Interaction of D0-brane bound states and Ramond-Ramond photons

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We consider the problem of the interaction between a D0-brane bound state and one-form Ramond-Ramond (RR) photons using the world-line theory. Based on the fact that in the world-line theory the RR gauge fields depend on the matrix coordinates of D0-branes, the gauge fields also appear as matrices in the formulation. At the classical level, we derive the Lorentz-like equations of motion for D0-branes, and it is observed that the center of mass is colorless with respect to the SU(N) sector of the background. Using the path integral method, the perturbation theory for the interaction between the bound state and the RR background is developed. Qualitative considerations show that the possibility of the existence of a map between the world-line theory and the non-Abelian gauge theory is very considerable.

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I. INTRODUCTION

In recent years a great deal of attention has been paid to the formulation and study of field theories in noncommutative spaces. Apart from the abstract mathematical interest, the physical motivation for this has been the natural appearance of noncommutative spaces in string theory. Correspondingly, it is understood that string theory involves some kinds of noncommutativity; two important examples are (1) the coordinates of the bound states of N Dp-branes [1], which are represented by $N\times N$ Hermitian matrices [2], and (2) the longitudinal directions of Dp-branes in the presence of a Neveu-Schwarz (NS) B-field background, which appears to be noncommutative [3,4], as seen by the ends of open strings [5]. In the second example, the coordinates in the longitudinal directions of the Dp-branes act as operators and satisfy the algebra

$$[\hat{x}^{\mu},\hat{x}^{\nu}]=i\,\theta^{\mu\nu}, \tag{1.1}$$

where $\theta^{\mu\nu}$ is a constant antisymmetric tensor. There have been many attempts in the recent literature to study different aspects of field theories defined in these kinds of noncommutative spaces. As one point, we mention that the above algebra is satisfied just by $(\infty\times\infty)$-dimensional matrices, and consequently the noncommutativities concerned should be assumed in all (regions) of the space. Also, since there is a nonzero expectation value for the tensor field of $\langle B^{\mu\nu}\rangle=(\theta^{-1})^{\mu\nu}$ [4], in these spaces generally one should expect violation of Lorentz invariance.

As we recalled above, there is another kind of noncommutativity concerning the coordinates of D-brane bound states, which from now on we call the "matrix coordinates." In contrast to the case related to the algebra (1.1), for the case of D-brane bound states, we have noncommutativity for finite dimensional matrices, and thus the noncommutativity of coordinates is not extended to all of the space. In this case the noncommutativity is "confined" just inside the bound state; to put it simply, the noncommutativity is not seen by an observer far from the bound state. In contrast with the case of infinite extension of noncommutativity, we call this kind confined noncommutativity.

In this picture, the natural question is, how can we know about the structure of confined noncommutativity? Since the noncommutativity of the bound state is confined, as in any other similar situation known in physics, the answer to the above question is gained by analyzing and studying the response of the substructure of the bound state to external probes. In this respect one may consider two kinds of the external probe, (1) another D-brane, or (2) the quanta of external fields, like gravitons or photons of the form fields. To be specific, let us consider the special case of D0-branes. Using another D0-brane as a probe of a system of D0-branes is a familiar example from studies related to the matrix model conjecture of M theory [6]. In the matrix model picture, since D0-branes are already assumed to be supergravitons of 11-dimensional supergravity theory in the light-cone gauge, the problem at hand is in fact nothing but "probing" the bound state by another individual graviton. In the matrix model, the high amount of supersymmetry, together with the specific form of the commutator potential of the matrix coordinates, help to calculate the elements of the S matrix for various scattering processes. The important peculiarity of this case is that, in these kinds of investigation, one uses noncommutativity (by things like the commutator potential) to study the effective theory of D0-branes, rather than analyzing the "structure" of confined noncommutativity itself [7]. In other words, generally in this case one ignores the internal dynamics inside the bound state (as target), and essentially considers only the relative dynamics of the target and other D0-brane(s) as probe(s).

In this work we want to discuss the basic elements of using the second kind of probe mentioned above (i.e., external fields) to find information about the structure of confined noncommutativity. As will be clear throughout the paper, the language used in this kind of probe is much closer to the field theory formulation of the problem in comparison with the approach in which the probe is viewed as another D0-

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brane. To continue, we need to know the dynamics of the bound state of D0-branes in different backgrounds. Because of the nature of the matrix coordinates, the formulation of the dynamics of D0-branes in the background of gravity and various form fields is a nontrivial question. Some of the most important progress in this direction was made in [8,9]. Here we use the results of [8,9], restricting ourselves to the simplest case of zero NS B field and a flat metric, but a nonzero one-form Ramond-Ramond (RR) field. Although the framework we use here comes from the Dp-branes of string theory, it is useful to consider the more general case in arbitrary space-time dimensions $d+1$. Also, as the first step, we consider the bosonic partners only.

