In quest of neutrino masses at $O(\mathrm{eV})$ scale

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Neutrino oscillation and tritium beta decay experiments taken simultaneously into account are able to access the so far imperceptible absolute neutrino masses at the electronvolt level. The neutrino mass spectrum derived in this way is independent of the nature of neutrinos (Dirac or Majorana). Furthermore, the lack of neutrinoless double beta decay gives additional constraints on the Majorana neutrino mass spectrum. A case of three neutrinos is examined. The influence of different solutions to the solar neutrino deficit problem on the results is discussed. Apart from the present situation, four qualitatively distinct experimental situations which are possible in the future are investigated: when the two decay experiments give only upper bounds on effective neutrino masses, when either one of them gives a positive result, and when both give positive results. The discussion is carried out by taking into account the present experimental errors of relevant neutrino parameters as well as their much more precise expected estimations (e.g., by ν factories). It is shown in which cases the upgraded decay experiments simultaneously with neutrino oscillation data may be able to fix the absolute scale of the neutrino mass spectrum, answer the question of the neutrino nature, and put some light on *CP* phases in the lepton sector.

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I. INTRODUCTION

The problem of the neutrino mass spectrum and its nature is the most important issue in the lepton part of the standard model. What new information can we obtain from the last experimental results, and what are the future perspectives? Three kinds of experiments play a fundamental role in answering this question. Two are traditional and have been known for years: beta decay and neutrinoless double beta decay $(\beta\beta)_{0\nu}$ of nuclei. Already Fermi [1] in 1934 and Furry [2] in 1939 realized that both processes are important for the neutrino mass and nature. The third type constitutes the neutrino oscillation experiments. These are responsible for anomalies observed in solar [3], atmospheric [4], and Liquid Scintillation Neutrino Detector (LSND) [5] experiments. Though trials of alternative explanations of the observations exist [6], they require much more sophisticated assumptions (such as, for example, the breaking of the equivalence principle, breaking of the special theory of relativity, the neutrino decay with lifetime much below expectations or huge neutrino magnetic moments) and give much poorer fits to the data [7].[1]

Solar, atmospheric and LSND experiments probe the neutrino oscillation hypothesis with three disconnected ranges of δm^2 parameters ($\delta m^2 \simeq 10^{-10}-10^{-5}\,\mathrm{eV}^2$ for solar neutrinos, $\delta m^2 \simeq 10^{-3}-10^{-2}\,\mathrm{eV}^2$ for atmospheric neutrinos, and $\delta m^2 \simeq 0.1-10\,\mathrm{eV}^2$ for the LSND experiment). The situation seems to be clear and in favor of the neutrino oscillation hypothesis for the atmospheric neutrino data analysis (agreement among different experiments). Also, the solar neutrino deficit is quite well explained by neutrino oscillations, but at present no unique solution for the oscillation parameters exists. As far as LSND is concerned, the situation is currently not clear at all [13]. The LSND results, if confirmed, would imply a fourth, sterile neutrino. Here we consider mass scenarios with three neutrinos only.

As there are definitely two scales of δm^2, $\delta m^2_{\mathrm{atm}} \gg \delta m^2_{\mathrm{sol}}$, two possible neutrino mass spectra must be considered (Fig. 1). The first, known as normal mass hierarchy (A_3) where $\delta m^2_{\mathrm{sol}} = \delta m^2_{21} \ll \delta m^2_{32} \approx \delta m^2_{\mathrm{atm}}$ and the second, inverse mass hierarchy spectrum (A_3^{inv}) with $\delta m^2_{\mathrm{sol}} = \delta m^2_{21} \ll -\delta m^2_{31} = \delta m^2_{\mathrm{atm}}$, $\delta m^2_{ij} = m^2_i - m^2_j$. Both schemes are not distinguishable by present experiments. There is hope that next

[1]There are also some astrophysical and cosmological arguments which shed some light on the neutrino masses. One of them comes from the analysis relating the cosmic microwave background temperature fluctuations to the present large scale structure formation. It depends strongly on the accepted cosmological model of the Universe and for three light neutrinos gives $m_\nu \leqslant 1.8(0.6)$ eV for any value of the cosmological density Ω_m ($\Omega_m = 0.3$) [8]. Another bound comes from the observation of ultrahigh cosmic rays. The so-called *Z*-burst model [9] gives $m_\nu \in (0.1-1)$ eV. Though the above numbers are very impressive (and better than the present tritium β decay bound), they depend on additional assumptions connected to the interpretation of astrophysical data and we will not include them in the present analysis (see Refs. [10] and [12] for a discussion which includes the cosmological data).

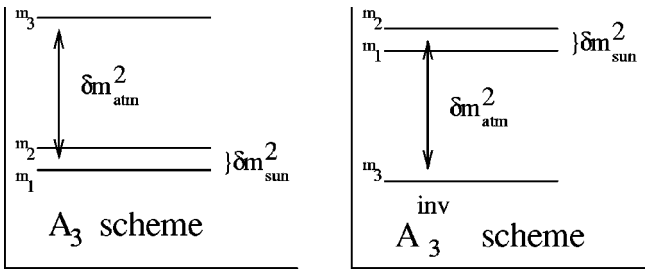

FIG. 1. Two possible mass spectra which can describe the oscillation data. Scheme A_3, normal mass hierarchy, has a small gap between m_1 and m_2 to explain the oscillation of solar neutrinos and a larger gap for the atmospheric neutrinos ($\delta m^2_{\rm sol}=\delta m^2_{21}\ll\delta m^2_{32}\simeq\delta m^2_{\rm atm}$; $m_1<m_2\ll m_3$). In the inverse mass hierarchy scheme $A_3^{\rm inv}$, $-\delta m^2_{31}=\delta m^2_{\rm atm}\gg\delta m^2_{\rm sol}\approx\delta m^2_{21}$ The mass of the lightest neutrino $(m_\nu)_{\rm min}=m_1$ in the A_3 schemes and $(m_\nu)_{\rm min}=m_3$ in the $A_3^{\rm inv}$ scheme.

long base line experiments (e.g., MINOS, ICARUS) and/or neutrino factories will do that.[2] Such schemes are the basic ones. As the neutrino mass spectrum is determined by the mass of the lightest neutrino $(m_\nu)_{\rm min}$, other possible neutrino mass schemes known in the literature as ''quasidegenerate,'' ''partial mass hierarchy,'' or ''partial inverted mass hierarchy'' [15,16] are considered automatically in the paper [$(m_\nu)_{\rm min}$ in the range from zero up to around 2.2 eV is taken into account].

The oscillation experiments are able to find differences of mass squares δm^2 (not the absolute masses separately) and absolute values of some of the mixing matrix elements $|U_{ei}|$ (presently no information on *CP* phases is available).

Different combinations of masses and U_{ei}'s are measured in tritium β and $(\beta\beta)_{0\nu}$ decays. Taking these data together we can probe the absolute neutrino masses. Such an analysis has been partially done in different contexts in Refs. [12,15–18].

Here, our main motivation is to answer the following questions: when, how precisely, and under what circumstances the absolute neutrino masses can be determined. As can be expected, the answer depends crucially on the mass of the lightest neutrino $(m_\nu)_{\rm min}$. For $(m_\nu)_{\rm min}$ above approximately 0.3 eV (the exact value which is discussed later on, depends on the precision of the neutrino oscillation parameters' determination) we can expect that the upgraded tritium β decay experiments together with the oscillation data will be able to determine the absolute neutrino masses m_i independently of the neutrino character. If $(m_\nu)_{\rm min}\leqslant 0.3\,{\rm eV}$, the $(\beta\beta)_{0\nu}$ decay gives some chance to determine $(m_\nu)_{\rm min}$. We discuss the conditions required for this to happen. Two future scenarios are considered. In the first case neutrinos are Majorana particles and the effective Majorana mass is determined by the $(\beta\beta)_{0\nu}$ experiment. In the second case the nature of neutrinos is not known and we will still have only a bound on $\langle m_\nu\rangle$. In this case there are circumstances when the combined results from $(\beta\beta)_{0\nu}$, tritium β decay, and oscillation experiments are able to exclude the Majorana nature of the neutrino. In our numerical analysis, special attention is paid to the influence of present and future experimental errors on the absolute neutrino mass determination. It is shown that the expected improvements from incoming ν factories will provide additional severe constraints on the neutrino masses.

The paper is organized as follows. In the next section, the experimental status of neutrino oscillation searches, β tritium and $(\beta\beta)_{0\nu}$ decays, is shortly reviewed. Expected improvements of the precision of parameters' determination are listed. In Sec. III basic analytical formulas which are used in the neutrino absolute mass search are presented. Section IV includes a discussion of numerical results. Four possible future scenarios mentioned in the Abstract and their consequences for the determination of the neutrino mass spectrum are analyzed. The paper ends with conclusions.

II. MAIN NEUTRINO DATA AND THEIR PRESENT AND FUTURE EXPERIMENTAL PRECISION

A global analysis of the solar, atmospheric and reactor neutrino data determines five parameters: three mixing angles $[\Theta_{12},\Theta_{13},\Theta_{23}(0<\Theta_{ij}<\pi/2)]$ and two mass square differences ($\pm\delta m^2_{31}=\delta m^2_{\rm atm}>0,\pm\delta m^2_{21}=\delta m^2_{\rm sol}>0$).

