Meson spectrum and analytic confinement

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The basic dynamic properties of two-particle bound states of ''quarks'' and ''gluons'' are investigated within a simple relativistic quantum field model with the Yukawa interaction. Provided with an analytic confinement of the constituent particles and small coupling constant, this model explains qualitatively the experimental evidence: the quarks and gluons are confined, the final bound states are stable, massless gluons constitute the glueballs and the Regge trajectories of ''mesonic'' orbital excitations are asymptotically linear.

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I. INTRODUCTION

At the present time, QCD is commonly regarded as the true theory of strong interactions describing all processes in the hadron world, including mesonic spectroscopy [1]. However, being a nonlinear theory with a local color gauge symmetry, QCD is quite complicated from the computational point of view, and the conventional methods of calculation require great effort in making additional assumptions and ideas. In contrast with QED, simple and reliable methods of calculations are still missing in QCD. From our point of view, any acceptable description of quarks and gluons and their hadronization on large distances, where the confinement of quarks and gluons takes place, directly depends on the structure of the QCD vacuum, and this structure is not well established yet. In other words, the propagators of quarks and gluons on large distances are quite far from those given by standard Dirac and Klein-Gordon equations. Generally, one may expect that a theoretical description of colorless hadrons considered as bound states of quarks and gluons, when the confinement is taken into account and an averaging over all nonobservable color degrees of freedom is performed, can lead to a physical picture, where the quarks and gluons are realized in the form of some phenomenological ''bricks.'' We suppose that a successful guess of the structure of these ''bricks'' in the confinement region can result, particularly, in a qualitatively correct description of the basic features of the meson spectrum. Our guess is that the analytic confinement realizes these ''bricks.''

In particle physics there exist several models based on the idea of analytic confinement. The quark confinement model [2] treats light hadrons as collective colorless excitations of quark-gluon interactions while the analytic quark confinement is provided by averaging over gluon backgrounds. The analytic form of the form factor providing the quark confinement is a phenomenological function unique for all processes in the low-energy physics. This model reproduces the low-energy relations of chiral theory in the case of zero momentum and allows one to obtain more sophisticated hadron characteristics such as slope parameters and formfactors.

An approach based on the assumption that QCD vacuum is realized by the self-dual homogeneous vacuum gluon field which is the classical solution of the Yang-Mills equations was developed in [3,4]. According to Leutwyler [5], this gluon configuration is stable over local quantum fluctuations, and can lead to the quark and gluon confinement as well as a necessary chiral symmetry breaking. Hereby, propagators of quarks and gluons in this field are entire analytic functions in the p^2-complex plane, i.e., the analytic confinement takes place. The spectrum and weak decays of light mesons, their excited states, heavy quarkonia and heavy-light mesons, several regimes for masses and decay constants were found in complete agreement with experimental data. This approach contains a minimal set of parameters: the gauge coupling constant, the strength of the vacuum field and the quark masses. Thus, the self-dual homogeneous gluon field leading to the analytic confinement can be considered a good candidate to realize the QCD vacuum. One can say that existing models with analytic confinement describe satisfactorily the experimental evidence.

However, real calculations of different amplitudes in particle physics require to take into account simultaneously the confinement as well as some quantum characteristics associated with color, flavor and spin within a chiral symmetry breaking. Besides, analytic calculations within these approaches are quite cumbersome, so that the problem arises ''not to see the forest among trees.'' In addition, it is necessary to note that there exists a prejudice to the idea of the analytic confinement (see, for example, [6]). Therefore, it seems reasonable to consider simple quantum field models in order to investigate qualitatively just ''pure'' effects due to analytic confinement.

The present paper is aimed to clarify the role of the analytic confinement in properties of hadrons, the bound states of quarks and gluons by considering a simple relativistic quantum field model. Particularly, we explain qualitatively and semiquantitatively the basic features of experimentally observed meson spectra analyzed recently in [7].

The hadron spectroscopy as the theory of bound states of quarks, and the phenomenology of the Regge trajectories (RTs) are important and interdependent subjects of investigation in particle physics (see, e.g., [8–10]). The basic characteristics of mesons considered as bound states of quarks and gluons (in contrast to the relations of the SU_3 flavor symmetry) can be roughly listed as follows: quarks and gluons are confined (nonobservable); glueballs are bound states of

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massless gluons and completely relativistic systems; the RTs of different families of mesonic orbital excitations are asymptotically linear and their slopes differ insignificantly. Therefore, the slope of RTs may be a universal parameter dictated by the general nature of quark-gluon interaction.

Obviously, these characteristics are hardly obtained in the framework of any local quantum field theory, where the constituent particles, the quarks and gluons, are described by the standard Dirac and Klein-Gordon equations. From common point of view, the confinement plays the main role in understanding and explaining this picture. The point is how to realize mathematically the conception of confinement within a specific theoretical formalism?

The standard QCD calculations leading to linear RTs of hadrons are based on (i) a nonlinear QCD gluon dynamics with a particular infrared behavior of the gluon propagator and (ii) a three-dimensional reduction of the relativistic Bethe-Salpeter equation. This results in a linear increasing potential between quarks in three-dimensional space (see, e.g., [11]). This infrared singular behavior is commonly interpreted as quark confinement.

In reality, the modern picture is more complicated (see, for example, [12,13]), but we do not discuss the details here. Note only that it is necessary to overcome some mathematical problems caused by the singularity of the gluon kernel and an ambiguously defined choice of particular reduction of the relativistic two-body Bethe-Salpeter equation (see, e.g., [14]).

