Nonleptonic Ω^- decays and the Skyrme model

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Nonleptonic Ω^- decay branching ratios are estimated by means of the QCD enhanced effective weak Hamiltonian supplemented by the SU(3) Skyrme model used to estimate the nonperturbative matrix elements. The model has only one free parameter, namely, the Skyrme charge e, which is fixed through the experimental values of the octet-decuplet mass splitting Δ and the axial vector coupling constant g_A. The whole scheme is equivalent to the one that works well for nonleptonic hyperon decays. The ratios of the calculated amplitudes are in agreement with experiment. However, the absolute values are about twice as large if short-distance corrections and only ground intermediate states are included.

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Recently both s- and p-wave nonleptonic hyperon decay amplitudes were quite successfully reproduced by the SU(3) extended Skyrme model with the QCD enhanced effective weak Hamiltonian [1]. The decay amplitudes were described through the current-algebra commutator, the ground-state baryon pole terms, and factorizable contributions. The nonperturbative quantities, i.e., the baryon four-quark operator matrix elements, were estimated using the SU(3) Skyrme model. For the s-wave hyperon decay amplitudes, correct relative signs and absolute magnitudes were obtained. For the p waves all relative signs were correct, with their relative magnitudes roughly following the experimental data. As far as the absolute magnitudes are concerned, the poorest agreement between theory and experiment was within a factor of 2. One is thus faced with an obvious question: could an analogous approach work equally well for the Ω^- nonleptonic weak decays? Such a question should be considered in connection with the accurate measurements of the Ω^- decay branching ratios. The experimental value for the Ω^- mean lifetime is [2]

$$\tau_{\text{experiment}}(\Omega^-) = 82.1 \pm 1.1 \ \ \text{ps},$$

from which the values of the p-wave amplitudes have been extracted.

In the Skyrme model, baryons emerge as soliton configurations of the field U of pseudoscalar mesons [3,4]. Extension of the model to the strange sector [5–8] is done by an isospin embedding of the static hedgehog ansatz into an SU(3) matrix that is the subject of a time dependent rotation

$$U(\vec{r},t) = A(t)\begin{pmatrix} \exp[i\vec{\tau}\cdot\vec{n}F(r)] & 0 \\ 0 & 1 \end{pmatrix} A^\dagger(t) \qquad (1)$$

by a collective coordinate matrix $A(t) \in \text{SU}(3)$, which defines the generalized velocities $A^\dagger(t)\dot{A}(t) = (i/2)\Sigma_{\alpha=1}^{8}\lambda_\alpha \dot{a}^\alpha$, and the profile function $F(r)$ is interpreted as a chiral angle that parametrizes the soliton. The collective coordinates a^α are canonically quantized to generate the states that possess the quantum numbers of the physical strange baryons. In order to account for a nonzero strange quark mass the appropriate chiral symmetry breaking (SB) terms should be included. In this work, however, following [1], we shall neglect the SB effects since they are much smaller than the uncertainties coming from other sources (such as, for example, the values of the c_i coefficients in the effective Hamiltonian, or factorization). Moreover, the SU(3) symmetry is used in the course of the calculations of the decay amplitudes, so for consistency the nonperturbative matrix elements have to be evaluated in the same approximation scheme.

Our goal is to see whether the effective weak Hamiltonian and the SU(3) Skyrme model are able to predict the nonleptonic Ω^- decay amplitudes (Ω_K):$\Omega^- \to \Lambda K^-$, (Ω^-_-):$\Omega^- \to \Xi^0\pi^-$, and (Ω^-_0):$\Omega^- \to \Xi^-\pi^0$ following the method of Refs. [1,9]. To this end we shall employ the standard model effective Hamiltonian and the minimal number of couplings concept of the Skyrme model to estimate the nonperturbative matrix elements of the four-quark operators [1,10] and the axial vector current form factor for the decuplet-octet transition. This approach uses only one free parameter, i.e., the Skyrme charge e. In order to avoid an unnecessary numerical burden, throughout this report we use the arctan ansatz for the Skyrme profile function $F(r)$ [7,11], which allows us to calculate the pertinent overlap integrals analytically with an accuracy of the order of a few percent with respect to the exact numerical results.

