Leptogenesis and low-energy observables

M. S. Berger* and Kim Siyeon†
Physics Department, Indiana University, Bloomington, Indiana 47405
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We relate leptogenesis in a class of theories to low-energy experimental observables: quark and lepton masses and mixings. With reasonable assumptions motivated by grand unification, one can show that the CP-asymmetry parameter takes a universal form. Furthermore the dilution mass is related to the light neutrino masses. Overall, these models offer an explanation for a lepton asymmetry in the early universe.

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I. INTRODUCTION

Recent compelling evidence for neutrino oscillations has accelerated work on formulating theoretical models for fermion masses and mixings. The current data indicate that there are most likely two large mixing angles and one small one in the lepton sector. The first large mixing angle arises in the atmospheric neutrino data, while it is becoming increasingly likely that the solar neutrino data are described by a Mikheyev-Smirnov-Wolfenstein-(MSW-) type oscillation with a large mixing angle (LMA) [1–4]. On the other hand the CHOOZ experiment [5] gives an upper bound on the third mixing angle. A best fit [6] for the atmospheric neutrino data and the LMA solution for the solar neutrino data is

$$\Delta m_{32}^2 = 3.2\times 10^{-3}\ \ \mathrm{eV}^2, \tag{1}$$

$$\sin^2 2\theta_{23} = 1.000, \tag{2}$$

$$\Delta m_{21}^2 = 3.2\times 10^{-5}\ \ \mathrm{eV}^2, \tag{3}$$

$$\sin^2 2\theta_{12} = 0.75,\ \ \tan^2\theta_{12} = 0.33. \tag{4}$$

The observations of neutrino mixing and the measured values for the differences in mass-squareds make very plausible the existence of heavy Majorana neutrinos, ν_{M_i}. These neutrinos can naturally be very heavy since they are standard model gauge singlets and their masses are not connected to the breaking of the electroweak symmetry. These heavy Majorana neutrinos existed in the early universe and can have CP-violating decay modes. Therefore the heavy neutrinos are natural candidates for producing a lepton asymmetry via out-of-equilibrium decays. This asymmetry produced in the early universe is recycled into a baryon asymmetry by sphaleron transitions which violated both baryon number and lepton number. The resulting baryon asymmetry is the same order of magnitude as the original lepton asymmetry [7].

In the mass basis where the right-handed Majorana mass matrix M_R is diagonal the asymmetry in heavy neutrino decays

$$\epsilon_i = \frac{\Gamma(\nu_{M_i}\to lH_2) - \Gamma(\nu_{M_i}\to l^c H_2^c)}{\Gamma(\nu_{M_i}\to lH_2) + \Gamma(\nu_{M_i}\to l^c H_2^c)}, \tag{5}$$

is given by [8–13]

$$\epsilon_i = \frac{3}{16\pi v_2^2}\frac{1}{(\mathcal{N}^\dagger\mathcal{N})_{ii}}\sum_{n\neq i}\mathrm{Im}[(\mathcal{N}^\dagger\mathcal{N})_{ni}^2]\frac{M_i}{M_n}, \tag{6}$$

where $\mathcal{N}$ is the neutrino Dirac mass matrix in a weak basis. The masses M_i are the three eigenvalues of the heavy Majorana mass matrix and v_2 is the vacuum expectation value (VEV) of the Higgs boson giving Dirac masses to the neutrinos and up-type quarks. M_1 is the mass of the lightest of the three heavy Majorana neutrinos, and Eq. (6) is an approximate formula valid for $M_n \gg M_i$. When this is the case, the lepton asymmetry is generated by the decays of the lightest Majorana neutrino, ν_{M_1}.