In the present paper we show that there exists another possible mechanism explaining the above mentioned characteristics of mesonic spectra, particularly, the properties of RTs. In doing so, we use a simple relativistic quantum-field model of two scalar particles (the prototypes of constituent "quarks" and intermediate "gluons") with the analytic confinement. Our approach is based on the following assumptions: the analytic confinement takes place; the interaction is described by a Yukawa-type Lagrangian; the coupling constant binding the "quarks" with "gluons" is small; final bound "hadron" states of "quarks" are described by the relativistic Bethe-Salpeter equation in one-"gluon" exchange mode without using any 3D reduction.

In addition we demonstrate a mathematical sketch of calculations of two-body bound-state spectrum within the Bethe-Salpeter equation in the weak-coupling regime. In doing so, we use simple relativistic models based on physically transparent hypotheses, which can be treated by simple analytic methods. We believe that the analytic confinement is the basic underlying principle leading to a qualitatively correct description of main characteristics of meson spectra. In any case our models represent certain theoretical interest because they clarify the underlying physical principles of the meson spectrum.

II. YUKAWA TYPE MODELS WITH ANALYTIC CONFINEMENT

Our aim is to investigate the role of the analytic confinement in the meson spectroscopy by omitting quantum degrees of freedom such as the spin, color and flavor. We consider a simple system, a Yukawa model of two interacting scalar fields $\Phi(x)$ and $\varphi(x)$ described by the following Lagrangian in the Euclidean space-time:

$$\mathcal{L}(x) = -\Phi^{+}(x) S^{-1}(\Box)\Phi(x) - \frac{1}{2}\varphi(x) D^{-1}(\Box)\varphi(x) - g\Phi^{+}(x)\Phi(x)\varphi(x), \quad (1)$$

where coupling constant g is supposed sufficiently small.

We postulate that the analytic confinement takes place here. It means that the Fourier transforms of propagators of confined particles Φ and φ are entire analytic functions in the complex p^2-plane, so $S^{-1}(p^2)$ and $D^{-1}(p^2)$ have no zero at any finite complex p^2. Hence, the equations for the free fields

$$S^{-1}(\Box)\Phi(x) = 0, \quad D^{-1}(\Box)\varphi(x) = 0 \quad (2)$$

result only in the trivial solutions $\Phi(x) \equiv 0$ and $\varphi(x) \equiv 0$. We call this property *analytic confinement*, i.e., the corresponding particles exist only in virtual states [2,6]. One can say that these fields describe constituent particles, i.e., $\Phi(x)$ and $\varphi(x)$ represent scalar "quarks" and scalar "gluons," respectively.

Below we deal with two specific versions of the analytic confinement.

(1) In the first simplest model we consider pure Gaussian exponents for the "quark" and "gluon" propagators:

$$S(x_1 - x_2) = S(\Box)\,\delta(x_1 - x_2) = \frac{\Lambda^2}{(4\pi)^2} e^{-(1/4)\Lambda^2 (x_1 - x_2)^2},$$

$$\tilde{S}(p^2) = \frac{1}{\Lambda^2} e^{-p^2/\Lambda^2}, \quad (3)$$

$$D(x_1 - x_2) = D(\Box)\,\delta(x_1 - x_2) = \frac{\Lambda^2}{(4\pi)^2} e^{-(1/4)\Lambda^2 (x_1 - x_2)^2},$$

$$\tilde{D}(p^2) = \frac{1}{\Lambda^2} e^{-p^2/\Lambda^2},$$

where the only parameter $1/\Lambda$ represents the "radius" or scale of confinement. From a physical point of view this model is important because the eigenfunctions and eigenvalues of the relativistic Bethe-Salpeter equation within one-particle exchange approximation can be found explicitly and the obtained RTs are purely linear. In some sense, this model can be considered a "relativistic oscillator" because the exact solution possesses equidistant spectra resulting in pure linear RTs. We call this case *the virton model* [15].

(2) The second model implies that there exists a certain dynamical mechanism generating analytic confinement of standard particles with initial masses m and 0. So we introduce the second parameter, a "quark" mass m. The propagators are given in more realistic forms [16]:

$$S(x_1-x_2)=\left(\frac{\Lambda}{4\pi}\right)^2\int_0^1\frac{d\alpha}{\alpha^2}e^{-(m^2/\Lambda^2)\alpha-[\Lambda^2(x_1-x_2)^2/4\alpha]},$$

$$\widetilde{S}(p^2)=\frac{1}{p^2+m^2}(1-e^{-(p^2+m^2)/\Lambda^2}),$$

$$D(x_1-x_2)=\frac{1}{(2\pi)^2x^2}e^{-\Lambda^2(x_1-x_2)^2/4},$$

$$\widetilde{D}(p^2)=\frac{1}{p^2}(1-e^{-p^2/\Lambda^2}). \tag{4}$$

In the deconfinement limit $\Lambda\to 0$ this model allows one to obtain the conventional propagators of massive and massless scalar particles. Within this model we can analyze the influence of the mass parameter $\nu=m/\Lambda$ on the behavior of the meson spectrum. We call this case *the scalar confinement model*. We show that this model describes qualitatively well dynamic characteristics of meson spectra.

Note that both these models realize the ‘‘quark’’ and ‘‘gluon’’ confinement only. Other important quantum characteristics as color, flavor and spin with an appropriate chiral broken symmetry are not considered.

III. TWO-PARTICLE BOUND STATES

‘‘Two-quark’’ bound states can be found in the following way. Let us consider the partition function

$$Z=\int\int\int\delta\Phi\,\delta\Phi^+\,\delta\phi e^{-(\Phi^+S^{-1}\Phi)-(1/2)(\varphi D^{-1}\varphi)-g(\Phi^+\Phi\varphi)}. \tag{5}$$

This partition function is written in the quark and gluon variables. Our aim is to rewrite Z in terms of ‘‘hadron’’ fields in order to realize so-called *quark-hadron duality*.

