Long-term future of extragalactic astronomy

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If the current energy density of the universe is indeed dominated by a cosmological constant, then high-redshift sources will remain visible to us only until they reach some finite age in their rest frame. The radiation emitted beyond that age will never reach us due to the acceleration of the cosmic expansion rate, and so we will never know what these sources look like as they become older. As a source image freezes on a particular time frame along its evolution, its luminosity distance and redshift continue to increase exponentially with observation time. The higher the current redshift of a source is, the younger it will appear as it fades out of sight. For the popular set of cosmological parameters, I show that a source at a redshift $z_0 \sim 5-10$ will only be visible up to an age of $\sim 4-6$ billion years. Arguments relating the properties of high-redshift sources to present-day counterparts will remain indirect even if we continue to monitor these sources for an infinite amount of time. These sources will not be visible to us when they reach the current age of the universe.

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Recent observations of the microwave background and type Ia supernovae indicate that the universe is flat and its expansion is currently dominated by a cosmological constant [1–7]. The cosmological scale factor may be just entering an exponential expansion phase (similar to inflation) during which all comoving observers will eventually lose causal contact with each other.

The qualitative implications of the future exponential expansion were discussed in the literature [8,9]. It was recognized that an asympototically de Sitter universe of this type possesses a moving light cone out of which all distant sources will eventually exit [10,11]. In similarity to the case of a black hole, one can define an event horizon [12] out to which events can in principle be seen by us (for definitions of other surfaces see [12,13,11]). In an asympototically de Sitter universe, the event horizon asymptotes to a fixed proper distance from us [12,11] above which cosmological events will never be visible to us. Because of the exponential expansion of the cosmological scale factor, all sources which follow the Hubble expansion will eventually exit (in their own reference frame) out of our event horizon. The further a source is away from us, the earlier it exits.

But since our universe was matter-dominated earlier in its history, the number of cosmological sources visible to us has been increasing steadily with cosmic time until recently. It is therefore interesting to examine quantitatively what will happen in the future to the images of all the currently visible sources as a function of their current redshifts. In this paper I will show quantitatively that as a result of the acceleration in the cosmic expansion, all high-redshift sources will fade out of our sight at a finite age (similarly to a source which is infalling through the horizon of a black hole). This implies that we will never be able to see their image as they get older. I will then calculate the *maximum visible age* of a cosmological source as a function of its currently measured redshift. For concreteness, I adopt the present-day density parameter values of $\Omega_M = 0.3$ for matter and $\Omega_\Lambda = 0.7$ for the cosmological constant.

Calculation. The line element for a flat universe is given by $ds^2 = c^2 dt^2 - a^2(t)(dr^2 + r^2 d\Omega)$, where $a(t)$ is the scale factor. Photon trajectories satisfy $ds = 0$, and so the comoving distance of a source that emits radiation at a cosmic time $t_{\rm em}$ and is observed at the current age of the universe t_0 is given by

$$r = \int_{t_{\rm em}}^{t_0} \frac{c\,dt}{a(t)}. \tag{1}$$

If the source continues to emit at a later time $t'_{\rm em}$, then this radiation will be observed by us at a future time t'_0. Since the source maintains its comoving coordinate,

$$r = \int_{t_{\rm em}}^{t_0} \frac{c\,dt}{a(t)} = \int_{t'_{\rm em}}^{t'_0} \frac{c\,dt}{a(t)}, \tag{2}$$

or equivalently

$$\int_{t_0}^{t'_0} \frac{dt}{a(t)} = \int_{t_{\rm em}}^{t'_{\rm em}} \frac{dt}{a(t)}. \tag{3}$$

In terms of the conformal time, $\eta(t) \equiv \int_0^t dt'/a(t')$, Eq. (3) is equivalent to the condition $[\eta(t'_0) - \eta(t_0)] = [\eta(t'_{\rm em}) - \eta(t_{\rm em})]$. The question of whether this equality can be satisfied for an arbitrary value of the source age, $t'_{\rm em}$, depends on the future evolution of the scale factor $a(t)$. It is easy to see that as long as $0 < d \ln a/dt < 1$, this equality can be satisfied for an arbitrary value of $t'_{\rm em}$. This is the case, for example, in a matter-dominated universe where $a \propto t^{2/3}$. However, in a de Sitter universe the scale factor grows exponentially and so the integrand on the left hand side of Eq. (3) saturates at a finite value even as $t'_0 \to \infty$. This implies that there is a maximum intrinsic age, $t'_{\rm em}$, over which the source is visible to us. Emission after the source reaches this age

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will never be observable by us[1] (unless the vacuum energy density which makes up the cosmological constant decays). The maximum visible age obviously depends on $t_{\rm em}$ or the currently measured source redshift, z_0, which is given by the relation $a(t_{\rm em})=(1+z_0)^{-1}$.