The size of the lepton asymmetry generated by ν_{M_1} decays is also strongly dependent on the size of a mass parameter sometimes called the dilution mass defined as

$$\tilde{m}_1 = \frac{(\mathcal{N}^\dagger\mathcal{N})_{11}}{M_1}. \tag{7}$$

This parameter controls (a) the decay width of the lightest right-handed Majorana neutrino ν_{M_1} since

$$\Gamma_{\nu_{M_1}} = \frac{1}{8\pi}(\mathcal{N}^\dagger\mathcal{N})_{11}\frac{M_1}{v_2^2}, \tag{8}$$

as well as (b) the amount of dilution caused by lepton number violating scattering: the resulting lepton asymmetry depends critically on the parameter $\tilde{m}_1$ because it governs the size of the most important Yukawa coupling in the $\Delta L = 2$ scattering processes, as has been shown in detail in numerical calculations [7,9,11,14,15]. These two constraints bound the possible values of $\tilde{m}_1$ such that a sufficient asymmetry is produced to agree with observation. The generated lepton asymmetry Y_L is defined in terms of the number densities of the leptons and antileptons as well as the entropy density as

$$Y_L = \frac{n_L - n_{\bar{L}}}{s} = \kappa\frac{\epsilon_1}{g^*}, \tag{9}$$

*Electronic address: berger@gluon.physics.indiana.edu
†Electronic address: siyeon@gluon.physics.indiana.edu

where g^* is the number of light (effective) degrees of freedom in the theory, and κ is a dilution factor that can be reliably calculated by solving the full Boltzmann equations.

It has been shown [9,11] that a CP-violation parameter $\epsilon_1 \sim 10^{-6}$ and a dilution mass $\tilde{m}_1$ in the range of the light neutrino masses can produce the sufficient amount of leptogenesis to account for the observed baryon asymmetry. From the definition of the dilution mass in Eq. (7) it is clear that the dilution mass will indeed be related to the light neutrino masses in most models. It is a nontrivial occurrence that the amount of baryon asymmetry of the universe is obtained from a recycling of the leptogenesis that naturally occurs via Majorana neutrino decays.

Suppose one starts in a basis where M_R is diagonal with eigenvalues M_j, and suppose the matrix M_R is connected to the light neutrino mass matrix m_ν by a seesaw mechanism,

$$M_R = \mathcal{N}^T m_\nu^{-1} \mathcal{N}. \tag{10}$$

One can then define mixing matrices $U_{L,R}^{(N)}$ and V_L that diagonalize N and m_ν, respectively:

$$\mathcal{N} = U_L^{(N)} N_{diag} U_R^{(N)\dagger}, \tag{11}$$

$$\mathcal{N}^T = U_R^{(N)*} N_{diag} U_L^{(N)T}, \tag{12}$$

$$m_\nu^{-1} = V_L^* m_{diag}^{-1} V_L^\dagger. \tag{13}$$

With these transformation matrices defined, M_j can be written in terms of mass eigenvalues and mixings of $(m_\nu, \mathcal{N})$:

$$M_j = \sum_k \sum_l m_k^{-1} (V_L^\dagger U_L^{(N)})_{kl}^2 N_l^2 U_{Rlj}^{(N)\dagger 2} \tag{14}$$

where N_l are the diagonal elements of N_{diag}. The unitary transformation $U_R^{(N)}$ diagonalizes $\mathcal{N}^\dagger \mathcal{N}$ as $\mathcal{N}^\dagger \mathcal{N} = U_R^{(N)} N_{diag}^2 U_R^{(N)\dagger}$, then

$$(\mathcal{N}^\dagger \mathcal{N})_{1j} = \sum_k N_k^2 U_{R1k}^{(N)} U_{Rjk}^{(N)*}. \tag{15}$$

By inverting M_j in Eq. (14), the mass eigenvalues of m_ν can be expressed in terms of V_L, mixing angles and eigenvalues of Dirac mass matrix, and Majorana neutrino masses,

$$m_k = \sum_j \sum_l (V_L^\dagger U_L^{(N)})_{kl}^2 N_l^2 U_{Rlj}^{(N)\dagger 2} M_j^{-1}. \tag{16}$$