Integration over φ results in

$$Z=\int\int\delta\Phi\,\delta\Phi^+e^{-(\Phi^+S^{-1}\Phi)+(g^2/2)(\Phi^+\Phi D\Phi^+\Phi)}. \tag{6}$$

Let us introduce a complete orthonormal system $\{U_Q(y)\}$:

$$\int dy\,U_Q(y)U_{Q'}(y)=\delta_{QQ'},$$

$$\sum_Q U_Q(y)U_{Q'}(y')=\delta(y-y'), \tag{7}$$

where $Q=\{n,l,\{\mu\}\}$ is a set of radial n, orbital l and magnetic $\{\mu\}=(\mu_1,\ldots,\mu_l)$ quantum numbers. Then, the term $L_2[\Phi]=(\Phi^+\Phi D\Phi^+\Phi)$ can be rewritten

$$\begin{aligned}L_2[\Phi]&=\frac{g^2}{2}\int\int dx_1\,dx_2\,\Phi^+(x_1)\Phi(x_1)D(x_1-x_2)\\&\quad\times\Phi^+(x_2)\Phi(x_2)\\&=\frac{g^2}{2}\int dx\int\int dy_1\,dy_2\sqrt{D(y_1)}J(x,y_1)\,\delta(y_1-y_2)\\&\quad\times\sqrt{D(y_2)}J^+(x,y_2)\\&=\frac{g^2}{2}\sum_Q\int dx\,J_Q(x)J_Q(x),\end{aligned} \tag{8}$$

with $x_1=x+y/2,\quad x_2=x-y/2$ and

$$J(x,y)=\Phi^+\left(x+\frac{1}{2}y\right)\Phi\left(x-\frac{1}{2}y\right)=\Phi^+(x)e^{(y/2)\overleftrightarrow{\partial}}\Phi(x),$$

$$J^+(x,y)=J(x,-y),\quad J_Q(x)=\Phi^+(x)V_Q(\overleftrightarrow{\partial})\Phi(x), \tag{9}$$

$$J_Q^+(x)=J_Q(x),\quad V_Q(\overleftrightarrow{\partial})=i^l\int dy\sqrt{D(y_1)}U_Q(y)e^{(y/2)\overleftrightarrow{\partial}},$$

where $V_Q(\overleftrightarrow{\partial})$ is a nonlocal vertex.

By using the Gaussian functional representation we write

$$\begin{aligned}e^{L_2[\Phi]}&=e^{(g^2/2)\Sigma_Q\int dx\,J_Q(x)J_Q(x)}\\&=\int\prod_Q\delta B_Q e^{-(1/2)\Sigma_Q(B_QB_Q)+g\Sigma_Q(B_QJ_Q)}.\end{aligned}$$

Substituting this representation into Eq. (6) and by integrating over Φ we obtain

$$\begin{aligned}Z&=\int\prod_Q\delta B_Q\exp\left\{-\frac{1}{2}\sum_Q(B_QB_Q)\right.\\&\quad\left.-\mathrm{Tr}\ln(1-gB_QV_QS)\right\}\\&=\int\prod_Q\delta B_Q\exp\left\{-\frac{1}{2}\sum_{QQ'}(B_Q[\delta_{QQ'}-\alpha\Pi_{QQ'}]B_{Q'})\right.\\&\quad\left.+W_I[gB]\right\},\end{aligned} \tag{10}$$

where

$$W_I[gB]=-\mathrm{Tr}\left[\ln(1-gB_QV_QS)+\frac{g^2}{2}B_QV_QSB_{Q'}V_{Q'}S\right]$$

is a functional describing interactions of fields B_Q. Polarization operator $\alpha\Pi_{QQ'}$ in the one-loop approximation reads

$$\alpha\Pi_{QQ'}(z)=\int\int dy_1\,dy_2\,U_Q(y_1)\,\alpha\Pi(z;y_1,y_2)U_{Q'}(y_2),$$

$$\alpha\Pi(z;y_1,y_2)=g^2\sqrt{D(y_1)}S\left(z+\frac{y_1-y_2}{2}\right)S\left(z-\frac{y_1-y_2}{2}\right)\times\sqrt{D(y_2)},$$

where $z=x_1-x_2$ and $\alpha=(g/4\pi\Lambda)^2$. Its Fourier transform reads

$$\alpha\tilde{\Pi}_{QQ'}(p)=\int\int dy_1\,dy_2\,U_Q(y_1)\,\alpha\tilde{\Pi}_p(y_1,y_2)U_{Q'}(y_2),$$

$$\alpha\tilde{\Pi}_p(y_1,y_2)=g^2\sqrt{D(y_1)}\int\frac{dk}{(2\pi)^4}e^{-ik(y_1-y_2)}\tilde{S}\left(k+\frac{p}{2}\right)\times\tilde{S}\left(k-\frac{p}{2}\right)\sqrt{D(y_2)}. \quad (11)$$

Suppose, the orthonormal system $\{U_Q(y)\}$ diagonalizes the kernel in Eq. (11). It means that we solve the eigenvalue problem

$$\int dy'\,\alpha\tilde{\Pi}_p(y,y')U_Q(y)=E_Q(-p^2)U_Q(y), \quad (12)$$

where $E_Q(-p^2)=E_{nl}(-p^2)$, i.e., the eigenvalues are degenerated over the magnetic quantum numbers $\{\mu\}$. We stress that the Bethe-Salpeter kernel in Eq. (12) is real and symmetric, therefore, variational methods can be applied for its further evaluation.

