Influence of an external chromomagnetic field on color superconductivity

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We study the competition of quark-antiquark and diquark condensates under the influence of an external chromomagnetic field modeling the gluon condensate and its dependence on the chemical potential and temperature. As our results indicate, an external chromomagnetic field might produce remarkable qualitative changes in the picture of the color superconducting (CSC) phase formation. This concerns, in particular, the possibility of a transition to the CSC phase and diquark condensation at finite temperature.

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I. INTRODUCTION

Low energy (large distance) effects in QCD can only be studied by approximate (nonperturbative) methods in the framework of various effective models or in terms of lattice calculations. At the present time, one of the most popular QCD-like effective theories is the well-known Nambu–Jona-Lasinio (NJL) model [1], which is a relativistic quantum field theory with four-fermion interactions. The physics of light mesons (see e.g. [2] and references therein), diquarks [3,4] and meson-baryon interactions [5–7] based on dynamical chiral symmetry breaking can be effectively described by NJL chiral quark models. Moreover, NJL models are widely used in nuclear physics and astrophysics (neutron stars) for the investigation of quark matter [8] to construct alternative models of electroweak interactions [9] and in cosmological applications [10]. Moreover, its $(2+1)$-dimensional analogue serves as a satisfactory microscopic theory for several effects in the physics of high-temperature superconductors [11].

The NJL model displays the same symmetries as QCD. So it can be successfully used for simulating some of the QCD vacuum properties under the influence of external conditions such as temperature T and chemical potential μ [12]. The role of such considerations significantly increases especially in the cases where numerical lattice calculations are not admissible in QCD, i.e. at nonzero density and in the presence of external electromagnetic fields [13,14]. Recently, it was shown in the framework of a $(2+1)$-dimensional NJL model that an arbitrary small external magnetic field induces spontaneous chiral symmetry breaking (χSB) even under conditions when the interaction between fermions is arbitrarily weak [15]. Later it was shown that this phenomenon (called the magnetic catalysis effect) has a rather universal character and gets its explanation on the basis of the dimensional reduction mechanism [16]. (The recent reviews [17] consider the modern status of the magnetic catalysis effect and its applications in different branches of physics.)

As an effective theory for low energy QCD, the NJL model does not contain any dynamical gluon fields. Such a nonperturbative feature of the real QCD vacuum, as the nonzero gluon condensate $\langle F^a_{\mu\nu}F^{a\mu\nu}\rangle \equiv \langle FF\rangle$ can, however, be mimicked in the framework of NJL models with the help of external chromomagnetic fields. In particular, for a QCD-motivated NJL model with gluon condensate (i.e., in the presence of an external chromomagnetic field) and finite temperature, it was shown that a weak gluon condensate plays a stabilizing role for the behavior of the constituent quark mass, the quark condensate, meson masses and coupling constants for varying temperature [18]. Then, in a series of papers, devoted to the NJL model with gluon condensate, it was shown that an external chromomagnetic field, similar to the ordinary magnetic field, serves as a catalyzing factor in the fermion mass generation and dynamical breaking of chiral symmetry as well [19]. The basis for this phenomenon is the effective reduction of the space dimensionality in the presence of external chromomagnetic fields [20].

There exists the exciting idea proposed more than twenty years ago [21–23] that at high baryon densities a colored diquark condensate $\langle qq\rangle$ might appear. In analogy with ordinary superconductivity, this effect was called color superconductivity (CSC). The CSC phenomenon was investigated in the framework of the one-gluon exchange approximation in QCD [24], where the colored Cooper pair formation is predicted self-consistently at extremely high values of the chemical potential $\mu \gtrsim 10^8$ MeV [25]. Unfortunately, such baryon densities are not observable in nature and not accessible in experiments (the typical densities inside the neutron stars or in the future heavy ion experiments correspond to $\mu \sim 500$ MeV). The possibility for the existence of the CSC phase in the region of moderate densities was proved quite recently (see, e.g., the papers in [26–29] as well as the review articles in [30] and references therein). In these papers it was shown on the basis of different effective theories for low energy QCD (instanton model, NJL model, etc.) that the diquark condensate $\langle qq\rangle$ can appear already at a rather moderate baryon density ($\mu\sim400$ MeV), which can possibly be detected in the future experiments on ion-ion collisions. Since quark Cooper pairing occurs in the color antitriplet channel, the nonzero value of $\langle qq\rangle$ means that, apart from the electromagnetic $U(1)$ symmetry, the color $SU_c(3)$ should be spontaneously broken down inside the CSC phase as well. In the framework of NJL models the CSC phase formation has generally be considered as a dynamical competition between diquark $\langle qq\rangle$ and usual quark-antiquark condensation $\langle\bar{q}q\rangle$. However, the real QCD vacuum is characterized in addition by the appearance of a gluon condensate $\langle FF\rangle$ as well, which might change the generally accepted conditions for the CSC observation. In particular, one would expect that, similarly to the case of quark-antiquark condensation, the process of diquark condensation might be induced by external chromomagnetic fields. For a $(2+1)$-dimensional quark model, this was recently demonstrated in [31]. There, a $SU(2)_L\times SU(2)_R$ chirally symmetric $(2+1)$-dimensional NJL model with three colored quarks of two flavors was considered at zero T,μ. It was shown that in this case for arbitrary fixed values of coupling constants there exists a critical value of the external chromomagnetic field at which a CSC second order phase transition is induced in the system.[1] Since the two-flavored QCD_3 and the considered NJL model are not in the same universality class of theories [QCD_3 with $N_f=2$ has a higher flavor symmetry $SU(4)$], the obtained results are intrinsic to real QCD_4 rather than to QCD_3. Indeed, our recent investigations on the basis of a $(3+1)$-dimensional NJL model [32,33] and $\mu=0$ show that some types of sufficiently strong external chromomagnetic fields may catalyze the diquark condensation.

As argued above, CSC might occur inside neutron stars and possibly become observable in ion-ion collisions, i.e., at nonzero baryon densities. Taking into account the fact that at finite chemical potential the magnetic generation of dynamical χSB qualitatively differs from the $\mu=0$ case [14], one might expect analogous effects for CSC, too. For this reason, the investigation of the chromomagnetic generation of CSC under the influence of a finite chemical potential (finite particle density) is a very interesting and actual physical problem.

The aim of the present paper is to study the influence of external conditions such as chemical potential, temperature and especially of the gluon condensate (as modeled by external color gauge fields) on the phase structure of quark matter with particular emphasis on its CSC phase. To this end, we shall extend our earlier analysis of the chromomagnetic generation of CSC at $\mu=0$ [31–33] to the case of a $(3+1)$-dimensional NJL type model with finite chromomagnetic field, temperature and chemical potential presenting a generalization of the free field model of [29].

The paper is organized as follows. In Secs. II and III the extended NJL model under consideration is presented, and its effective potential ($\equiv$ thermodynamic potential) at nonzero external chromomagnetic field, chemical potential and temperature is obtained in the one-loop approximation. This quantity contains all the necessary information about the quark and diquark condensates of the theory. In Sec. IV the phase structure of the model is discussed on the basis of numerical investigations of the global minimum point of the effective potential. As our main result, it is shown that the external chromomagnetic field can induce the transition to the CSC phase and diquark condensation even at finite temperature. Thereby, the characteristics of the CSC phase can significantly change in dependence on the strength of the chromomagnetic field. Finally, Sec. V contains a summary and discussion of the results. Some details of the effective potential calculation are relegated to an Appendix.

[1]Strictly speaking, the CSC is induced by those components of external chromomagnetic fields which can stay massless inside the CSC phase.