One of the questions which can be addressed in this direction is about the nature of the effective field theory that captures the interaction between the bound state of D0-branes and the "photons" of the one-form RR field. To be more specific, it will be interesting to derive the effective vertex function for the interaction of a one-form RR photon with the incoming and outgoing D0-branes. These kinds of questions, and in particular the question of the amplitudes of which field theory may correspond to the amplitudes derived by the world-line theory of D0-branes in a RR background, constitute some parts of the discussion of this paper.[1]

The world-line formulation we will use in this work is very much like that of the matrix model conjecture; in particular, it is in the nonrelativistic limit. To approach the Lorentz covariant formulation, following the finite-N interpretation of [11], it is reasonable to interpret things in the discrete light-cone quantization (DLCQ) framework. This point of view should also be kept for the correspondence we consider with an effective field theory for the interacting theory of D0-branes and photons.

The organization of the remaining parts of this paper is as follows. In Sec. II, based on [8,9], we review the main aspects of the world-line formulation of the dynamics of D0-brane bound states in nontrivial backgrounds. These include the equations of motion of D0-branes in a one-form background, and also the symmetry aspects of the world-line formulation. In Sec. III, by using the path integral method, we quantize the D0-brane theory. In particular, we write down the expression of the propagator in the first order of perturbation, which can be converted to the amplitudes of the scattering processes by an arbitrary external source. Section IV is devoted to the conclusion and discussion.

The discussions and ideas in this paper stem from previous work in [12] and [13]. In particular, the problem we consider in this work was interpreted in [13] as the world-line formulation of "electrodynamics on matrix space." Also, the subject of probing confined noncommutativity is mentioned briefly in the last part of [13].

II. DYNAMICS OF D0-BRANES IN ONE-FORM RR BACKGROUND

A. First look: Dp-branes in general background

It is known that the transverse coordinates of bound states of N Dp-branes are represented by $N\times N$ Hermitian matrices rather than numbers [2]; see the review [14]. Because of the nature of matrix coordinates, the formulation of the dynamics of Dp-branes in the background of gravity and various form fields is a nontrivial question. Some of the most important progress in this direction is in [8,9]. In [8], by taking the T duality of string theory as the guiding principle, an action for the dynamics of the bound states of Dp-branes in a nontrivial background is proposed. The proposed bosonic action for the bound state of N Dp-branes (in units where $2\pi l^2=1$) is the sum of

$$S_{\rm BI}=-T_p\int d^{p+1}\sigma\,{\rm Tr}\Big(e^{-\phi}\sqrt{-\det[P\{E_{IJ}+E_{Ii}(Q^{-1}-\delta)^{ij}E_{jJ}\}+F_{IJ}]\det(Q^i_j)}\Big), \tag{2.1}$$

$$S_{\rm CS}=\mu_p\int {\rm Tr}\left[P\left\{e^{i\mathbf{i}_\Phi\mathbf{i}_\Phi}\left(\sum C^{(n)}e^B\right)\right\}e^F\right], \tag{2.2}$$

with the following definitions [8]:

$$E_{\mu\nu}\equiv G_{\mu\nu}+B_{\mu\nu},\quad Q^i_j\equiv\delta^i_j+i[\Phi^i,\Phi^j]E_{kj},$$
$$\mu,\nu=0,...,9,\quad I,J=0,...,p,\quad i,j=p+1,...,9. \tag{2.3}$$

In the above, $G_{\mu\nu}$ and $B_{\mu\nu}$ are the metric and NS B field, respectively, and Φ^i are world-volume scalars and $N\times N$ Hermitian matrices that describe the position of the Dp-branes in the transverse directions. The $C^{(n)}$ is an n-form RR field, while F_{IJ} is the U(N) field strength. In this action, $P\{\cdots\}$ denotes the pullback of the bulk fields to the world volume of the Dp-branes, and Tr is the trace on the gauge group. $\mathbf{i}_{\mathbf{v}}$ denotes the interior product with a vector $\mathbf{v}$; for example, $\mathbf{i}_\Phi$ acts on the two-form $C^{(2)}=\frac{1}{2}C^{(2)}_{ij}dx^idx^j$ as

$$\mathbf{i}_\Phi C^{(2)}=\Phi^iC^{(2)}_{ij}dx^j,$$

[1] The reader can refer to [10], as an attempt to interpret the quantized propagation of D0-branes while they are interacting with each other via the commutator potential, like the Feynman graphs of a field theory in the light-cone gauge.