The evolution of the scale factor is determined by the Friedmann equation

$$\frac{1}{a}\frac{da}{dt}=H_0\left(\frac{\Omega_M}{a^3}+\Omega_\Lambda\right)^{1/2}, \tag{4}$$

where $\Omega_M+\Omega_\Lambda=1$. Equations (1) and (4) admit analytic solutions [15–21] for $r(t_{\rm em},t_0)$ in terms of an incomplete elliptic integral, and for the scale factor in the form

$$a(t)=\left(\frac{\Omega_M}{1-\Omega_M}\right)^{1/3}\left(\sinh\left(\frac{3}{2}\sqrt{1-\Omega_M}H_0t\right)\right)^{2/3}. \tag{5}$$

Reference [22] provides a simple fitting formula for $\eta(a)=[r(0)-r(a)]$. The luminosity and angular diameter distances at any future time t_0' are given by $d_L=\{a^2(t_0')/a(t_1')\}r$ and $d_A=a(t_1')r$, respectively [19]. The source redshift evloves as $z=\{a(t_0')/a(t_1')\}-1$.

Figure 1 shows the emission time, $t_{\rm em}'$, as a function of the future observing time, t_0'. All time scales are normalized by the inverse of the current Hubble expansion rate, $H_0=(\dot{a}/a)|_{t_0}$. Clearly, as the current source redshift increases, its maximum visible age in the future (i.e. the asymptotic value of $t_{\rm em}'$ for $t_0'\to\infty$) decreases. Typically, the maximum emission time $t_{\rm em}'$ is much longer than the current emission time $t_{\rm em}$, and so only sources that are steady over many Hubble times at their current redshift are suitable for this discussion.

The microwave background anisotropies, for example, do not possess the above property since they were generated over a narrow temporal interval around the time of recombination, $t_{\rm rec}$ (corresponding to $z_0\sim1000$). Hence, the comoving distance of their last scattering surface will increase with the advance of cosmic time, $r_{\rm rec}(t_0')=\int_{t_{\rm rec}}^{t_0'}dt/a(t)$, and we will be seeing spatial regions that were more distant from us at $t_{\rm rec}$. Eventually, $r_{\rm rec}$ will approach a constant value $\sim4.4cH_0^{-1}$ at $t_0'\gtrsim4H_0^{-1}$ and the background anisotropy pattern on the sky will freeze. Since the comoving scale associated with the first acoustic peak of the anisotropies is $\sim100h_{0.7}^{-1}$ Mpc and the asymptotic value of $r_{\rm rec}$ is different from its preset value by $1.14cH_0^{-1}=4.9h_{0.7}^{-1}$ Gpc, we will be able to sample only ~50 independent realizations of the density fluctuation mode corresponding to the first peak. This implies that the cosmic variance of the first acoustic peak would be at best reduced by a factor of $\sim\sqrt{50}=7.1$ relative to its value today. The statistics improve, of course, for modes with shorter wavelengths.

[1]The observed redshift of the source diverges exponentially as $t_0'\to\infty$ and so does the luminosity distance (see Fig. 3). Hence, the flux received from the source declines exponentially with increasing observing time t_0'. As the image of the source fades away it stays frozen at a fixed time along its evolution. This situation is qualitatively analogous to the observed properties of a source falling through the event horizon of a Schwarzschild black hole [14].

