Final-state phases in doubly Cabibbo-suppressed charmed meson nonleptonic decays

Cheng-Wei Chiang*
Enrico Fermi Institute and Department of Physics, University of Chicago, 5640 S. Ellis Avenue, Chicago, Illinois 60637 and HEP Division, Argonne National Laboratory, 9700 S. Cass Avenue, Argonne, Illinois 60439

Jonathan L. Rosner†
Enrico Fermi Institute and Department of Physics, University of Chicago, 5640 S. Ellis Avenue, Chicago, Illinois 60637
(Received 8 November 2001; published 28 January 2002)

Cabibbo-favored nonleptonic charmed particle decays exhibit large final-state phase differences in $\bar{K}\pi$ and $\bar{K}^*\pi$ but not $\bar{K}\rho$ channels. It is of interest to know the corresponding pattern of final-state phases in doubly Cabibbo-suppressed decays, governed by the $c\to du\bar{s}$ subprocess. An experimental program is outlined for determining such phases via measurements of rates for $D\to K^*\pi$ and $K(\rho,\omega,\phi)$ channels and for determining the interference between bands in Dalitz plots. Such a program is feasible at planned high-intensity sources of charmed particles.

DOI: 10.1103/PhysRevD.65.054007 PACS number(s): 13.25.Ft, 11.30.Hv, 14.40.Lb

I. INTRODUCTION

The observation of direct CP violation in decays of particles containing heavy (c,b) quarks requires two or more channels differing in both strong and weak phases. Whereas the weak phases can be anticipated within the standard model based on the Cabibbo-Kobayashi-Maskawa matrix, the strong phases must in general be extracted from experiment. This is particularly so in the case of charmed particle decays, where phases in some channels have been shown to be large. (For particles containing b quarks, schemes for calculating such phases have been proposed recently [1,2].)

In Cabibbo-favored decays of charmed particles, governed by the subprocess $c\to su\bar{d}$, the pattern of final-state phases differs from channel to channel. In the decays $D\to\bar{K}\pi$ and $D\to\bar{K}^*\pi$, the final states with isospins $I=1/2$ and $I=3/2$ have relative phases close to 90°, while in $D\to\bar{K}\rho$, the $I=1/2$ and $I=3/2$ final states have relative phases close to zero. This behavior has been traced using an SU(3) flavor analysis [3] to a sign flip in the contribution of one of the amplitudes contributing to the $\bar{K}\rho$ processes in comparison with its contribution to the other two.

The corresponding final-state phases for doubly Cabibbo-suppressed charmed particle decays, governed by the subprocess $c\to du\bar{s}$, are of interest for several reasons. First, they are needed whenever one wishes to study CP asymmetries in such decays. Such asymmetries are not expected in the standard model, but the low rate for such processes makes them especially sensitive in their CP asymmetries to non-standard contributions. Second, the question of whether final-state phases are the same in CP-conjugate states such as $K^+\pi^-$ and $K^-\pi^+$ [4–7] is of current interest in interpreting D^0–$\bar{D}^0$ mixing results. Proposals for shedding light on this question include using the correlations between D^0 and $\bar{D}^0$ at the $\psi(3770)$ [8], and assuming relations among phase shifts in different $K^*\pi$ channels with the same isospin [9].

It is easy to determine relative final-state phases in Cabibbo-favored D decays since there are three charge states (such as $D^0\to K^-\pi^+$, $D^0\to\bar{K}^0\pi^0$, and $D^+\to\bar{K}^0\pi^+$) and only two independent amplitudes. The amplitudes for the three processes thus form a triangle in the complex plane as a result of the definite isospin of the $c\to su\bar{d}$ subprocess: $\Delta I=\Delta I_3=1$. We shall refer to such decays as "right-sign" decays. In contrast, the subprocess $c\to du\bar{s}$ governing doubly Cabibbo-suppressed decays, which we shall call "wrong-sign" decays, has $\Delta I_3=0$ and either $\Delta I=0$ or $\Delta I=1$. There are four charge states (e.g., $D^0\to K^+\pi^-$, $D^0\to K^0\pi^0$, $D^+\to K^+\pi^0$, and $D^+\to K^0\pi^+$) and three isospin amplitudes (two with $I=1/2$ and one with $I=3/2$), so that the amplitudes form a quadrangle. Without additional assumptions or information, one cannot learn relative phases.