It is well known that the nonleptonic weak decays of baryons can be reasonably well described in the framework of the standard model [9,12]. The starting point in such an analysis is the effective weak Hamiltonian in the form of the current $\otimes$ current interaction, enhanced by quantum chromodynamics (QCD), i.e., obtained by integrating out the heavy-quark and W-boson fields,

$$H_w^{\text{eff}}(\Delta S=1) = \sqrt{2}G_F V_{ud}^* V_{us} \sum_{i=1}^{4} c_i O_i , \qquad (2)$$

where G_F is the Fermi constant and $V_{ud}^* V_{us}$ are the Cabibbo-Kobayashi-Maskawa matrix elements. The coefficients c_i are the well known Wilson coefficients [12], most recently evaluated in Ref. [13], and the O_i are the familiar four-quark operators [12,13]. For the purpose of this work we neglect the so called penguin operators since their contributions are proven to be small [12,13]. We are using the Wilson coefficients from Ref. [12]: $c_1=-1.90-0.61\zeta$, $c_2=0.14+0.020\zeta$, $c_3=c_4/5$, $c_4=0.49+0.005\zeta$, with $\zeta=V_{td}^*V_{ts}/V_{ud}^*V_{us}$. Without QCD short-distance corrections, the Wilson coefficients would have the following values: $c_1=-1$, $c_2=1/5$, $c_3=2/15$, $c_4=2/3$. In this paper we simply consider both possibilities and compare the resulting amplitudes.

The techniques used to describe nonleptonic Ω^- decays ($3/2^+\rightarrow 1/2^+ + 0^-$ reactions involve only p and d waves) are known as the modified current-algebra approach. The general form of the decay amplitude reads

$$\langle \pi(q)B'(p')|H_w^{\rm eff}|B(p)\rangle=\overline{\mathcal{U}}(p')[\mathcal{B}+\gamma_5\mathcal{C}]q^\mu\mathcal{W}_\mu(p). \tag{3}$$

Here $\mathcal{U}(p')$ denotes a regular spinor while $\mathcal{W}_\mu(p)$ is the Rarita-Schwinger spinor. The parity-conserving amplitudes $\mathcal{B}$ correspond to the p-wave and the parity-violating amplitudes $\mathcal{C}$ correspond to the d-wave Ω^- decays.

The decay probability $\Gamma(3/2^+\rightarrow 1/2^+ + 0^-)$ reads

$$\Gamma=\frac{|\mathbf{p}'|^3}{12\pi m_\Omega}[(E'+m_f)|\mathcal{B}|^2+(E'-m_f)|\mathcal{C}|^2],$$

$$|\mathbf{p}'|^2=[(m_\Omega^2-m_f^2+m_\phi^2)/2m_\Omega]^2-m_\phi^2,$$

$$E'=(m_\Omega^2+m_f^2-m_\phi^2)/2m_\Omega. \tag{4}$$

Here m_f denotes the final baryon mass and m_ϕ is the mass of the emitted meson. Obviously the parity-violating amplitude $\mathcal{C}$ (d wave) is multiplied in Γ by an unfavorable factor ($E'-m_f$). In our calculation framework, the amplitude $\mathcal{C}$ can receive contributions only from the pole diagrams. As these pole diagrams contain negative-parity $1/2^-$ and $3/2^-$ baryon resonances, the sum of baryon masses instead of the difference appears in the denominator. Parity-violating amplitudes are thus suppressed by a large factor. If the vertices in both $\mathcal{B}$ and $\mathcal{C}$ pole contributions are of the same order of magnitude, then the decay of Ω^- is almost parity conserving. This conclusion is the same as the one drawn in Ref. [14].

We calculate the parity-conserving p-wave Ω^- decay amplitudes $\mathcal{B}$ using the so-called tree-diagram approximation at the particle level. This means that in this work we take into account all possible tree diagrams involving baryons and mesons, i.e., factorizable and pole diagrams.

All $\mathcal{B}$ amplitudes receive contributions from the pole diagrams, as shown in Fig. 1, and they are as follows:

$$B_{\mathcal{P}}(\Omega_K)=\frac{g_{\Omega^-K^-\Xi^0}a_{\Lambda\Xi_0}}{m_{\Xi^0}-m_\Lambda}-\frac{g_{\Xi*^-K^-\Lambda}a_{\Xi*^-\Omega^-}}{m_{\Omega^-}-m_{\Xi*^-}},$$

$$B_{\mathcal{P}}(\Omega_-^-)=-\frac{g_{\Xi^0\Xi*^-\pi^-}a_{\Xi*^-\Omega^-}}{m_{\Omega^-}-m_{\Xi*^-}},$$

