Effects of lepton number violating interactions on $t\bar{t}$ production at the Next Linear Collider

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We discuss the effects of lepton number violating interactions, namely, *R*-parity violation and leptoquarks, on top-quark pair production at the upcoming e^+e^- linear colliders. The effects of SU(2) singlet, doublet, and triplet leptoquark interactions are investigated. The *R*-parity violating minimal supersymmetric standard model also allows certain kinds of lepton number violating interactions which are the same as singlet leptoquarks with left-handed interactions. We have calculated the cross ssection of $e^+e^- \to t\bar{t}$ in the presence of the above interactions. With conservative values of lepton number violating coupling strengths we get an enhancement of the top-quark pair production cross section in all of the above cases.

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I. INTRODUCTION

It is well known that lepton number conservation in the standard model (SM) is an accidental symmetry. It is a mere outcome of particle content and the gauge structure of the SM. In many extensions of the SM, lepton number violating interactions occur in a natural way. The minimal supersymmetric standard model (MSSM) without *R* parity [1] and nonsupersymmetric theories with leptoquarks [2] are well cited examples of it. The key feature of these theories, relevant for the following analysis, is the presence, in their spectrum, of a scalar (leptoquark) which couples to a quark and a lepton at the same time. Leptoquarks arise in many models of extended gauge symmetry including grand unified theories. In many of these models, vector leptoquarks can also arise. The gauged vector leptoquarks are superheavy. Their mass is related to the scale of spontaneous breaking of the lepton number. On the other hand, interactions involving the nongauged vector leptoquarks are nonrenormalizable. Several interesting phenomenological analyses have been carried out considering both of these interactions. In this article, we will focus on how these scalar leptoquark [a class that also includes the squarks in *R*-parity violating supersymmetry (SUSY)] interactions can modify the top-quark pair production cross section significantly at the next generation e^+e^- colliders. The choice of this particular process has several advantages. The foremost is the copious production of top-quark pairs at these machines. Also, the cleaner environment of leptonic colliders helps one to make precision studies such as measuring more accurately deviations from the SM expectation, if they exist. One of the major goals of these e^+e^- machines is to measure the top-quark interactions to a high level of precision [3]. The measurement of the lepton number violating couplings involving light quarks (mainly of the first generation) and leptons can also be done at hadron colliders by studying processes such as Drell-Yan pair production of leptons. But at a hadronic machine, the couplings in which we will be interested in our analysis can be probed only in the decays of the heavy quarks. Although the production cross section of a heavy quark pair is huge at a hadron collider, the presence of competing QCD backgrounds may interfere in such precise measurements.

Baryon (*B*) and lepton (*L*) number violating processes involving the top-quark have been investigated by several authors. For example, the effects of *B* and *L* violation in top-quark production at hadronic colliders have been analyzed in Ref. [4] in the context of *R*-parity violating SUSY. People have extensively studied the single top-quark production [5] and decay of the top-quark [6] mediated by *R*-parity violating interactions. The effects of *R*-parity violation on the top mass have been discussed in [7]. So much attention has already been given to the top-quark phenomenology [8] in the context of *R*-parity violation. Although leptoquark interactions have similarities with those of *R*-parity violating SUSY, in some cases the chiral structure of the relevant couplings differs. Much attention has also been paid to leptoquark phenomenology. Apart from direct leptoquark searches at future lepton and hadron colliders [9], the effects of these interactions have been studied in the context of neutrinoless double beta decay [10], the muon anomalous magnetic moment [11], and, needless to mention, to explain the DESY *ep* collider HERA anomaly [12]. The indirect effects of leptoquark interactions have also been investigated in the context of e^+e^-, $e\gamma$, and hadronic colliders [13]. In this paper we will try to discuss, in some detail, how these lepton number violating couplings can affect the pair production and decay of the heaviest quark. This has been studied previously in [14] in a slightly different manner. Using a polarized electron beam in e^+e^- collision, constraints are derived, in the above reference, on the leptoquark mass and couplings by comparing (and then doing a χ^2 analysis) the angular distribution of leptoquark mediated processes with that of the pure SM. It was shown in this reference that a 1 TeV e^+e^- collider will be more efficient than a 500 GeV machine in exploring/excluding the parameter space of leptoquark interactions. We will focus on this point more later. People have also considered the effects of vector leptoquarks on $t\bar{t}$ production from e^+e^- collision [15]. The authors of Ref. [15] also used polarized e^- beams to differentiate the vector leptoquark interactions from the SM. They presented the variation of the total number of $t\bar{t}$ events with vector leptoquark mass as-

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TABLE I. Different kinds of leptoquark interaction relevant for our analysis. *R*-parity violating MSSM interaction in Eq. (2) corresponds to the left-handed (proportional to $\lambda_{13}^{(1)}$) interaction of Φ_1.