II. ASSUMPTIONS

It is well known that one must make theoretical assumptions about the structure of the neutrino masses and mixings to make progress in ascertaining whether leptogenesis is viable. For example the source of CP violation responsible for producing the CP-violating decays of heavy Majorana neutrinos (and hence giving rise to leptogenesis) does not have to be related to the CP violation that might be measurable at low-energy experiments in the future [16,17]. An extensive study of the weak-basis CP invariants in models with three iso-singlet neutrinos is given in Ref. [18]. If one makes the assumption of single right-handed neutrino dominance, then the low energy neutrino observables and the leptogenesis predictions decouple entirely [19]. On the other hand, in certain classes of grand unified theories previously unconstrained parameters become related to observables. For example, in models with a left-right symmetry, the right-handed mixing angles can be related the left-handed ones that enter into low-energy experiments [20]. In this section we list our theoretical assumptions about the underlying grand unified theory. Many authors have discussed leptogenesis in the context of grand unified theories [21–36]; our emphasis here is on making the most general assumptions that allow us to relate low-energy observables like masses and mixing angles to the required lepton asymmetry that can ultimately account for the baryon asymmetry of the universe.

[A1] We assume that the Dirac mass matrices $\mathcal{N}$ and $\mathcal{U}$ are symmetric, and $\mathcal{N} \sim \mathcal{U}$.[1] This similarity between the neutrino Dirac mass matrix and the up-quark mass matrix is motivated by grand unified theories.

[A2] The mixing angles contained in the transformation matrices that diagonalize the neutrino Dirac mass matrix $\mathcal{N}$ are related to the eigenvalues[2] $s_{ij} \sim \sqrt{N_i/N_j}$. In general these mixing angles cannot be larger than $\sqrt{N_i/N_j}$, but can in principle be smaller. The s_{ij} being suppressed compared to $\sqrt{N_i/N_j}$ might occur, for example, if some elements of $\mathcal{N}$ are suppressed or zero. So the result of our second assumption is that there is no such suppression or cancellation in the Dirac neutrino matrix.

The crucial features that follow from our two assumptions listed above are (a) the neutrino Dirac mass matrix has eigenvalues that mimic the large hierarchy that exists in the up-quark sector, and (b) the mixing angles s_{ij} are fixed to be of some definite size related to the up-type quark masses, e.g. $s_{13} \sim \sqrt{N_1/N_3} \sim \sqrt{m_u/m_t}$. These two results will be important in arriving at the relatively simple results that follow.

[A3] Our approach does not allow us to determine the CP-violating phase that enters into the parameter ϵ_1 in Eq. (6). We simply assume that phases are of order one, and there is no suppression arising from unnaturally small parameters.

A standard parametrization of the unitary transformation involving three angles and a phase is

[1] We use the notation $\sim$ to denote that entries are the same size to leading order in all small quantities such as small mass ratios or small mixing angles.

[2] We use the shorthand notation $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.

$$U=\begin{pmatrix}1&0&0\\0&c_{23}&s_{23}\\0&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&0&s_{13}e^{-i\delta}\\0&1&0\\-s_{13}e^{i\delta}&0&c_{13}\end{pmatrix}\begin{pmatrix}c_{12}&s_{12}&0\\-s_{12}&c_{12}&0\\0&0&1\end{pmatrix}$$
$$=\begin{pmatrix}c_{13}c_{12}&s_{12}c_{13}&s_{13}e^{-i\delta}\\-s_{12}c_{23}-s_{23}s_{13}c_{12}e^{i\delta}&c_{23}c_{12}-s_{23}s_{13}s_{12}e^{i\delta}&s_{23}c_{13}\\s_{23}s_{12}-s_{13}c_{23}c_{12}e^{i\delta}&-s_{23}c_{12}-s_{13}s_{12}c_{23}&c_{23}c_{13}\end{pmatrix}. \tag{17}$$