$$\mathbf{i_\Phi i_\Phi} C^{(2)} = \Phi^i \Phi^j C^{(2)}_{ij} = \tfrac{1}{2}[\Phi^i,\Phi^j] C^{(2)}_{ij}. \tag{2.4}$$

Therefore $(\mathbf{i_\Phi})^2 C^{(n)}=0$ for the commutative case, i.e., for one Dp-brane.

Some comments on the above action are in order.

(i) All the derivatives in the longitudinal directions should actually be covariant derivatives, i.e., $\partial_I \to D_I = \partial_I + i[A_I, \,]$ [15]. This point is true also for the pullback quantities.

(ii) The pullback quantities depend on the transverse directions of the Dp-branes only via their functional dependence on the world-volume scalars Φ^i [16]. Since the matrix coordinates Φ do not commute with each other, the problem of ordering ambiguity is present. Following previous arguments, it is proposed that the coordinates Φ appear in the background fields by the "symmetrization prescription" [8–19]. The symmetrization on coordinates can be obtained by the so-called non-Abelian Taylor expansion. The non-Abelian Taylor expansion for an arbitrary function $f(\Phi^i,\sigma^I)$ is given by

$$\begin{aligned} f(\Phi^i,\sigma^I) &\equiv f(x^i,\sigma^I)|_{x\to\Phi} = \exp[\Phi^i \partial_{x^i}] f(x^i,\sigma^I)|_{x=0} \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \Phi^{i_1}\cdots\Phi^{i_n} (\partial_{x^{i_1}}\cdots\partial_{x^{i_n}}) f(x^i,\sigma^I)|_{x=0.} \end{aligned} \tag{2.5}$$

In the above expansion the symmetrization is recovered via the symmetric property of the derivatives inside the term $(\partial_{x^{i_1}}\cdots\partial_{x^{i_n}})$.

(iii) This action involves a single Tr, and this Tr should be calculated by the symmetrization prescription for the noncommutative quantities F_{IJ}, $D_I\Phi^i$, and $i[\Phi^i,\Phi^j]$ [20].[2]

To become more familiar with the terms in the action of Dp-branes, let us consider the special case $p=0$ of D0-branes, in which the world volume consists of only the time direction, $\sigma^0=t$. The dynamics of D0-branes in the background of the metric $G_{\mu\nu}(x,t)$, the one-form RR field $C^{(1)}_\mu(x^\nu)\equiv A_\mu(x,t)$, and zero NS B field (without being precise about the indices and coefficients) in the lowest orders is given by an action like [8,9]

$$\begin{aligned} S = \int dt\, \mathrm{Tr}\bigg(& \frac{m}{2} G_{ij}(\Phi,t) D_t\Phi^i D_t\Phi^j \\ & + qG_{ij}(\Phi,t)A^i(\Phi,t)D_t\Phi^j - qA_0(\Phi,t) \\ & - qG_{0i}(\Phi,t)D_t\Phi^i A_0(\Phi,t) + mG(\Phi,t)G(\Phi,t)[\Phi,\Phi]^2 \\ & + [1-G_{00}(\Phi,t)] + \cdots \bigg), \end{aligned} \tag{2.6}$$

in which $D_t = \partial_t + i[a_t(t), \,]$ acts as convariant derivative on the world line, and we have set the charge $\mu_0 = q$. In the above, the functional dependence on the matrix coordinates of D0-branes should be understood. Finally, we have the action (2.6), which can be interpreted as the world-line formulation of the dynamics of D0-branes in nontrivial backgrounds.

[2] There is a stronger prescription, with symmetrization between all noncommutative objects F_{IJ}, $D_I\Phi^i$, and $i[\Phi^i,\Phi^j]$ and the individual Φ's appearing in the functional dependences of the pullback fields [8,21]. We will not use this one in our future discussion for the case of D0-branes, with no essential change in the conclusions.