Leptoquark type	Coupling	$SU(3)_c \times SU(2)_L \times U(1)_Y$
Φ_1	$[\lambda_{ij}^{(1)}\overline{Q_{Lj}^c}L_{Li}+\tilde{\lambda}_{ij}^{(1)}\overline{u_{Rj}^c}e_{Ri}]\Phi_1$	$(\bar{3},1,2/3)$
Φ_2	$[\lambda_{ij}^{(2)}\overline{Q}_{Lj}e_{Ri}+\tilde{\lambda}_{ij}^{(2)}\overline{u}_{Rj}L_{Li}]\Phi_2$	$(3,2,7/3)$
Φ_3	$\lambda_{ij}^{(3)}\overline{Q_{Lj}^c}L_{Li}\Phi_3$	$(\bar{3},3,2/3)$

suming the leptoquark couplings to e and t of order unity. Although the structures of the vector leptoquark interactions are different from those of the scalar leptoquarks, qualitatively the variation of the production cross section with leptoquark mass agrees with our results. In this article, we will concentrate on how the total cross section would change in the presence of such particles and how angular asymmetry in $t\bar{t}$ production and decay can be used to among discriminate the different types of leptoquark interaction. The plan of the rest of the article is as follows. In the next section, we will discuss the models briefly with special emphasis on the relevant couplings and the similarities and differences in two the models of our interest. The third section contains the result of our analysis followed by a conclusion in the last section.

II. RELEVANT INTERACTIONS

In this section we will discuss briefly the phenomenology of lepton number violating interactions in the context of $t\bar{t}$ production in e^+e^- collision. As we emphasized earlier, two main kinds of model that allow these interactions are the MSSM with *R*-parity violation and non-SUSY theories with leptoquarks. As has been noted in the literature, unless a discrete symmetry[1] is introduced by hand, the MSSM superpotential contains the following terms [16]:

$$W_{\not R}=\lambda_{ijk}\hat{L}_i\hat{L}_j\hat{E}_k^c+\lambda'_{ijk}\hat{L}_i\hat{Q}_j\hat{D}_k^c+\lambda''_{ijk}\hat{U}_i^c\hat{D}_j^c\hat{D}_k^c+\epsilon_i\hat{L}_i\hat{H}_2. \tag{1}$$

However, such a symmetry is *ad hoc*. So it is of interest to consider possible violation of this symmetry especially when it has some interesting experimental consequences in detecting the supersymmetric particles [17]. One can easily see that the first two and the last terms in the superpotential violate the lepton number/flavor explicitly while the third term breaks the baryon number. As we are interested in the top-quark pair production in electron positron annihilation, we will be interested in the second term. One can expand this piece in terms of the normal fields. This, in turn, yields (with many others) the relevant interactions involving a lepton and a quark along with a squark. One can easily write the interaction of our interest:

$$\mathcal{L}_{\lambda'}=-\lambda'_{13k}(\tilde{d}_R^k)^*(\overline{e_L})^c t_L+\text{H.c.} \tag{2}$$

Now we will turn our attention to the leptoquark interactions. The interactions necessary for our purpose are listed in a tabular form in Table I [18]. Here we have suppressed the SU(2) indices. One can very easily write the interactions relevant for our purpose involving e, t, and a particular leptoquark from Table I. Below we write the interaction Lagrangians separately for singlet, doublet, and triplet leptoquarks:[2]

$$\mathcal{L}_1=-[\lambda_{13}^{(1)}(\overline{e})^c P_L t+\tilde{\lambda}_{13}^{(1)}(\overline{e})^c P_R t]\phi_1+\text{H.c.},$$

$$\mathcal{L}_2=[\lambda_{13}^{(2)}\overline{t}P_L e-\tilde{\lambda}_{13}^{(2)}\overline{t}P_R e]\phi_2+\text{H.c.},$$

$$\mathcal{L}_3=\lambda_{13}^{(3)}(\overline{e})^c P_L t\phi_3+\text{H.c.} \tag{3}$$

There are some similarities and differences between the above interactions and that in Eq. (2). The triplet and the left-handed singlet (proportional to $\lambda_{13}^{(1)}$) have similar structures to the *R*-parity violating interaction. The charges of the leptoquarks in such cases are also the same as that of the squark involved in Eq. (2). At the same time ϕ_1 has a coupling with e and t which is right-handed in nature. This type of interaction is not allowed in SUSY. The SU(2) doublet leptoquark ϕ_2 has a similar kind of interaction to ϕ_1. The only difference is its electromagnetic charge, which is equal to $\frac{5}{3}$.

The operators, that will contribute to the top-quark pair production via e^+e^- annihilation follow very easily from the Lagrangian. They are given in Table II.