The right-handed and left-handed mixing matrices with small angles $(c_{ij}\approx 1)$ are

$$U_R^{(N)}\approx U_L^{(N)}\approx\begin{pmatrix}1&s_{12}&s_{13}\\-s_{12}-s_{23}s_{13}&1&s_{23}\\s_{23}s_{12}-s_{13}&-s_{23}-s_{13}s_{12}&1\end{pmatrix}, \tag{18}$$

where we will assume that phase $e^{i\delta}$ is not suppressed: δ is not close to 0 or π. For our purposes, we consider only the leading contributions to each element so that

$$|U_R^{(N)}|\sim|U_L^{(N)}|\sim\begin{pmatrix}1&s_{12}&s_{13}\\s_{12}&1&s_{23}\\s_{13}&s_{23}&1\end{pmatrix}. \tag{19}$$

III. NEUTRINO TRANSFORMATION

In general, we can write the transformation as

$$V_L\sim\begin{pmatrix}1&\Theta_{12}&\Theta_{13}\\-\Theta_{12}-\Theta_{23}\Theta_{13}&1&\Theta_{23}\\\Theta_{23}\Theta_{12}-\Theta_{13}&-\Theta_{23}-\Theta_{13}\Theta_{12}&1\end{pmatrix}. \tag{20}$$

We henceforth interpret the quantities Θ_{ij} as

$$\cos\Theta\sim 1,\quad \sin\Theta\sim 1,\quad \text{for large angles}$$
$$\cos\Theta\sim 1,\quad \sin\Theta\sim\Theta,\quad \text{for small angles.} \tag{21}$$

In other words, the matrix can be expressed in the same way in terms of Θ_{ij} if we are only interested in the order-of-magnitude size of the elements (including the ones on the diagonal which would only be of order one in general). The Maki-Nakagawa-Sakata (MNS) neutrino mixing matrix [37] is

$$U_{MNS}=U_L^{(E)\dagger}V_L, \tag{22}$$

where $U_L^{(E)}$ is the matrix that diagonalizes the charged lepton mass matrix. The constraints from reactor neutrino mixing data [5] imply that Θ_{13} must be small provided there is no cancellation among V_L and $U_L^{(E)}$. Retaining only information about the size of the individual elements, we may write Eq. (20) as follows:

$$V_L\sim\begin{pmatrix}1&\Theta_{12}&\Theta_{13}\\\max(\Theta_{12},\Theta_{23}\Theta_{13})&1&\Theta_{23}\\\max(\Theta_{23}\Theta_{12},\Theta_{13})&\Theta_{23}&1\end{pmatrix}, \tag{23}$$

with the entries interpreted according to Eq. (21).

IV. HEAVY MAJORANA NEUTRINO MASSES

Define the matrix

$$W_{kj}\equiv\sum_l (V_L^\dagger U_L^{(N)})^2_{kl}n_l^2U_{Rlj}^{(N)\dagger 2}, \tag{24}$$

where $n_i\equiv N_i/N_3$ are the ratios of the Dirac neutrino masses. The heavy Majorana neutrino masses are

$$M_j=N_3^2\sum_k m_k^{-1}W_{kj}, \tag{25}$$

and the light neutrino masses are given by Eq. (16) as

$$m_j=N_3^2\sum_k W_{jk}M_k^{-1}. \tag{26}$$

The factor $n_l^2U_{Rlj}^{(N)\dagger 2}$ in W_{kj} has the form

$$n_l^2U_{Rlj}^{(N)\dagger 2}\sim\begin{pmatrix}n_1^2&n_1^2s_{12}^2&n_1^2s_{13}^2\\n_2^2s_{12}^2&n_2^2&n_2^2s_{23}^2\\s_{13}^2&s_{23}^2&1\end{pmatrix}. \tag{27}$$