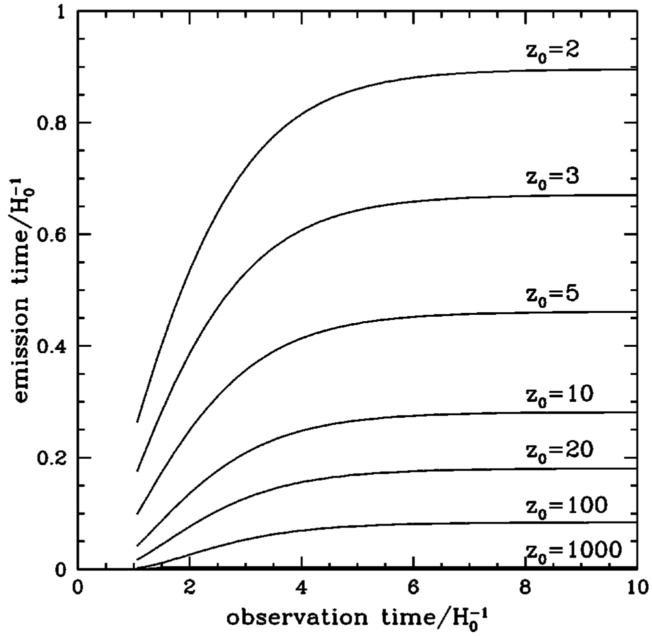

FIG. 1. Emission time as a function of future observation time for an $\Omega_M=0.3,\Omega_\Lambda=0.7$ universe. Time is measured in units of $H_0^{-1}=14h_{0.7}^{-1}$ Gyr, where $h_{0.7}\equiv(H_0/70$ km s^{-1} Mpc$^{-1})$. The current time is $t_0=0.96H_0^{-1}$. For any currently measured redshift z_0 of a source, there is a maximum intrinsic age up to which we can see that source even if we continue to monitor it indefinitely.

The upper panel of Fig. 2 shows the maximum visible age of a source (starting from the big bang) as a function of its currently measured redshift. The lower panel gives the corresponding redshift below which it will not be possible to identify a counterpart to the source in a current deep image of the universe, even if we continue to monitor this source indefinitely.

As the source image freezes on a particular time frame along its evolution (Fig. 1), its flux continues to decline and its redshift increases. Figure 3 shows the evolution of the luminosity and angular diameter distances (relative to their values today) as well as the source redshift as functions of observation time, for a source with a present-day redshift $z_0=5$. Although $d_L\propto\exp(2\sqrt{1-\Omega_M}H_0t)$ and $z\propto\exp(\sqrt{1-\Omega_M}H_0t)$ diverge exponentially at $t_0'\gg t_0$, the angular diameter distance d_A approaches a constant value. A source with a constant intrinsic size at $z_0=5$ will occupy in the distant future a fixed angular size on the sky, which is ~3.3 times larger than its angular size today.

Discussion. The quantitative results of this work are summarized in Figs. 1–3. In an asymptotically de Sitter universe, we can see sources up to the time (given in Fig. 1) when they crossed our event horizon. Figure 2 implies that a source at a redshift $z_0=5$ will only be visible to us up to an age of $\sim6.4h_{0.7}^{-1}$Gyr. Thus, we will never be able to observe the evolution of this source and identify its counterpart in a map that we have taken today of the universe at a redshift $z_0<0.8$, even if we continue to monitor this source indefi-

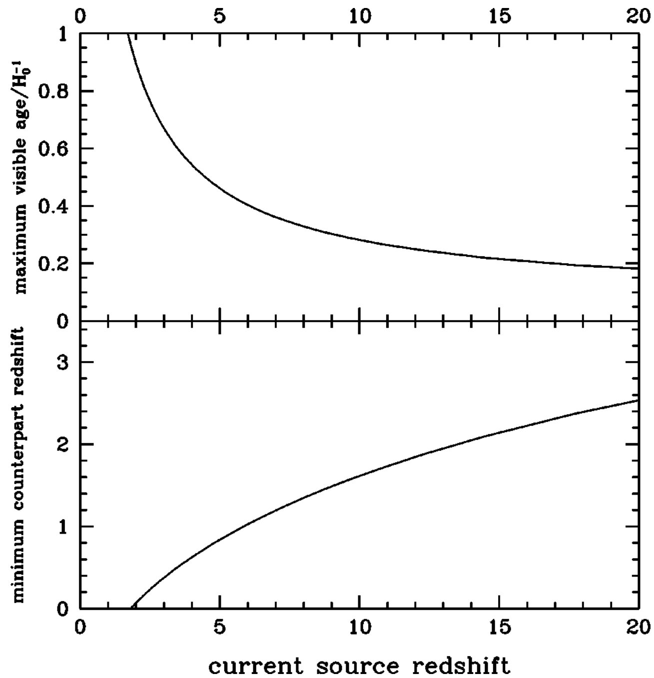

FIG. 2. The upper panel shows the maximum *visible age* of a source (in units of $H_0^{-1}=14h_{0.7}^{-1}$ Gyr) as a function of its currently measured redshift, z_0. The lower panel shows the redshift at which the age of the universe equals this maximum *visible age* of the source. This is the minimum redshift for which it will be possible, in principle, to identify a counterpart to the source in a current deep image of the universe. The counterparts of all sources at $z_0<1.8$ can be traced to the present time.