Apart from the SM *s*-channel diagram (mediated by γ or Z), one has to calculate an extra diagram mediated by the squark or leptoquarks (see Fig. 1) due to these lepton number violating interactions. Looking at the Lagrangians, one can easily check that in *R*-parity violating contributions, one vertex is proportional to P_L and the other is proportional to P_R, while in the leptoquark mediated contributions P_L or P_R can arise in both the vertices.

[1]This symmetry is called R symmetry. R is defined as $(-1)^{3(B-L)-2S}$. All the SM fields have $R=1$ and all the SUSY partners have $R=-1$. Apart from ruling out both B and L violating interactions, this symmetry has the additional consequence of rendering the lightest superparticle absolutely stable.

[2]In Eq. (3) and Table II below, one should not confuse the λ couplings with that in Eq. (2). The λ couplings here have more similarities with the λ' coupling in Eq. (1).

TABLE II. Different types of operator contributing to the process $e^+e^-\rightarrow t\overline{t}$, made from interactions in Eqs. (2), (3). *R*-parity violating MSSM corresponds to case 1. For the first two cases *i* can be 1 or 3.

1	Squark, singlet/triplet leptoquark (left-handed)	$\lvert\lambda_{13}^{(i)}\rvert^2(\overline{t}P_R e^c\,\overline{e^c}P_L t)\widehat{\phi_i\phi_i^*}$ (RL)
2	Singlet leptoquark (right-handed)	$\lvert\tilde{\lambda}_{13}^{(i)}\rvert^2(\overline{t}P_L e^c\,\overline{e^c}P_R t)\widehat{\phi_i\phi_i^*}$ (LR)
3	Singlet leptoquark (right-left)	$\lvert\lambda_{13}^{(1)}\tilde{\lambda}_{13}^{(1)}\rvert(\overline{e^c}P_\alpha t\,\overline{t}P_\alpha e^c)\widehat{\phi_1\phi_1^*}$ $\alpha=L,R$ (LL,RR)
4	Doublet leptoquark (left)	$\lvert\lambda_{13}^{(2)}\rvert^2(\overline{t}P_L e\,\overline{e}P_R t)\widehat{\phi_2\phi_2^*}$ (LR)
5	Doublet leptoquark (right)	$\lvert\tilde{\lambda}_{13}^{(2)}\rvert^2(\overline{t}P_R e\,\overline{e}P_L t)\widehat{\phi_2\phi_2^*}$ (RL)
6	Doublet leptoquark (right-left)	$\lvert\lambda_{13}^{(2)}\tilde{\lambda}_{13}^{(2)}\rvert(\overline{t}P_\alpha e\,\overline{e}P_\alpha t)\widehat{\phi_2\phi_2^*}$ $\alpha=L,R$ (LL,RR)

For the sake of completeness, we write down the expressions for the amplitudes, arising due to the different types of interaction listed in Table II, along with the SM:

$$\mathcal{M}_{SM}=-\frac{1}{s-m_V^2+im_V\Gamma_V}[\overline{v}(p_1)\gamma_\mu(a_e+b_e\gamma_5)u(p_2)]$$
$$\times[\overline{u}(p_3)\gamma^\mu(a_t+b_t\gamma_5)v(p_4)],$$
$$\mathcal{M}_{LQ}^{S/T}=\frac{|\lambda|^2,|\tilde{\lambda}|^2,\lambda\tilde{\lambda}}{t-m_\phi^2}[\overline{u}(p_3)P_iu(p_1)][\overline{v}(p_2)P_jv(p_4)],$$
$$\mathcal{M}_{LQ}^{D}=\frac{|\lambda|^2,|\tilde{\lambda}|^2,\lambda\tilde{\lambda}}{t-m_\phi^2}[\overline{u}(p_3)P_iu(p_2)][\overline{v}(p_1)P_jv(p_4)]. \tag{4}$$

The first ($\mathcal{M}_{SM}$) of the above equations stands for the two SM *s*-channel diagrams. For the photon-exchange diagram, $b_e=b_t=0$, $a_e=-e$, $a_t=\frac{2}{3}e$, and $m_V=\Gamma_V=0$. For the *Z*-exchange diagram, $a_e=(g/\cos\theta_W)(-\frac{1}{4}+\sin^2\theta_W)$, $b_e=g/4\cos\theta_W$, $a_t=(g/\cos\theta_W)(\frac{1}{4}-\frac{2}{3}\sin^2\theta_W)$, and $b_t=-g/4\cos\theta_W$. The next two expressions $\mathcal{M}_{LQ}^{S/T}$ and $\mathcal{M}_{LQ}^{D}$ are for singlet/triplet and doublet leptoquark mediated diagrams respectively. p_1, p_2, p_3, and p_4 are the momenta of e^+, e^-, t, and $\overline{t}$. The Mandelstum variables are defined as $s=(p_1+p_2)^2$ and $t=(p_1-p_3)^2$ for singlet/triplet and $t=(p_2-p_3)^2$ for doublet leptoquarks. The amplitudes for leptoquark mediated diagrams are proportional to $|\lambda|^2$ when $P_i=P_R$ and $P_j=P_L$, to $|\tilde{\lambda}|^2$ when $P_i=P_L$ and $P_j=P_R$, and to $\tilde{\lambda}\lambda$ when both are P_L or P_R. Following Tables I and II, the triplet leptoquark contribution can only be proportional to $|\lambda|^2$. The other cases do not arise for triplet leptoquark mediation.