II. THE MODEL

Let us first give several (very approximative) arguments motivating the chosen structure of our QCD-motivated extended NJL model introduced below. For this aim, consider two-flavor QCD with nonzero chemical potential and color group $SU_c(N_c)$ and decompose the gluon field $\mathcal{A}^a_\nu(x)$ into a condensate background ("external") field $A^a_\nu(x)$ and the quantum fluctuation $a^a_\nu(x)$ around it, i.e. $\mathcal{A}^a_\nu(x)=A^a_\nu(x)+a^a_\nu(x)$. By integrating in the generating functional of QCD over the quantum field $a^a_\nu(x)$ and further "approximating" the nonperturbative gluon propagator by a δ function, one arrives at an effective local chiral four-quark interaction of the NJL type describing low energy hadron physics in the presence of a gluon condensate. Finally, by performing a Fierz transformation of the interaction term, one obtains a four-fermionic model with $(\bar{q}q)$ and (qq) interactions and an external condensate field $A^a_\mu(x)$ of the color group $SU_c(N_c)$ given by the following Lagrangian:[2]

$$L=\bar{q}\left[\gamma^\nu\left(i\partial_\nu+gA^a_\nu(x)\frac{\lambda^a}{2}\right)+\mu\gamma^0\right]q+\frac{G_1}{2N_c}[(\bar{q}q)^2+(\bar{q}i\gamma^5\vec{\tau}q)^2]+\frac{G_2}{N_c}[i\bar{q}_c\varepsilon(i\lambda^b_{as})\gamma^5q][i\bar{q}\varepsilon(i\lambda^b_{as})\gamma^5q_c]. \quad (1)$$

It is necessary to note that in order to obtain realistic estimates for masses of vector–axial-vector mesons and diquarks in extended NJL-type models [3], we have to allow for independent coupling constants G_1,G_2, rather than to consider them related by a Fierz transformation of a current-current interaction via gluon exchange. Clearly, such a procedure does not spoil chiral symmetry.

[2]The most general four-fermion interaction would include additional vector and axial-vector $(\bar{q}q)$ as well as pseudoscalar, vector and axial-vector-like (qq) interactions. For our goal of studying the effect of chromomagnetic catalysis for the competition of quark and diquark condensates, the interaction structure of Eq. (1) is, however, sufficiently general.

In Eq. (1) g denotes the gluon coupling constant, μ is the quark chemical potential, $q_c = C\bar{q}^t$, $\bar{q}_c = q^t C$ are charge-conjugated spinors, and $C = i\gamma^2\gamma^0$ is the charge conjugation matrix (t denotes the transposition operation). In what follows we assume $N_c = 3$ and replace the antisymmetric color matrices λ^b_{as} (with a factor i) by the antisymmetric ϵ^b operator. Moreover, summation over repeated color indices $a = 1, \ldots, 8$; $b = 1,2,3$ and Lorentz indices $\nu = 0,1,2,3$ is implied. The quark field $q \equiv q_{i\alpha}$ is a flavor doublet and color triplet as well as a four-component Dirac spinor, where $i = 1,2$; $\alpha = 1,2,3$. (Latin and Greek indices refer to flavor and color indices, respectively; spinor indices are omitted.) Furthermore, we use the notations $\lambda^a/2$ for the generators of the color $SU_c(3)$ group appearing in the covariant derivative as well as $\vec{\tau} \equiv (\tau^1, \tau^2, \tau^3)$ for Pauli matrices in the flavor space; $(\varepsilon)^{ik} \equiv \varepsilon^{ik}$, $(\epsilon^b)^{\alpha\beta} \equiv \epsilon^{\alpha\beta b}$ are totally antisymmetric tensors in the flavor and color spaces, respectively. Clearly, the Lagrangian (1) is invariant under the chiral $SU(2)_L \times SU(2)_R$ and color $SU_c(3)$ groups.

Next, let us for a moment suppose that in Eq. (1) $A^a_\mu(x)$ is an arbitrary classical gauge field of the color group $SU_c(3)$. (The following investigations do not require the explicit inclusion of the gauge field part of the Lagrangian.) The detailed structure of $A^a_\mu(x)$ corresponding to a constant chromomagnetic gluon condensate will be given below.

The linearized version of the model (1) with auxiliary bosonic fields has the following form:

$$\begin{aligned}\tilde{L} = {} & \bar{q}\left[\gamma^\nu\left(i\partial_\nu + gA^a_\nu(x)\frac{\lambda_a}{2}\right) + \mu\gamma^0\right]q - \bar{q}(\sigma + i\gamma^5\vec{\tau}\vec{\pi})q \\ & - \frac{3}{2G_1}(\sigma^2 + \vec{\pi}^2) - \frac{3}{G_2}\Delta^{*b}\Delta^b - \Delta^{*b}[iq^t C\varepsilon\epsilon^b\gamma^5 q] \\ & - \Delta^b[i\bar{q}\varepsilon\epsilon^b\gamma^5 C\bar{q}^t]. \end{aligned} \tag{2}$$

The Lagrangians (1) and (2) are equivalent, as can be seen by using the equations of motion for bosonic fields, from which it follows that

$$\Delta^b \sim iq^t C\varepsilon\epsilon^b\gamma^5 q, \quad \sigma \sim \bar{q}q, \quad \vec{\pi} \sim i\bar{q}\gamma^5\vec{\tau}q. \tag{3}$$

Clearly, the σ and $\vec{\pi}$ fields are color singlets. Besides, the (bosonic) diquark field Δ^b is a color antitriplet and a (isoscalar) singlet under the chiral $SU(2)_L \times SU(2)_R$ group. Note further that the σ, Δ^b, are scalars, but the $\vec{\pi}$ are pseudoscalar fields. Hence, if $\sigma \neq 0$, then chiral symmetry of the model is spontaneously broken, whereas $\Delta^b \neq 0$ indicates the dynamical breaking of both the color and electromagnetic symmetries of the theory.

In the one-loop approximation, the effective action for the boson fields which is invariant under the chiral (flavor) as well as color and Lorentz groups is expressed through the path integral over quark fields:

$$\exp(iS_{\rm eff}(\sigma,\vec{\pi},\Delta^b,\Delta^{*b},A^a_\mu)) = N'\int[d\bar{q}][dq] \times \exp\left(i\int \tilde{L}d^4x\right),$$

where

$$S_{\rm eff}(\sigma,\vec{\pi},\Delta^b,\Delta^{*b},A^a_\mu) = -N_c\int d^4x\left[\frac{\sigma^2+\vec{\pi}^2}{2G_1} + \frac{\Delta^b\Delta^{*b}}{G_2}\right] + \tilde{S}, \tag{4}$$

N' is a normalization constant. The quark contribution to the partition function is here given by

$$\begin{aligned} Z_q &= \exp(i\tilde{S}) \\ &= N'\int[d\bar{q}][dq]\exp\left(i\int[\bar{q}\mathcal{D}q + \bar{q}\mathcal{M}\bar{q}^t + q^t\bar{\mathcal{M}}q]d^4x\right). \end{aligned} \tag{5}$$

In Eq. (5) we have used the following notations:

$$\mathcal{D} = D + \gamma^\mu gA^a_\mu(x)\frac{\lambda^a}{2}; \quad D = i\gamma^\mu\partial_\mu - \sigma - i\gamma^5\vec{\pi}\vec{\tau} + \mu\gamma^0,$$

$$\bar{\mathcal{M}} = -i\Delta^{*b}C\varepsilon\epsilon^b\gamma^5, \quad \mathcal{M} = -i\Delta^b\varepsilon\epsilon^b\gamma^5 C, \tag{6}$$

where D is the Dirac operator in the coordinate, spinor and flavor spaces, whereas $\mathcal{D}$, $\mathcal{M}$ and $\bar{\mathcal{M}}$ are in addition operators in the color space, too. Let us next assume that in the ground state of our model $\langle\Delta^1\rangle = \langle\Delta^2\rangle = \langle\vec{\pi}\rangle = 0$ and $\langle\sigma\rangle$, $\langle\Delta^3\rangle \neq 0$.[3] Obviously, the residual symmetry group of such a vacuum is $SU_c(2)$ whose generators are the first three generators of the initial $SU_c(3)$. Now suppose that in this frame the constant external chromomagnetic field, simulating the presence of a gluon condensate $\langle FF\rangle = 2H^2$, has the following form $H^a = (H^1, H^2, H^3, 0, \ldots, 0)$. Furthermore, due to the residual $SU_c(2)$ invariance of the vacuum, one can put $H^1 = H^2 = 0$ and $H^3 \equiv H$.