The right-sign amplitude triangle for two final-state pseudoscalar mesons is related by a U-spin transformation [10] $(d\leftrightarrow s)$ to a corresponding triangle involving the two wrong-sign D^0 decays (to $K^+\pi^-$ and $K^0\pi^0$) and the decay $D_s\to K^0K^+$ [7]. However, the final states involving K^0 cannot be distinguished from the much-more-copious right-sign final states involving $\bar{K}^0$. If one replaces a K^0 by a K^{*0}, one can learn its flavor by its decay to $K^+\pi^-$. However, in the case of D decays to a vector meson and a pseudoscalar meson, the U-spin transformation turns out not to give a useful relation because of the lack of symmetry under interchange of the two final particles. One can estimate final-state phases for the wrong-sign $D\to K\pi$ decays with the help of information about direct-channel resonances and form factors [7].

Using the wrong-sign decays $D\to K^*\pi$, for which one can determine the flavor of the K^* for all four charge states, Golowich and Pakvasa [9] obtained a constraint sufficient to specify relative phases of amplitudes (given measurements of all four rates) by assuming that the final-state phases in the two $I=1/2$ $K^*\pi$ amplitudes are equal. Since this assumption is risky for a highly inelastic channel such as $K^*\pi$ at the

*Email address: chengwei@hep.uchicago.edu
†Email address: rosner@hep.uchicago.edu

mass of the D, we seek an alternative method which employs only experimental data. We have found such a method which relies upon interference of K^* bands in the $K^+\pi^-\pi^0$ Dalitz plot. In the course of this study, we find that all the relative phases of wrong-sign D decay amplitudes with one pseudoscalar meson P and one vector meson V in the final state can be specified using just $K\pi\pi$ and $KK\overline{K}$ final states. These predictions can then be checked in cases where a π^0 is replaced by an η or η'.

We begin in Sec. II with a decomposition of amplitudes for $D\to PP$ and $D\to PV$ final states. We point out relations among these in Sec. III, and discuss experimental prospects for testing them in Sec. IV. Section V concludes.

II. AMPLITUDE DECOMPOSITIONS

We can categorize decay amplitudes according to the topology of Feynman diagrams [11]: (1) a color-favored tree amplitude T, (2) a color-suppressed tree amplitude C, (3) an exchange amplitude E, and (4) an annihilation amplitude A. E only contributes to D^0 decays, and A only to Cabibbo-favored D_s^+ decays and Cabibbo-suppressed D^+ decays. The Cabibbo-favored non-leptonic two-body decays are governed by the subprocess $c\to su\overline{d}$ involving the weak coupling $V_{cs}^*V_{ud}$, while the doubly Cabibbo-suppressed ones are governed by the subprocess $c\to du\overline{s}$ involving the weak coupling $V_{cd}^*V_{us}$. We use notation introduced in Ref. [12] for PV decays in which a subscript denotes the meson (P or V) containing the spectator quark.

We can decompose the decay amplitudes both in terms of their topological characters and in terms of isospin structure. We use the following quark content and phase conventions [11]: *charmed mesons*: $D^0=-c\overline{u}$, $D^+=c\overline{d}$, $D_s^+=c\overline{s}$; *pseudoscalar mesons*: $\pi^+=u\overline{d}$, $\pi^0=(d\overline{d}-u\overline{u})/\sqrt{2}$, $\pi^-=-d\overline{u}$, $K^+=u\overline{s}$, $K^0=d\overline{s}$, $\overline{K}^0=s\overline{d}$, $K^-=-s\overline{u}$, $\eta=(s\overline{s}-u\overline{u}-d\overline{d})/\sqrt{3}$, $\eta'=(u\overline{u}+d\overline{d}+2s\overline{s})/\sqrt{6}$; and *vector mesons*: $\rho^+=u\overline{d}$, $\rho^0=(d\overline{d}-u\overline{u})/\sqrt{2}$, $\rho^-=-d\overline{u}$, $\omega=(u\overline{u}+d\overline{d})/\sqrt{2}$, $K^{*+}=u\overline{s}$, $K^{*0}=d\overline{s}$, $\overline{K}^{*0}=s\overline{d}$, $K^{*-}=-s\overline{u}$, $\phi=s\overline{s}$.

The wrong-sign (WS) D decays are listed in Tables I and II, where SU(3) flavor symmetry is assumed. We distinguish the amplitudes obtained through $I=1$ and $I=0$ currents by superscripts 1 and 0 on the amplitudes $A_{1/2}$ and $B_{1/2}$. We list the isospin decompositions only for $K\pi$ and $K^*\pi$ modes. It is the amplitudes $B_{1/2}^1$ and $B_{1/2}^0$ which were assumed to have the same strong phases in Ref. [9]. As mentioned, we make no such assumption. For some of the other decays we list simplified expressions which arise from assuming relations between different E or A amplitudes. As in Ref. [3], we omit contributions of flavor topologies in which η and η' exchange no quark lines with the rest of the diagram, and couple through their SU(3)-singlet components. This assumption, which goes beyond a purely SU(3)-based analysis, appeared to give a self-consistent description in the case of most right-sign (RS) decays with the exception of $D_s^+\to\rho^+\eta'$. We shall see that it can be tested in the case of WS decays, since the individual T, C, E, and A amplitudes can be predicted independently of modes involving η and η'.