Then, the polarization operator in Eq. (11) reads

$$\alpha\tilde{\Pi}_{QQ'}(p)=E_Q(-p^2)\,\delta_{QQ'}\,. \quad (13)$$

Note that diagonalization (13) is nothing else but the solution of the Bethe-Salpeter equation in the one-boson exchange approximation. The standard form of the Bethe-Salpeter may be obtained, if one introduces in Eq. (12) new functions $U_Q(y)=\sqrt{D(y)}\Psi_Q(y)$ and goes to the momentum space (see, for example, [17]).

By introducing a Gaussian measure defined by

$$G_Q^{-1}(x_1-x_2)=[1-E_Q(\Box)]\,\delta(x_1-x_2), \quad p^2=-\Box$$

we rewrite the partition function (10) in the final form

$$Z=\int\prod_Q\,\delta\tilde{B}_Q e^{-(1/2)\Sigma_Q(B_QG_Q^{-1}B_Q)+W_I[gB]}. \quad (14)$$

We stress that this representation is completely equivalent to the initial one (5). It is a mathematical realization of the quark-hadron duality in the model under consideration. From physical point of view, we pass on from the world containing fields Φ and ϕ to the world of bound states $\{B_Q\}$. The field variables $\{B_Q\}$ can be interpreted as fields of particles with quantum numbers $Q=\{nl\}$ and masses M_Q, if the Green function $\tilde{G}_Q(p^2)=1/[1-E_Q(-p^2)]$ has a simple pole in the Minkowski space ($p^2=-M_Q^2$). The mass of two-particle bound states are defined by the equation

$$1=E_Q(M_Q^2). \quad (15)$$

Formally, $G_Q^{-1}(-\Box)$ defines the kinetic term of the field B_Q. To go to its standard form, we expand it in the vicinity of $p^2=-M_Q^2$ as follows:

$$1-E_Q(-p^2)=Z_Q(p^2+M_Q^2)+O[(p^2+M_Q^2)^2],$$

$$Z_Q=-E_Q'(-M_Q^2)>0.$$

The positive constant Z_Q provides the renormalization of the wave function of the field B_Q. We rewrite the kinetic and interaction parts in terms of the renormalized fields $\tilde{B}_Q(p)=Z_Q^{-1/2}\tilde{\mathcal{B}}_Q(p)$ as follows:

$$(\tilde{B}_Q^+(p)[1-E_Q(-p^2)]\tilde{B}_Q(p))=(\tilde{\mathcal{B}}_Q^+(p)[(p^2+M_Q^2)+O[(p^2+M_Q^2)^2]]\tilde{\mathcal{B}}_Q(p)),$$

$$W_I[gB]=W_I[g_{\rm eff}\mathcal{B}], \quad g_Q^{\rm eff}=gZ_Q^{-1/2}=\frac{g}{\sqrt{-E_Q'(-M_Q^2)}}>0. \quad (16)$$

The functional $W_I[g_{\rm eff}\mathcal{B}]$ describes all "strong interactions" of the "mesons" $\mathcal{B}_Q$. In addition it should be stressed that the effective coupling constant $g_Q^{\rm eff}$ in Eq. (16), defining the strength of boson interactions does not explicitly depend on the initial coupling constant g because of relation $E_Q'(-M_Q^2)\sim g$.

IV. THE VIRTON MODEL

Due to the pure Gaussian character of the propagators in this model, the polarization kernel (11) becomes quite simple:

$$\alpha\tilde{\Pi}_p(y,y')=\alpha\left(\frac{\Lambda^2}{8\pi}\right)^2e^{-p^2/2\Lambda^2}K(y,y'),$$

$$K(y,y')=e^{-(\Lambda^2/4)(y^2-yy'+y'^2)}. \quad (17)$$

Explicit diagonalization of kernel $K(y,y')$ on $\{U_Q(y)\}$ results in the eigenvalues

$$\kappa_Q=\kappa_{nl}=\kappa_0\left(\frac{1}{2+\sqrt{3}}\right)^{2n+l}, \quad \kappa_0=\left(\frac{8\pi}{\Lambda^2(2+\sqrt{3})}\right)^2. \quad (18)$$

Corresponding eigenfunctions $U_Q(y)$ are given in Appendix A.

Therefore, the mass spectrum of two-particle bound states can be found explicitly

$$M_Q^2 = M_{nl}^2 = 2\Lambda^2 \ln\frac{\alpha_c}{\alpha} + (2n+l)2\Lambda^2 \ln(2+\sqrt{3}),$$

$$\alpha_c = (2+\sqrt{3})^2. \tag{19}$$

Thus, a pure Gaussian form of analytic confinement (3) leads to the linear and parallel RTs. The slope of RTs is defined only by the scale of the confinement region Λ and does not depend on α and other dynamic constants. Bound states exist for $\alpha < \alpha_c$. If $\alpha \ll \alpha_c$, the size of the confinement region is remarkably larger than the Compton length of any bound state

$$r_{\rm conf} \sim \frac{1}{\Lambda} \gg \frac{1}{M_Q} \sim l_Q .$$

In other words, all physical particles described by the fields $B_Q(x)$ and all physical transformations involving them take place inside the confinement region.

V. THE SCALAR CONFINEMENT MODEL

In order to solve the eigenvalue problem (12) we will use the variational principle because the kernel $\tilde{\Pi}_p(y,y')$ is real and symmetric. For further simplicity we consider only the orbital excitations, i.e., $n=0$ and $Q=\{0,l,\{\mu\}\}$.