Now we make use of our assumptions [A1] and [A2] that allow us to compare the relative sizes of the n_i and the mixing angles s_{ij}. Specifically we have that $s_{ij}\sim\sqrt{n_i/n_j}$ as well as $n_i\ll n_j$ for $i<j$ so that

$$n_1^2U_{R1j}^{(N)\dagger 2}\ll n_2^2U_{R2j}^{(N)\dagger 2}\ll U_{R3j}^{(N)\dagger 2}. \tag{28}$$

We henceforth refer to this condition as "third-generation dominance." In fact if, as we have assumed, the hierarchy in the Dirac masses for neutrinos is as strong as it is for the up quark Dirac masses, as one might expect in a grand unified theory, then the smallness of n_1 and n_2 suppresses all other contributions to W_{kj} relative to the dominant contribution coming from $(V_L^\dagger U_L^{(N)})^2_{k3}$ and $U_{R3j}^{(N)\dagger 2}$. So we arrive at the following factored form for the matrix:

$$W_{kj}\sim\begin{pmatrix}(V_L^\dagger U_L^{(N)})_{13}^2\\(V_L^\dagger U_L^{(N)})_{23}^2\\(V_L^\dagger U_L^{(N)})_{33}^2\end{pmatrix}(s_{13}^2\,s_{23}^2\,1). \tag{29}$$

Finally we can write the Majorana masses in the following way

$$(M_1,M_2,M_3)\sim N_3^2\widetilde{W}_3(s_{13}^2,s_{23}^2,1), \tag{30}$$

where

$$\widetilde{W}_3=\sum_k m_k^{-1}(V_L^\dagger U_L^{(N)})_{k3}^2. \tag{31}$$

The result in Eq. (30) indicates that, based on our assumptions, the mass ratios of the Majorana masses are related to the mixing angles s_{i3} and are independent of the light neutrino mixings which appear only in the overall factor $\widetilde{W}_3$. This result follows from the third-generation dominance Eq. (28) which is related to the large hierarchy in the Dirac neutrino masses that is inherited from the large hierarchy in the experimentally measured up-quark masses. On the other hand, the light neutrino masses under the third-generation condition are given by Eq. (26) as

$$(m_1,m_2,m_3)\sim\frac{N_3^2}{M_3}((V_L^\dagger U_L^{(N)})_{13}^2,(V_L^\dagger U_L^{(N)})_{23}^2,(V_L^\dagger U_L^{(N)})_{33}^2). \tag{32}$$

So the mass ratios of the light neutrinos can be expressed in terms of the left-handed mixing angles.

V. LEPTOGENESIS

In this section we utilize the simple form for the mass ratios of the heavy Majorana neutrino masses found in the last section to derive a simple formula for the CP-asymmetry parameter ϵ_1 in Eq. (6). The couplings give

$$(\mathcal{N}^\dagger\mathcal{N})_{1j}=N_3^2\sum_k n_k^2 U_{R1k}^{(N)}U_{Rjk}^{(N)*}, \tag{33}$$

where the dominant contribution is given in this case by $k=3$,

$$(\mathcal{N}^\dagger\mathcal{N})_{1j}\sim N_3^2 U_{R13}^{(N)}U_{Rj3}^{(N)*}. \tag{34}$$

As with the Majorana masses, third-generation dominance implies that simple expressions exist for

$$[(\mathcal{N}^\dagger\mathcal{N})_{11},(\mathcal{N}^\dagger\mathcal{N})_{12},(\mathcal{N}^\dagger\mathcal{N})_{13}]\sim N_3^2 s_{13}[s_{13},s_{23},1]. \tag{35}$$