TABLE I. Amplitudes for WS decay modes of charmed mesons to two pseudoscalar mesons.

Mode	A_{topology}	A_{isospin}
$D^0\to K^+\pi^-$	$T+E$	$\frac{1}{3}(A_{3/2}-A_{1/2}^1)-\frac{1}{\sqrt{3}}A_{1/2}^0$
$D^0\to K^0\pi^0$	$\frac{1}{\sqrt{2}}(C-E)$	$\frac{\sqrt{2}}{3}A_{3/2}+\frac{1}{3\sqrt{2}}A_{1/2}^1+\frac{1}{\sqrt{6}}A_{1/2}^0$
$D^0\to K^0\eta$	$\frac{1}{\sqrt{3}}C$	
$D^0\to K^0\eta'$	$-\frac{1}{\sqrt{6}}(C+3E)$	
$D^+\to K^0\pi^+$	$C+A$	$\frac{1}{3}(A_{3/2}-A_{1/2}^1)+\frac{1}{\sqrt{3}}A_{1/2}^0$
$D^+\to K^+\pi^0$	$\frac{1}{\sqrt{2}}(T-A)$	$\frac{\sqrt{2}}{3}A_{3/2}+\frac{1}{3\sqrt{2}}A_{1/2}^1-\frac{1}{\sqrt{6}}A_{1/2}^0$
$D^+\to K^+\eta$	$-\frac{1}{\sqrt{3}}T$	
$D^+\to K^+\eta'$	$\frac{1}{\sqrt{6}}(T+3A)$	
$D_s^+\to K^+K^0$	$T+C$	

III. AMPLITUDE RELATIONS

The RS $D\to\overline{K}^*\pi$ decays give the sum rule

$$A(D^0\to K^{*-}\pi^+)+\sqrt{2}A(D^0\to\overline{K}^{*0}\pi^0)-A(D^+\to\overline{K}^{*0}\pi^+)=0, \tag{1}$$

which forms a triangle in the amplitude complex plane. This triangle, and corresponding ones for $D\to\overline{K}\pi$ and $D\to\overline{K}\rho$, have been used to obtain relative phases between the unique $I=1/2$ and $I=3/2$ amplitudes contributing to each set of processes [3,13].

The sum rules for WS $D\to PP$ decays [14],

$$3\sqrt{2}A(K^+\pi^0)+4\sqrt{3}A(K^+\eta)+\sqrt{6}A(K^+\eta')=0, \tag{2}$$

$$3\sqrt{2}A(K^0\pi^0)-4\sqrt{3}A(K^0\eta)-\sqrt{6}A(K^0\eta')=0, \tag{3}$$

allow one to form triangles. In terms of amplitudes of different topologies, these are, respectively,

$$3(T-A)-4T+(T+3A)=0, \tag{4}$$

$$3(C-E)-4C+(C+3E)=0. \tag{5}$$

TABLE II. Amplitudes for WS decay modes of charmed mesons to one vector meson and one pseudoscalar meson.