For the solar neutrino problem several analyses have been carried out so far allowing mixings among 2, 3, or 4 neutrinos [19–21]. The results differ slightly, nevertheless, in the 3ν scenario four solutions for the solar neutrino deficit are still acceptable at the 95% C.L. [21]. The first one, is the small mixing angle solution (SMA) [Mikheyev-Smirnov-Wolfenstein (MSW)] with $\sin^2 2\Theta\doteq 0.001-0.01$. The three remaining solutions [large mixing angle (LMA) MSW low-mass, (LOW) MSW and quasivacuum oscillation (QVO)] include large mixing angles, namely, $\sin^2 2\Theta\geqslant 0.55$. In these cases a maximal mixing $\sin^2 2\Theta=1$ is still acceptable. The present situation and future expectations are summarized in Table I. The matter enhanced solution of the solar neutrino problem is accepted for $\delta m^2_{21}>0$ only. The sign of δm^2_{31} cannot yet be determined, so two schemes are considered (Fig. 1). Incoming long baseline experiments and especially neutrino factories should be able to distinguish between these two schemes.

The mixing angles given in Table I enter the effective neutrino mass formulas, which can be written as

$$m_\beta\equiv\left[\sum_{i=1}^{3}|U_{ei}|^2m_i^2\right]^{1/2} \tag{1}$$

for the tritium β decay, and

$$\langle m_\nu\rangle=\left|\sum_i U_{ei}^2m_i\right| \tag{2}$$

for the $(\beta\beta)_{0\nu}$ decay.

In both schemes U_{ei}'s are given by

$$U_{e1}=\cos\Theta_{12}\cos\Theta_{13},\quad U_{e2}=\sin\Theta_{12}\cos\Theta_{13},$$

[2]According to a recent analysis of the neutrino spectrum from the SN1987A [14], the $A_3^{\rm inv}$ scheme is disfavored for $|U_{e3}|^2>0.001$.

TABLE I. The allowed ranges of neutrino parameters from global analysis altogether with expected future improvements (taken from Gonzales-Garcia *et al.* in [20]). In three central columns minimum and maximum are given at 90% C.L. Future improvements on parameters are mainly connected to LMA MSW solutions and accelerator physics (MINOS, ICARUS, OPERA projects, ν factories).

		min.	best fit	max.	future improvements
$\tan^2\Theta_{13}$		0	0.005	0.055	$\lvert\Delta\Theta_{13}\rvert\sim 10^{-2}$ [36] $\sim 10^{-4}$ [37]
$\delta m^2_{32}[\times 10^3\ \mathrm{eV}^2]$		1.4	3.1	6.1	$\lvert\Delta(\delta m^2_{23})\rvert\sim 10\%$ acc. [36] $\sim 1\%$ [38]
$\tan^2\Theta_{23}$		0.39	1.4	3.0	$\lvert\Delta(\sin^2 2\Theta_{23})\rvert\sim 5\%$ acc. [36] $\sim 1\%$ [37]
δm^2_{21} [eV^2]	LMA$\times 10^5$	~ 1.6	3.3	~ 20	$\lvert\Delta(\delta m^2_{21})\rvert\sim 10\%$ acc. [36]
	LOW$\times 10^8$	~ 0.08	9.6	~ 30	
	SMA$\times 10^6$	~ 4	5.1	~ 9	
$\tan^2\Theta_{12}$	LMA	0.2	0.36	~ 1	$\lvert\Delta(\sin^2 2\Theta_{12})\rvert\sim 0.1$ [36]
	LOW-QVO	0.2	0.58	3	
	SMA	$\sim 10^{-4}$	6.8×10^{-4}	$\sim 2\times 10^{-3}$	

$$|U_{e3}| = \sin\Theta_{13}. \tag{3}$$

The experimental data at the end of the Curie plot in the tritium β decay provides the upper limit on the effective electron neutrino mass m_β.

The present best limit is given by the Mainz Collaboration [22]

$$m_\beta < \kappa' = 2.2\ \mathrm{eV}. \tag{4}$$

A second collaboration from Troitsk gives similar results [23]

$$m_\beta < 2.5\ \mathrm{eV}. \tag{5}$$

The groups from Mainz, Troitsk, Karlsruhe, and Fulda have presented a project [24] for a new experiment (KATRIN), which should improve the existing limit by a factor of ten, so within 6–7 years m_β should reach $m_\beta \sim 0.3$ eV.

The effective neutrino mass $\langle m_\nu\rangle$ in Eq. (2) is extracted from the decay half lifetime of even-even nuclei [25]

$$[T^{1/2}(\beta\beta)_{0\nu}]^{-1} = |M_{\mathrm{nucl}}|^2 \times (\text{phase space integral}) \times \frac{\langle m_\nu\rangle^2}{m_e^2} + \cdots. \tag{6}$$

Ellipses represent the other, different from direct light Majorana neutrino exchange mechanisms which can contribute to $(\beta\beta)_{0\nu}$ decay (e.g., mechanisms with heavy neutrinos or supersymmetric particles [26]). The identification of various mechanisms responsible for the neutrinoless double beta decay, as well as the precise calculation of the nuclear matrix elements is a very difficult task, e.g., the nuclear matrix element $|M_{\mathrm{nucl}}|^2$ has been calculated by several groups and the results differ among them roughly by a factor of 3 [25–27].

The present limit on the effective light neutrino mass is [28]

$$\langle m_\nu\rangle < \kappa = 0.2\ \mathrm{eV}. \tag{7}$$

Several new experiments are considered which will be able to further increase the sensitivity of the $\langle m_\nu\rangle$ measurement [29–31] though the best limit is planned to be obtained by the GENIUS experiment. In its first stage of running, GENIUS with 1 ton of ^{76}Ge should be able to reach a sensitivity of $\langle m_\nu\rangle \sim 0.02$ eV, later with 10 tons of ^{76}Ge, a sensitivity of the order of $\langle m_\nu\rangle \sim 0.006$ eV will be available [32].

III. DIRAC AND MAJORANA NEUTRINO MASSES IN THE A_3 AND A_3^{inv} SCHEMES: ANALYTICAL FORMULAS

Here we summarize the key expressions, used to determine the absolute neutrino masses. It is known that the electron energy distribution in the β decay of nuclei and flavor oscillations do not distinguish between Dirac and Majorana neutrinos.

(i) *Oscillation experiments*. Since in both neutrino mass schemes

$$(m_\nu)^2_{\max} = (m^2_\nu)_{\min} + \delta m^2_{\mathrm{sol}} + \delta m^2_{\mathrm{atm}}, \tag{8}$$

the oscillation experiments alone give [33]

$$(m_\nu)_{\max} \geqslant \sqrt{\delta m^2_{\mathrm{sol}} + \delta m^2_{\mathrm{atm}}} \tag{9}$$

and

$$|m_i - m_j| \geqslant \sqrt{\delta m^2_{\mathrm{sol}} + \delta m^2_{\mathrm{atm}}}. \tag{10}$$

(ii) *Tritium β decay*. From the effective neutrino mass formula Eq. (1) we can find a double inequality

$$(m_\nu)_{\min} \leqslant m_\beta \leqslant (m_\nu)_{\max}. \tag{11}$$

Currently, we have only a bound on m_β Eq. (4), that gives a limit on the absolute neutrino mass

$$(m_\nu)_{\min} \leqslant \kappa' \tag{12}$$

without any limits on $(m_\nu)_{\max}$.

(iii) *The tritium m_β decay together with neutrino oscillation data*. From Eq. (1) we can find the relations

$$m_\beta^2 = (m_\nu)_{\min}^2 + \Omega_{\text{scheme}} \tag{13}$$

and

$$(m_\nu)_{\max}^2 = m_\beta^2 + \Lambda_{\text{scheme}}, \tag{14}$$

where Ω and Λ are scheme dependent quantities and in both schemes A_3 and A_3^{inv} are given by

$$\Omega(A_3) = (1-|U_{e1}|^2)\,\delta m_{\text{sol}}^2 + |U_{e3}|^2\,\delta m_{\text{atm}}^2, \tag{15}$$

$$\Lambda(A_3) = |U_{e1}|^2\,\delta m_{\text{sol}}^2 + (1-|U_{e3}|^2)\,\delta m_{\text{atm}}^2, \tag{16}$$

$$\Omega(A_3^{\text{inv}}) = (1-|U_{e3}|^2)\,\delta m_{\text{atm}}^2 + |U_{e1}|^2\,\delta m_{\text{sol}}^2, \tag{17}$$

$$\Lambda(A_3^{\text{inv}}) = (1-|U_e|^2)\,\delta m_{\text{sol}}^2 + |U_{e3}|^2\,\delta m_{\text{atm}}^2. \tag{18}$$

From Eqs. (4),(8),(13),(14) better limits on $(m_\nu)_{\min}$ and $(m_\nu)_{\max}$ follow:

$$(m_\nu)_{\min} \leqslant \sqrt{(\kappa')^2 - \Omega_{\text{scheme}}^{\min}}, \tag{19}$$

$$\sqrt{\delta m_{\text{sol}}^2 + \delta m_{\text{atm}}^2} \leqslant (m_\nu)_{\max} \leqslant \sqrt{(\kappa')^2 + \Lambda_{\text{scheme}}^{\max}}, \tag{20}$$

where $\Omega_{\text{scheme}}^{\min}$ and $\Lambda_{\text{scheme}}^{\max}$ are the allowed minimal and maximal values given by Eqs. (15)–(18).