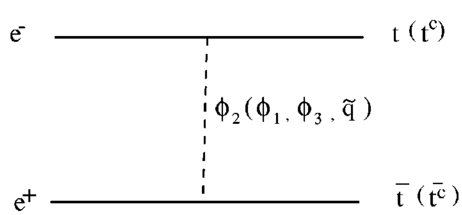

FIG. 1. Feynman diagram for the process $e^+e^-\rightarrow t\overline{t}$ in R_p violating SUSY or leptoquarks.

Now let us discuss the experimental bounds on the relevant couplings. The *R*-parity violating contribution is proportional to the coupling λ'_{13k}, where *k* is the generation index. We will consider only one *R*-parity violating coupling to be nonzero at a time. Looking at the literature [19], one can check easily that the coupling λ'_{132} is the most loosely constrained [16].[3] So we will use this particular coupling in the following analysis. This implies that the exchanged squark in Fig. 1 is the supersymmetric partner of the *s* quark. The same constraints would also exactly apply on the left-handed singlet ($\lambda_{13}^{(1)}$) and triplet ($\lambda_3^{(13)}$) leptoquark couplings to *e* and *t*. The product of the couplings $\lambda_{13}^{(i)}\tilde{\lambda}_{13}^{(i)}$ in these two cases is unconstrained. The SU(2) doublet leptoquark couplings $\lambda_{13}^{(2)}$ and $\tilde{\lambda}_{13}^{(2)}$ (left and right) are individually constrained from the e^+e^- partial decay width of the *Z* boson [20]. It is interesting to observe that the left-handed couplings are more stringently constrained than their right-handed counterparts. The numerical values of the upper bounds on the left-handed couplings of the SU(2) doublet leptoquarks are comparable with the upper bounds obtained for *R*-parity violating coupling strengths. There is no upper bound on the product of the left- and right-handed leptoquark couplings. So we may take their values as free parameters, keeping in mind that the value should be perturbatively viable.

III. DISCUSSION OF THE RESULTS

We will discuss in this section the numerical results from our analysis. We have only estimated the Born level diagrams corresponding to the operators in Table II. All the coupling constants scale with the scalar mass. In the case of *R*-parity violation, λ'_{132} scales linearly with $\tilde{t}_L$ mass. This particular coupling is constrained to be less than 0.28 for a 100 GeV $\tilde{t}_L$ mass [19]. As we discussed earlier, this bound applies equally to the left-handed singlet and triplet leptoquark couplings. We will also use the same values for $\tilde{\lambda}_{13}^{(1)}$ $(=0.3)$ and the product $\tilde{\lambda}_{13}^{(1)}\lambda_{13}^{(1)}(=0.09)$ as there are no phenomenological bounds available for those. Again, for numerical values of the couplings involving doublet leptoquarks we follow Ref. [20]. For a 100 GeV scalar, the upper bound for left- ($\lambda_{13}^{(2)}$) or right-type ($\tilde{\lambda}_{13}^{(2)}$) coupling is almost the same (and is nearly equal to 0.4). While the upper bound

[3]This particular coupling is constrained from the forward-backward asymmetry in e^+e^- collision.