Mode	$A_{\rm topology}$	$A_{\rm isospin}$
$D^0 \to K^{*+}\,\pi^-$	T_P+E_V	$\frac{1}{3}(B_{3/2}-B^1_{1/2})-\frac{1}{\sqrt{3}}B^0_{1/2}$
$D^0 \to K^{*0}\,\pi^0$	$\frac{1}{\sqrt{2}}(C_P-E_V)$	$\frac{\sqrt{2}}{3}B_{3/2}+\frac{1}{3\sqrt{2}}B^1_{1/2}+\frac{1}{\sqrt{6}}B^0_{1/2}$
$D^+ \to K^{*0}\,\pi^+$	C_P+A_V	$\frac{1}{3}(B_{3/2}-B^1_{1/2})+\frac{1}{\sqrt{3}}B^0_{1/2}$
$D^+ \to K^{*+}\,\pi^0$	$\frac{1}{\sqrt{2}}(T_P-A_V)$	$\frac{\sqrt{2}}{3}B_{3/2}+\frac{1}{3\sqrt{2}}B^1_{1/2}-\frac{1}{\sqrt{6}}B^0_{1/2}$
Mode	$A_{\rm topology}$	$A_{\rm simplified}$
$D^0 \to \phi\, K^0$	$-E_V$	
$D^0 \to \rho^-\, K^+$	T_V+E_P	T_V-E_V
$D^0 \to \rho^0\, K^0$	$\frac{1}{\sqrt{2}}(C_V-E_P)$	$\frac{1}{\sqrt{2}}(C_V+E_V)$
$D^0 \to \omega\, K^0$	$-\frac{1}{\sqrt{2}}(C_V+E_P)$	$-\frac{1}{\sqrt{2}}(C_V-E_V)$
$D^0 \to K^{*0}\,\eta$	$\frac{1}{\sqrt{3}}(C_P-E_P+E_V)$	$\frac{1}{\sqrt{3}}(C_P+2E_V)$
$D^0 \to K^{*0}\,\eta'$	$-\frac{1}{\sqrt{6}}(C_P+2E_P+E_V)$	$-\frac{1}{\sqrt{6}}(C_P-E_V)$
$D^+ \to \phi\, K^+$	A_V	
$D^+ \to \rho^+\, K^0$	C_V+A_P	C_V-A_V
$D^+ \to \rho^0\, K^+$	$\frac{1}{\sqrt{2}}(T_V-A_P)$	$\frac{1}{\sqrt{2}}(T_V+A_V)$
$D^+ \to \omega\, K^+$	$\frac{1}{\sqrt{2}}(T_V+A_P)$	$\frac{1}{\sqrt{2}}(T_V-A_V)$
$D^+ \to K^{*+}\,\eta$	$-\frac{1}{\sqrt{3}}(T_P-A_P+A_V)$	$-\frac{1}{\sqrt{3}}(T_P+2A_V)$
$D^+ \to K^{*+}\,\eta'$	$\frac{1}{\sqrt{6}}(T_P+2A_P+A_V)$	$\frac{1}{\sqrt{6}}(T_P-A_V)$
$D_s^+ \to K^{*+}\, K^0$	T_P+C_V	
$D_s^+ \to K^{*0}\, K^+$	T_V+C_P	

The sum rules

$$A(K^+\pi^-)+\sqrt{2}A(K^0\pi^0)=A(K^0\pi^+)+\sqrt{2}A(K^+\pi^0)$$

$$=\sqrt{3}[A(K^0\eta)-A(K^+\eta)]=A(K^+K^0) \qquad (6)$$

give triangles all sharing one side. This can be seen from the decomposed amplitudes

$$(T+E)+(C-E)=(C+A)+(T-A)=T+C. \qquad (7)$$

We also find from these WS $D \to PP$ modes the following relations:

$$|T|^2=3|A(K^+\eta)|^2, \qquad (8)$$

$$|C|^2=3|A(K^0\eta)|^2, \qquad (9)$$

$$|A|^2=\frac{1}{2}[|A(K^+\pi^0)|^2+|A(K^+\eta')|^2]-|A(K^+\eta)|^2, \qquad (10)$$

$$|E|^2=\frac{1}{2}[|A(K^0\pi^0)|^2+|A(K^0\eta')|^2]-|A(K^0\eta)|^2, \quad (11)$$

$$\cos\delta_{TC}=\frac{1}{2|T||C|}[|A(K^+K^0)|^2-3|A(K^+\eta)|^2 -3|A(K^0\eta)|^2], \quad (12)$$

$$\cos\delta_{TA}=\frac{1}{2|T||A|}\left[2|A(K^+\eta)|^2+\frac{1}{2}|A(K^+\eta')|^2 -\frac{3}{2}|A(K^+\pi^0)|^2\right], \quad (13)$$

$$\cos\delta_{CE}=\frac{1}{2|C||E|}\left[2|A(K^0\eta)|^2+\frac{1}{2}|A(K^0\eta')|^2 -\frac{3}{2}|A(K^0\pi^0)|^2\right], \quad (14)$$

$$\cos\delta_{TE}=\frac{1}{2|T||E|}\left\{|A(K^+\pi^-)|^2-3|A(K^+\eta)|^2 -\frac{1}{2}[|A(K^0\pi^0)|^2+|A(K^0\eta')|^2]+|A(K^0\eta)|^2\right\}, \quad (15)$$

$$\cos\delta_{CA}=\frac{1}{2|C||A|}\left\{|A(K^0\pi^+)|^2-3|A(K^0\eta)|^2 -\frac{1}{2}[|A(K^+\pi^0)|^2+|A(K^+\eta')|^2] +|A(K^+\eta)|^2\right\}. \quad (16)$$

Therefore, knowing the absolute value of the decay amplitudes one could completely determine the above triangles. However, all decays involving a K^0 will be overwhelmed by Cabibbo-favored decays involving a $\overline{K}^0$, with no way to distinguish between them since one detects only a K_S. Thus in practice one is able to determine only $|T|$, $|A|$, and δ_{TA}, which is still a useful piece of information relevant to final-state interactions. We shall discuss the prospects for this determination in Sec. IV.