B. Action of D0-branes in one-form background

In the following consideration in this work, we take the special case of the dynamics of D0-branes in the background of a one-form RR field $(A_0(x,t), A_i(x,t))$, in a flat metric and zero NS B field. Consequently, the low energy bosonic action of N D0-branes, after restoring the string length l, is given by

$$\begin{aligned} S_{\mathrm{D0}} = \int dt\, \mathrm{Tr}\bigg(& \frac{1}{2} m D_t X_i D_t X^i + qD_t X^i A_i(X,t) - qA_0(X,t) \\ & + m\frac{[X^i,X^j]^2}{4(2\pi l^2)^2} + \cdots \bigg), \end{aligned} \tag{2.7}$$

in which we have slightly changed the notation for matrix coordinates from $(2\pi l^2)\Phi^i$ to X^i, with the usual expansion

$$X^i = X^i_a T^a, \quad i=1,...,d, \quad a=0,1,...,N^2-1, \tag{2.8}$$

with T^a as the basis for the Hermitian matrices [i.e., the generators of U(N)]. Although Dp-branes of string theory exist in the critical dimensions $D=10$ (or 26), for the case of D0-branes it will be useful to consider the more general case in arbitrary spatial dimensions d. We recall that the gauge fields appear in the action through their functional dependence on symmetrized products of the matrix coordinates X. The action (2.7) can be interpreted as the world-line formulation of electrodynamics on the matrix space [13]. We mention also that in this action the degrees of freedom are enhanced from d in ordinary space to $d\times N^2$ in the space with matrix coordinates.

The original theory, which may be called the bulk theory, is invariant under the usual U(1) transformations such as

$$A_\mu(x,t) \to A'_\mu(x,t) = A_\mu(x,t) - \partial_\mu \Lambda(x,t), \quad \mu = 0.1,\ldots,d. \tag{2.9}$$

In the world-line theory, the transformation takes the form

$$\begin{aligned} A_i(X,t) &\to A'_i(X,t) = A_i(X,t) + \delta_i \Lambda(X,t), \\ A_0(X,t) &\to A'_0(X,t) = A_0(X,t) - \partial_t \Lambda(X,t), \end{aligned} \tag{2.10}$$

in which δ_i is the functional derivative $\delta/\delta X^i$. Consequently, one obtains

$$\begin{aligned} \delta S_{\mathrm{D0}} \sim q \int dt\, \mathrm{Tr}\{ & \partial_t \Lambda(X,t) + \dot{X}^i \delta_i \Lambda(X,t) \\ & + ia_t[X^i, \delta_i\Lambda(X,t)]\} \sim q\int dt\, \mathrm{Tr}\bigg(\frac{d\Lambda(x,t)}{dt} \\ & + ia_t[X^i,\delta_i\Lambda(X,t)] \bigg) \sim 0. \end{aligned} \tag{2.11}$$

In the above, the first term is a surface term, and the second term vanishes by the symmetrization prescription [12].[3]

The equations of motion for the X's and a_t by the action (2.7), ignoring for the moment the commutator potential $[X_i,X_j]^2$, are found to be [12,13]

$$mD_tD_tX_i=q[E_i(X,t)+\underbrace{D_tX^jB_{ji}(X,t)}],\qquad(2.12)$$

$$m[X_i,D_tX^i]=q[A_i(X,t),X^i],\qquad(2.13)$$

with the following definitions:

$$E_i(X,t)\equiv-\delta_iA_0(X,t)-\partial_tA_i(X,t),\qquad(2.14)$$

$$B_{ji}(X,t)\equiv-\delta_jA_i(X,t)+\delta_iA_j(X,t).\qquad(2.15)$$

In Eq. (2.12), the symbol $\underbrace{D_tX^jB_{ji}(X,t)}$ denotes the average over all of positions of D_tX^j between the X's of $B_{ji}(X,t)$. The above equations for the X's are like the Lorentz equations of motion, with the exceptions that two sides are $N\times N$ matrices, and the time derivative ∂_t is replaced by its covariant counterpart D_t [15].