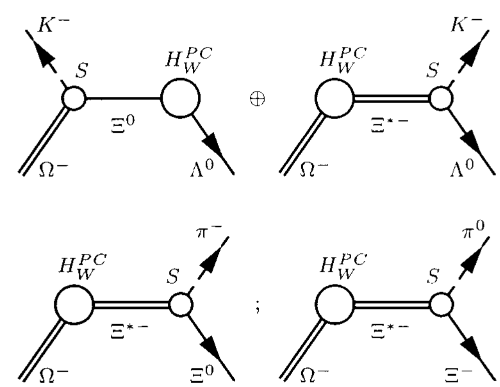

FIG. 1. Pole diagrams. The double lines are $3/2^+$ resonances. The full lines are hyperons and the dashed lines are mesons. Weak and strong vertices are indicated.

$$B_{\mathcal{P}}(\Omega_0^-)=-\frac{g_{\Xi^-\Xi*^-\pi^0}a_{\Xi*^-\Omega^-}}{m_{\Omega^-}-m_{\Xi*^-}}. \tag{5}$$

The decuplet-octet-meson strong coupling constants determined from the experimental value of the decay rate $\Delta^{++}\rightarrow p\pi^+$ and by using SU(3) relations are as follows:

$$g_{\Delta^{++}\pi^+p}=g_{\Xi^0\Omega^-K^-}=-g_{\Omega^-\Xi^-\pi^0}=\sqrt{2}g_{\Lambda^0\Xi*^-K^-}$$
$$=\sqrt{3}g_{\Xi^0\Xi*^-\pi^-}=\sqrt{6}g_{\Xi^-\Xi*^-\pi^0}=15.75\ \ \mathrm{GeV}^{-1}. \tag{6}$$

Note that the strong couplings $g_{\Delta N\pi}$ and $g_{NN\pi}$ are related by $g_{\Delta N\pi}=\frac{3}{2}g_{NN\pi}$, which follows from the $1/N_c$ expansion without other assumptions [4]. This relation is in excellent agreement with experiment and with the evaluation of $g_{NN\pi}$ and $g_{\Delta N\pi}$ in the Skyrme model [4]. Note also that $g_{NN\pi}$ and g_A^{NN} are related by the Goldberger-Treiman relation, which is also true in the Skyrme model. So altogether we can reproduce the experimental value of $g_{\Delta N\pi}$ if we use the Skyrme model parameters that reproduce g_A^{NN}. However, consistently following the approach of Ref. [1], we are using the Skyrme model to estimate only the unknown matrix elements. Therefore, it is natural to use the experimental value $g_{\Delta^{++}\pi^+p}=15.75\ \mathrm{GeV}^{-1}$ in the calculations of the baryon poles (5).

The baryon-pole $B_{\mathcal{P}}$ amplitudes contain weak matrix elements defined as $a_{BB'}=\mathcal{G}\langle B'|\Sigma_{i=1}^4 c_iO_i^{PC}|B\rangle$, where the constant $\mathcal{G}=\sqrt{2}G_FV_{ud}^*V_{us}$. The important parts of $a_{BB'}$ are the four-quark operator matrix elements, which are nonperturbative quantities. This is the first point of this work at which the Skyrme model is used.

The factorizable contributions to p waves [only for $(\Omega_{-,0}^-)$ amplitudes] are calculated by inserting the vacuum states; it is therefore a factorized product of two current matrix elements, where the octet-decuplet matrix element of the axial vector current reads

$$\langle \Xi(p')|A^\mu|\Omega^-(p)\rangle=g_A^{\Xi\Omega}(q^2)\overline{\mathcal{U}}_\Xi(p')\mathcal{W}_\Omega^\mu(p).$$

Summing over all factorizable contributions gives the following expressions for the amplitudes:

$$B_{\mathcal{S}}(\Omega^-_-)=\frac{1}{3\sqrt{2}}\mathcal{G}f_\pi g_A^{\Xi^0\Omega^-}[c_1-2(c_2+c_3+c_4)],$$

$$B_{\mathcal{S}}(\Omega^-_0)=\frac{1}{6}\mathcal{G}f_\pi g_A^{\Xi^-\Omega^-}[c_1-2(c_2+c_3-2c_4)]. \tag{7}$$

The $g_A^{\Xi\Omega}$ represents the form factor of the spatial component of the axial vector current between $\Xi^{(^0_-)}$ and Ω^- states. This is the second point of this paper where the Skyrme model enters the calculation. Note that $|B_{\mathcal{S}}(\Omega^-_0)|\ll|B_{\mathcal{S}}(\Omega^-_-)|$ due to the helicity suppression enhanced by the QCD corrections.

The total theoretical amplitudes are

$$\mathcal{B}_{\rm theory}(q^2)=B_{\mathcal{P}}(q^2)+B_{\mathcal{S}}(q^2), \tag{8}$$

where the relative signs between pole and factorizable contributions are determined via SU(3) and the generalized Goldberger-Treiman relations.