Some remarks about the structure of the external chromomagnetic fields $A^a_\nu(x)$ used in Eq. (1) are needed. From this moment on, we assume $A^a_\nu(x)$ in such a form that the only nonvanishing components of the corresponding field strength tensor $F^a_{\mu\nu}$ are $F^3_{12} = -F^3_{21} = H = \text{const}$. The above homogeneous chromomagnetic field can be generated by the following vector-potential:

$$A^3_\nu(x) = (0,0,Hx^1,0); \quad A^a_\nu(x) = 0 \quad (a \neq 3), \tag{7}$$

which defines the well known Matinyan-Savvidy model of the gluon condensate in QCD [34].

In QCD the physical vacuum may be interpreted as a region split into an infinite number of domains with mac-

[3] If $\langle\vec{\pi}\rangle \neq 0$ then one would have spontaneous breaking of parity. For strong interactions parity is, however, a conserved quantum number, justifying the assumption $\langle\vec{\pi}\rangle = 0$.

roscopic extension [35]. Inside each such domain there can be excited a homogeneous background chromomagnetic field, which generates a nonzero gluon condensate $\langle FF\rangle \neq 0$. (Averaging over all domains results in a zero background chromomagnetic field, hence color as well as Lorentz symmetries are not broken.)[4]

In order to find nonvanishing condensates $\langle\sigma\rangle$ and $\langle\Delta^3\rangle$, we should calculate the effective potential, whose global minimum point provides us with these quantities. Suppose that [apart from the external vector-potential $A^a_\mu(x)$ (7)] all boson fields in $S_{\rm eff}$ (4) do not depend on space-time. In this case, by definition, $S_{\rm eff}=-V_{\rm eff}\int d^4x$, where

$$V_{\rm eff}=\frac{3(\sigma^2+\vec{\pi}^2)}{2G_1}+\frac{3\Delta^b\Delta^{*b}}{G_2}+\tilde{V};\quad \tilde{V}=-\frac{\tilde{S}}{v},\quad v=\int d^4x. \tag{8}$$

Due to our assumption on the vacuum structure, we put $\Delta^{1,2}\equiv 0$, as well as $\vec{\pi}=0$. Then, taking into account the form of the vector-potential (7), one can easily see that the functional integral for $\tilde{S}$ in Eq. (5) is factorized

$$\begin{aligned}Z_q&=\exp(i\tilde{S}(\sigma,\Delta))\\&=N'\int[d\bar{q}_3][dq_3]\exp\left(i\int\bar{q}_3\tilde{D}q_3d^4x\right)\end{aligned} \tag{9}$$

$$\begin{aligned}&\times\int[d\bar{Q}][dQ]\exp\left(i\int[\bar{Q}\tilde{\mathcal{D}}Q\right.\\&\left.+\bar{Q}M\bar{Q}^t+Q^t\bar{M}Q]d^4x\right),\end{aligned} \tag{10}$$

where $\Delta\equiv\Delta^3$, q_3 is the quark field of color 3 and $Q\equiv(q_1,q_2)^t$ is the doublet, composed from quark fields of the colors 1,2. Moreover, $\tilde{D}=D|_{\vec{\pi}=0}$ [D is presented in Eq. (6)] and

$$\begin{aligned}&\tilde{\mathcal{D}}=\tilde{D}+\gamma^\mu gA^3_\mu(x)\frac{\sigma_3}{2};\quad \bar{M}=-i\Delta^*C\varepsilon\tilde{\epsilon}\gamma^5,\\&M=-i\Delta\varepsilon\tilde{\epsilon}\gamma^5C.\end{aligned} \tag{11}$$

In Eq. (11) σ_3, $\tilde{\epsilon}$ are matrices in the two-dimensional color subspace, corresponding to the $SU_c(2)$ group:

$$\sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\quad \tilde{\epsilon}=\begin{pmatrix}0&1\\-1&0\end{pmatrix}.$$

Clearly, the integration over q_3 in Eq. (9) yields det $\tilde{D}$.

Defining $\Psi^t=(Q^t,\bar{Q})$ and introducing the matrix-valued operator

$$Z=\begin{pmatrix}2\bar{M}, & -\tilde{\mathcal{D}}^t\\ \tilde{\mathcal{D}}, & 2M\end{pmatrix},$$

the Gaussian integral over $\bar{Q}$ and Q in (10) can be rewritten in compact matrix notation and evaluated as

$$\int[d\Psi]e^{(i/2)\int\Psi^tZ\Psi d^4x}=\det{}^{1/2}Z. \tag{12}$$

Then, by using in Eq. (12) the general formula

$$\det\begin{pmatrix}A, & B\\ \bar{B}, & \bar{A}\end{pmatrix}=\det[-\bar{B}B+\bar{B}A\bar{B}^{-1}\bar{A}]=\det[\bar{A}A-\bar{A}B\bar{A}^{-1}\bar{B}],$$

one obtains the result

$$\begin{aligned}\exp(i\tilde{S}(\sigma,\Delta))&=N'\det(\tilde{D})\det{}^{1/2}[4M\bar{M}+M\tilde{\mathcal{D}}^tM^{-1}\tilde{\mathcal{D}}]\\&=N'\det[(i\hat{\partial}-\sigma+\mu\gamma^0)]\det{}^{1/2}\Bigg[4|\Delta|^2\\&\quad+\left(-i\hat{\partial}-\sigma+\mu\gamma^0-g\hat{A}^3\frac{\sigma_3}{2}\right)\\&\quad\times\left(i\hat{\partial}-\sigma+\mu\gamma^0+g\hat{A}^3\frac{\sigma_3}{2}\right)\Bigg].\end{aligned} \tag{13}$$

Recall that the operator under the first det-symbol in Eq. (13) acts only in the flavor, coordinate and spinor spaces, whereas the operator under the second det-symbol acts in the two-dimensional color subspace, too.

III. THE GENERAL CASE $\mu\neq 0$, $T\neq 0$, $H\neq 0$

A. The effective potential

First of all, let us calculate the effective action from Eq. (13) at zero temperature T. It is convenient to rewrite the second determinant in Eq. (13) in the form

$$\begin{aligned}\det[4M\bar{M}+M\tilde{\mathcal{D}}^t(M)^{-1}\tilde{\mathcal{D}}]&=\det[4|\Delta|^2+\mu^2-p_0^2+\sigma^2\\&\quad-(\overline{\gamma\nabla})^2-2\mu\gamma^0(\sigma+\overline{\gamma\nabla})]\end{aligned} \tag{14}$$

where the p^0-momentum space representation and $\overline{\gamma\nabla}=\gamma_k(i\partial_k+gA^3_k\sigma_3/2)$, $(k=1,2,3)$ have been used. Similarly to quantum electrodynamics, it is easily seen that the operator $\mathcal{H}\equiv\gamma^0(\sigma+\overline{\gamma\nabla})$ is the Hamiltonian for quarks with color indices $\alpha=1,2$ and flavor $i=1,2$ in the background vector-potential (7). Its eigenvalues are $\pm\varepsilon_{\{n\}}$, where $\varepsilon_{\{n\}}=\sqrt{\sigma^2+p_3^2+gH(n+1/2)-gH\zeta/2}$, and corresponding eigenstates are denoted by $\Phi^\pm_{\{n\}p_2i\alpha}$. The set of quark quantum numbers in the background field are defined as follows: $\{n\}\equiv\{n=0,1,2,\ldots;\ -\infty<p_3<+\infty;\ \zeta=\pm 1\}$, $i,\alpha=1,2$, $-\infty<p_2<\infty$. Each of the eigenvalues $\pm\varepsilon_{\{n\}}$ for $\mathcal{H}$ is evidently fourfold degenerate with respect to flavor and color quantum numbers i,α. It is also degenerate with respect to

[4]Strictly speaking, our following calculations refer to some given macroscopic domain. The obtained results turn out to depend on color and rotational (Lorentz) invariant quantities only, and are independent of the concrete domain.