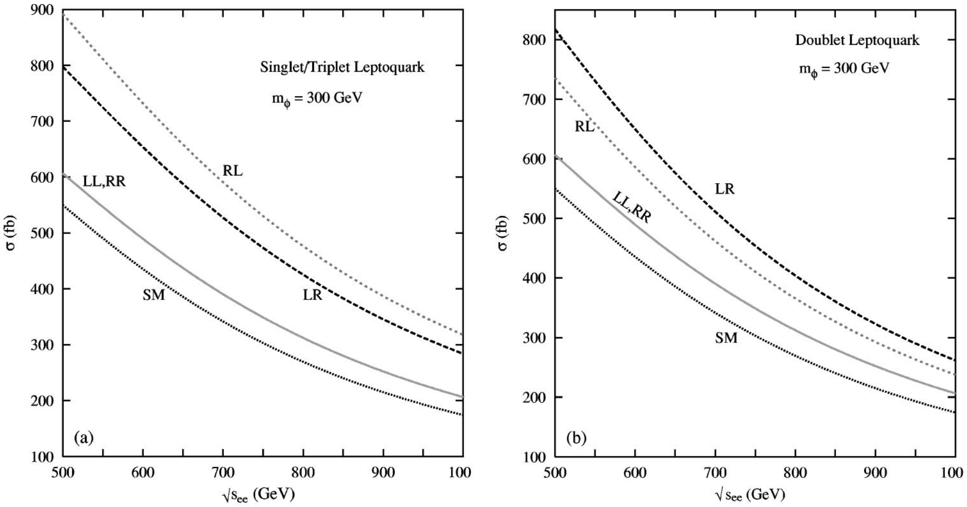

FIG. 2. Variation of top-pair production cross section in the presence of (a) singlet and triplet leptoquarks and *R*-parity violating interactions and (b) doublet leptoquarks with e^+e^- center-of-mass energy $\sqrt{s_{ee}}$. Leptoquark mass (m_ϕ) is fixed at 300 GeV. For comparison we have also plotted the pure SM contribution. The different lines are for different kinds of interactions. Legends follow from Table II. The curve marked RL in (a) corresponds to *R*-parity violating SUSY.

on the $\lambda_{13}^{(2)}$ coupling is not very sensitive to the leptoquark mass, the upper bound on the other one rises quite fast with increasing scalar mass. We will use the same values as before (as for the singlet and triplet leptoquarks) for these couplings, which makes our estimate conservative.

In Figs. 2 and 3 we present the numerical estimates of the cross sections. We do not consider any higher order corrections to the process of our interest. Higher order corrections are important [21]. In the case of the SM, inclusion of higher order effects increase the cross section significantly. The aim of this paper is to show the enhancement of the total cross section (of $t\bar{t}$ production) over its SM value, when one includes the lepton number violating interactions arising from leptoquarks or *R*-parity violation. We have calculated the cross section at center-of-mass energies away from the $t\bar{t}$ threshold. Around the center-of-mass energy of 350 GeV ($\sim 2m_t$), threshold effects are very important [22], and we wanted to avoid this extra complication. But this does not detract from the essence of our analysis.

In Fig. 2(a), we plotted the variation of the total cross section of top-quark pair production with center-of-mass energy for singlet and triplet leptoquarks. For the purpose of illustration, we present the cross section with one value of scalar mass (say 300 GeV, which is well above the bounds quoted by the CDF and D0 Collaborations [23] from the Tevatron search limits for squarks and leptoquarks) and setting the value of all the couplings ($\lambda_i, \tilde{\lambda}_i, i=1,2,3$) at, say, 0.3. There are several cases of interest, following Table II. The LL and RR types of interaction do not interfere with the SM contribution. It is also worth mentioning here that the *LL*, *RR*, and *LR* lines in Fig. 2(a) come from the singlet leptoquarks only. The others, namely, LR and RL, interfere constructively with the SM. For comparison, we plotted the pure SM contribution as well. It is clear from the figures that the presence of any one kind of lepton number violating interaction increases the $t\bar{t}$ cross section over its SM value. It is worth mentioning that the *R*-parity violating MSSM contribution corresponds to the RL case of Fig. 2(a). Incidentally, this case shows the maximum enhancement. The MSSM with or without *R*-parity conservation is one of the strongest contenders for physics beyond the SM which we expect to see at the next generation of colliding machines. So any enhancement of the top-quark cross section at e^+e^- linear colliders may be a positive signal of this kind of scenario. The LR case is also interesting to observe. Here also the enhancement is rather prominent. Finally, the LL or RR,