We can see that the knowledge of m_β together with the oscillation parameters gives a simple way to determine the absolute neutrino masses. If the neutrino happens to be a Dirac particle, then this will be the only way to determine its mass. If neutrinos are Majorana particles the bound on $\langle m_\nu \rangle$ applies and additional constraints follow.

(iv) *Neutrinoless double beta decay*. For three neutrinos in the A_3 and A_3^{inv} schemes we have

$$\langle m_\nu \rangle_{A_3} = \big| |U_{e1}|^2 (m_\nu)_{\min}^2 + |U_{e2}|^2 e^{2i\phi_2} \sqrt{(m_\nu)_{\min}^2 + \delta m_{\text{sol}}^2} + |U_{e3}|^2 e^{2i\phi_3} \sqrt{(m_\nu)_{\min}^2 + \delta m_{\text{sol}}^2 + \delta m_{\text{atm}}^2} \big| \tag{21}$$

and

$$\langle m_\nu \rangle_{A_3^{\text{inv}}} = \big| |U_{e1}|^2 \sqrt{(m_\nu)_{\min}^2 + \delta m_{\text{atm}}^2} + |U_{e2}|^2 e^{2i\phi_2} \times \sqrt{(m_\nu)_{\min}^2 + \delta m_{\text{atm}}^2 + \delta m_{\text{sol}}^2} + |U_{e3}|^2 e^{2i\phi_3} (m_\nu)_{\min}^2 \big|. \tag{22}$$

The three parameters used above, $(m_\nu)_{\min}$ and two Majorana *CP* violating phases ϕ_1 and ϕ_2 are unknown. We are not able to predict the value of $\langle m_\nu \rangle$ as a function of $(m_\nu)_{\min}$ but a range

$$(\langle m_\nu \rangle_{\min}, \langle m_\nu \rangle_{\max}), \tag{23}$$

can be obtained [11,12].

At present only the upper bound on $\langle m_\nu \rangle$ [Eq. (7)] is known. This result allows us to estimate the minimal mass of the lightest neutrino $(m_\nu)_{\min}$. In the future, if the $(\beta\beta)_{0\nu}$ experiment gives a positive result and a value $\langle m_\nu \rangle = \kappa \pm \Delta\kappa$ is found, the problem of the neutrino mass determination depends on the relation between the range given in Eq. (23) and the value of κ. Such a possibility will be discussed later.

IV. ABSOLUTE NEUTRINO MASSES: NUMERICAL RESULTS

A. Dirac or Majorana case

From oscillation experiments we can only state that the mass of the heaviest neutrino must be larger than 0.04 eV [Eq. (9), Table I]

$$(m_\nu)_{\max} \geqslant 0.04\,\text{eV}, \tag{24}$$

and the difference between any two neutrino masses is smaller than 0.08 eV

$$|m_i - m_j| < 0.08\,\text{eV}. \tag{25}$$

These results depend on the precision of δm_{atm}^2 [Eqs. (9),(10)]. This means that a future improvement in the determination of δm_{atm}^2 (up to 1%, see Table I) will result in a substantial improvement of these bounds.

From the tritium β decay [Eqs. (4),(12)] we can find that the mass of the lightest neutrino must be smaller than 2.2 eV

$$(m_\nu)_{\min} < 2.2\,\text{eV}, \tag{26}$$

which together with the bound Eq. (25) gives limits on the masses of each neutrino separately

$$m_i \leqslant 2.2\,\text{eV}, \quad i = 1,2,3. \tag{27}$$

Equations (24),(25),(27) establish the present knowledge of the neutrino masses independently of their nature. For Dirac neutrinos there is no better sources of information. In the future, the $^3_1$H decay supplemented by the oscillation data will be able to reconstruct the Dirac or Majorana neutrino mass spectrum up to small values of $(m_\nu)_{\min}$. This can be done quite precisely. From Eq. (13) it follows that the relative error of $(m_\nu)_{\min}$ is given by

$$\frac{\Delta(m_\nu)_{\min}}{(m_\nu)_{\min}} = \frac{m_\beta}{(m_\nu)_{\min}^2}\Delta m_\beta + \frac{1}{2(m_\nu)_{\min}^2}\Delta(\Omega_{\text{scheme}}). \tag{28}$$

The part of $\Delta(m_\nu)_{\min}$ which comes from the uncertainties of neutrino oscillation parameters is very small:

$$\Delta(\Omega_{A_3}) = 3.4 \times 10^{-4}, \tag{29}$$

$$\Delta(\Omega_{A_3^{\text{inv}}}) = 29.4 \times 10^{-4}, \tag{30}$$

and in the range of the KATRIN experiment $[(m_\nu)_{\min} \sim 0.3\,\text{eV}$ [24] the error becomes negligible

$$\frac{\Delta(\Omega_{A_3})}{2(m_\nu)^2_{\min}} \approx 0.2\%, \tag{31}$$

$$\frac{\Delta(\Omega_{A_3^{\rm inv}})}{2(m_\nu)^2_{\min}} \approx 1.6\%. \tag{32}$$

The errors increase with decreasing $(m_\nu)_{\min}$, e.g., for $(m_\nu)_{\min}=0.13\,$eV the error is 1% (A_3 scheme). Future improvements in the determination of neutrino oscillation parameters will decrease this error substantially, e.g., using the estimations from the last column of Table I,

$$\frac{\Delta(\Omega_{A_3})}{2(m_\nu)^2_{\min}} \approx 1\% \quad \text{for} \quad (m_\nu)_{\min}=0.02 \ \text{eV}. \tag{33}$$

As we can see the main error comes from Δm_β which will be under control for $m_\beta \geqslant 0.3\,$eV. Since in this case the error connected to uncertainties of the oscillation parameters is below 1%, the tritium β decay together with the oscillation experiments would be the ideal place for the neutrino mass spectrum reconstruction as long as $(m_\nu)_{\min}>0.3\,$eV. If neutrinos are Dirac particles and their masses are below this scale, then the absolute neutrino mass determination seems to be out of reach, unless some new methods of direct neutrino mass measurements are developed.

B. Majorana case

Currently, the bound on the effective Majorana mass $\langle m_\nu \rangle$ is one order of magnitude better than on m_β [compare Eqs. (4) and (7)]. Moreover, there are really impressive plans to get $\langle m_\nu \rangle \approx 0.006\,$eV in $(\beta\beta)_{0\nu}$ experiments. Will they be able to get down with a sensitivity of $(m_\nu)_{\min}$ to the meV scale? The situation seems to be very promising, however, $\langle m_\nu \rangle$ depends on the Majorana phases which can lead to large cancellations. For this reason, the range of possible $\langle m_\nu \rangle$ values [Eq. (23)] can be very wide. This range depends also very crucially on the U_{ei} mixing matrix elements which are not known with a satisfactory precision [see Table I, Eq. (3)]. The reactor experiments which determine U_{e3} are not enough precise, namely [34],

$$|U_{e3}|^2 \leqslant 0.04. \tag{34}$$

The maximum of $\langle m_\nu \rangle$ is stable and depends mostly on θ_{13}

$$\langle M_\nu \rangle_{\max} = (\cos^2\theta_{12} m_1 + \sin^2\theta_{12} m_2)\cos^2\theta_{13} + m_3 \sin^2\theta_{13}, \tag{35}$$

so for various regions of masses there is

$$\begin{aligned} A_3:\ \langle m_\nu \rangle_{\max} &\approx m_3 \sin^2\theta_{13}, \quad m_1 \ll m_2 \ll m_3, \\ &\approx m_1 \cos^2\theta_{13} + m_3 \sin^2\theta_{13}, \\ &\quad m_1 \approx m_2 \ll m_3, \\ &\approx m_1, \quad m_1 \approx m_2 \approx m_3, \end{aligned} \tag{36}$$

$$\begin{aligned} A_3^{\rm iuv}:\ \langle m_\nu \rangle_{\max} &\approx \cos^2\theta_{13} m_1, \quad m_1 \approx m_2 \gg m_3, \\ &\approx m_1, \quad m_1 \approx m_2 \approx m_3. \end{aligned}$$

The formula which gives the minimal value of $\langle m_\nu \rangle$ is much more complicated and strongly depends on the solar mixing angle θ_{12}. If $\theta_{12} \approx \pi/4$, cancellations among all three terms in $\langle m_\nu \rangle$ are possible and $\langle m_\nu \rangle_{\min}$ can be negligible small. If $\theta_{12} \neq \pi/4$, one of two terms $|U_{e1}|^2 m_1$, $|U_{e2}|^2 m_2$ dominates the cancellation is not complete, $\langle m_\nu \rangle_{\min}>0$. To see it let us take a large value of $(m_\nu)_{\min}$ $[(m_\nu)_{\min} \gg \delta m^2_{\rm atm}]$, then in both schemes

$$\begin{aligned} \langle m_\nu \rangle_{\min} &\approx (m_\nu)_{\min}(\cos^2\theta_{13}|\cos^2\theta_{12} - \sin^2\theta_{12}| - \sin^2\theta_{13}) \\ &= (m_\nu)_{\min}(\epsilon \cos^2\theta_{13} - \sin^2\theta_{13}), \end{aligned} \tag{37}$$

where the parameter ϵ is introduced[3]

$$\epsilon = |\cos^2\theta_{12} - \sin^2\theta_{12}| = \sqrt{1 - \sin^2 2\Theta_{12}}. \tag{38}$$

This new parameter measures the deviation of the Θ_{12} angle from its maximal value ($\theta_{12}=\pi/4$) and is quite suitable for our discussion.