According to Eq. (15), the mass of the bound state is determined by the following variational equation:

$$1 = \alpha\epsilon_l\left(\frac{M_l}{2\Lambda},\frac{m}{\Lambda}\right) = \max_{\Psi_Q}\sum_{\{\mu\}}\int\int dy_1\,dy_2\,\Psi_Q(y_1) \times \alpha\Pi_p(y_1,y_2)\Psi_Q(y_2),$$

$$p^2 = -M_l^2 . \tag{20}$$

Note, the variational optimization gives an upper bound to the mass M_l^2 because for $M_l^2 > 0$

$$\alpha\epsilon_l\left(\frac{M_l}{2\Lambda},\frac{m}{\Lambda}\right) \leq E_l(M_l^2).$$

Let us introduce a normalized trial wave function:

$$\Psi_{l\{\mu\}}(x,a) = C_l T_{l\{\mu\}}(x)\sqrt{D(x)}e^{-(\Lambda^2/4)ax^2}, \tag{21}$$

$$C_l = \Lambda^{l+1}\sqrt{\frac{(1+2a)^{l+1}}{2^l(l+1)!}},$$

$$\sum_{\{\mu\}}\int dx|\Psi_{l\{\mu\}}(x,a)|^2 = 1,$$

where a is a variational parameter. The four-dimensional spherical orthogonal harmonics $T_{l\{\mu\}}(x)$ are defined in Appendix A. We suppose that the test function in Eq. (21) should be a good guess to the exact one because the kernel (11) is proportional to $\sqrt{D(y)}$ and $S(y)$ is of the Gaussian type.

Further we use the following relation:

$$\tilde{\Phi}_{l\{\mu\}}(k,a) \equiv i^l\int dx\, e^{-ikx}\sqrt{D(x)}\Psi_{l\{\mu\}}(x,a)$$

$$= \frac{C_l}{(2\pi)^l}T_{l\{\mu\}}(k)\int d^{4+2l}Y\, e^{-iKY}D(Y)e^{-aY^2},$$

where $K,Y \in \mathbf{R}^{4+2l}$, $k^2 = K^2$ and the rotational symmetry $D(y^2) = D(Y^2)$ has been taken into account. Then, one obtains [16]

$$\sum_{\mu}\tilde{\Phi}_{l\{\mu\}}(k,a)\tilde{\Phi}_{l\{\mu\}}(k,a) = \frac{C_l^2 k^{2l}(l+1)}{2^{4+3l}}\left[\int_0^{u_0} du\, u^l e^{-uk^2/4}\right]^2 = 1,$$

$$u_0 = \frac{4}{\Lambda^2(1+a)}. \tag{22}$$

Substituting Eqs. (4), (21), and (22) into Eq. (20) and after some calculations we arrive at

$$1 = g^2\max_a\int\frac{dk}{(2\pi)^4}\sum_{\{\mu\}}\tilde{\Phi}_{l\{\mu\}}(k,a)\tilde{S}\left(k+\frac{p}{2}\right)\tilde{S}\left(k-\frac{p}{2}\right) \times\tilde{\Phi}_{l\{\mu\}}(k,a)$$

$$= \frac{\alpha}{l!}\max_c\left\{[4c(1-c)]^{l+1}\int\int_0^1 dt\,ds \times e^{(\mathcal{M}_l^2-\nu^2)(t+s)}R_l(t,s,\chi_l)\right\}, \tag{23}$$

where $p = (iM_l,0,0,0)$ and

$$R_l(t,s,\chi_l) = \int\int_0^1 du\,dv\, e^{-\chi_l^2/b}(uw)^l F_l(b,\chi_l),$$

$$F_l(b,\chi_l) = \frac{1}{\pi^2}e^{\chi_l^2/b}\int d^4k\, k^{2l}e^{-k^2b-kp(t-s)}$$

$$= e^{\chi_l^2/b}\left(-\frac{\partial}{\partial b}\right)^l\left[\frac{1}{b^2}e^{-\chi_l^2/b}\right],$$

$$\nu = \frac{m}{\Lambda},\quad \mathcal{M}_l = \frac{M_l}{2\Lambda},\quad \chi_l^2 = \mathcal{M}_l^2(t-s)^2,$$

$$b = t+s+2c(u+w).$$

Variational equation (23) defines the relation between parameters M_l, α, ν, and l.

In the deconfinement limit $\Lambda \to 0$ our variational estimation results in a qualitatively correct behavior (for details see Appendix B) of the final bound-state mass

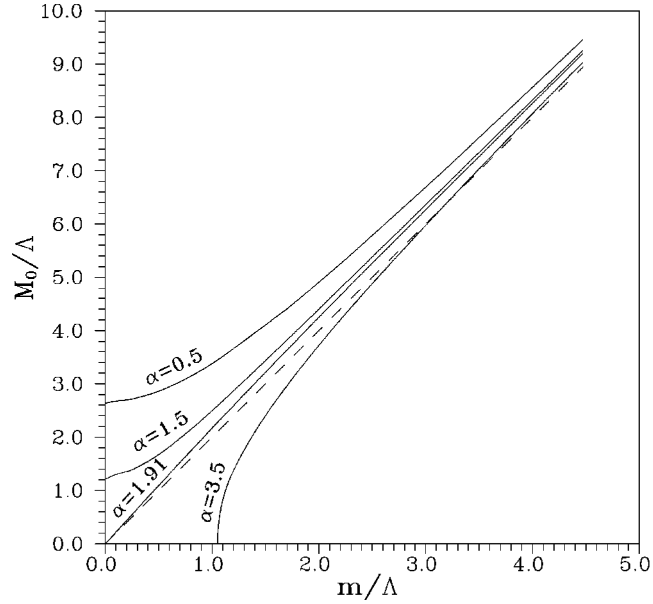

FIG. 1. The mass $\mathcal{M}_0 = M_0/\Lambda$ of the two-particle ground state ($l=0$) as a function of the mass $\nu = m/\Lambda$ of the "constituent" particle. Dashed line corresponds to the case when $M_0 = 2m$. The physical picture takes place only for relative weak coupling constant $\alpha < \alpha_c = 1.9149\ldots$.

$$M_0 = 2m - \frac{\alpha_0^2}{2} mK + O(\alpha_0^4), \quad \alpha_0 = \left(\frac{g}{4\pi m}\right)^2,$$

$$K = 0.6403\ldots,$$

i.e., we get the standard nonrelativistic (the coupling constant α_0 is small) behavior for a bound state under the Coulomb potential.