the quantum number p_2, which quasiclassically characterizes the charged particle center of orbit position in an external uniform magnetic field. Since

$$\mathcal{H}\mathcal{H}\Phi^{\pm}_{\{n\}p_2 i\alpha}=[\sigma^2-(\overline{\gamma\nabla})^2]\Phi^{\pm}_{\{n\}p_2 i\alpha}=\varepsilon^2_{\{n\}}\Phi^{\pm}_{\{n\}p_2 i\alpha},$$

one can easily conclude that in the basis $\Phi^{\pm}_{\{n\}p_2 i\alpha}$ the operator in the determinant (14) is diagonal. Moreover, its diagonal matrix elements are equal to $4|\Delta|^2-p_0^2+(\mu\pm\varepsilon_{\{n\}})^2$. Upon multiplying these quantities, one can find the determinant from Eq. (14). In a similar way, it is possible to calculate the first determinant from Eq. (13). Hence, taking into account the relation Det $O=\exp(\mathrm{Tr}\ln O)$, and following the standard procedure (see, e.g., [36]), the following expression for $\tilde{V}$ is obtained from Eqs. (13) and (14) (omitting an infinite σ- and Δ-independent constant):

$$\tilde{V}=-\frac{\tilde{S}}{v}=iN_f\int\frac{dp_0}{2\pi}\Bigg\{\sum_{\{p\}_0,\pm}\ln((E_p\pm\mu)^2-p_0^2)+A\sum_{\{n\}\pm}\ln(4|\Delta|^2-p_0^2+(\varepsilon_{\{n\}}\pm\mu)^2)\Bigg\},\qquad(15)$$

where $\{p\}_0$ denotes the set of quark quantum numbers for vanishing background field $(\{p\}_0\equiv\{-\infty<p_1,p_2,p_3<+\infty\})$, and $E_p=\sqrt{p^2+\sigma^2}$. The factor N_f in front of the integral in Eq. (15) is the result of summation over flavor indices $i=1,\ldots,N_f$, whereas the degeneracy factor $A\equiv gH/(8\pi^2)$ is due to the integration over the momentum p_2 and summation over the color indices $\alpha=1,2$. Moreover, $\Sigma_{\{p\}_0}\equiv\int d^3p/(2\pi)^3$, $\Sigma_{\{n\}}\equiv\int dp_3/(2\pi)\Sigma_{n,\zeta}$.

In the case of finite temperature $T=1/\beta>0$ the corresponding expression for $\tilde{V}_T$ can be obtained from Eq. (15) by means of the following replacements:

$$\int\frac{dp_0}{2\pi}(\cdots)\to iT\sum_l(\cdots);$$

$$p_0\to i\omega_l\equiv 2\pi iT(l+1/2);\quad l=0,\pm1,\pm2,\ldots,$$

where ω_l is the Matsubara frequency. Hence,

$$\tilde{V}_T=-N_fT\sum_{l=-\infty}^{l=\infty}\Bigg\{\sum_{\{p\}_0,\pm}\ln((E_p\pm\mu)^2+\omega_l^2)+A\sum_{\{n\},\pm}\ln(4|\Delta|^2+\omega_l^2+(\varepsilon_{\{n\}}\pm\mu)^2)\Bigg\}.\qquad(16)$$

In order to transform Eq. (16), let us first perform the summation over the Matsubara frequencies. It is evident that

$$\sum_l\ln(\omega_l^2+\Omega^2)=\sum_l\int_{1/\beta^2}^{\Omega^2}da^2\frac{1}{\omega_l^2+a^2}+\sum_l\ln\left(\omega_l^2+\frac{1}{\beta^2}\right),\qquad(17)$$

where Ω stands, according to Eq. (16), for $\sqrt{(E_p\pm\mu)^2}$ or $\sqrt{4|\Delta|^2+(\varepsilon_{\{n\}}\pm\mu)^2}$, i.e. $\Omega\geqslant0$. Note that we can neglect the contribution from the last term in Eq. (17), since it does not depend on σ and Δ. The first term in Eq. (17) can be presented in the following form (see Appendix):

$$\sum_l\int_{1/\beta^2}^{\Omega^2}da^2\frac{1}{\omega_l^2+a^2}=2\ln\mathrm{ch}(\Omega\beta/2)+\mathrm{const}=\Omega\beta+2\ln(1+e^{-\Omega\beta})+\mathrm{const}.\qquad(18)$$

Performing the summation over Matsubara frequencies in the second term in Eq. (16), and taking into account the degeneracy of the quark spectrum ε_n in the chromomagnetic field with respect to combination of quantum numbers n and ζ, we can use the following expression for the energy spectrum: $\varepsilon_n=\sqrt{gHn+p_3^2+\sigma^2}$, where $n=0,1,2,\ldots$ is the Landau quantum number, and $-\infty<p_3<\infty$. Then, summing over the spin quantum number $\zeta=\pm1$, we have to account for the fact that for the ground state with $n=0$ only one spin projection $\zeta=-1$ is possible. Hence, a factor $\alpha_n=2-\delta_{n0}$ should be included in the final expression. As for the summation over Matsubara frequencies in the first term in Eq. (16), it is necessary to take into account the fact that the function (18) is even with respect to the variable Ω. Finally, we thus arrive at the following result for the thermodynamic potential:

$$V_{H\mu T}(\sigma,\Delta)=N_c\left(\frac{\sigma^2}{2G_1}+\frac{|\Delta^2|}{G_2}\right)-2N_f\int\frac{d^3p}{(2\pi)^3}(N_c-2)\{E_p+T\ln[(1+e^{-\beta(E_p-\mu)})(1+e^{-\beta(E_p+\mu)})]\}$$
$$-N_fA\sum_{n=0}^{\infty}dp_3\alpha_n\{\sqrt{(\varepsilon_n-\mu)^2+4|\Delta|^2}+\sqrt{(\varepsilon_n+\mu)^2+4|\Delta|^2}$$
$$+2T\ln[(1+e^{-\beta\sqrt{(\varepsilon_n-\mu)^2+4|\Delta|^2}})(1+e^{-\beta\sqrt{(\varepsilon_n+\mu)^2+4|\Delta|^2}})]\}.\qquad(19)$$

For convenience, expressions are again written in terms of N_f and N_c even though in the following we will be concerned only with $N_f=2$ and $N_c=3$.

B. Regularization

First of all, let us subtract from Eq. (19) an infinite constant in order that the effective potential obeys the constraint $V_{H\mu T}(0,0)=0$. After this subtraction the effective potential still remains UV divergent. This divergency could evidently be removed by introducing a simple momentum cutoff $|p|<\Lambda$. Instead of doing this, we find it convenient to use another regularization procedure. To this end, let us recall that all UV divergent contributions to the subtracted potential $V_{H\mu T}(\sigma,\Delta)-V_{H\mu T}(0,0)$ are proportional to powers of meson and/or diquark fields σ, Δ. So, one can insert some momentum-dependent form factors in front of composite σ and Δ fields in order to regularize the UV behavior of integrals and sums.[5]

It is clear by now that we are going to study the effects of an external chromomagnetic condensate field in the framework of the NJL-type model (1), which in addition to two independent coupling constants G_1,G_2 includes regularizing meson (diquark) form factors. Of course, it would be a very hard task to study the competition of χSB and CSC for arbitrary values of coupling constants G_1,G_2 and any form factors. Thus, in order to restrict this arbitrariness and to be able to compare our results (at least roughly) with other approaches, we find it convenient to investigate the phase structures of the model (1) at $H=0$ and $H\neq 0$ only for some fixed values of G_1,G_2 and some simple expressions for meson or diquark form factors (for simplicity, meson or diquark form factors are chosen to be equal). Let us choose the form factors[6]

$$\phi=\frac{\Lambda^4}{(\Lambda^2+\vec{p}^{\,2})^2}, \quad \phi_n=\frac{\Lambda^4}{(\Lambda^2+p_3^2+gHn)^2}, \tag{20}$$

which have to be included in the energy spectra by a corresponding multiplication of the σ, Δ fields:

$$E_p^r=\sqrt{\vec{p}^{\,2}+\phi^2\sigma^2}, \quad \varepsilon_n^r=\sqrt{gHn+p_3^2+\phi_n^2\sigma^2},$$
$$|\Delta^2|\to\phi_n^2|\Delta^2|. \tag{21}$$