As θ_{13} is small [Eq. (34)], for degenerate neutrino masses $\langle m_\nu \rangle_{\min}$ depends crucially on ϵ. If $\epsilon \to 1$ (which is realized for SMA MSW solution)

$$\langle m_\nu \rangle_{\min} \approx (m_\nu)_{\min} \cos 2\theta_{13}, \tag{39}$$

and the spread of the region $\Delta\langle m_\nu \rangle = \langle m_\nu \rangle_{\max} - \langle m_\nu \rangle_{\min}$ for a given value of $(m_\nu)_{\min}$ is small (see Figs. 2, 3)

$$\frac{\Delta\langle m_\nu \rangle}{(m_\nu)_{\min}} = 2\sin^2\theta_{13} < 0.08. \tag{40}$$

For the LMA and LOW-QVO solutions of the solar neutrino problem $\epsilon \ll 1$. In this case strong cancellations in $\langle m_\nu \rangle_{\min}$ occur and values $\langle m_\nu \rangle_{\min} \approx 0$, even for large $(m_\nu)_{\min}$, are not excluded. Also the spread of the region $\Delta\langle m_\nu \rangle$ is substantial

$$\frac{\Delta\langle m_\nu \rangle}{(m_\nu)_{\min}} \approx \frac{\langle m_\nu \rangle_{\max}}{(m_\nu)_{\min}} \to 1. \tag{41}$$

The relations mentioned above are depicted in Figs. 4–6. Now we will discuss the results gathered in Figs. 2–6 in more details.

1. Majorana neutrinos and SMA MSW

Figures 2, 3 show the allowed $\langle m_\nu \rangle$ range for the SMA MSW solution. The solid lines represent $\langle m_\nu \rangle_{\max}$ and $\langle m_\nu \rangle_{\min}$ for the best fit parameters. The shaded and hashed regions correspond to uncertainties of the oscillation parameters (Table I) for $\langle m_\nu \rangle_{\min}$ and $\langle m_\nu \rangle_{\max}$, respectively.

[3]The formula [Eq. (37)] is valid only for $\epsilon > \tan^2\theta_{13}$. For smaller values of ϵ, $\langle m_\nu \rangle_{\min}=0$ [12].

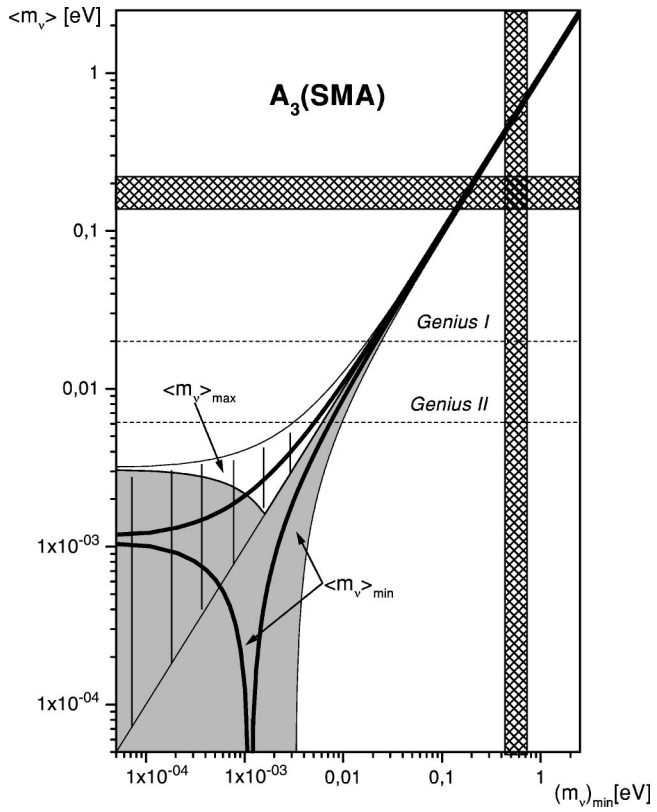

FIG. 2. The value of $\langle m_\nu\rangle_{\max}$ and $\langle m_\nu\rangle_{\min}$ as function of $(m_\nu)_{\min}$ for SMA MSW scenario and A_3 neutrino mass scheme. The shaded and hashed regions correspond to the allowed ranges of neutrino oscillation parameters (Table I) for $\langle m_\nu\rangle_{\min}$ and $\langle m_\nu\rangle_{\max}$, respectively. The solid lines correspond to the best fit values of neutrino oscillation parameters. The experimental bound on $\langle m_\nu\rangle$ planed by GENIUS I and GENIUS II are depicted (dashed, horizontal lines). The vertical band correspond to the possible range of $(m_\nu)_{\min}$ determined by the tritium β decay experiment.

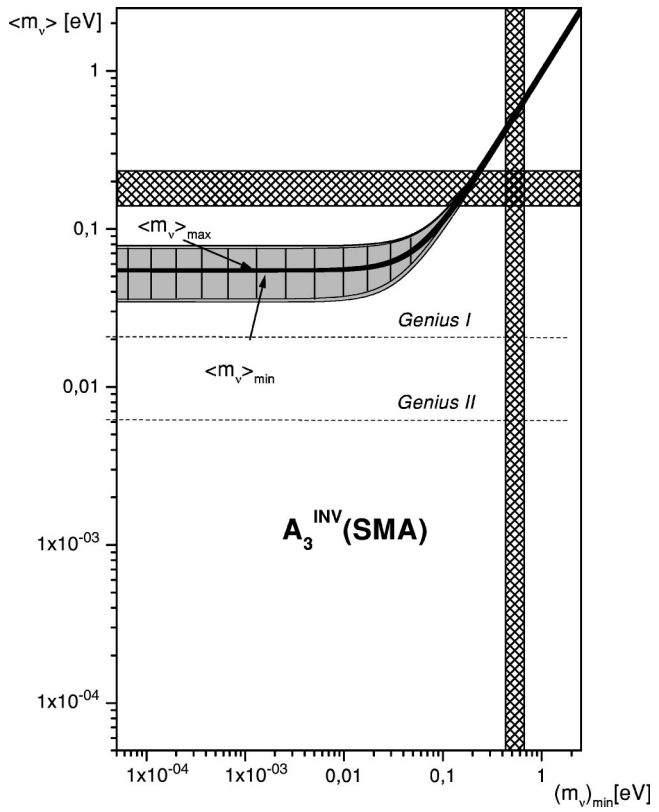

FIG. 3. The value of $\langle m_\nu\rangle_{\max}$ and $\langle m_\nu\rangle_{\min}$ as function of $(m_\nu)_{\min}$ for SMA MSW scenario and $A_3^{\rm inv}$ neutrino mass scheme. The shaded and hashed regions correspond to the allowed ranges of neutrino oscillation parameters (Table I) for $\langle m_\nu\rangle_{\min}$ and $\langle m_\nu\rangle_{\max}$, respectively. The solid lines correspond to the best fit values of neutrino oscillation parameters.

In the A_3 scheme (Fig. 2) the present bound on $\langle m_\nu\rangle$ [Eq. (7)] implies the largest value of $(m_\nu)_{\min}$

$$(m_\nu)_{\min}<0.2\ \text{eV}, \tag{42}$$

and from Eqs. (8), (24)

$$0.04\leqslant(m_\nu)_{\max}\leqslant 0.21\ \text{eV}. \tag{43}$$

Future bounds on $\langle m_\nu\rangle_{\rm exp}$, inferred from $(\beta\beta)_{0\upsilon}$ experiments, have chance to give a stringent limit on neutrino masses. For example, (1) if $\langle m_\nu\rangle<0.02\,$eV (GENIUS 1t), then

$$(m_\nu)_{\min}<0.024\ \text{eV}\Rightarrow 0.04\ \text{eV}\leqslant(m_\nu)_{\max}\leqslant 0.063\ \text{eV}, \tag{44}$$

(2) if $\langle m_\nu\rangle<0.006\,$eV (GENIUS 10t), then

$$(m_\nu)_{\min}<0.01\ \text{eV}\Rightarrow 0.04\ \text{eV}\leqslant(m_\nu)_{\max}\leqslant 0.059\ \text{eV}. \tag{45}$$

It is also possible to find Majorana neutrino masses if the $(\beta\beta)_{0\nu}$ decay is observed and a value $\langle m_\nu\rangle_{\rm exp}\in(0.006-0.2)\,$eV is inferred. We can see from Fig. 2 that the range $\Delta\langle m_\nu\rangle$ is up to $(m_\nu)_{\min}\approx 0.015\,$eV reasonably narrow and the knowledge of $\langle m_\nu\rangle_{\rm exp}$ gives a chance to determine $(m_\nu)_{\min}$ with a good precision. For instance, if $\langle m_\nu\rangle_{\rm exp}\approx 0.006\,$eV then $(m_\nu)_{\min}\sim(3-10)\times 10^{-3}\,$eV, the determination of smaller values of $(m_\nu)_{\min}\ll\sqrt{\delta m^2_{\rm atm}}$ for the hierarchical mass spectrum is impossible with the present oscillation parameters uncertainties.