In order to estimate the matrix elements entering Eqs. (5) and (7), we take the SU(3) extended Skyrme Lagrangian [8,10,15]

$$\mathcal{L}=\mathcal{L}_\sigma+\mathcal{L}_{\rm Sk}+\mathcal{L}_{\rm SB}+\mathcal{L}_{\rm WZ}, \tag{9}$$

where $\mathcal{L}_\sigma$, $\mathcal{L}_{\rm Sk}$, $\mathcal{L}_{\rm SB}$, and $\mathcal{L}_{\rm WZ}$ denote the σ-model, Skyrme, symmetry breaking (SB), and Wess-Zumino (WZ) terms, respectively. Their explicit form can be found in Refs. [8, 15]. In this work we will use the SU(3) extended Lagrangian (9) in the limit of $f_K=f_\pi$. Then our new set of parameters $\hat{x}$, β', and δ', determined from the masses and decay constants of the pseudoscalar mesons [8], is $\hat{x}=2m_K^2/m_\pi^2-1$, $\beta'=0$, $\delta'=m_\pi^2 f_\pi^2/4$. Owing to the presence of the δ' term in the classical mass $E_{\rm cl}$, the dimensionless size of the soliton x_0' becomes a function of e, f_π, and δ':

$$x_0'^2=\frac{15}{8}[1+\sqrt{1+30\delta'/e^2f_\pi^4}]^{-1}. \tag{10}$$

Note that $\hat{x}$ is responsible for the baryon mass splittings and the admixture of higher representations in the baryon wave functions. However, as explained above, in this work we use the SU(3) symmetric baryon wave functions in the spirit of the perturbative approach to SB. Indeed, we have shown in Ref. [1] that the SB effects through the weak operators in the decay amplitudes are very small.

Our fitting procedure is as follows. Since the coupling f_π is equal to its experimental value, the only remaining free parameter is the Skyrme charge e. The value $e\approx4$ was successfully adjusted to the mass difference of the low-lying $1/2^+$ and $3/2^+$ baryons (Table 2.1 of [8]). This value of e was next employed to evaluate the static properties of baryons.

For the evaluation of the nonleptonic Ω^- decays, the most important baryon static property is the octet-decuplet mass splitting Δ. Another important quantity is the axial vector coupling constant $g_A^{\rm pn}$. We compute these quantities by using the arctan ansatz in the SU(3) extension of the Skyrme Lagrangian (9). By fixing Δ and $g_A^{\rm pn}$ to their experimental values, we obtain $e=4.21$ and 3.41, respectively. In further calculations of the Ω^- decay amplitudes, we use the mean value $e=3.81$, as in the case of nonleptonic hyperon decays [1]. As in the latter case this introduces approximately 15% uncertainty in the decay amplitudes, which are dominated by $\Phi^{\rm SK}$ [see Eq. (13) below], which scales as $1/e$.

We proceed with the computation of the axial vector current form factor $g_A^{\Xi\Omega}$ in the Skyrme model using the arctan ansatz:

$$g_A^{\Xi^-\Omega^-}=g_A^{\Xi^0\Omega^-}\equiv g_A^{\Xi\Omega}(x_0')=\tfrac{2}{7}\sqrt{15}g_A^{\rm pn}(x_0'), \tag{11}$$

where $g_A^{\rm pn}(x_0')$ is given in Eq. (10) of Ref. [1].

Let us recapitulate the results of Ref. [1], where we calculated the matrix element of the product of two $(V-A)$ currents between the octet states using the Clebsch-Gordan decomposition [10,15]:

$$\langle B_2|\hat{O}^{(\rm SK)}|B_1\rangle=\Phi^{\rm SK}\times\sum_R C_R, \tag{12}$$

where $\Phi^{\rm SK}$ is a dynamical constant and C_R denotes the pertinent sum of the SU(3) Clebsch-Gordan coefficients in the intermediate representation R. The same holds for the WZ current. The total matrix element is then simply a sum $\langle\hat{O}_i^{(\rm SK)}+\hat{O}_i^{(\rm WZ)}\rangle$, with $i=1,...,4$. The quantities Φ are given by the overlap integrals of the profile function in Ref. [1].