Note that with the choice of simple form factors (20) our expression for the thermodynamic potential at $H=0$ formally coincides with the corresponding expression of Ref. [28] obtained for an NJL type model with instanton-induced four-fermion interactions. In particular, by a suitable choice of coupling constants G_1,G_2, we will later "normalize" our phase portraits for $H=0$ to the curves of this paper in order to illustrate the influence of a nonvanishing chromomagnetic field.[7]

As a result, instead of Eq. (19) we shall deal with the following regularized potential $V_r(\sigma,\Delta)$:

$$V_r(\sigma,\Delta)=V_0-2N_{\rm f}(N_{\rm c}-2)\int_{-\infty}^{\infty}V_1\frac{d^3p}{(2\pi)^3}$$
$$-\frac{gHN_{\rm f}}{8\pi^2}\int_{-\infty}^{\infty}\sum_{n=0}^{\infty}\alpha_nV_2dp_3, \tag{22}$$

where $V_0=N_{\rm c}(\sigma^2/2G_1+|\Delta^2|/G_2)$,

$$V_1=E_p^r+T\ln[(1+e^{-\beta(E_p^r+\mu)})(1+e^{-\beta(E_p^r-\mu)})],$$
$$V_2=\sqrt{(\varepsilon_n^r-\mu)^2+4|\Delta|^2\phi_n^2}+\sqrt{(\varepsilon_n^r+\mu)^2+4|\Delta|^2\phi_n^2}$$
$$+2T\ln[(1+e^{-\beta\sqrt{(\varepsilon_n^r-\mu)^2+4|\Delta|^2\phi_n^2}})$$
$$\times(1+e^{-\beta\sqrt{(\varepsilon_n^r+\mu)^2+4|\Delta|^2\phi_n^2}})] \tag{23}$$

and E_p^r, ε_n^r are given in Eq. (21). Despite the Λ modification, the expression (22) contains yet UV-divergent integrals. However, as it was pointed out from the very beginning, we shall numerically study the subtracted effective potential, i.e., the quantity $V_r(\sigma,\Delta)-V_r(0,0)$, which has no divergences.

In the next section the dependency of the global minimum point of the regularized potential (22) on the external parameters H,μ,T will be investigated.

IV. NUMERICAL DISCUSSIONS

In the previous section we have chosen the form factors as in Eq. (20) in order to roughly normalize our numerical calculations at $H\neq 0$ on the results obtained at $H=0$ in [28]. Comparing the effective potential (22) at $gH=0$ with the

[5]A suitable physical motivation for such form factors follows in the framework of nonlocal NJL type models based on the one-gluon-exchange approximation to QCD with nontrivial gluon propagator. In particular, in Ref. [37] it was shown that the arising exponential form factors for composite mesons, as obtained from the solution of the (nonlocal) Bethe-Salpeter (BS) equation, make the quark loop expansion including meson (diquark) vertices convergent. In this case, there is no need for introducing a sharp momentum cutoff as in the local NJL model. Hence, the introduction of smoothing form factors in our expressions of an approximate local NJL model may be interpreted as a regularization procedure taking some effects of the originally nonlocal current-current interaction afterwards into account.

[6]The application of the smooth meson form factors (20) leads in a natural way to a suppression of higher Landau levels, which is of particular use here. Hence, this regularization scheme is particularly suitable for the manifestation of the (chromo)magnetic catalysis effect of dynamical symmetry breaking. Indeed, the (chromo)magnetic catalysis effect and the underlying mechanism of dimensional reduction are closely related to the infrared dominance of the lowest Landau level with $n=0$ [17,20].

[7]It is necessary to underline that in our case the meson-diquark form factors (20) mimick solutions of the BS equation for some nonlocal four-fermion interaction arising from the one-gluon exchange approach to QCD. Contrary to this, the instantonlike form factor used in [28] has another physical nature. It appears as quark zero mode wave function in the presence of instantons [26].

corresponding one from that paper (denoting their respective diquark field and coupling constants by a tilde), we see that these quantities coincide if $2\Delta=\tilde{\Delta}$, $G_1=2N_c\tilde{G}_1$ and $G_2=N_c\tilde{G}_2$. Using further the numerical ratio of coupling constants from Ref. [28], we obtain in our case the following relation:

$$G_2=\frac{3}{8}G_1. \tag{24}$$

Now, let us perform the numerical investigation of the global minimum point (GMP) of the potential (22) for form factors and values of coupling constants as given by Eqs. (20) and (24), respectively. It was supposed earlier in some papers (see e.g. [38]) that quantitative features of the color superconducting phase transition might indeed depend on the value of the form factor parameter Λ. So, in order to check the Λ dependence of our results, we perform the investigations for three different values of the cutoff, $\Lambda=0.6$ GeV, 0.8 GeV and 1 GeV. For each value of Λ, the corresponding value of G_1 is selected from the requirement that the GMP of the function $V^r_{H\mu}(\sigma,\Delta)$ at $T=\mu=H=0$ is at the point $\sigma=0.4$ GeV, $\Delta=0$ in agreement with phenomenological results and [28]. Then, the value of G_2 is fixed by the relation (24). This yields, for example, $G_1\Lambda^2=2N_c 6.47$ at $\Lambda=0.8$ GeV, $G_1\Lambda^2=2N_c 6.16$ at $\Lambda=1$ GeV, etc.

Note further that in the case of zero temperature we have

$$V_1|_{T=0}=E^r_p+(\mu-E^r_p)\theta(\mu-E^r_p),$$

$$V_2|_{T=0}=\sqrt{(\varepsilon^r_n-\mu)^2+4|\Delta|^2\phi_n^2}+\sqrt{(\varepsilon^r_n+\mu)^2+4|\Delta|^2\phi_n^2}.$$

In order to study the phase structure of the model using numerical methods, the summation over n in Eq. (22) is limited by a maximum value $n_{max}=(2.5\Lambda)^2/gH$, where other terms of the series can be neglected due to their smallness.

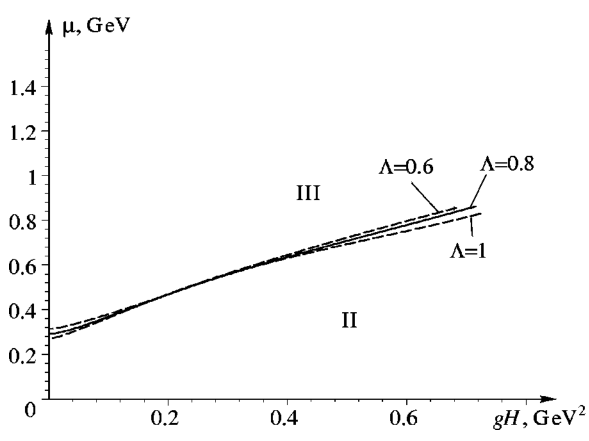

FIG. 1. Phase portrait of the model in terms of variables (μ,gH) at $T=0$ for three values of the cutoff parameter $\Lambda=0.6,0.8,1$ GeV. Regions II, III describe the phase with broken chiral symmetry $(\sigma\neq0,\Delta=0)$ and the color superconducting phase $(\sigma=0,\Delta\neq0)$, respectively.

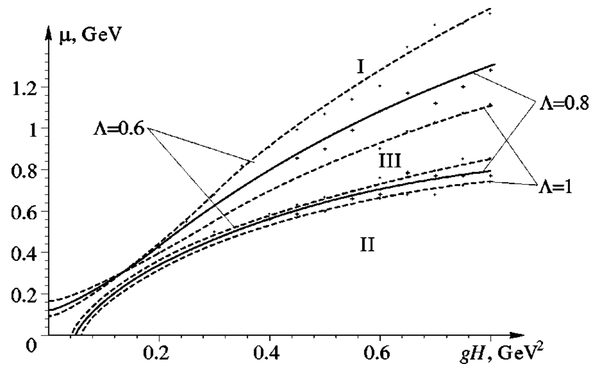

FIG. 2. Phase portrait of the model in terms of variables (μ,gH) at $T=0.15$ GeV for three cutoff values $\Lambda=0.6,0.8,1$ GeV. The included phase I is the symmetrical one $(\sigma=0,\Delta=0)$.