The resulting CP-asymmetry parameter in Eq. (6) can now be expressed to leading order as

$$\epsilon_1\sim\frac{3}{16\pi}\frac{N_3^2}{v_2^2}\mathrm{Im}\left[\frac{(\mathcal{N}^\dagger\mathcal{N})_{12}^2}{(\mathcal{N}^\dagger\mathcal{N})_{11}}\frac{M_1}{M_2}+\frac{(\mathcal{N}^\dagger\mathcal{N})_{13}^2}{(\mathcal{N}^\dagger\mathcal{N})_{11}}\frac{M_1}{M_3}\right] \tag{36}$$

and one arrives at the simple result

$$\epsilon_1\sim 10^{-1}s_{13}^2\sim 10^{-1}\frac{m_u}{m_t}, \tag{37}$$

where we have used Eqs. (30) and (35). We have also used $N_3\sim v_2$ since the largest Yukawa coupling in the neutrino Dirac mass matrix is similar ($\mathcal{N}\sim\mathcal{U}$) to the top quark Yukawa coupling which is close to one. One can understand that the contribution involving the mixing angle s_{13}^2 is the leading contribution in the following way: The dominant contribution to leptogenesis comes from the decay of the lightest Majorana neutrino ($i=1$) and the dominant Yukawa couplings occur in the third generation ($j=3$). One obtains an acceptable amount of baryon asymmetry if $\epsilon_1\sim 10^{-6}$; this indeed results if $s_{13}\sim\sqrt{m_u/m_t}$.

The dilution mass defined in Eq. (7) can be expressed as

$$\widetilde{m}_1\sim\frac{N_3^2 s_{13}^2}{N_3^2\widetilde{W}_3 s_{13}^2}=\widetilde{W}_3^{-1}, \tag{38}$$

using the third-generation dominance that results from assumptions [A1] and [A2]. Given the expression for $\widetilde{W}_3$ in Eq. (31) it is clear that the dilution mass is related in all cases to the light neutrino masses. This is precisely the range of dilution mass that gives a large asymmetry as has been pointed out many times before as an attractive and natural feature of the leptogenesis scenario.

We now proceed to examine some special cases for the size of the dilution mass. Assumptions [A1] and [A2] allow us to identify the sizes of the mixing angles in the the mixing matrix $U_L^{(N)}$. For example $s_{23}\sim\sqrt{m_c/m_t}$. So Eq. (19) can be written as

$$U_L^{(N)}\sim\begin{pmatrix}1 & s_{12} & s_{13}\\ s_{12} & 1 & s_{23}\\ s_{13} & s_{23} & 1\end{pmatrix}. \tag{39}$$

Recall that the left-handed mixing angles are similar to the right-handed mixing angles according to our assumptions. Using Eq. (23) we have that

$$(V_L^\dagger U_L^{(N)})_{k3}^2\sim\begin{pmatrix}\max(s_{13}^2,\Theta_{12}^2 s_{23}^2,\Theta_{23}^2\Theta_{12}^2,\Theta_{23}^2\Theta_{13}^2 s_{23}^2,\Theta_{13}^2)\\ \max(s_{23}^2,\Theta_{23}^2)\\ 1\end{pmatrix}. \tag{40}$$

These elements together with Eqs. (31) and (38) allow one to determine the dilution mass. The quantities s_{ij} are all small compared with one since they have been related to the (left-handed) Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, but the Θ_{ij} might or might not be small. From the CHOOZ data [5] we know that the mixing angle Θ_{13} must be small as long as there is no unnatural cancellation between this angle and the one involved in converting the weak basis to the mass basis for the charged leptons, cf. Eq. (22). One can relate the dilution mass in Eq. (38) to the light neutrino masses using Eqs. (31), (32), and (40). The mass ratios between light neutrinos are

$$\frac{m_i}{m_j} \sim \frac{(V_L^\dagger U_L^{(N)})_{i3}^2}{(V_L^\dagger U_L^{(N)})_{j3}^2}. \tag{41}$$