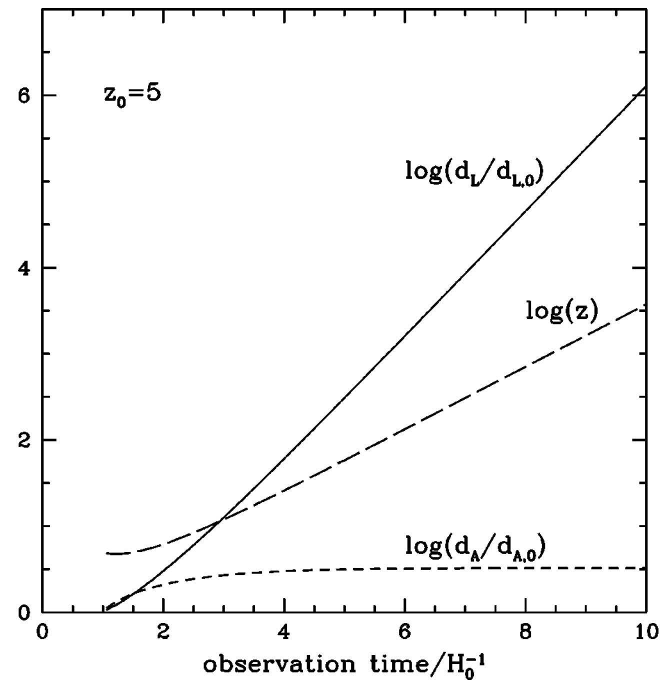

FIG. 3. Future evolution of the luminosity and angular diameter distances $[d_L(t_0'),d_A(t_0')]$ relative to their values today $[d_{L,0}\equiv d_L(t_0),d_{A,0}\equiv d_A(t_0)]$ and the observed redshift $z(t_0')$ for a source with a present-day redshift $z_0=5$.

nitely. This is because the age of the currently observed universe at $z_0\lesssim 0.8$ exceeds $6.6h_{0.7}^{-1}$Gyr. In other words, arguments relating the properties of high-redshift sources to counterparts in the present-day universe will forever remain indirect. Similarly, any light signal that we send out today will not be able to reach all sources with current redshifts $z_0\gtrsim 1.8$ (see Fig. 2).

The *visible age* limit becomes stricter for flux–limited observations where the maximum value of t_0' is constrained by the requirement that the luminosity distance will not exceed some value (see Fig. 3). While the flux limit may depend on technological advances in instrumentation, the visible age limit derived in this paper for $t_0'\to\infty$ is absolute.

The results illustrated in Figs. 1–3 might change only if the vacuum energy density, ρ_V, which makes up the cosmological constant would decrease significantly over the next few Hubble times or $\sim 5\times 10^{10}$ years [8,25]. The exponential expansion phase will not occur if the vacuum energy density would eventually vanish. Although this behavior is possible in the case of a rolling scalar field or "quintessence" [23,24], it requires that the equation of state of the corresponding "dark energy" would deviate significantly from the $p_V=-\rho_V c^2$ relation that characterizes the pressure p_V of a true cosmological constant. A past deviation as small as $\lesssim 10\%$ from this relation is measurable by forthcoming projects, such as the proposed Supernova/Acceleration Probe (SNAP) mission[2] which intends to monitor $\lesssim 2000$ type Ia supernovae across the sky per year and determine their luminosity distances up to a redshift $z_0\sim 1.5$ with high precision.

As long as ρ_V will remain nearly constant, the prospects for extragalactic astronomy in the long-term future appear grim.[3] In contrast to a matter-dominated universe [27], the statistics of visible sources in a Λ-dominated universe are getting worse with the advance of cosmic time. Within $\lesssim 10^{11}$ years, we will be able to see only those galaxies that are gravitationally bound to the local group of galaxies, including the Virgo cluster and possibly some parts of the local supercluster (where the global overdensity in a sphere around Virgo is larger than a few). All other sources of light will fade away beyond detection and their fading image will be frozen at a fixed age.

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[2]http://snap.lbl.gov/

[3]As far as individual objects (such as planets, stars, or galaxies) are concerned, their evolution into the much longer term future of the universe has been discussed in detail in the literature (see Ref. [26] and references therein).

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