First of all, it should be remarked that, as in paper [28] at $gH=0$, a mixed phase of the model was not found for $H\neq0$, i.e., for a wide range of parameters μ,H,T we did not find a global minimum point of the potential (22), at which $\sigma\neq0$, $\Delta\neq0$. The results of our numerical investigations of the GMP of $V_r(\sigma,\Delta)$ are graphically represented in the set of Figs. 1–6, where the notations I, II and III are used for the symmetric phase, for the phase with chiral symmetry breaking and for the CSC phase, respectively. For the points from region I the GMP of the potential lies at $\sigma=0$, $\Delta=0$. In region II we have a phase with broken chiral symmetry, corresponding to the GMP of the potential at $\sigma\neq0$, $\Delta=0$. Finally, the color superconducting phase with the GMP of the potential at $\sigma=0$, $\Delta\neq0$, corresponds to the points from region III of these figures.

In Fig. 1, one can see the phase portrait of the model in terms of μ,gH at $T=0$ for each of the above mentioned values of the cutoff Λ. The boundary between phases III and II is practically Λ-independent and represents a first order phase transition curve $\mu_{cr}(gH)$. It is necessary to note also that for each value of Λ and a fixed value of gH there is a critical chemical potential $\tilde{\mu}_c(H)$, at which the GMP is transformed from a point of type III to a symmetric point of type

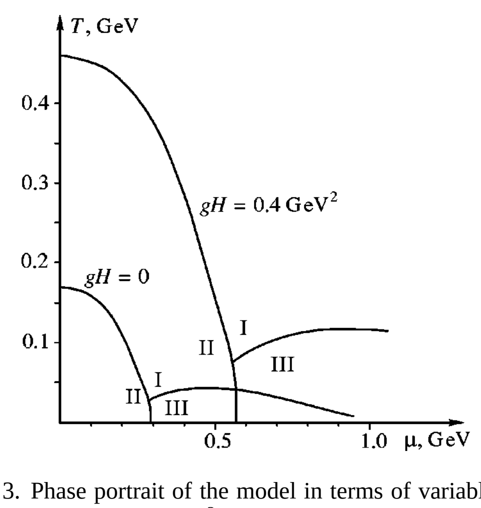

FIG. 3. Phase portrait of the model in terms of variables (T,μ) at $gH=0$ and $gH=0.4$ GeV^2 for $\Lambda=0.8$ GeV.

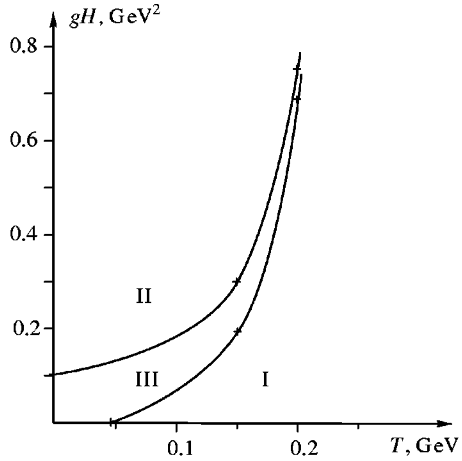

FIG. 4. Phase portrait of the model in terms of variables (gH,T) at $\mu=0.4$ GeV and $\Lambda=0.8$ GeV .

I. However, this phase transition is remarkably Λ-dependent. Indeed, even in the simplest case with $H=0$ we have $\tilde{\mu}_c(0)=1$ GeV at $\Lambda=0.6$ GeV, $\tilde{\mu}_c(0)=1.3$ GeV at $\Lambda=0.8$ GeV, $\tilde{\mu}_c(0)=1.65$ GeV at $\Lambda=1$ GeV. It is well known from the one-gluon exchange approximation in QCD [24,25] that CSC can exist even at enormously high values of the chemical potential $\mu \gtrsim 10^8$ MeV. So the above mentioned NJL framed transition from phase III to phase I looks, in the case $T=0$, rather like an artifact of the regularization procedure.[8] Thus, since this phase transition turns out to be unphysical, it is not shown in Fig. 1. The critical curves of this figure are obtained by interpolation in the most simple manner, i.e., by a second order polynomial, of numerical points lying at $gH>0.2$ GeV^2 (for technical reasons, such points are explicitly shown in Fig. 2, rather than in Fig. 1). Earlier, in the papers [14] the model (1) at $G_2=0$ and in the presence of an external magnetic field was considered. As shown there, for small values of the magnetic field strength the critical curves, as well as various thermodynamical and dynamical parameters of the system, demonstrate oscillating behavior. In order to make more accurate interpolations and to ascertain whether or not analogous oscillations appear in the present case (i.e., for the critical curves in Figs. 1, 2) as well, one should make an enormous amount of numerical calculations, which proved to be rather difficult to accomplish. Due to this, we can make only a conjecture of an oscillating behavior of the critical curves judging from the positions of the points we have really calculated. Note further that in the region of low chromomagnetic fields, $gH<0.2$ GeV^2, we have extrapolated the critical curves to the known points at $gH=0$.

In Fig. 2, the phase portrait of the model in terms of μ,gH at $T=0.15$ GeV is presented for each value of parameter $\Lambda=0.6$ GeV, 0.8 GeV, and 1 GeV. One can see that the boundary between II and III phases (a critical curve of a first order phase transition) only slowly changes with varying Λ. The second order phase transition from the CSC phase III to the symmetric phase I, which for finite T is now supposed to really exist, also has a weak Λ dependence for $gH<0.2$ GeV^2. At greater values of gH the boundary between III and I phases has, however, a stronger Λ dependence. The phase diagrams in the (T,μ) plane for $gH=0$ and $gH=0.4$ GeV^2 are schematically represented in Fig. 3. The phase diagram in the (T,gH) plane for $\mu=0.4$ GeV is represented in Fig. 4. For both figures we choose $\Lambda=0.8$ GeV, for simplicity. It is necessary to point out that at $gH=0$ the numerical results of Figs. 1–4 coincide with those obtained in [28] at $\Lambda=0.8$ GeV. Moreover, it should be emphasized that in all the above mentioned figures, a second order phase transition takes place at the boundary of the region I. At the boundary between regions II and III a first order phase transition takes place.

Let us for a moment fix the value of the chemical potential and temperature at varying values of gH. In this case, in

[8]Note that in [38] it was also claimed that in the NJL model at high enough μ the diquark condensate vanishes, which is the consequence of the regularization by a form factor. In this region, it might be necessary to use another approximation for the CSC investigation.

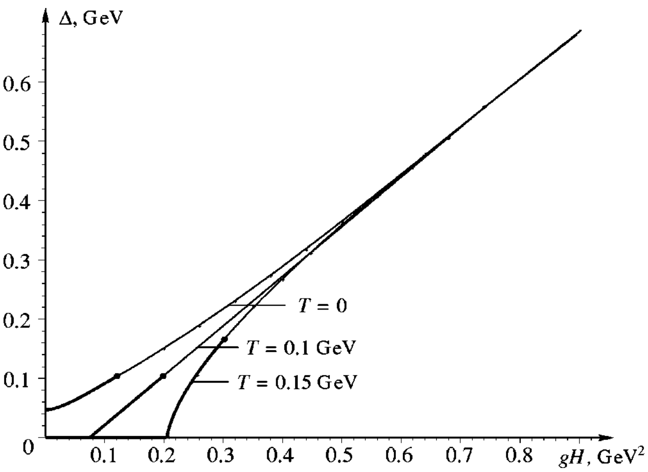

FIG. 5. Diquark condensate as a function of an external chromomagnetic field for $\mu=0.4$ GeV, $\Lambda=0.8$ GeV and three values of the temperature, $T=0$, $T=0.1$ and $T=0.15$ GeV.