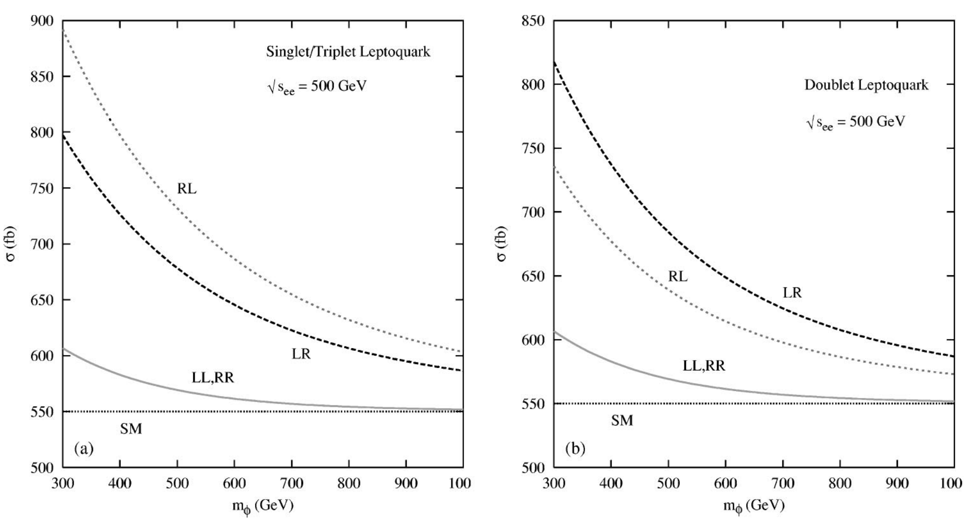

FIG. 3. Variation of top-quark pair production cross section in the presence of (a) singlet and triplet leptoquarks and *R*-parity violating interactions and (b) doublet leptoquarks with leptoquark mass m_ϕ. For comparison we have also plotted the pure SM contribution which is independent of m_ϕ. The e^+e^- center-of-mass energy $\sqrt{s_{ee}}$ is fixed at 500 GeV. The different lines are for different kinds of interaction. Legends follow from Table II. The curve marked by RL in (a) corresponds to *R*-parity violating SUSY.

which can arise only from leptoquark interactions (this is also true for the LR case), enhance the total cross section by 10% or so over the entire range of center-of-mass energy we have considered.

The plots in Fig. 2(b) are for doublet leptoquarks. The structures of the interactions, here, are little different from those of the singlet case. One can easily see, comparing Figs. 2(a) and 2(b), that the contributions are nearly the same for both cases. Here again the LL or RR types of interaction do not interfere with the SM. The enhancement of the $t\bar{t}$ cross section is also exactly the same in magnitude as in the singlet case with LL or RR interaction.

Now let us consider the variation of the cross section with leptoquark mass. For this purpose, we fixed the center-of-mass energy of the e^+e^- system at 500 GeV. One can easily see that the leptoquark (or squark) mass acts as the scale of the new physics we are interested in. This particular feature is reflected in Figs. 3(a) and 3(b) where we plotted the variation of the total cross section with m_ϕ. As m_ϕ increases all the cross sections are converging to the SM value, indicating the decoupling nature of the leptoquark interactions at higher energies.

From the above discussion, it is evident that the presence of lepton number violating couplings may enhance the total rate of top-quark pair production in electron-positron annihilation. The absence of any such increase in $t\bar{t}$ cross section at the future e^+e^- machines would help us to constrain the parameter space of the theories that allow such interactions. As we emphasized, there can be several types of such interaction. Now it is important to consider how one can differentiate those if at any experiment such an enhancement is detected. The different chiral structures of the interactions point to the fact that the angular distribution may be helpful. The most useful signal of top-quark pair production comes when one top quark decays semileptonically and the other decays hadronically. The cleaner environment of an electron-positron collider enables us to reconstruct the scattering angle from the hadronically decaying top quark. So we have tried to compare the angular distributions of the pure SM case with those of the leptoquark case. At lower $\sqrt{s_{ee}}$, there is very little difference between these cases. At higher center-of-mass energies (~ 1 TeV), the angular distribution in the leptoquark cases become less asymmetric in $\cos\theta$ (θ is the scattering angle) than in the SM. To quantify this we calculate the forward-backward asymmetry A_{FB}, defined as

$$A_{FB}=\frac{\sigma_B-\sigma_F}{\sigma_B+\sigma_F} \tag{5}$$

where $\sigma_B=\int_{-1}^{0}[d\sigma/d(\cos\theta)]d(\cos\theta)$ and $\sigma_F=\int_0^1[d\sigma/d(\cos\theta)]d(\cos\theta)$.

We have plotted this asymmetry with e^+e^- center-of-mass energy in Fig. 4 for the singlet/triplet leptoquark interactions, along with the SM. As expected, for the SM, A_{FB} grows with increasing center-of-mass energy. From the figure it is evident that, although at lower energies A_{FB} for all four cases remain very close to the SM value, at higher energies the angular distributions for the leptoquark mediated cases become less asymmetric. This in turn reduces A_{FB} in all these cases from the SM value. The forward-backward asymmetries for the LL and RR cases come out to be equal. At higher energies also the values of A_{FB} for different kinds of leptoquark interaction remain very close to each other. So one needs a large number of clean background-free events (which looks possible in the next generation e^+e^- machines) to differentiate these scenarios. Once again we will try to compare our results with that obtained in Ref. [14] in a qualitative manner. According to this work, a 1 TeV electron-positron collider will explore a larger area in leptoquark parameter space than a 500 GeV machine. When one looks at the total cross sections [see Fig. 2(a) and Fig. 2(b)], one can see that at higher center-of-mass energies the differences between the SM cross section and those of different leptoquark (+SM) mediated processes are less than the differences at lower energies. But when we look at the forward-backward asymmetries at different energies, it is evident that at higher energies the differences between the SM case and the leptoquarks are higher than those evaluated at smaller center-of-mass energies. So comparison of the forward-backward asymmetry (which is also the reflection of the angular distribution of the processes) will be more efficient at higher energies to discriminate the leptoquark models from the SM, which is in consonance with the results in Ref. [14].