An equation of motion similar to Eq. (2.12) is considered in [23,24] as part of the similarities between the dynamics of D0-branes and bound states of quarks–QCD strings in a baryonic state [23–25]. The point is that the dynamics of the bound state center of mass (c.m.) is not affected directly by the non-Abelian sector of the background, i.e., the c.m. is "white" with respect to the SU(N) sector of U(N). The c.m. coordinates and momenta are defined by

$$X^i_{\text{c.m.}}\equiv\frac{1}{N}\mathrm{Tr}X^i,\quad P^i_{\text{c.m.}}\equiv\mathrm{Tr}P^i,\qquad(2.16)$$

where we are using the convention $\mathrm{Tr}\mathbf{1}_N=N$. To specify the net charge of a bound state, as an extended object, its dynamics should be studied in zero magnetic and uniform electric fields, i.e., $B_{ji}=0$ and $E_i(X,t)=E_{0i}$.[4] Since the fields are uniform, they do not involve X matrices, and contain just the U(1) part. In other words, under gauge transformations E_{0i} and $B_{ji}=0$ transform to $\tilde{E}_i(X,t)=V^\dagger(X,t)E_{0i}V(X,t)=E_{0i}$ and $\tilde{B}_{ji}=0$. Thus the action (2.7) yields the following equation of motion:

$$(Nm)\ddot{X}^i_{\text{c.m.}}=NqE^i_{0(1)},\qquad(2.17)$$

in which the subscript (1) emphasises the U(1) electric field. So the c.m. interacts directly only with the U(1) part of U(N). From the string theory point of view, this observation is based on the simple fact that the SU(N) structure of D0-branes arises just from the internal degrees of freedom inside the bound state.

The world-line formulation we have here is very similar to the matrix model conjecture; in particular, it is in the nonrelativistic limit. For the case of the dynamics of a charged particle with ordinary coordinates, we can see easily that the light-cone dynamics have a form similar to the one we have in action (2.7); see the Appendix of [24]. To approach the Lorentz covariant formulation, following the finite-N interpretation of [11], it is reasonable to interpret things here in the DLCQ framework. This should also be applied in considering the correspondence of the effective field theory and the interacting theory of D0-branes–photons.

C. Symmetry transformations

Actually, the action (2.7) is invariant under the transformations

$$X^i\rightarrow\tilde{X}^i=U^\dagger X^iU,$$
$$a_t(t)\rightarrow\tilde{a}_t(X,t)=U^\dagger a_t(t)U-iU^\dagger\partial_tU,\qquad(2.18)$$

with $U\equiv U(X,t)$ as an arbitrary $N\times N$ unitary matrix; in fact, under these transformations one obtains

$$D_tX^i\rightarrow\tilde{D}_t\tilde{X}^i=U^\dagger D_tX^iU,\qquad(2.19)$$

$$D_tD_tX^i\rightarrow\tilde{D}_t\tilde{D}_t\tilde{X}^i=U^\dagger D_tD_tX^iU.\qquad(2.20)$$

Now, in the same spirit as for the previously introduced U(1) symmetry of Eq. (2.10), one finds the symmetry transformations

$$X^i\rightarrow\tilde{X}^i=U^\dagger X^iU,$$
$$a_t(t)\rightarrow\tilde{a}_t(X,t)=U^\dagger a_t(t)U-iU^\dagger\partial_tU,$$
$$A_i(X,t)\rightarrow\tilde{A}_i(X,t)=U^\dagger A_i(X,t)U+iU^\dagger\delta_iU,$$
$$A_0(X,t)\rightarrow\tilde{A}_0(X,t)=U^\dagger A_0(X,t)U-iU^\dagger\partial_tU,\qquad(2.21)$$

in which we assume that $U\equiv U(X,t)=\exp(-i\Lambda)$ is arbitrary up to the condition that $\Lambda(X,t)$ is totally symmetrized in the X's. The above transformations in the gauge potentials are similar to those of non-Abelian gauge theories, and we mention that this is just the consequence of the enhancement of the degrees of freedom from numbers (x) to matrices (X). In other words, we are faced with a situation in which "the rotation of fields" is generated by "the rotation of coordinates." The above observation on the gauge symmetry associated with D0-brane matrix coordinates is not a new one, and we already know another example of this kind in noncommutative gauge theories; see [13]. In addition, the case we see here for D0-branes may be considered as the finite-N version of the relation between gauge symmetry transformations and transformations of matrix coordinates [26].

[3] A general proof of the invariance of the full Chern-Simons action was reported recently in [22].

[4] In a non-Abelian gauge theory a uniform electric field can be defined up to a gauge transformation, which is quite adequate for identification of white (singlet) states.