The WS $D\to K^*\pi$ decays give the sum rule

$$\begin{aligned}A(K^{*+}\pi^-)+\sqrt{2}A(K^{*0}\pi^0)&=A(K^{*0}\pi^+)+\sqrt{2}A(K^{*+}\pi^0)\\ &=(T_P+E_V)+(C_P-E_V)=(C_P+A_V)+(T_P-A_V)\\ &=T_P+C_P,\end{aligned} \quad (17)$$

which forms a quadrangle in the complex plane, as shown in Fig. 1.

Knowing the lengths of the four sides in a quadrangle does not fix the shape; one still needs information about relative angles among the sides. In principle such information could be obtained from other sum rules involving any two of the decay modes related to the sides of the quadrangle in which we are interested. However, these were searched for in Ref. [7], and no such triangle sum rule exists for these WS decays.

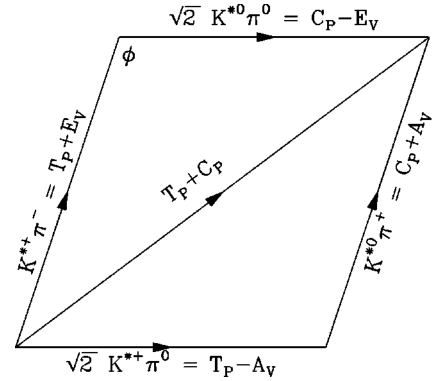

FIG. 1. Quadrangle illustrating amplitude relations for $D\to K^*\pi$ decays. The other diagonal (not shown) corresponds to the combination E_V+A_V.

Fortunately, one can use interference between the two K^* bands on the Dalitz plot for $D^0\to K^+\pi^-\pi^0$, a final state recently reported by the CLEO Collaboration [15], to measure the relative phase ϕ between the amplitudes for $D^0\to K^{*+}\pi^-$ and $D^0\to K^{*0}\pi^0$. This method is analogous to the use of the decay $D^0\to K_S\pi^+\pi^-$ in which the interference between $K^{*+}\pi^-$ and $K^{*-}\pi^+$ bands provides direct information on the relative strong phase difference between the two channels [16,17]. Once the angle ϕ in Fig. 1 is specified, the shape of the quadrangle is fixed up to a folding about the diagonal. However, this is still not sufficient to specify each individual amplitude T_P, C_P, E_V, or A_V.

One way to help resolve the above ambiguity is to compare the WS quadrangle with the RS triangle [Eq. (1)]. Denote the relative phase between $D^0\to K^{*-}\pi^+$ and $D^0\to K^{*+}\pi^-$ by θ_0, that between $D^+\to\overline{K}^{*0}\pi^+$ and $D^+\to K^{*+}\pi^0$ by θ_+, and that between $D^0\to K^{*-}\pi^+$ and $D^+\to\overline{K}^{*0}\pi^+$ by ψ. θ_0 can be obtained by analyzing the K^{*+} and K^{*-} bands in the Dalitz plot of the final state $D^0\to K_S\pi^+\pi^-$; θ_+ can be similarly measured from the Dalitz plot of $D^+\to K_S\pi^+\pi^0$. With ψ given by the RS triangle, the relative phase between $D^0\to K^{*+}\pi^-$ and $D^+\to K^{*+}\pi^0$ is then $\psi\pm|\theta_0|\pm|\theta_+|$. Therefore, except for singular cases, the angle between the left and bottom sides of the quadrangle in Fig. 1 can be determined.

One also makes further progress by assuming [3] that (1) $A_P=-A_V$ and/or (2) $E_P=-E_V$. These assumptions are valid if these amplitudes involve an intermediate quark-antiquark state [18].

If only $A_P=-A_V$ is imposed, several of the expressions for D^+ decays are simplified. We find $A(K^{*+}\pi^0)=\sqrt{3}A(K^{*+}\eta')$ and the following sum rules:

$$A(K^{*0}K^+)-\sqrt{2}A(\omega K^+)-A(K^{*0}\pi^+)=0, \quad (18)$$

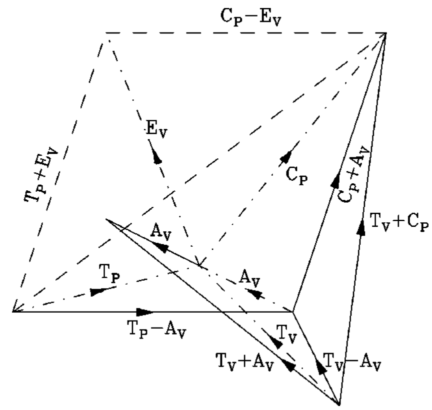

FIG. 2. Amplitude triangles illustrating amplitude relations between $D^+ \to K^* \pi$ decays and other D^+ or D_s^+ decays. The dot-dashed lines represent the individual amplitudes.