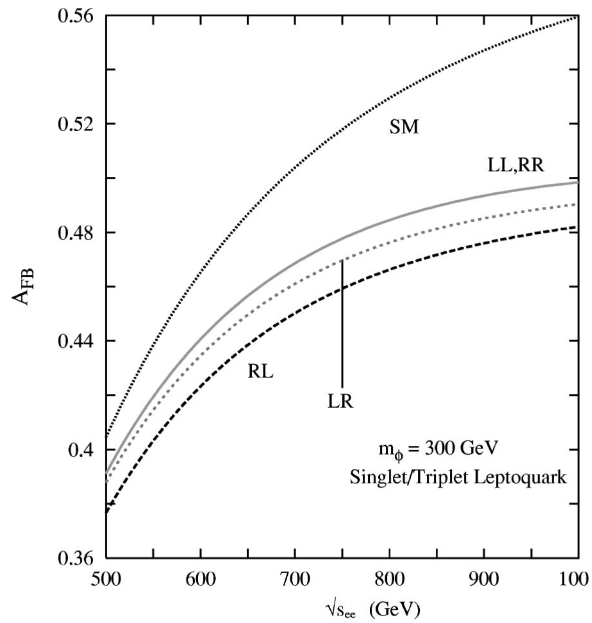

FIG. 4. Variation of forward-backward asymmetry A_{FB} in the presence of singlet and triplet leptoquarks and R-parity violating interactions with e^+e^- center-of-mass energy. For comparison we have also plotted the pure SM contribution. The leptoquark mass is fixed at 300 GeV. The different lines are for different kinds of interactions. Legends follow from Table II. The curve marked by RL corresponds to R-parity violating SUSY.

For the doublet leptoquarks, there are no qualitative differences in A_{FB} from the singlet case. Numerically, for different types of doublet leptoquark interaction (LL,RL,LR, etc.) A_{FB} values differ very little from those in the corresponding singlet/triplet cases. We do not present them here.

Finally, we want to make some comments about the top-quark decay mediated via these new interactions. As we assume this particular coupling (involving e, t, and a scalar leptoquark, i.e., $\lambda'_{132}, \lambda_{13}^{(i)}$, or $\tilde{\lambda}_{13}^{(i)}$) to be nonzero, the top-quark decay width to $b e \nu_e$ could also be modified. We have not written the relevant interactions involving a b quark, a neutrino, and a leptoquark. Looking at the interactions in Ref. [18], one can easily check that this particular decay cannot be mediated via the SU(2) doublet leptoquarks. The operators (apart from the SM contribution mediated by the W boson) contributing to this process can be written as

$$\text{singlet or triplet:}\quad |\lambda_{13}^{(i)}|^2 (\overline{e^c}\, P_L\, t)(\overline{\nu_e}\, P_R\, b^c)\widehat{\phi_i \phi_i^*},$$

$$\text{singlet:}\quad \lambda_{13}^{(1)} \tilde{\lambda}_{13}^{(1)} (\overline{e^c}\, P_R\, t)(\overline{\nu_e}\, P_R\, b^c)\widehat{\phi_1 \phi_1^*}. \tag{6}$$

R-parity violating SUSY corresponds to the first of Eq. (6). There can be other decay modes, but as long as we confine ourselves to the specific coupling that we have used so far this is the only one. We have calculated the decay widths corresponding to the cases in Eq. (6). With the values of couplings and leptoquark masses we have used before, the width comes out to be very nearly equal to the SM value. This looks surprising because with the same values of the parameters we get rather good enhancement in $t\bar{t}$ production. The smallness of the new physics contribution can be attributed to the fact that the dominant contributions to the amplitudes corresponding to Eqs. (6), are proportional to $m_t m_b$, while in the case of the top-quark pair production these are proportional to m_t^2. So the top-quark semileptonic branching ratio (to as electron) is barely changed in the presence of these new interactions, unless the couplings are big enough.