In the case of $A_3^{\rm inv}$ scheme (Fig. 3), the shaded and hashed regions which describe the uncertainty in the determination of $\langle m_\nu\rangle_{\min}$ and $\langle m_\nu\rangle_{\max}$ are almost identical and narrow. From the present limit on $\langle m_\nu\rangle$ [Eq. (7)] it follows that

$$\langle m_\nu\rangle<0.2\ \text{eV}\Rightarrow(m_\nu)_{\min}<0.22\ \text{eV}. \tag{46}$$

Future bounds on $\langle m_\nu\rangle$ up to $\langle m_\nu\rangle_{\rm exp}\approx 0.04\,$eV, still give the upper limit on $(m_\nu)_{\min}$. If the bound on $\langle m_\nu\rangle$ is smaller (GENIUS I) the scheme $A_3^{\rm inv}$ is excluded for Majorana neutrinos.

The A_3 scheme can not be excluded in this way, even for very small $\langle m_\nu\rangle_{\rm exp}$. However, we can also consider a hypothetical situation. Let us imagine that the $^3_1$He decay measurement gives the mass of $(m_\nu)_{\min}$ in the region (0.4–0.7) eV (see Fig. 2). At the same time the GENIUS I experiment will moves the limit to $\langle m_\nu\rangle<0.02\,$eV. So, from the second

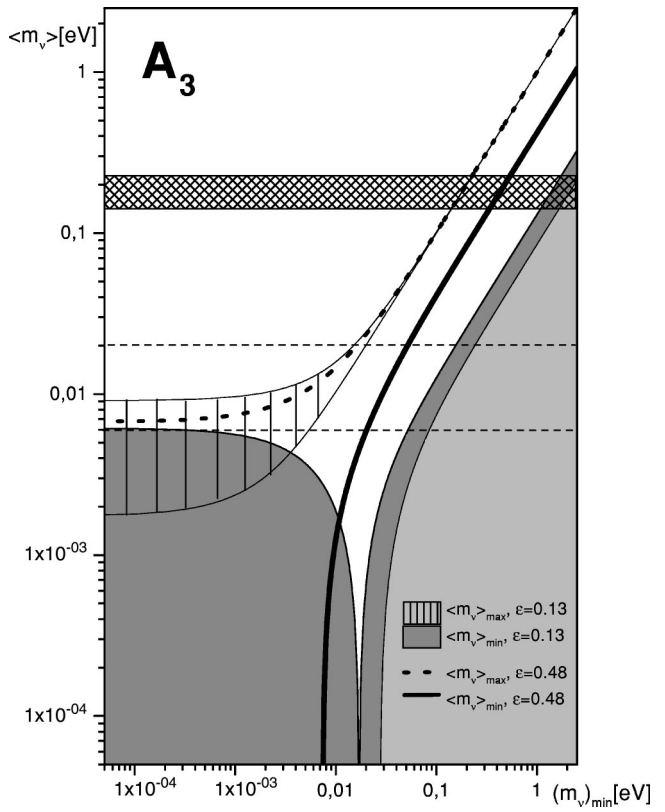

FIG. 4. The value of $\langle m_\nu\rangle_{\max}$ and $\langle m_\nu\rangle_{\min}$ as function of $(m_\nu)_{\min}$ for LMA (LOW-QVO) MSW scenario and A_3 neutrino mass scheme. Shaded areas shows $\langle m_\nu\rangle_{\min}$ for $\epsilon=0.13$ and $\delta m^2_{\rm atm}, |U_{e3}|^2, \delta m^2_{\rm sol}$ parameters in a full range of their present possible values without error of ϵ (dark shaded) and with this error (light shaded) (see Table I). Hashed region shows $\langle m_\nu\rangle_{\max}$ with $\delta m^2_{\rm atm}, |U_{e3}|^2, \delta m^2_{\rm sol}$ parameters also in a full range of their present possible values. Horizontal band corresponds to $\langle m_\nu\rangle$ as planed by GENIUS I (with some anticipated error). The thick solid (dashed) line correspond to $\langle m_\nu\rangle_{\min}$ ($\langle m_\nu\rangle_{\max}$) and $\epsilon=0.48$. This time neutrino oscillation parameters are taken with their best values (Table I).

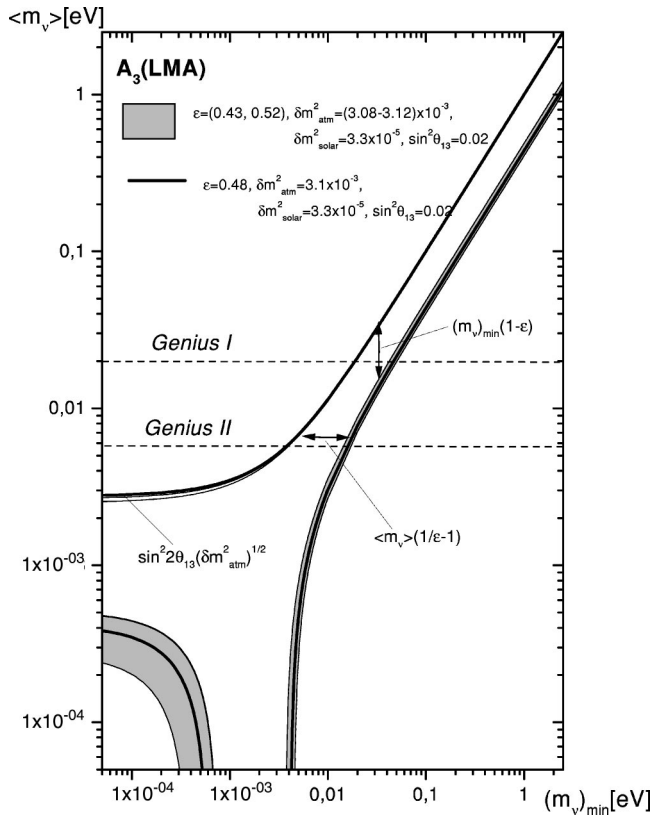

FIG. 5. The value of $\langle m_\nu\rangle_{\max}$ and $\langle m_\nu\rangle_{\min}$ as function of $(m_\nu)_{\min}$ for LMA MSW scenario and A_3 neutrino mass scheme, $\epsilon=0.48$. This time, opposite to the case of Figs. 4, 6 the anticipated error of 10% in future $\sin^2 2\Theta_{\rm sol}$ determination is included, $\epsilon \simeq (0.43-0.52)$. Expected improvement in $\delta m^2_{\rm atm}, |U_{e3}|^2, \delta m^2_{\rm sol}$ parameters determination is also taken into account. See the last column in Table I and the text for details.

information it follows, that $(m_\nu)_{\min}$ is smaller than 0.024 eV [Eq. (44)], which is in evident conflict with the ${}^3_1$H decay measurements. There is only one obvious conclusion in this case. Neutrinos cannot be Majorana particles, they must have a Dirac nature.

We can see that the SMA MSW solution gives a crucial information about the Majorana neutrino mass spectrum independently of future $(\beta\beta)_{0\nu}$ experiments giving a bound, or finding a finite value for $\langle m_\nu\rangle_{\rm exp}$. Unfortunately, among the four possible solutions of the solar neutrino problem, the SMA gives presently the worst goodness of fit [20].

2. Majorana neutrinos and LMA, LOW-QVO solutions: A_3 scheme

In Figs. 4, 6 regions of $\Delta\langle m_\nu\rangle$ as function of $(m_\nu)_{\min}$ for various ϵ values are shown. $\epsilon=0.13$ corresponds to value of $\sin^2 2\Theta_{\rm sol}=0.98$ ($\tan^2\Theta_{\rm sol}=0.77$) while $\epsilon=0.48$ ($\sin^2 2\Theta_{\rm sol}=0.77$) is present best fit value (see Table I). The dark shaded area shows the influence of the uncertainties connected to the present neutrino oscillation parameters' determination (δm^2_{21}, δm^2_{32}, $\sin^2\Theta_{13}$ in Table I) on $\langle m_\nu\rangle_{\min}$ for $\epsilon=0.13$. At 95% C.L. $\epsilon=0$ is possible and $\langle m_\nu\rangle_{\min}$ reaches zero also for higher values of $(m_\nu)_{\min}$ (light shaded region). The hashed region describes the influence of the present oscillation parameters' uncertainties on $\langle m_\nu\rangle_{\max}$ (once more with constant ϵ).

Let us now consider two situations, first when a future new bound on $\langle m_\nu\rangle_{\rm exp}<\kappa\in(0.2-0.006)$eV is obtained and second when some value $\langle m_\nu\rangle_{\rm exp}=k+\Delta\kappa$ is definitely found.