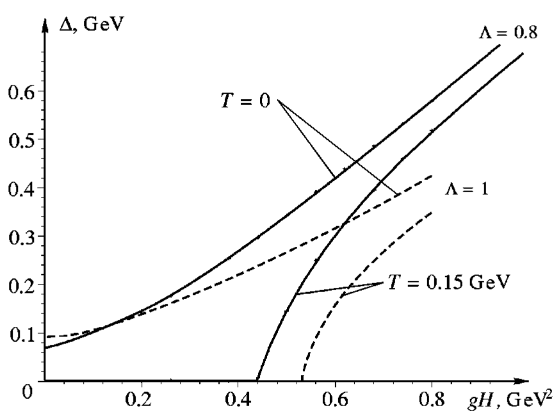

FIG. 6. Diquark condensate as a function of an external chromomagnetic field for $\mu=0.8$ GeV and two values of the temperature, $T=0$, and $T=0.15$ GeV and for two values of the cutoff parameter $\Lambda=0.8,1$ GeV.

Figs. 1 and 2, one will have a straight line parallel to the gH axis. In particular, if $\mu=0.4$ GeV and $T=0$, in Fig. 1 this line originates at $gH=0$ in the CSC phase III. At some (critical) value $(gH)_c\approx 0.1$ GeV^2 it crosses the line $\mu_{cr}(gH)$ and then, at yet greater values of gH, it passes through the phase II. Accordingly, at $gH<(gH)_c$ the GMP of the effective potential lies at the point $(\sigma=0,\Delta\neq 0)$, where Δ is equal to the diquark condensate in the true stable vacuum, whereas at $gH>(gH)_c$ the point $(0,\Delta\neq 0)$ ceases to be a GMP. In this case it is only a local minimum point, so that $\Delta\neq 0$ corresponds to a metastable ground state of the system [for the stable ground state at $gH>(gH)_c$ the GMP is of the form $\sigma\neq 0,\Delta=0$]. Thus, the value $(gH)_c$ is the so-called evaporation point for the diquark condensate.

In Fig. 5, the diquark condensate Δ is depicted as a function of gH for three values of the temperature at $\mu=0.4$ GeV and $\Lambda=0.8$ GeV (in this figure, due to problems with distinguishing closely positioned points for different cutoff values, we restricted ourselves to plotting curves for only one cutoff value $\Lambda=0.8$ GeV.). Thick curves correspond to a stable diquark condensate [the point $(0,\Delta\neq 0)$ is the global minimum of the effective potential], and thin curves correspond to a quasistable diquark condensate (this is a local minimum).

Here we should note that recent investigations yield the following value of the QCD gluon condensate at $T=\mu=0$: $gH\approx 0.6$ GeV^2 [39]. It was shown in [40] that in the framework of a quark-meson model at ordinary nuclear density ρ_0 the gluon condensate decreases by no more than six percent, compared with its value at zero density. At densities $3\rho_0$ the value of $\langle FF\rangle$ decreases by fifteen percent. This means that for values of the chemical potential $\mu<1$ GeV the gluon condensate is a slowly decreasing function vs μ. Taking in mind this fact, one can draw an important conclusion from our numerical analysis: At $H=0$, $T=0$ and $\mu=0.4$ GeV there should exist the CSC phase (see [28]). However, if the condensate value $gH\approx 0.5$ GeV^2 is taken into account at $\mu=0.4$ GeV, then our model consideration concludes that the CSC phase does not exist at $T=0$ for such a large value of the gluon condensate (cf. Figs. 4,5).[9] Assuming that our results would remain valid also for more realistic condensate fields, this would seemingly render it difficult to observe a CSC phase for $T=0$ at $\mu=0.4$ GeV.

Notice, however, that our results change at finite temperature. First, if one takes $T=0.1$ GeV and $\mu=0.4$ GeV, then at sufficiently small values of gH there is a symmetric phase of the theory, where both $\langle\bar{q}q\rangle$ and $\langle qq\rangle$ condensates are zero (see Fig. 5). However, at the point $gH\approx 0.1$ GeV^2 there is a second order phase transition to the CSC phase (here only $\langle qq\rangle\neq 0$), and at the point $gH\approx 0.2$ GeV^2 the external field destroys the CSC in favor of the chiral phase, where $\langle\bar{q}q\rangle\neq 0$ and $\langle qq\rangle=0$. A similar behavior is observed at $T=0.15$ GeV and $\mu=0.4$ GeV (see Fig. 5). Secondly, if one takes $T=0.15$ GeV and $\mu\geqslant 0.6$ GeV, then at sufficiently small values of gH there is a symmetric phase of the theory, where both $\langle\bar{q}q\rangle$ and $\langle qq\rangle$ condensates are zero (see Fig. 2). However, at the point $gH\geqslant 0.3$ GeV^2 there is a second order phase transition to the CSC phase, where only $\langle qq\rangle\neq 0$. Hence, in some cases the gluon condensate can induce the CSC (the so-called chromomagnetic catalysis effect of CSC).

This effect is further illustrated in Fig. 6, where the diquark condensate vs gH is depicted at $\mu=0.8$ GeV for two temperatures $T=0$, $T=0.15$ GeV and two values of $\Lambda=0.8$ GeV and $\Lambda=1$ GeV. One can easily see that the value of $\Delta(gH)$ for $T=0.15$ GeV is identically zero for $gH\leqslant 0.44$ GeV^2 with $\Lambda=0.8$ GeV, and for $gH\leqslant 0.52$ GeV^2 with $\Lambda=1$ GeV, i.e., for each value of Λ, there exists a critical value of gH at which the nonzero diquark condensate is generated.

Finally, let us make some additional remarks concerning the diquark condensate at $\mu=0.8$ GeV. As it follows from our numerical analysis at $T=0$ (see Figs. 1,6), for $\mu=0.8$ GeV, $gH=0$, the GMP of the effective potential corresponds to the CSC phase with a stable diquark condensate $\Delta\leqslant 0.1$ GeV. However, assuming that the value of the gluon condensate $gH\approx 0.4$ GeV^2 would hold for the above nonvanishing chemical potential, one would get a value of the diquark condensate $\Delta\gtrsim 0.2$ GeV, which is significantly larger in magnitude, than at $gH=0$. It follows from Fig. 6 that the diquark condensate noticeably depends on Λ. One could suppose that this is due to the rather high value of the considered chemical potential, $\mu=0.8$ GeV. However, as our calculations show, a similar Λ dependence of the diquark condensate occurs for smaller values of μ as well. These results confirm the conclusions of the paper [38] obtained at $H=0$ that the value of the diquark condensate varies with Λ, if the form factor regularization is used in the NJL model. In contrast, the points of the boundary between II and III phases do not show a remarkable Λ dependence (see Figs. 1,2).

As a general conclusion, we see that taking into account an external chromomagnetic field at least in the form as considered in the model above, might, in principle, lead to remarkable qualitative and quantitative changes in the picture of the diquark condensate formation, obtained in the framework of NJL models at $H=0$. This concerns, in particular, the possibility of a transition to the CSC phase and diquark condensation at finite temperature. Clearly, a detailed quantitative discussion would, however, require having additional information on the gluon condensate as a function of the temperature and chemical potential and on a possible μ and gH dependence of the cutoff parameter Λ.[10] A further interesting generalization could be to extend this kind of approach to inhomogeneous background field configurations [42]. This concerns, in particular, the non-Abelian condensate fields of the stochastic vacuum model of QCD realizing Wilson's area law of confinement [43].

[9]Recently, a similar prediction, namely that nonperturbative gluon fluctuations might be strong enough to destroy the CSC, was done in [41], but in a rather qualitative form.

[10]Such a dependence might, for example, arise, if one identifies the cutoff in the local 4-fermion model with the effective gluon mass in the gluon propagator.

V. SUMMARY AND CONCLUSIONS

In the present paper the influence of different physical factors on the phase structure of the two-flavor NJL model (1) with two independent structures of four-quark interactions has been considered. This model is adequate for the description of the low energy physics of two-flavor QCD both in $q\bar{q}$ and qq channels. In the papers [27–29] it was shown that in QCD-motivated type of models (1) with $H=0$ the new color superconducting (CSC) phase can exist for moderate values of the chemical potential (baryon density). As generalization of the "free field" NJL model of Ref. [29], we have, in particular, taken into account such nonperturbative features of the real QCD vacuum as the nonzero gluon chromomagnetic condensate $\langle F^a_{\mu\nu}F^{a\mu\nu}\rangle \equiv 2H^2$, which in the framework of a NJL model can be simulated by an external chromomagnetic field. The recent estimates give the following fixed value $gH_{phys} \approx 0.6$ GeV^2 [39] for the gluon condensate in QCD at $\mu, T=0$. Despite this fact, we considered it useful to treat H as a free external parameter of the model.