A. The lowest state

Let us consider the lowest state with $l=0$. The equation of the bound state becomes

$$\epsilon_0(\mathcal{M}_0,\nu) = \max_c \left\{ 4c(1-c) \int\int_0^1 dt\,ds \int\int_0^1 du\,dv \times \frac{e^{-\nu^2(t+s)+\mathcal{M}_0^2((t+s)-(t-s)^2/[t+s+2c(u+v)])}}{[t+s+2c(u+v)]^2} \right\} = \frac{1}{\alpha}. \tag{24}$$

We have analyzed Eq. (24) at different regimes of parameters α, m, and Λ and have solved it numerically for $\mathcal{M}_0$. Some of the obtained results are represented in Fig. 1.

By analyzing our results we can conclude the following remarks:

(1) There exists a critical coupling constant $\alpha_c = 1.9149\ldots$ obeying the equation

$$\epsilon_0(0,0) = \frac{1}{\alpha_c}. \tag{25}$$

It means that there may exist a bound state with $m=0$, $M_0=0$, i.e., massless "gluons" are able to produce a massless "hadron" bound state.

(2) If $\alpha \leqslant \alpha_c$ the mass of the "hadron" bound state obeys the inequality $M_0 \geqslant 2m$ for $\forall m \geqslant 0$. Particularly, for $\alpha < \alpha_c$ there exist states with $M_0 > 0$ for $m=0$, i.e., massless "gluons" can produce massive "hadron" bound states—the "glueballs." For heavy "quarks" ($m \gg \Lambda$) one obtains an asymptotical behavior

$$M_0^2 = 4m^2 + \frac{\Lambda^2}{2}\ln\left(\frac{m}{\Lambda}\right) + O(1). \tag{26}$$

(3) If the coupling strength exceeds the critical value $\alpha > \alpha_c$ the physical condition $M_0^2 \geqslant 0$ results in the requirement $m \geqslant m_c$, where

$$\epsilon_0\left(0, \frac{m_c}{\Lambda}\right) = \frac{1}{\alpha} < \frac{1}{\alpha_c}. \tag{27}$$

In other words, for a fixed $\alpha > \alpha_c$ the mass of the "quark" should exceed the critical value m_c in order to constitute physically meaningful bound states. Particularly, there exist massless "hadrons" $M_0=0$ constituted of two massive "quarks" with $m=m_c$. This kind of "mass annihilation" does not coincide with conventional physical conception.

Thus, we conclude that the value of the coupling constant $\alpha = (g/4\pi\Lambda)^2$ plays a crucial role in formulation of the final two-particle bound states and there exist two physically different pictures.

If $\alpha < \alpha_c$, there exist physically allowed bound states with masses $M_0 > 2m$. Particularly, glueballs exist as massive bound states of massless constituent particles.

If $\alpha > \alpha_c$, there exists a critical mass of the constituent particle m_c, so that bound states can exist only for $m > m_c$. Therefore, a massless meson as a bound state of two massive quarks can exist. But, any glueballs cannot exist at all because $0 \neq m > m_c$.

Therefore, we can conclude that a physically reasonable picture can be realized within our model only for relatively small coupling constant $\alpha < \alpha_c$.

B. Orbital excitations

In general case, formula (23) defines the mass of an orbital excitation M_l as function of input partameters: the coupling constant α, the mass of constituent "quark" m and the confinement scale Λ at any given orbital quantum number l.

As mentioned above, we believe that the scalar confinement model grasps the basic characteristics of meson spectrum, it especially should be effective in describing the orbital excitations because the latters are determined mainly by interactions on large distances, where detailing of the quark-gluon interaction are not so important. Therefore, we are able to evaluate the confinement scale Λ and the coupling constant α by applying Eq. (23) to a set of experimental data on

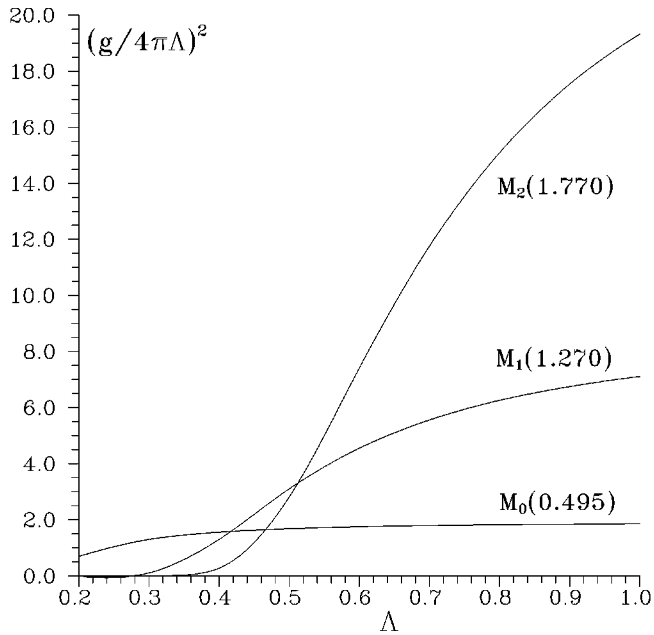

FIG. 2. The dependence $\Lambda=\Lambda(\alpha)$ evaluated from the mass equation for three different two-particle bound states with $l=0$, $M_0=0.495$ GeV; $l=1$, $M_1=1.270$ GeV; and $l=2$, $M_2=1.770$ GeV. Here we use quark masses $m_u=0.010$ GeV and $m_s=0.100$ GeV.