No signal for $(\beta\beta)_{0\nu}$ in future. The information on the Majorana neutrino masses can be inferred only if $\langle m_\nu\rangle_{\min}\neq 0$ which is equivalent to $\epsilon>\tan^2\theta_{13}$. In this case the condition $\langle m_\nu\rangle_{\min}<\kappa$ gives nontrivial bounds on $(m_\nu)_{\min}$ [Eq. (37)]

$$(m_\nu)_{\min}<\frac{\kappa}{\cos^2\theta_{13}(\epsilon-\tan^2\theta_{13})}. \tag{47}$$

For small values of ϵ such a bound can be less restrictive than the one obtained from the ${}^3_1$H decay. We would like to concentrate on the possibility that the future solar neutrino experiments give the value $\epsilon\gg\tan^2\theta_{13}$. Then

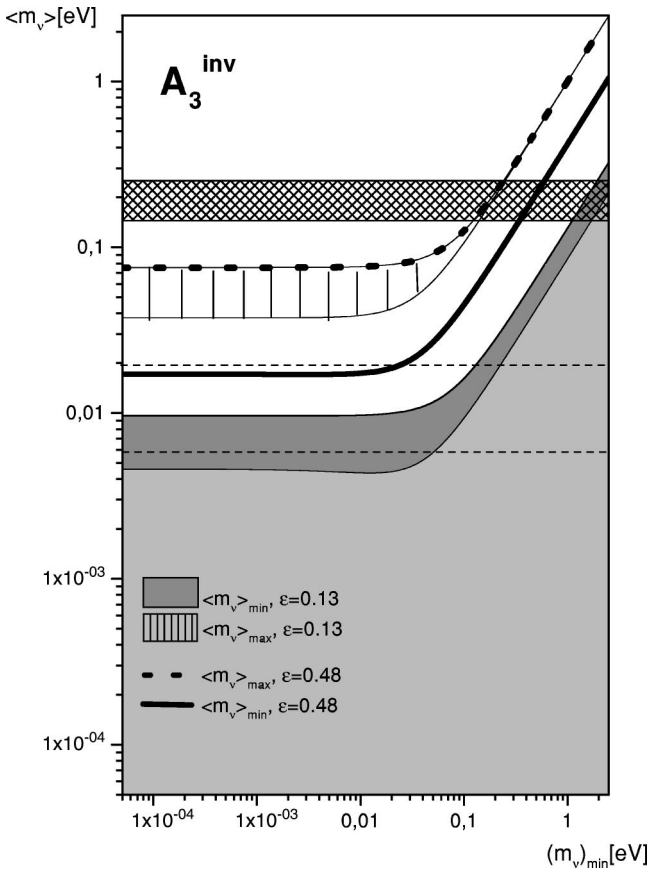

FIG. 6. The value of $\langle m_\nu\rangle_{\max}$ and $\langle m_\nu\rangle_{\min}$ as function of $(m_\nu)_{\min}$ for LMA (LOW-QVO) MSW scenario and $A_3^{\rm inv}$ neutrino mass scheme. Shaded area shows $\langle m_\nu\rangle_{\min}$ for $\epsilon=0.13$ and $\delta m^2_{\rm atm}, |U_{e3}|^2, \delta m^2_{\rm sol}$ parameters in a full range of their present possible values (see Table I). Hashed area shows the same for $\langle m_\nu\rangle_{\max}$. Horizontal band corresponds to $\langle m_\nu\rangle$ as planed by GENIUS I (with some anticipated error). The thick solid (dashed) line correspond to $\langle m_\nu\rangle_{\min}$ ($\langle m_\nu\rangle_{\max}$) and $\epsilon=0.48$. This time neutrino oscillation parameters are taken with their best values (Table I).

$$(m_\nu)_{\min}\leqslant\frac{\kappa}{\epsilon}. \tag{48}$$

If, $\epsilon=0.13(0.48)$ (Fig. 4) then present experimental bound on $\langle m_\nu\rangle$ gives the upper limit on $(m_\nu)_{\min}$,

$$(m_\nu)_{\min}<2.2(0.4)\ \text{eV}, \tag{49}$$

which is better than the tritium β decay [Eq. (26)] for a larger value of ϵ. If $\langle m_\nu\rangle_{\rm exp}<0.006$ eV, then $(m_\nu)_{\min}<0.022$ eV ($\epsilon=0.48$), and $(m_\nu)_{\min}<0.085$ eV ($\epsilon=0.13$).

So, if $(\beta\beta)_{0\nu}$ is not found, we do not know if neutrinos are Majorana particles. If they indeed are then limits on neutrino masses can be found which are improving for increasing ϵ.

Positive signal for $\langle\beta\beta\rangle_{0\nu}$ in future. From the $\langle\beta\beta\rangle_{0\nu}$ measurement we know that

$$\langle m_\nu\rangle_{\rm exp}=\kappa\pm\Delta\kappa. \tag{50}$$

Obviously, in this case consistency requires

$$\langle m_\nu\rangle_{\max}>\kappa-\Delta\kappa, \tag{51}$$

from which interesting informations on the Majorana neutrino masses can be obtained, even for small values of $\epsilon\to 0$. This situation was already considered in Refs. [15], [16], so we will not analyze it in detail. We only add some numbers which follow from Fig. 4. The measured values of $\langle m_\nu\rangle_{\rm exp}\geqslant 0.01$ eV are able to bound $(m_\nu)_{\min}$ from below, e.g., $\langle m_\nu\rangle_{\rm exp}\approx 0.03$ eV gives $(m_\nu)_{\min}\geqslant 0.025$ eV. Depending on the measured values of $\langle m_\nu\rangle_{\rm exp}$ various mass schemes can be excluded or allowed. For instance, if $\langle m_\nu\rangle_{\rm exp}\leqslant 0.01$ eV then $(m_\nu)_{\min}$ can be very small and the hierarchical mass spectrum is allowed. For $\langle m_\nu\rangle\geqslant 0.03$ eV only the degenerate spectrum will be acceptable.

A new situation occurs if $\langle m_\nu\rangle_{\rm exp}\geqslant 0.01$ eV and $\epsilon\gg\tan^2\theta_{12}$. Then from $\langle m_\nu\rangle_{\min}$ an upper limit on $(m_\nu)_{\min}$ can also be found. It means that a finite range of possible values of $(m_\nu)_{\min}$ can be derived

$$(m_\nu)_{\min}\in[(m_\nu)^{\min}_{\min},(m_\nu)^{\max}_{\min}], \tag{52}$$

where

$$(m_\nu)^{\min}_{\min}=\kappa-\Delta\kappa,$$

$$(m_\nu)^{\max}_{\min}=(\kappa+\Delta\kappa)\frac{1}{(1+\epsilon)\cos^2\theta_{13}-1}. \tag{53}$$

The uncertainty of the $(m_\nu)_{\min}$ determination

$$\Delta(m_\nu)_{\min}=(m_\nu)^{\max}_{\min}-(m_\nu)^{\min}_{\min} \tag{54}$$

decreases with increasing ϵ

$$\frac{\Delta(m_\nu)_{\min}}{\kappa}=\frac{2-(1+\epsilon)\cos^2\theta_{13}}{(1+\epsilon)\cos^2\theta_{13}-1}+\frac{\Delta\kappa}{\kappa}\frac{(1+\epsilon)\cos^2\theta_{13}}{(1+\epsilon)\cos^2\theta_{13}-1}$$

$$\xrightarrow{\theta_{13}\to 0}\left(\frac{1}{\epsilon}-1\right)+\frac{\Delta\kappa}{\kappa}\left(\frac{1}{\epsilon}+1\right). \tag{55}$$

From Fig. 4 and for $\epsilon=0.13$ we can read (neglecting $\Delta\kappa$)

$$\langle m_\nu\rangle_{\rm exp}=0.2\ \text{eV}\Rightarrow(m_\nu)_{\min}\in(0.2-2.2)\ \text{eV}, \tag{56}$$

$$\langle m_\nu\rangle_{\rm exp}=0.02\ \text{eV}\Rightarrow(m_\nu)_{\min}\in(0.012-0.2)\ \text{eV}. \tag{57}$$

Moreover, if $\langle m_\nu\rangle_{\rm exp}<0.01$ eV we can say only that $(m_\nu)_{\min}<0.02-0.05$ eV.

So, for LMA and LOW-QVO solutions, the knowledge of $\langle m_\nu\rangle_{\rm exp}$ is able to restrict the Majorana neutrino mass spectrum, as long as $\epsilon\geqslant\tan^2\Theta_{13}$. The range of $(m_\nu)_{\min}$ [Eq. (54)] depends on ϵ and $\Delta\kappa$ and is smaller for larger values of ϵ.

In Fig. 5 the region $(\langle m_\nu\rangle_{\min},\langle m_\nu\rangle_{\max})$ is shown for the case $\epsilon=0.48$ ($\sin^2 2\Theta_{\rm sol}=0.77$) where anticipated, much smaller errors on $\delta m^2_{\rm atm}$, $|U_{e3}|^2$, and $\delta m^2_{\rm sol}$ compared to the present ones are taken into account (see Table I for details).

The expected 10% error of $\sin^2 2\Theta_{\rm sol}$ is included. The uncertainty of $\langle m_\nu\rangle_{\rm max}$ is now almost invisible. For $\langle m_\nu\rangle_{\rm min}$ two separated regions of nonzero $\langle m_\nu\rangle$ are present. The light shaded region as in Fig. 4 does not appear at all. The shape of the $\langle m_\nu\rangle_{\rm min}$ depends on the value of ϵ and this presented in Fig. 5 is typical of non-negligible values of ϵ. We can see that for the experimental bound $\langle m_\nu\rangle_{\rm exp}<\kappa$ the upper limit of $(m_\nu)_{\rm min}$ can be easily found. The same is true of the case $\langle m_\nu\rangle_{\rm exp}=\kappa\pm\Delta\kappa$ where the range of possible $(m_\nu)_{\rm min}$ can be found again. There is only one important modification. If the future bound or the experimental value of $\langle m_\nu\rangle_{\rm exp}$ will be smaller than 0.001 eV then the mass $(m_\nu)_{\rm min}$ of Majorana neutrinos will be limited from below and, this time, also from above. For instance, if $\langle m_\nu\rangle_{\rm exp}<2\times10^{-4}$ eV, then $(m_\nu)_{\rm min}\in(1.0\times10^{-4}\ {\rm eV},5.0\times10^{-3}\ {\rm eV})$.