The operators responsible for top-quark decay [Eq. (6)] have a distinctly different structure from the SM case. Although the total width shows a little enhancement over the SM value, it would be interesting to see how the angular distribution of the decay products differs from the latter. As we pointed out, the cleanest signal for top-quark pair production comes when one top-quark decays semileptonically and the other decays hadronically. We have calculated the angular distribution of the e^+ coming from the top-quark decay keeping the full spin correlation between the top-quark production and decay, in the presence of leptoquark interactions as well as the SM. From the angular distribution one can easily calculate the forward-backward asymmetry of the e^+ (A_{FB}^l). For the purpose of illustration, we have presented the result of our analysis for singlet/triplet leptoquarks in Fig. 5. We have chosen the first of Eqs. (6) to calculate the top-quark decay matrix element. Figure 5 clearly shows the difference in A_{FB}^l between the SM and leptoquark interaction over the energy range we have considered. Despite the fact that these new interactions (with the coupling strength we have considered) cannot change the top semileptonic branching ratio to a significant extent, the angular asymmetries still play a crucial role in discriminating these effects from the SM. With the ballpark values of the $t\bar{t}$ cross sections at these energies [see Fig. 2(a)] and with the projected e^+e^- luminosities, one can easily detect these asymmetries. A comparison of Fig. 5 with Fig. 4, reveals that the A_{FB} in $t\bar{t}$ production differs from the A_{FB}^l over the whole range of center-of-mass energy. This can be accounted for by the chiral structure of the decay matrix element which plays a crucial role in determining the angular distribution of the top-quark decay products.

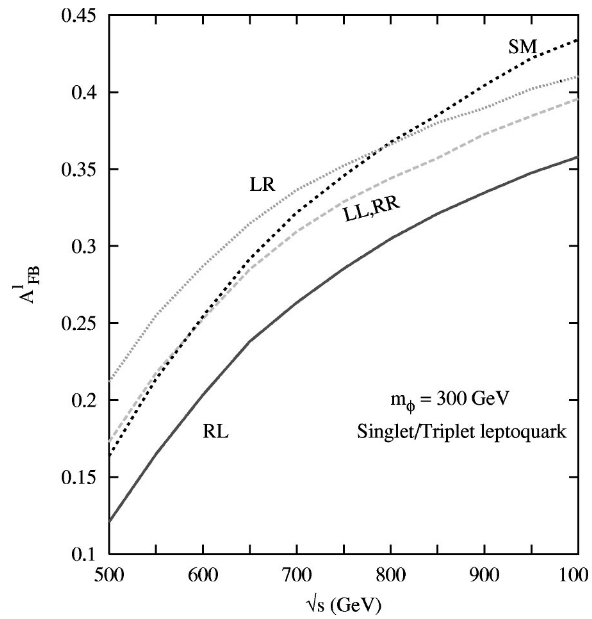

FIG. 5. Variation of e^+ (coming from t-quark decay) forward-backward asymmetry A_{FB}^l in the presence of singlet and triplet leptoquarks and R-parity violating interactions with e^+e^- center-of-mass energy. For comparison we have also plotted the pure SM contribution. The leptoquark mass is fixed at 300 GeV. The different lines are for different kinds of interactions. Legends follow from Table II. For top-quark decay we have used the first of Eq. (6). The curve marked by RL corresponds to R-parity violating SUSY.

The R-parity violating MSSM allows the t quark to decay to a left-handed selectron ($\tilde{e}_L$) and a b quark via the same λ'_{132} coupling. $\tilde{e}_L$ will in turn decay to an electron and the lightest neutralino ($\tilde{\chi}_1^0$). $\tilde{\chi}_1^0$ is no longer stable and will decay to an s quark, ν_e, and b quark. This has been discussed in detail in Ref. [24]. This decay will lead to three jets (including one b quark), an electron, and missing energy originating from a neutrino. So R-parity violation can be separated from non-SUSY leptoquarks by this kind of top-quark decay signal.

IV. CONCLUSION

To summarize, we show that the presence of lepton number violating interactions can enhance the top-quark pair production cross section in electron-positron annihilation at the next generation linear collider machines. We have considered different kinds of leptoquark interaction. R-parity violating interactions involving one lepton and two quark superfields belong to one of these cases. Non-SUSY theories with leptoquarks allow both left- and right-handed couplings involving a scalar leptoquark, a top quark, and an electron. We have estimated the cross sections in all the cases separately.

With moderate values of these lepton number violating Yukawa couplings one gets quite good enhancement of the total cross section over the SM value. Depending on the e^+e^- center-of-mass energy and leptoquark mass, enhancement varies from a few percent to 60%. With higher values of the leptoquark mass the cross section converges to the SM value. This clearly points to the fact that these interactions are decoupling in nature at higher energies. We have also considered the effects of this coupling on the top-quark semileptonic decay. The top-quark decay width changes very little after inclusion of these new interactions. Forward-backward asymmetry in top-quark pair production and top-quark decay may be used to differentiate these lepton number violating interactions from the SM and among themselves at higher center-of-mass energies. This will need a large sample of $t\bar{t}$ events, which looks feasible at the next generation e^+e^- linear colliders.

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