Since in the CSC phase the original $SU_c(3)$ symmetry of the theory is spontaneously broken down to $SU_c(2)$, five color gauge bosons acquire masses. The corresponding external fields are expelled from the CSC phase (Meissner effect). However, the other three "color isospin" gauge bosons remain massless, in accordance with the residual $SU_c(2)$ symmetry of the vacuum. Clearly, the corresponding external fields may then penetrate into the CSC phase. It is just the influence of these types of external chromomagnetic fields on the formation of CSC which was studied in our previous paper [31] in the framework of a $(2+1)$-dimensional NJL model for vanishing chemical potential and temperature. There it was shown that chromomagnetic fields may induce the CSC phase transition. In the present paper, the chromomagnetic generation of CSC has been studied in the framework of a $(3+1)$-dimensional NJL model for finite chemical potential and temperature. The vector-potential of the external chromomagnetic field was chosen to be of the Matinyan-Savvidy form (7) and lies in the algebra of the residual $SU_c(2)$ group, too.

The coupling constants G_1, G_2 of our model (1) are considered as free independent parameters. In our numerical estimates, we found it convenient to use the relation (24) in order to "normalize" our calculations at $H\neq 0$ on the known results at $H=0$ [28]. However, we hope that our qualitative conclusions remain also valid for values of G_1, G_2 in some neighborhood of Eq. (24). The results of numerical investigations of the effective potential (22) are presented in a set of Figs. 1–6, where phase diagrams for the extended NJL model (1) in terms of μ, T, H as well as the behavior of diquark condensates versus gH are shown.

First of all we should note that the form factor regularization of the NJL model (1) is used throughout in the present paper. As shown in [38], in this case even at $H=0$ the features of CSC depend on the value of the form factor parameter Λ. So, in order to clarify the corresponding situation at $H\neq 0$, we have used three different values of $\Lambda=0.6$ GeV, 0.8 GeV, and 1 GeV in the regularization scheme under consideration. One can see that phase transitions between the chirally broken and CSC phases have rather weak Λ dependence (see Figs. 1, 2), whereas the diquark condensate values noticeably depend on Λ (see, e.g., Fig. 6).

The main conclusion of our investigations is that the inclusion of an external chromomagnetic field can significantly change the phase portrait, obtained at $H=0$. Indeed, at $H=T=0$ the values of the chemical potential corresponding to the CSC phase approximately lie in the interval $\mu > 0.3$ GeV (see Fig. 1). If the external chromomagnetic field of the type (7) is switched on at $\mu=0.4$ GeV, then at $gH_c \approx 0.1$ GeV^2 there is a transition of the system from CSC to a phase, where only chiral symmetry is broken down. Thus, at $gH_{phys}\approx 0.6$ GeV^2, $\mu=0.4$ GeV and $T=0$ the CSC cannot be observed at all. However, if $T=0$ and the chemical potential is fixed at $\mu=0.8$ GeV, then at least for all values $0\leqslant gH \leqslant 0.6$ GeV^2 one can observe the CSC phase in which the diquark condensate $\Delta(H)$ is nonzero. It is worth mentioning that in this case the function $\Delta(H)$ is monotonically increasing (see Fig. 6) and the value of the diquark condensate at gH_{phys} turns out to be significantly greater in magnitude than at vanishing H.[11]

Finally, one should note that at $\mu\neq 0, T\neq 0$ the external chromomagnetic field can induce the CSC phase transition. For example, at $T=0.15$ GeV and $\mu=0.8$ GeV there is a symmetric phase of the theory in which $\sigma=\Delta=0$ (both chiral and diquark condensates are zero) for sufficiently small values of gH (see Fig. 2). However, at some critical point gH_c, a phase transition of the second order from the symmetric to the CSC phase is induced by the external chromomagnetic field. Notice that the CSC induction by some types of external chromomagnetic fields was observed in the framework of a NJL model at zero μ, T (see [31–33]). The present analysis shows that this effect takes place at some nonzero values of μ, T, too, which in principle could be important for heavy ion-ion collisions taking place at nonzero temperature. Clearly, a somewhat unpleasant feature of the above NJL approach, which requires some caution, is the Λ dependence of some of the results. Nevertheless, we believe that the above results are interesting and may serve as a starting point for further investigations of this issue.

ACKNOWLEDGMENTS

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[11]We roughly suppose throughout the present paper that at $(\mu,T)\neq 0$ the real gluon condensate is the same as at $(\mu,T)=0$. However, using a given μ,T dependency of the gluon condensate, it would be possible to extract physical information about the CSC phase using our phase diagrams in Figs. 1–4 and plots of $\Delta(H)$ functions in Figs. 5,6.

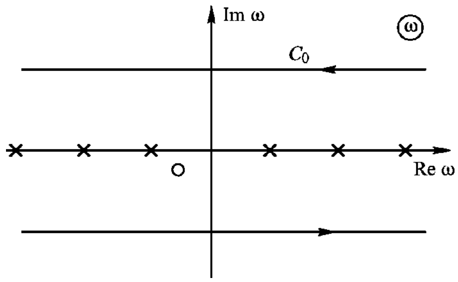

FIG. 7. Integration path C_0 used in Eq. (A1) of the Appendix.

APPENDIX

Let us sketch the calculation of the integral in Eq. (18) of the text. Evidently, it can be rewritten as a contour integral

$$\sum_l \int_{1/\beta^2}^{\Omega^2} da^2 \frac{1}{\omega_l^2+a^2} = -\int_{1/\beta^2}^{\Omega^2} da^2 \int_{C_0} \frac{d\omega}{2\pi i} \frac{1}{\omega^2+a^2} \times \frac{\beta}{2} \mathrm{tg} \frac{\beta\omega}{2}, \tag{A1}$$

where $\mathrm{tg}(\beta\omega/2)$ has poles inside the integration path C_0 (see Fig. 7):

$$\frac{\beta\omega}{2} = \pm\frac{\pi}{2}(2l+1), \quad l=0,1,2,\dots. \tag{A2}$$

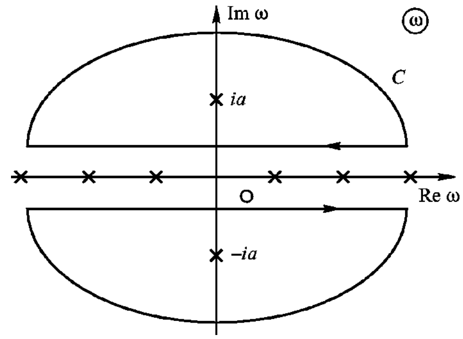

FIG. 8. Integration path C used in Eq. (A3) of the Appendix.

The integral over C_0 is equal to the integral along the contour C (Fig. 8), and hence

$$\begin{aligned} -\int_{C_0} \frac{d\omega}{2\pi i} \int_{1/\beta^2}^{\Omega^2} \frac{da^2}{a^2+\omega^2} \frac{\beta}{2} \mathrm{tg} \frac{\beta\omega}{2} &= -\int_{1/\beta^2}^{\Omega^2} da^2 \int_C \frac{d\omega}{2\pi i} \frac{\beta}{2} \left(\frac{1}{\omega-ia} - \frac{1}{\omega+ia} \right) \times \frac{1}{2ia} \mathrm{tg} \frac{\beta\omega}{2} \\ &= \beta \int_{1/\beta}^{\Omega} da \, \mathrm{th} \frac{\beta a}{2} = 2 \ln \mathrm{ch} \frac{\beta a}{2} \Bigg|_{a=1/\beta}^{a=\Omega} = 2 \ln \mathrm{ch} \frac{\Omega\beta}{2} + \mathrm{const}. \end{aligned} \tag{A3}$$

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