For larger values of ϵ ($\epsilon\sim0.5$) it can also happened that the accepted range of $(m_\nu)_{\rm min}$ found from tritium β decay is in conflict with a bound given by $(\beta\beta)_{0\nu}$ decay. The situation is practically the same as in the SMA case and the conclusions concerning the neutrino nature are the same (see previous discussion). From such a unique scenario follows that neutrinos are Dirac particles.

Now we would like to comment on the *CP* phases. The effect of two unknown *CP* Majorana phases disappears if $\epsilon\to1$ (SMA). So for large ϵ the information about the *CP* phases is lost. If, however, ϵ is small ($\epsilon\sim0.1-0.5$) and $\langle m_\nu\rangle_{\rm exp}=\kappa\pm\Delta\kappa$, $m_\beta=\kappa'\pm\Delta\kappa'$ are found with a good precision, some insight into the *CP* symmetry is possible. Comparing both bands $\langle m_\nu\rangle\in(\kappa-\Delta\kappa,\kappa+\Delta\kappa)$ and $m_\beta\in(\kappa'-\Delta\kappa',\kappa'+\Delta\kappa')$ with the $(\langle m_\nu\rangle_{\rm min},\langle m_\nu\rangle_{\rm max})$ region allowed by the oscillation data is a check of internal consistency of the theory. With precise data the crossing of the three regions can be used to specify the values of the *CP* breaking Majorana phases [Eqs. (21), (22)]. If the two bands $\langle m_\nu\rangle$ and m_β cross the oscillation region near $\langle m_\nu\rangle_{\rm max}$ then two phases are equal $\phi_1=\phi_2\approx n\pi$. This means that all three Majorana neutrinos have the same *CP* parity $\eta_{CP}=+i$ and the symmetry is conserved. If the two bands cross the oscillation region near $\langle m_\nu\rangle_{\rm min}$, once more the *CP* symmetry is satisfied and $\eta_{CP}(\nu_1)=-\eta_{CP}(\nu_2)=-\eta_{CP}(\nu_3)=i$. Finally, if all three regions cross somewhere in between, the phases ϕ_1 and ϕ_2 are nontrivial and the *CP* symmetry is broken. We can also imagine the situation that all three regions do not cross in the same place. This would be a signal that the theory with three light Majorana neutrinos is not consistent.

3. Majorana masses and LMA, LOW-QVO solutions: $A_3^{\rm inv}$ scheme

The same analysis as before can be done for the $A_3^{\rm inv}$ scheme. For degenerate masses [$(m_\nu)_{\rm min}\geqslant0.2$ eV], two functions $\langle m_\nu\rangle_{\rm min}$ and $\langle m_\nu\rangle_{\rm max}$ are exactly the same as in the A_3 scheme [Eqs. (35), (37)]. So conclusions concerning the determination of the Majorana neutrino masses are the same. The behavior of the functions $\langle m_\nu\rangle_{\rm min(max)}$ is different for small values of $(m_\nu)_{\rm min}$. As can be seen from Figs. 3, 6 $\langle m_\nu\rangle_{\rm min}$ never vanishes if $\epsilon\neq0$. The minimal value of $\langle m_\nu\rangle_{\rm min}$ is proportional to ϵ, namely,

$$\langle m_\nu\rangle_{\rm min}[(m_\nu)_{\rm min}\approx0]\approx\epsilon\cos^2\theta_{13}\sqrt{\delta m^2_{\rm atm}}\approx\epsilon\times0.04. \tag{58}$$

The minimal value of $\langle m_\nu\rangle_{\rm max}$ does not depend on ϵ and

$$\langle m_\nu\rangle_{\rm max}[(m_\nu)_{\rm min}\approx0]\approx\cos^2\theta_{13}\sqrt{\delta m^2_{\rm atm}}\approx0.08. \tag{59}$$

So, if in future the $(\beta\beta)_{0\nu}$ decay gives a bound $\langle m_\nu\rangle_{\rm exp}<\kappa$, then the scheme $A_3^{\rm inv}$ has to be rejected (for Majorana neutrinos) or neutrinos are Dirac particles when

$$\epsilon\cos^2\theta_{13}\sqrt{\delta m^2_{\rm atm}}>\kappa. \tag{60}$$

If, on the other hand, a finite value of $\langle m_\nu\rangle_{\rm exp}=\kappa\pm\Delta\kappa$ is found then three scenarios are possible: (1) $\kappa-\Delta\kappa>\cos^2\theta_{13}\sqrt{\delta m^2_{\rm atm}}$. The lightest neutrino masses $(m_\nu)_{\rm min}$ can be bounded from below; (2) $\epsilon\cos^2\theta^2_{13}\sqrt{\delta m^2_{\rm atm}}-\Delta\kappa<\kappa<\cos^2\theta_{13}\sqrt{\delta m^2_{\rm atm}}+\Delta\kappa$, the mass $(m_\nu)_{\rm min}$ is weakly bounded to the region $(m_\nu)_{\rm min}\leqslant0.05$ eV; and finally (3) $\kappa+\Delta\kappa<\epsilon\cos^2\theta_{13}\sqrt{\delta m^2_{\rm atm}}$. The scheme $A_3^{\rm inv}$ is excluded.

V. CONCLUSIONS

Atmospheric and solar neutrino experiments give strong evidence that neutrinos have nonzero masses and that they mix. However, these experiments alone are not able to determine the absolute neutrino masses. All other terrestrial experiments are consistent with the assumption that neutrinos are massless [12,35]. Only tritium β and $(\beta\beta)_{0\nu}$ decays are sensitive to neutrino masses at the O(eV) level and the confirmation of their existence at this scale seems to be just around the corner. However, even there only the combination of neutrino masses can be determined. We have considered whether and how precisely the present and future experimental data can determine the single absolute neutrino masses. With the present experimental precision we have found

$$|m_i-m_j|<0.08\ \ {\rm eV},\ \ i,j=1,2,3,$$

$$m_i<2.2\ \ {\rm eV},$$

$$\max(m_1,m_2,m_3)>0.04\ \ {\rm eV}. \tag{61}$$

In the future tritium beta decay altogether with oscillation experiments are the best options to reconstruct the absolute values of neutrino masses, independently of whether they are Dirac or Majorana particles. The relative error which comes from the uncertainty of the oscillation parameters is very small and has no influence on the neutrino mass determination. The results depend uniquely on the precision to which m_β can be determined. That is why this procedure is effective for neutrino masses above $\sim$0.2–0.3 eV. This will be a challenge for future experiments.

If neutrinos are Majorana particles additional information can be inferred from the neutrinoless double beta decay, independently if a nonzero $(\beta\beta)_{0\nu}$ decay rate is found or not. There is only one difference, in the second case we have no experimental confirmation that they really have the Majorana nature. The precision depends on the solution of the solar

neutrino problem. Solutions with smaller $\sin^2 2\theta_{\rm sol}$ are better for the neutrino mass determination. For the SMA solution ($\epsilon \simeq 1$) we found

$$0 \leqslant \min(m_i) \leqslant 0.2 \ \text{eV},$$

$$0.04 \ \text{eV} \leqslant \max(m_i) \leqslant 0.21 \ \text{eV} \tag{62}$$

which is a much stringer bound than Eq. (61). If the SMA solution is confirmed by the future data, the next generation of the $(\beta\beta)_{0\nu}$ experiments has a good chance to find neutrino masses as small as $(m_\nu)_{\rm min} \approx 0.015\,\text{eV}$. Unfortunately, the SMA scenario is presently not a favored solution of the solar neutrino problem.

The neutrino mass determination in the case of the solar neutrino anomaly with small ϵ (LMA, LOW-QVO) is more complicated. First of all $\langle m_\nu \rangle_{\rm min} = 0$ for $\epsilon \leqslant \tan^2 \Theta_{13}$ and the upper limit on $(m_\nu)_{\rm min}$ cannot be obtained (the lower limit is given). If, however, $\epsilon > \tan^2 \theta_{13}$ then the derivation of some useful upper bounds is possible. We have found the analytical bound on $(m_\nu)_{\rm min}$ [Eq. (47)] given by the experimental limit on $\langle m_\nu \rangle_{\rm exp}$ and the parameter ϵ. We have also found the uncertainty in the $(m_\nu)_{\rm min}$ determination $\Delta(m)_{\rm min}$ as function of $\langle m_\nu \rangle_{\rm exp}$ and the two oscillation parameters ϵ, θ_{13}.

It can happen in future that the discovery of $(m_\nu)_{\rm min}$ from the tritium β decay will be in conflict with the bound on $(m)_{\rm min}$ derived from the $(\beta\beta)_{0\nu}$ decay. This is the unique situation where the Dirac character of neutrinos could be confirmed. For smaller values of ϵ and a good experimental precision to which $\langle m_\nu \rangle$ and m_β can be determined, some insight into *CP* symmetry violation or *CP* eigenvalues of neutrinos is possible.

