delta m_d \Delta t)], \quad (19)$$

where

$$S_{\pi\pi} \equiv \frac{2 \text{Im}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2}, \quad C_{\pi\pi} \equiv \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2} \quad (20)$$

and

$$\lambda_{\pi\pi} \equiv e^{-2i\beta} \frac{A(\bar{B}^0 \rightarrow \pi^+ \pi^-)}{A(B^0 \rightarrow \pi^+ \pi^-)}. \quad (21)$$

Here

$$A(B^0 \rightarrow \pi^+ \pi^-) \simeq -(|T|e^{i\delta_T}e^{i\gamma} + |P|e^{i\delta_P}e^{-i\beta}),$$

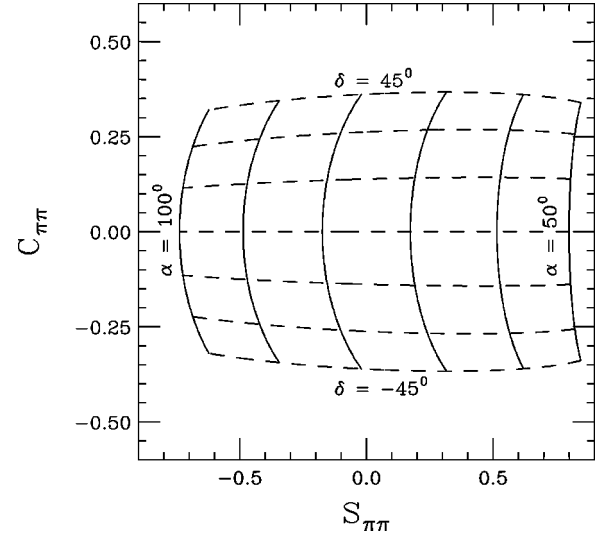


FIG. 7. Relation between  $S_{\pi\pi}$  and  $C_{\pi\pi}$  for fixed values of  $\alpha$  (solid curves) and  $\delta$  (dashed curves). The values of  $\alpha$  range in steps of  $10^\circ$  from  $50^\circ$  (right) to  $100^\circ$  (left); those of  $\delta$  range in steps of  $15^\circ$  from  $-45^\circ$  (bottom) to  $45^\circ$  (top). Here  $|P/T|=0.26$  has been assumed.

$$A(\bar{B}^0 \rightarrow \pi^+ \pi^-) \simeq -(|T|e^{i\delta_T}e^{-i\gamma} + |P|e^{i\delta_P}e^{i\beta}), \quad (22)$$

where  $\delta_T$  and  $\delta_P$  are strong phases of the tree and penguin amplitudes. To first order in  $|P/T|$ , using  $\beta + \gamma = \pi - \alpha$  and defining  $\delta \equiv \delta_P - \delta_T$ , we then have

$$\lambda_{\pi\pi} \simeq e^{2i\alpha} \left( 1 + 2i \frac{|P|}{|T|} e^{i\delta} \sin \alpha \right). \quad (23)$$

In the limit of small  $|P/T|$  and vanishing final-state phase  $\delta$ , the  $S_{\pi\pi}$  term is just  $\sin(2\alpha_{\text{eff}})$ , where

$$\alpha_{\text{eff}} \simeq \alpha + \left| \frac{P}{T} \right| \sin \alpha. \quad (24)$$

A plot of this relation for  $|P/T|=0.26\pm 0.08$  is shown in Fig. 6. The BaBar Collaboration [10] has reported  $S_{\pi\pi} = 0.03_{-0.56}^{+0.53} \pm 0.11$  on the basis of  $30.4 \text{ fb}^{-1}$ . The corresponding central and  $\pm 1\sigma$  values of  $\alpha_{\text{eff}}$  and  $\alpha$  are shown as the solid and dashed lines on the figure.

To first order in  $|P/T|$ , the  $C_{\pi\pi}$  term may be written

$$C_{\pi\pi} \simeq 2|P/T| \sin \delta \sin \alpha. \quad (25)$$

The BaBar Collaboration's value [10]  $C_{\pi\pi} = -0.25_{-0.47}^{+0.45} \pm 0.47$  is consistent with zero, as one might expect for a small final-state phase  $\delta$ . This measurement in the future will serve mainly to constrain  $\delta$ , given the limited range expected for  $|P/T|$  and  $\sin \alpha$ . Such a constrained value of  $\delta$  will then be useful in interpreting the flavor-averaged branching ratio (3) in terms of the tree-penguin interference discussed previously. The combined measurements of the flavor-averaged  $B^0\rightarrow\pi^+\pi^-$  branching ratio and the coefficients  $S_{\pi\pi}$  and  $C_{\pi\pi}$ , when combined with independent determinations of  $|T|$  and  $|P|$ , should allow us to dispense with the assump-

tions that the final-state phase  $\delta$  is small and that the weak phase of  $P$  is dominated by the top quark in the loop.

An example is shown in Fig. 7 of how  $S_{\pi\pi}$  and  $C_{\pi\pi}$  measurements can be used to constrain  $\alpha$  and  $\delta$ . Values extracted from such plots can then be checked for consistency with Eq. (16) to check our assumption that the phase and magnitude of  $P$  is dominated by the top quark.

## VII. CONCLUSIONS

We have discussed rate and spectrum requirements in  $B \rightarrow \pi l \nu_l$  decays needed to reduce errors in the tree-amplitude contribution  $T$  to  $B^0 \rightarrow \pi^+ \pi^-$ . Better knowledge of  $T$  can be combined with an estimate of the penguin amplitude  $P$  to see if destructive tree-penguin interference is occurring in  $B^0 \rightarrow \pi^+ \pi^-$ , and to evaluate the correction to the time-dependent  $CP$  asymmetry parameters  $S_{\pi\pi}$  and  $C_{\pi\pi}$ . Present data lead to the estimate  $|P/T| = 0.26 \pm 0.08$  but substantial improvement will be possible once the semileptonic rate and spectrum (particularly near  $q^2=0$ ) are better measured. We

have estimated that at least 470  $B \rightarrow \pi l \nu$  events (about 4.4 times the present sample size at Belle) are needed to reduce the error on  $T$  to 5.5%. For  $\alpha$  near  $90^\circ$  we predict  $\alpha_{\text{eff}} - \alpha \simeq (15 \pm 5)^\circ$ . Destructive tree-penguin interference in  $B^0 \rightarrow \pi^+ \pi^-$  could be significant if  $\alpha$  were closer to the lower limit of about  $56^\circ$  allowed by the present analysis. The form factor  $F_+(q^2)$  measured in  $B \rightarrow \pi l \nu_l$  also can be helpful in estimating the “wrong-sign” amplitude in  $B \rightarrow D^* \pi$  decays [42].

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