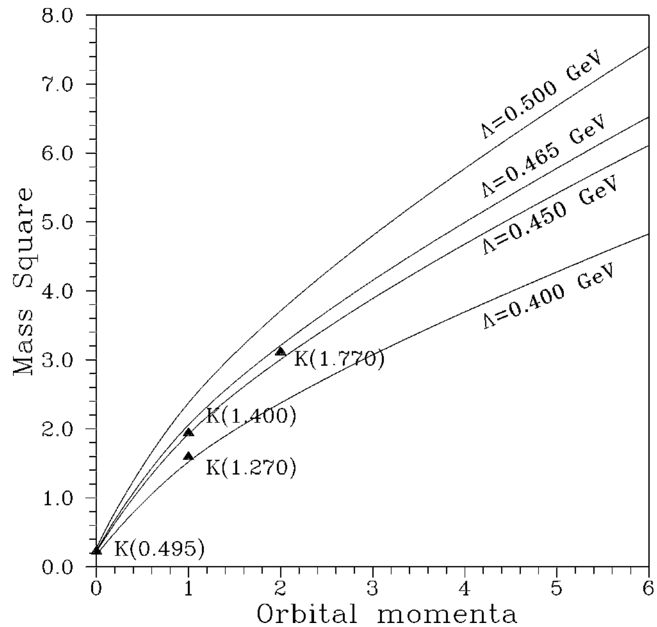

FIG. 3. The Regge trajectories of the two-particle bound states calculated for $\alpha=1.7$ at different values of Λ to compare with experimental evidence (triangles) of the K-meson family. Hereby, we plot both $K(1.270)$ and $K(1.400)$ at $l=1$ because the RPP assignment table lists K_{1B} as a mixture of these states.

the RTs. Note, the pion RT is not suitable for our consideration because the lowest π-meson has anomalously small mass caused by the mechanism of the broken chiral symmetry, which is absent in the model under consideration. So, we choose the K-meson family of orbital excitations $\{K(0.495),K(1.270)/K(1.400),K(1.770)\}$ with $l=\{0,1,2\}$. Here and below all masses are given in GeV. Since K-mesons consist of $u(d)$ and s quarks with different masses m_u and m_s, we modify formula (23) as follows:

$$1=\frac{\alpha}{l!}\max_c\left\{[4c(1-c)]^{l+1}\int\int_0^1 dt\,ds\right.$$
$$\left.\times e^{-(\nu_u^2 t+\nu_s^2 s)+(t+s)\mathcal{M}_l^2}R_l(t,s,\chi_l)\right\}, \qquad (28)$$

$$\nu_u=\frac{m_u}{\Lambda},\qquad \nu_s=\frac{m_s}{\Lambda}.$$

Thus, we solve the problem by finding α and Λ for given m_u and m_s and M_l by using data on the K-meson family. For each member of this family we have obtained the dependence $\Lambda=\Lambda(\alpha)$ at fixed "constituent quark" masses $m_u=0.010$ and $m_s=0.100$. The obtained curves $\Lambda=\Lambda(\alpha)$ are plotted in Fig. 2. We see that our input parameters α and Λ should be localized in relative short intervals to fit the kaon Regge trajectory, namely

$$\Lambda=0.4\text{–}0.5\ \text{GeV},\qquad \alpha=1.5\text{–}1.9. \qquad (29)$$

Our preliminary analyses performed for other meson families (π,K^*,ρ) indicate that this choice of our fundamental parameters is able to fit satisfactorily the experimentally observed mesonic Regge trajectories (see Table 2 in [18]). Note that these curves deform slightly when the initial "quark" masses vary in wide ranges: $m_u\in(0.010,0.100)$ and $m_s\in(0.100,0.450)$.

Further, the RTs or, the dependence of $M_l^2=M_l^2(l)$ on l for the K-meson family for $\alpha=1.7$, $m_u=0.010$, and $m_s=0.100$ at different values of $\Lambda\in(0.400,0.500)$ are plotted in Fig. 3. One can see that the RTs are far not linear for lower values of $l=0$–4, although the linearity occurs asymptotically for sufficiently large l. Besides, the curvature of these RTs and their slopes depend on Λ considerably. The asymptotical behavior of the RTs for large l can be obtained analytically and coincides with the exact solution of the Virton model (19) as follows:

$$M_l^2\sim l\,2\Lambda^2\ln(2+\sqrt{3})\qquad \text{for}\qquad l\to\infty. \qquad (30)$$

A recent analysis of experimental data shows (see [7]) that the RTs of different meson and baryon families are approximately linear and their slopes slightly deviate around a constant value, although the quark configurations and quantum numbers of these hadronic families are considerably different. Note, the analyzed experimental data in [7] are available for low orbital momenta $l=0$–3 only. Nevertheless, one can conclude that the slope of RTs weakly depends on specific details of hadron internal dynamics and may be considered as a universal characteristic which is dictated by the general properties of quark-gluon interactions. Precisely this qualitative picture takes place in our models with analytic confinement. Thus, we have sufficient grounds to claim that the analytic confinement realizes these general properties and leads to the approximate linearity of RTs for meson families.

In conclusion, the analytic confinement in the weak coupling regime explains qualitatively the main features of meson spectra. The authors understand that these simple models do not contain the real quantum degrees of freedom of quarks and gluons (color, flavor, spin) as well as the mechanism of the chiral symmetry breaking and, therefore, cannot pretend to describe quantitavely all details of the meson spectroscopy. The last remark: the obtained value of the coupling constant α in Eq. (29) is not relatively weak; however, our qualitative analysis shows that the introduction of N additional quark degrees of freedom leads to the substitution $\alpha\to N\alpha_s$ so that the "effective" value of the input coupling constant α_s decreases almost in N times. More careful consideration in this direction is the object of our next investigations.