$$\sqrt{2}A(\rho^0 K^+) - \sqrt{2}A(\omega K^+) - 2A(\phi K^+) = 0, \qquad (19)$$

$$\sqrt{3}A(K^{*+}\eta) + \sqrt{2}A(K^{*+}\pi^0) + 3A(\phi K^+) = 0. \qquad (20)$$

In terms of amplitudes, these read, respectively,

$$(T_V + C_P) - (T_V - A_V) - (C_P + A_V) = 0, \qquad (21)$$

$$(T_V + A_V) - (T_V - A_V) - 2A_V = 0, \qquad (22)$$

$$-(T_P + 2A_V) + (T_P - A_V) + 3A_V = 0. \qquad (23)$$

The first two of these are illustrated in Fig. 2. Measurement of the corresponding rates for $D_s \to K^{*0} K^+$ and $D^+ \to (\rho^0, \omega, \phi) K^+$ along with the four $D \to K^* \pi$ rates and the relative phase of $D^0 \to K^{*+}\pi^-$ and $D^0 \to K^{*0}\pi^0$ mentioned earlier can specify the individual amplitudes up to the discrete ambiguity associated with reflection about the dashed diagonal of the quadrangle. This ambiguity affects only the phase and magnitude of E_V with respect to the other amplitudes. Since we have not used Eq. (20) in this construction, we obtain a prediction for the amplitude $A(K^{*+}\eta)$. The residual ambiguity can be removed if one assumes a certain magnitude hierarchy among T, C, and E.

Under the assumption $A_P = -A_V$ we also find from the WS $D^+ \to VP$ modes the following relations:

$$|A_V|^2 = |A(\phi K^+)|^2, \qquad (24)$$

$$|T_V|^2 = |A(\rho^0 K^+)|^2 + |A(\omega K^+)|^2 - |A(\phi K^+)|^2, \qquad (25)$$

$$|T_P|^2 = 4|A(K^{*+}\eta')|^2 + |A(K^{*+}\eta)|^2 - 2|A(\phi K^+)|^2, \qquad (26)$$

$$\cos\delta_{T_V A_V} = \frac{1}{2|T_V||A_V|}[|A(\rho^0 K^+)|^2 - |A(\omega K^+)|^2], \qquad (27)$$

$$\cos\delta_{T_P A_V} = \frac{1}{2|T_P||A_V|}[|A(K^{*+}\eta)|^2 - 2|A(K^{*+}\eta')|^2 - |A(\phi K^+)|^2]. \qquad (28)$$

As in the WS $D^+ \to PP$ decays, we can learn both the magnitudes and the relative phases of the T and A amplitudes directly from decay rates involving observable final states.

If now $E_P = -E_V$ is assumed, some of the expressions in D^0 decays are simplified. One finds $A(K^{*0}\pi^0) = -\sqrt{3}A(K^{*0}\eta')$ and the following sum rules:

$$A(K^{*+}\pi^-) - \sqrt{2}A(\omega K^0) - A(K^{*+}K^0) = 0, \qquad (29)$$

$$\sqrt{2}A(\rho^0 K^0) + \sqrt{2}A(\omega K^0) + 2A(\phi K^0) = 0, \qquad (30)$$

$$\sqrt{3}A(K^{*0}\eta) - \sqrt{2}A(K^{*0}\pi^0) + 3A(\phi K^0) = 0. \qquad (31)$$

These have the following form in terms of amplitudes:

$$(T_P + E_V) + (C_V - E_V) - (T_P + C_V) = 0, \qquad (32)$$

$$(C_V + E_V) - (C_V - E_V) - 2E_V = 0, \qquad (33)$$

$$(C_P + 2E_V) - (C_P - E_V) - 3E_V = 0. \qquad (34)$$

For these modes, we obtain the following relations:

$$|E_V|^2 = |A(\phi K^0)|^2, \qquad (35)$$

$$|C_V|^2 = |A(\rho^0 K^0)|^2 + |A(\omega K^0)|^2 - |A(\phi K^0)|^2, \qquad (36)$$

$$|C_P|^2 = 4|A(K^{*0}\eta')|^2 + |A(K^{*0}\eta)|^2 - 2|A(\phi K^0)|^2, \qquad (37)$$

$$\cos\delta_{C_V E_V} = \frac{1}{2|C_V||E_V|}[|A(\rho^0 K^0)|^2 - |A(\omega K^0)|^2], \qquad (38)$$

$$\cos\delta_{C_P E_V} = \frac{1}{2|C_P||E_V|}[|A(K^{*0}\eta)|^2 - 2|A(K^{*0}\eta')|^2 - |A(\phi K^0)|^2]. \qquad (39)$$

These relations all suffer from the presence of a K^0 in at least one of their amplitudes, and contamination by the corresponding mode with $\overline{K^0}$ makes them unusable. However, the fact that with $E_P = -E_V$ we also have amplitudes for the observable processes $D^0 \to (\rho^- K^+, K^{*0}\eta, K^{*0}\eta')$, all of which involve E_V and amplitudes which have been previously specified, should allow the resolution of the last remaining discrete ambiguity except in singular cases.