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[1] E. Fermi, Z. Phys. **88**, 161 (1934).
[2] W. H. Furry, Phys. Rev. **56**, 1184 (1939); W. Grimus, Prog. Part. Nucl. Phys. **43**, 1 (1999).
[3] K. Lande, P. Wildenhain, R. Corey, M. Foygel, and J. Distel, Nucl. Phys. B (Proc. Suppl.) **91**, 50 (2001); E. Bellotti, *ibid.* **91**, 44 (2001); SAGE Collaboration, V. N. Gavrin, *ibid.* **91**, 36 (2001); Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **77**, 1683 (1996); W. Hampel *et al.*, Phys. Lett. B **447**, 127 (1999); SNO Collaboration, Q. R. Ahmad *et al.*, Phys. Rev. Lett. **87**, 071301 (2001).
[4] Super-Kamiokande Collaboration, S. Fukuda *et al.*, Phys. Rev. Lett. **86**, 5656 (2001).
[5] LSND Collaboration, A. Aguilar *et al.*, Phys. Rev. D **64** 112007 (2001).
[6] S. Pakvasa, Pramana, J. Phys. **54**, 65 (2000).
[7] P. Lipari and M. Lusignoli, hep-ph/9905229; M. Lusignoli, Nucl. Phys. B (Proc. Suppl.) **100**, 250 (2001).
[8] J. R. Primack, astro-ph/0007187.
[9] T. J. Weiler, Astropart. Phys. **11**, 303 (1999); D. Fargion, B. Mele, and A. Salis, Astrophys. J. **517**, 725 (1999); T. Weiler, Phys. Rev. Lett. **49**, 234 (1982); Z. Fodor, S. D. Katz, and A. Ringwald, hep-ph/0105064.
[10] H. Pas and T. J. Weiler, Phys. Rev. D **63**, 113015 (2001).
[11] F. Vissani, J. High Energy Phys. **06**, 022 (1999).
[12] M. Czakon, J. Gluza, J. Studnik, and M. Zrałek, Acta Phys. Pol. B **31**, 1365 (2000).
[13] S. J. Yellin, hep-ex/9902012; A. D. Santo, Int. J. Mod. Phys. A **6**, 4085 (2001).
[14] C. Lunardini and A. Y. Smirnov, Phys. Rev. D **63**, 073009 (2001); H. Minakata and H. Nunokawa, Phys. Lett. B **504**, 301 (2001).
[15] S. M. Bilenky, S. Pascoli, and S. T. Petcov, Phys. Rev. D **64**, 053010 (2001); **64**, 113003 (2001).
[16] H. V. Klapdor-Kleingrothaus, H. Pas, and A. Y. Smirnov, Phys. Rev. D **63**, 073005 (2001).
[17] S. T. Petcov and A. Y. Smirnov, Phys. Lett. B **322**, 109 (1994); S. M. Bilenky, A. Bottino, C. Giunti, and C. Kim, Phys. Rev. D **54**, 1881 (1996); S. M. Bilenky, C. Giunti, C. Kim, and S. Petcov, *ibid.* **54**, 4432 (1996); J. Hellmig and H. V. Klapdor-Kleingrothaus, Z. Phys. A **359**, 351 (1997); H. V. Klapdor-Kleingrothaus, J. Hellmig, and M. Hirsch, J. Phys. G **24**, 483 (1998); H. Minataka and O. Yasuda, Phys. Rev. D **56**, 1692 (1997); Nucl. Phys. **B523**, 597 (1998); S. M. Bilenky, C. Giunti, C. W. Kim, and M. Monteno, Phys. Rev. D **57**, 6981 (1998); hep-ph/9904328; T. Fukuyama, K. Matsuda, and H. Nishiura, Mod. Phys. Lett. A **13**, 2279 (1998); S. M. Bilenky, C. Giunti, and W. Grimus, hep-ph/9809368; S. Bilenky and C. Giunti, hep-ph/9904328; S. Bilenky, C. Giunti, W. Grimus, B. Kayser, and S. T. Petcov, Phys. Lett. B **465**, 193 (1999); V. Barger and K. Whisnant, *ibid.* **456**, 194 (1999); J. R. Ellis and S. Lola, *ibid.* **458**, 310 (1999); G. C. Branco, M. N. Rebelo, and J. I. Silva-Marcos, Phys. Rev. Lett. **82**, 683 (1999); C. Giunti, Phys. Rev. D **61**, 036002 (2000); R. Adhikari and G. Rajasekaran, *ibid.* **61**, 031301(R) (2000); K. Matsuda, N. Takeda, T. Fukuyama, and H. Nishiura, *ibid.* **62**, 093001 (2000); Y. Farzan, O. L. Peres, and A. Y. Smirnov, Nucl. Phys. **B612**, 59 (2001); W. Rodejohann, *ibid.* **B597**, 110 (2001).
[18] M. Czakon, J. Gluza, and M. Zralek, Phys. Lett. B **465**, 211 (1999).
[19] J. N. Bahcall, P. I. Krastev, and A. Y. Smirnov, J. High Energy Phys. **105**, 015 (2001); J. N. Bahcall, M. C. Conzalez-Garcia, and C. Pena-Garay, *ibid.* **08**, 014 (2001); M. C. Gonzalez-Garcia, P. C. de Holanda, C. Pena-Garay, and J. W. Valle, Nucl. Phys. **B573**, 3 (2000); A. de Gouvea, A. Friedland, and H. Murayama, Phys. Lett. B **490**, 125 (2000).
[20] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, and A. Palazzo, hep-ph/0104221; G. L. Fogli, E. Lisi, D. Montanino, and A. Palazzo, Phys. Rev. D **62**, 013002 (2000); M. C. Gonzalez-Garcia, M. Maltoni, C. Pena-Garay, and J. W. Valle, *ibid.* **63**, 033005 (2001).
[21] C. Giunti, M. C. Gonzalez-Garcia, and C. Pena-Garay, Phys. Rev. D **62**, 013005 (2000); V. Barger, S. Pakvasa, T. J. Weiler,

and K. Whisnant, *ibid.* **58**, 093016 (1998); V. D. Barger, D. Marfatia, and K. Whisnant, hep-ph/0106207.
[22] J. Bonn *et al.*, Nucl. Phys. B (Proc. Suppl.) **91**, 273 (2001).
[23] V. M. Lobashev *et al.*, Nucl. Phys. B (Proc. Suppl.) **91**, 280 (2001).
[24] For details see URI http://www-ik1.fzk.de/tritium; KATRIN Collaboration, A. Osipowicz *et al.*, hep-ex/0109033.
[25] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
[26] W. C. Haxton and G. J. Stephenson, Prog. Part. Nucl. Phys. **12**, 409 (1984); M. Doi, T. Kotani, and E. Takasugi, Prog. Theor. Phys. Suppl. **83**, 1 (1985); M. Hirsch, H. V. Klapdor-Kleingrothaus, and O. Panella, Phys. Lett. B **374**, 7 (1996); H. V. Klapdor-Kleingrothaus, L. Baudis, J. Hellmig, M. Hirsch, S. Kolb, H. Pas, and Y. Ramachers, Nucl. Phys. B (Proc. Suppl.) **70**, 242 (1999); H. Pas, M. Hirsch, and H. V. Klapdor-Kleingrothaus, Phys. Lett. B **459**, 450 (1999); H. Ejiri, Nucl. Phys. B (Proc. Suppl.) **91**, 255 (2001).
[27] V. D. Vergados, Phys. Rev. D **28**, 2887 (1983); C. Greub and P. Minkowski, Int. J. Mod. Phys. A **13**, 2363 (1998).
[28] L. Baudis *et al.*, Phys. Rev. Lett. **83**, 41 (1999).
[29] S. Pirro *et al.*, Nucl. Instrum. Methods Phys. Res. A **444**, 71 (2000).
[30] E. Fiorini, Phys. Rep. **307**, 309 (1998).
[31] H. Ejiri, J. Engel, R. Hazama, P. Krastev, N. Kudomi, and R. G. Robertson, Phys. Rev. Lett. **85**, 2917 (2000).
[32] GENIUS Collaboration, H. V. Klapdor-Kleingrothaus *et al.*, hep-ph/9910205.
[33] V. Barger, T. J. Weiler, and K. Whisnant, Phys. Lett. B **442**, 255 (1998).
[34] M. Apollonio *et al.*, Phys. Lett. B **466**, 415 (1999).
[35] M. Czakon, M. Zrałek, and J. Gluza, Acta Phys. Pol. B **30**, 3121 (1999).
[36] OPERA Collaboration, A. G. Cocco, Nucl. Phys. B (Proc. Suppl.) **85**, 125 (2000); MINOS Collaboration, K. Lang, Nucl. Instrum. Methods Phys. Res. A **461**, 290 (2001); ICARUS Collaboration, F. Cavanna *et al.*, Report No. LNGS-P21-99-ADD-2.
[37] V. Barger, hep-ph/0102052; A. Cervera, A. Donini, M. B. Gavela, J. J. Gomez Cadenas, P. Hernandez, O. Mena, and S. Rigolin, Nucl. Phys. **B579**, 17 (2000); **B593**, 731(E) (2000).
[38] V. Barger, S. Geer, R. Raja, and K. Whisnant, Phys. Rev. D **62**, 013004 (2000); S. Geer, hep-ph/0008155; B. Richter, hep-ph/0008222.