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APPENDIX A

Consider the kernel

$$K=K(x,y)=e^{-ax^2+2bxy-ay^2}, \quad a>b \tag{A1}$$

with

$$\mathrm{Tr}\,K=\int dy\,K(y,y)=\int dy\,e^{-2(a-b)y^2}=\frac{\pi^2}{4(a-b)^2}<\infty.$$

The eigenvalues with quantum numbers $Q=\{nl\{\mu\}\}=\{nl\{\mu_1\dots\mu_l\}\}$ and eigenfunctions of the problem

$$\int dy\,K(x,y)U_Q(y)=\kappa_Q U_Q(x)$$

can be solved explicitly. The eigenvalues are

$$\kappa_Q=\kappa_{nl}=\kappa_0\left(\frac{b}{a+\sqrt{a^2-b^2}}\right)^{2n+l}, \quad \kappa_0=\frac{\pi^2}{(a+\sqrt{a^2-b^2})^2}. \tag{A2}$$

The eigenfunctions are

$$U_Q=U_{nl\{\mu\}}(y)=N_{nl}T_{l\{\mu\}}(y)L_n^{(l+1)}(2\beta y^2)e^{-\beta y^2}. \tag{A3}$$

Here $L_n^{(l+1)}(x)$ are the Laguerre polynomials and

$$\beta=\sqrt{a^2-b^2}, \quad N_{nl}=\frac{\sqrt{2^l(l+1)}}{\pi}(2\beta)^{1+l/2}\sqrt{\frac{\Gamma(n+1)}{\Gamma(n+l+2)}}.$$

The functions

$$T_{l\{\mu\}}(y)=T_{l\{\mu\}}(n_y)|y|^l, \quad n_y=\frac{y}{|y|}, \quad |y|=\sqrt{y^2}$$

satisfy the conditions

$$T_{l\{\mu_1\mu_2\dots\mu_l\}}(n)=T_{l\{\mu_2\mu_1\dots\mu_l\}}(n),$$

$$T_{l\{\mu\mu\mu_3\dots\mu_l\}}(n)=0,$$

$$\sum_{\{\mu\}}T_{l\{\mu\}}(n_1)T_{l\{\mu\}}(n_2)=\frac{1}{2^l}C_l^1((n_1n_2)),$$

$$C_l^1(1)=l+1,$$

where $C_l^1(t)$ are the Gegenbauer polynomials and

$$\int dn\,T_{l\{\mu\}}(n)T_{l'\{\mu'\}}(n)=\delta_{ll'}\delta_{\{\mu\}\{\mu'\}}\frac{2\pi^2}{2^l(l+1)}.$$

Besides, the following relation takes place:

$$\int d^4y\,T_{l\{\mu\}}(y)F(y^2)e^{-iky}=\left(\frac{-i}{2\pi}\right)^l T_{l\{\mu\}}(k)J(k^2), \tag{A4}$$

$$J(k^2)=\int dY\,e^{-iKY}F(Y^2),$$

$$K,Y\in\mathbf{R}^{4+2l}, \quad k^2=K^2.$$

APPENDIX B

Let us consider the variational problem (23) for the lowest state $(n=l=0)$ in the deconfinement limit $\Lambda\to 0$. We have

$$4\alpha_0\left(\frac{m}{\Lambda}\right)^2\max_{0<c<1}\left\{c(1-c)\int\int_0^1 dt\,ds\,\times e^{-(m^2/\Lambda^2-M_0^2/4\Lambda^2)(t+s)}\times\int\int_0^1 du\,dv\frac{\exp\left\{-\frac{M_0^2}{4\Lambda^2}\frac{(t-s)^2}{t+s+2c(u+v)}\right\}}{[t+s+2c(u+v)]^2}\right\}=1. \tag{B1}$$

Here M_0 is the mass of the lowest bound state and the effective coupling constant is supposed small

$$\alpha_0=\left(\frac{g}{4\pi m}\right)^2\ll 1.$$

Going to the new variables

$$t=\frac{\Lambda^2}{2m^2}(x+y), \quad s=\frac{\Lambda^2}{2m^2}(x-y), \quad c=\frac{\Lambda^2}{m^2}\xi$$

one can rewrite Eq. (B1) in the limit $\Lambda \to 0$ (which exists if $M_0 < 2m$) as follows:

$$2\alpha_0 \max_\xi \left\{ \xi \int_0^\infty dx\, e^{-(1-M_0^2/4m^2)x} \int\int_0^1 \frac{du\,dv}{[x+2\xi(u+v)]^2} \times \int_{-x}^{x} dy\, e^{-(M_0^2/4m^2)(y^2/[x+2\xi(u+v)])} \right\} = 1.$$

If $\alpha_0 \ll 1$ then $1 - M_0/2m \ll 1$ and the main contribution to the integral over dx comes from large x, so that the inner integral over dy can be explicitly taken on the extended interval $\{-\infty,\infty\}$. Thus, we get

$$\frac{4m\alpha_0}{M_0}\sqrt{\frac{\pi}{1-M_0^2/4m^2}}C = 1, \tag{B2}$$

$$C = \max_{0<\xi<\infty} \left\{ \xi \int_0^\infty dx\, e^{-x} \times \int\int_0^1 \frac{du\,dv}{[x+2\xi(u+v)]^{3/2}} \right\} = 0.31923\ldots.$$

By solving Eq. (B2) one obtains the mass of the lowest two-particle bound state in the deconfinement limit $\Lambda \to 0$ as follows:

$$M_0 = 2m - \frac{\alpha_0^2}{2} mK + O(\alpha_0^4), \quad K = 2\pi C^2 = 0.6403\ldots.$$

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