One can investigate a number of cases. Without any fine-tuning one expects the angles Θ_{ij} to be of the same order as the angles s_{ij}. In that case (1), one obtains

$$m_2 \sim \frac{m_c}{m_t} m_3, \quad m_1 \sim \frac{m_u}{m_t} m_3, \tag{42}$$

from Eqs. (40) and (41). The dilution mass is $\tilde{m}_1 \sim m_3$ from Eqs. (31) and (38). This does not give good agreement with the experimental data since $m_2 \simeq \sqrt{\Delta m_{21}^2}$ is too small to reconcile it with the solar LMA data and atmospheric neutrino data $m_3 \simeq \sqrt{\Delta m_{32}^2}$. The neutrino masses inherit the large hierarchy from the up quark sector. The conclusion is that one needs some amount of fine-tuning to get masses in acceptable agreement with the solar LMA data.

(2) If one accepts some fine-tuning so that the mixing angle Θ_{23} is large and order one rather than similar to s_{23} and Θ_{12} remains small, then Eq. (40) reduces to

$$(V_L^\dagger U_L^{(N)})_{j3}^2 \sim \begin{pmatrix} \Theta_{12}^2 \\ 1 \\ 1 \end{pmatrix}. \tag{43}$$

Even in this case the determinant of the seesaw mass formula, Eq. (10), must satisfy

$$m_1 m_2 m_3 = m_u m_c m_t \left(\frac{m_t}{M_3}\right)^3. \tag{44}$$

Then since Θ_{23} is large one expects the mass eigenvalues to satisfy

$$m_2 \sim m_3 \sim \sqrt{m_c m_t}\left(\frac{m_t}{M_3}\right), \tag{45}$$

so that

$$m_1 \sim m_u \left(\frac{m_t}{M_3}\right). \tag{46}$$

The masses can be consistent with the LMA solar and atmospheric neutrino data. Then the dilution mass is given by $\tilde{m}_1 \sim m_2 \sim m_3$ from Eqs. (31) and (38) and is in an acceptable range.

(3) With additional fine-tuning both the mixing angles Θ_{12} and Θ_{23} can be made large. Eq. (40) reduces to

$$(V_L^\dagger U_L^{(N)})_{j3}^2 \sim \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{47}$$

The light neutrino masses are all the same order so that from Eq. (44) one gets

$$m_1 \sim m_2 \sim m_3 \sim (m_u m_c m_t)^{1/3}\left(\frac{m_t}{M_3}\right), \tag{48}$$

where m_i cannot be larger than m_j if $i<j$. Then the dilution mass is $\tilde{m}_1 \sim m_1$. This solution does not offer any explanation for a hierarchy in neutrino masses.

In all three cases the dilution mass $\tilde{m}_1$ lies roughly in the range spanned by light neutrino masses

$$m_1 \lesssim \tilde{m}_1 \lesssim m_3. \tag{49}$$

It should be understood here that the $\lesssim$ means that $\tilde{m}_1$ could be outside the upper and lower ends of the range by an order one parameter.

More generally, and outside the assumptions of this paper, one can consider the possibility that the charged lepton mass matrix contributes to large mixing for both the solar neutrino and atmospheric netrino oscillations or for either one, through the charged lepton transformation matrix $U_L^{(E)}$ via Eq. (22).

VI. SUMMARY

We have shown that based upon a limited number of reasonable assumptions about the neutrino sector motivated by grand unification, one obtains the universal expression in Eq. (37) for the dominant contribution to the CP-violation parameter ϵ_1 that determines the amount of leptogenesis in the early universe. Furthermore the dilution mass $\tilde{m}_1$ is expressed in terms of mixing angles in the light neutrino masses and it naturally falls in the range needed to explain the baryon asymmetry of the universe. While these assumptions are not required to obtain the necessary lepton asymmetry to explain the observed baryon asymmetry of the universe, they provide enough constraints to allow one to relate the CP violation in the heavy Majorana neutrino decays and the important Yukawa couplings of these heavy neutrinos to low-energy observables: fermion masses and mixing angles.

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