An analysis of SU(3) breaking based on the method of Ref. [7] may be able to provide direct information on relative strong phases in Cabibbo-favored and doubly Cabibbo-suppressed $D \to PV$ decays. One needs information on direct-channel resonances with $J^P = 0^-$, which is the only channel which can decay to the $J = 0$ PV state. A candidate for such a state around 1830 MeV (i.e., not far from the D mass) has been reported in the $K\phi$ channel [19] but needs confirmation.

IV. EXPERIMENTAL PROSPECTS

At present, the following WS modes are quoted by the Particle Data Group [22]:

$$\mathcal{B}(D^0 \to K^+\pi^-) = (1.46 \pm 0.30) \times 10^{-4},$$

$$\mathcal{B}(D^+ \to K^{*0}\pi^+) = (3.6 \pm 1.6) \times 10^{-4},$$

$$\mathcal{B}(D^+ \to \rho^0 K^+) = (2.5 \pm 1.2) \times 10^{-4},$$

$$\mathcal{B}(D^+ \to \phi K^+) < 1.3 \times 10^{-4} \qquad (\mathrm{CL} = 90\%)$$

(where CL is the confidence level). If one assumes that the amplitude T is dominant in PP modes, from the branching ratio of $D^0 \to K^+\pi^-$ one would infer $\mathcal{B}(D^+ \to K^+\pi^0) \simeq 1.8 \times 10^{-4}$ and $\mathcal{B}(D^+ \to K^+\eta) \simeq 1.2 \times 10^{-4}$. A substantial deviation from these expected values would indicate the importance of E and/or A contributions.

Since the peak cross section for $e^+e^- \to \psi(3770) \to D\bar{D}$ is about 10 nb and the foreseen integrated luminosity for a charm factory operating at this energy is about 3 fb^{-1}, one expects to collect 3×10^7 $D\bar{D}$ pairs, giving about 15 million D^0 ($\bar{D}^0$) and 15 million $D^+(D^-)$. With branching ratios of $O(10^{-4})$ for the WS decays, we would have ~ 3000 events for each type. The D^0 decays must be flavor-tagged through the study of the flavor of the opposite-side neutral D.

Tagging via the chain $D^{*+} \to \pi^+ D^0$ is possible if one operates at higher c.m. energy. Indeed, it is estimated that in CLEO II.V with 6 fb^{-1} on the $\Upsilon(4S)$ and 3 fb^{-1} in the continuum below the $\Upsilon(4S)$, 34 million charmed mesons were produced [16]. BaBar and Belle should be able to accumulate an even larger sample.

In the analysis of $D \to PV$ decays, one needs to analyze the branching ratios and resonant channel fractions of the set of three-body final states listed in Table III. Examples of recent progress in studying these states are noted in Refs. [15,17,20,21].

TABLE III. Summary of doubly Cabibbo-suppressed three-body modes required for extracting amplitudes in $D \to PV$ decays. All modes with a K^0 have $\bar{K}^0$ backgrounds. D^+ and D_s^+ modes with a K^+ are self-tagging.

	Final state	Branching ratio
D^0	$K^0\pi^+\pi^-$	
	$K^+\pi^-\pi^0$	$(6.0 \pm 1.0) \times 10^{-4}$ [15]
	$K^+\pi^-\eta$	
	$K^+\pi^-\eta'$	
D^+	$K^0\pi^+\pi^0$	
	$K^0\pi^+\eta$	
	$K^0\pi^+\eta'$	
	$K^+\pi^0\pi^0$	
	$K^+\pi^0\eta$	
	$K^+\pi^0\eta'$	
	$K^+\pi^-\pi^+$	$(6.8 \pm 1.5) \times 10^{-4}$ [22]; see also [20]
	$K^+K^+K^-$	$(1.41 \pm 0.27) \times 10^{-4}$ [21]
D_s^+	$K^0K^0\pi^+$	
	$K^0K^+\pi^0$	
	$K^+K^+\pi^-$	

V. CONCLUSIONS

As we have seen, doubly Cabibbo-suppressed ("wrong-sign" or WS) decays with a final neutral K meson in general suffer from overwhelming backgrounds of Cabibbo-favored ("right-sign" or RS) decays. It is thus preferable to extract information from decay modes with charged K mesons in the final states. We have shown that the amplitudes for the D^+ decay modes $K^+\pi^0, K^+\eta, K^+\eta'$ form a triangle in the complex plane. These charged D decays provide a good place to study the amplitudes $|T|$, $|A|$ and the relative strong phase $\cos\delta_{TA}$. It will be interesting to see whether in the case of WS D decays one still observes A and E with comparable amplitudes to T and C as in the RS decays [3]. It will also be useful to compare U-spin related RS and WS triangles to see whether they are similar, from which one could learn final state interaction patterns and U-spin breaking effects.