For the $\hat{O}_1$ operator, $R=8_{a,s}$ or 27. The matrix elements read

$$\begin{aligned}\langle\Lambda^0_{1/2}|\hat{O}_1|\Xi^0_{1/2}\rangle&=(18.14\Phi^{\rm SK}-20.41\Phi^{\rm WZ})\times10^{-3}\\&=(4.86|_{\rm SK}-0.09|_{\rm WZ})\times10^{-3}\ {\rm GeV}^3,\end{aligned} \tag{13}$$

$$\begin{aligned}\langle\Xi^{*-}_{3/2}|\hat{O}_1|\Omega^-_{3/2}\rangle&=(-10.60\Phi^{\rm SK}+12.89\Phi^{\rm WZ})\times10^{-3}\\&=(-2.84|_{\rm SK}+0.06|_{\rm WZ})\times10^{-3}\ {\rm GeV}^3.\end{aligned} \tag{14}$$

Note that we have omitted the static kaon fluctuations in all of our computations.

Our main goal was to learn how the approach of Refs. [1], [9], in which the Skyrme model is used to estimate the unknown nonperturbative matrix elements, applies to the Ω^- nonleptonic decays. We have systematically presented all possible tree diagrams, namely, factorizable contributions, the octet diagram [only for the (Ω_K) amplitude], and the decuplet pole diagrams. Numerical results presented in Table I are very encouraging. They are in satisfactory quantitative agreement with experimental data. In Table I both factorizable $[B_{\mathcal{S}}(q^2)]$ and pole amplitudes $[B_{\mathcal{P}}(q^2)]$ are displayed. A comparison of the total amplitudes (8) $\mathcal{B}_{\rm theory}(q^2)$ with experiment shows the following.

(a) The dynamics based on the pole diagrams supported by the Skyrme model leads to very good results for the rela-

TABLE I. The p-wave ($\mathcal{B}$) nonleptonic Ω^- decay amplitudes. Choices (off, on) correspond to the amplitudes without and with inclusion of short-distance corrections, respectively. For Ω_K ($\Omega^-_{0,-}$) amplitude $q^2=m_K^2$ (m_π^2).

Amplitude (10^{-6} GeV^{-1})		(Ω_K)	(Ω^-_-)	(Ω^-_0)
$B_{\mathcal{P}}^{(1/2^+)}(q^2)$	off	2.67	0	0
	on	5.51	0	0
$B_{\mathcal{P}}^{(3/2^+)}(q^2)$	off	1.57	1.28	0.91
	on	3.14	2.56	1.81
$B_{\mathcal{S}}(q^2)$	off	0	-0.27	-0.06
	on	0	-0.30	0.03
$\mathcal{B}_{\rm theory}(q^2)$	off	4.24	1.01	0.85
	on	8.65	2.26	1.84
$\mathcal{B}_{\rm exp}$ [2]		4.02	1.35	0.80

tive importance of various decay modes. However, all amplitudes are too large by about a factor of 2.

(b) The decay amplitude Ω_K, which does not contain factorizable contributions, has the largest pole contribution, since both pole diagrams are of approximately the same size and contribute constructively.

(c) Short-distance corrections to the effective weak Hamiltonian are proved to be important [12,13].

(d) In fact, too large values of the QCD-enhanced $B_{\mathcal{P}}(q^2)$ amplitudes are actually a welcome feature; namely, if one takes into account the higher $\Xi(3/2^+)$ resonances then the poles (5) will flip in sign, since their masses are larger than the Ω^- mass. The assumption that the vertices will be more or less the same as for the ground-state poles leads to internal cancellations between pole-diagram contributions [16,17]. Taking into account (a)–(c) and Ref. [17], we expect that this dynamical scheme would produce better agreement with experiment.

(e) We have found octet dominance, i.e., the 27-contaminations are small.

(f) We also find the dominance of the Skyrme Lagrangian currents over the WZ current. For $e\approx 4$, the WZ contribution to Eqs. (13), (14) is below 3%.

(g) As in the nonleptonic hyperon decays, the factorizable contributions, in the Skyrme model approach, turn out not to be important for the Ω^- nonleptonic decays.

(h) Finally, we found that the pole terms and the factorizable contributions have opposite signs.

We conclude that in general our approach provides a good description of the Ω^- decays. Obviously, not all details are under full control (e.g., m_s corrections are neglected); nevertheless, it seems the QCD-corrected weak Hamiltonian $H_w^{\rm eff}$, together with the inclusion of other possible types of contribution to the total amplitudes (K poles and/or factorization, higher baryon poles, etc.) supplemented by the Skyrme model, leads to the correct answer.

We hope that the present calculation, taken together with the analogous calculation of the nonleptonic hyperon decay amplitudes [1,9], will contribute to the understanding of the nonleptonic hyperon interactions. It is also a test of the application of the Skyrme model to the evaluation of nonperturbative quantities like axial vector coupling constants and the dimension-6 operator matrix elements between the different baryon states.

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