We also observed that without further assumptions, one could only form quadrangle relations from the amplitudes for $D \to PV$ decays. For example, the four $D \to K^*\pi$ amplitudes form a quadrangle. The relative phase between the neutral D amplitudes can be obtained by analyzing the $D^0 \to K^+\pi^-\pi^0$ Dalitz plot. This fixes the quadrangle up to a twofold ambiguity corresponding to folding about the diagonal. By further assuming $A_P = -A_V$, we can obtain three triangle relations and determine $|T_V|$, $|T_P|$, $|A_V|$, $\cos\delta_{T_V A_V}$, and $\cos\delta_{T_P A_V}$. The twofold quadrangle ambiguity can be resolved by assuming $E_P = -E_V$ and measuring the rate for $D^0 \to K^+\rho^-$. Many cross-checks of the method are possible by measuring further WS rates for three-body decays involving η or η' and by analyses of interferences between right-sign and wrong-sign $K^*\pi$ contributions to Dalitz plots.

ACKNOWLEDGMENTS

We thank David Asner for stimulating discussions and Michael Gronau, John Cumalat, and Harry Lipkin for helpful comments. This work was supported in part by the Department of Energy through Grants No. DE-FG02-90ER-40560 and W-31109-ENG-38.

[1] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Nucl. Phys. **B606**, 245 (2001).
[2] Y.Y. Keum, H.n. Li, and A.I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008 (2001).
[3] J.L. Rosner, Phys. Rev. D **60**, 114026 (1999).
[4] L.L. Chau and H.Y. Cheng, Phys. Lett. B **333**, 514 (1994).
[5] T.E. Browder and S. Pakvasa, Phys. Lett. B **383**, 475 (1996).
[6] A.F. Falk, Y. Nir, and A.A. Petrov, J. High Energy Phys. **12**, 019 (1999).
[7] M. Gronau and J.L. Rosner, Phys. Lett. B **500**, 247 (2001).
[8] M. Gronau, Y. Grossman, and J.L. Rosner, Phys. Lett. B **508**, 37 (2001).
[9] E. Golowich and S. Pakvasa, Phys. Lett. B **505**, 94 (2001).
[10] S. Meshkov, C.A. Levinson, and H.J. Lipkin, Phys. Rev. Lett. **10**, 361 (1963).
[11] M. Gronau, O.F. Hernandez, D. London, and J.L. Rosner, Phys. Rev. D **50**, 4529 (1994).
[12] A.S. Dighe, M. Gronau, and J.L. Rosner, Phys. Rev. D **57**, 1783 (1998).
[13] M. Suzuki, Phys. Rev. D **58**, 111504 (1998).
[14] H. J. Lipkin (private communication) has pointed out that the linear combination of η and η' which enter these sum rules is just the octet component, which should also be true for other choices of octet-singlet mixing for the η and η'.
[15] CLEO Collaboration, G. Brandenburg *et al.*, Phys. Rev. Lett. **87**, 071802 (2001).
[16] D. M. Asner (private communication).
[17] BABAR Collaboration, A. Palano, presented at Hadron 2001, IX International Conference on Hadron Spectroscopy, 2001, Protvino, Russia, SLAC report SLAC-PUB-9034, hep-ex/0111003.
[18] H.J. Lipkin, Phys. Lett. B **433**, 117 (1998); **445**, 403 (1999).
[19] Bari-Birmingham-CERN-Milan-Paris-Pavia Collaboration, T. Armstrong *et al.*, Nucl. Phys. **B221**, 1 (1983).
[20] FOCUS Collaboration, K. Stenson, presented at the 9th International Symposium on Heavy Flavor Physics, Caltech, Pasadena, California, 2001.
[21] FOCUS Collaboration, L. Moroni, presented at the International Europhysics Conference on High Energy Physics (EPS–HEP 99), Tampere, Finland, 1999. Published in *High Energy Physics 99: Proceedings*, edited by K. Huitu, H. Kurki-Suonio, and J. Maalampi (IOP, Bristol, 2000), p. 625. A more up-to-date branching ratio is in process (J. Cumalat, private communication).
[22] Particle Data Group, D.E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).