The behavior of Eqs. (2.12) and (2.13) under the gauge transformation (2.21) can be checked. Since the action is invariant under Eq. (2.21), it is expected that the equations of motion change covariantly. The left-hand side of Eq. (2.12) changes to $U^\dagger D_t D_t X U$ by Eq. (2.20), and therefore we should find the same change for the right-hand side. This is in fact the case, since for any function $f(X,t)$ under transformations (2.18) we have

$$f(X,t)\rightarrow\tilde{f}(\tilde{X},t)=U^\dagger f(X,t)U,$$

$$\delta_i f(X,t)\rightarrow\tilde{\delta}_i\tilde{f}(\tilde{X},t)=U^\dagger \delta_i f(X,t)U,$$

$$\partial_t f(X,t)\rightarrow\partial_t\tilde{f}(\tilde{X},t)=U^\dagger \partial_t f(X,t)U. \tag{2.22}$$

In conclusion, the definitions (2.14) and (2.15), lead to

$$E_i(X,t)\rightarrow\tilde{E}_i(\tilde{X},t)=U^\dagger E_i(X,t)U,$$

$$B_{ji}(X,t)\rightarrow\tilde{B}_{ji}(\tilde{X},t)=U^\dagger B_{ji}(X,t)U, \tag{2.23}$$

a result consistent with the fact that E_i and B_{ji} are functionals of X. We thus see that, in spite of the absence of the usual commutator term $i[A_\mu,A_\nu]$ of non-Abelian gauge theories, in our case the field strengths transform like non-Abelian ones. We recall that this is all a consequence of the matrix coordinates of D0-branes. Finally, for a similar reason to the vanishing of the second term of Eq. (2.11), both sides of Eq. (2.13) transform identically.

The last notable points are about the behavior of $a_t(t)$ and $A_0(X,t)$ under symmetry transformations (2.21). From the world-line theory point of view, $a_t(t)$ is a dynamical variable, but $A_0(X,t)$ should be treated as a part of the background; however, they behave similarly under transformations. Also, we see by Eq. (2.21) that the coordinate independence of $a_t(t)$, which is a consequence of dimensional reduction, should be understood up to a gauge transformation. In [12] a possible map between the dynamics of D0-branes and the semiclassical dynamics of charged particles in a Yang-Mills background was mentioned. It is worth mentioning that this possible relation might be an explanation for the above notable points [12].

III. QUANTUM THEORY IN ONE-FORM BACKGROUND

A. Some general aspects of bound state–photon interaction

Before presenting the formulation, it is useful to mention some general aspects of the problem at hand. First let us recall another representation of the symmetrization of the matrix coordinates. The other useful symmetric expansion is done by using the Fourier components of a function. To gain this Fourier expansion in matrix coordinates (we call it the non-Abelian Fourier expansion), one can simply interpret the derivatives of the usual coordinates ∂_{x^i} in Eq. (2.5) as momentum numbers ik_i. It is then not hard to see that for an arbitrary function $f(X,t)$ the non-Abelian Fourier expansion will be found to be

$$f(X,t)=\int d^dk\,\overline{f}(k,t)e^{ik_iX^i}, \tag{3.1}$$

in which $\overline{f}(k,t)$ are the Fourier components of the function $f(x,t)$ (i.e., the function in ordinary coordinates) which is defined by the known expression

$$\overline{f}(k,t)\equiv\frac{1}{(2\pi)^d}\int d^dx\,f(x,t)e^{-ik_ix^i}. \tag{3.2}$$

Since the momentum numbers k_i are ordinary numbers, and so commute with each other, the symmetrization prescription is automatically recovered in the expansion of the momentum eigenfunctions $e^{ik_iX^i}$. This picture of symmetrization for the matrix coordinates is similar to that we already know for Weyl ordering in phase space $(\hat{q},\hat{p})$, with $[\hat{q},\hat{p}]=i$.

Now, by using the symmetric expansion (3.1), we can imagine some general aspects of the interaction between D0-brane bound states and RR photons. We recall that the bound state of D0-branes is described by the action (2.7) after setting $A_\mu(x,t)\equiv 0$. We mention that the degrees of freedom still interact due to the commutator potential. By doing a simple dimensional analysis it can be shown that the size scale of the bound state for a finite number N of D0-branes is finite and is of the order of $\ell\sim m^{-1/3}\ell^{2/3}$ [27,24]. We recall that the action we are using comes from string perturbative calculations, and consequently we have for the size scale the further relation $\ell\ll l$ [27,24].

Before proceeding further, we should distinguish the dynamics of the c.m. from the internal degrees of freedom of the bound state. As mentioned before, the c.m. position and momentum of the bound state are represented by the U(1) sector of U(N)=SU(N)×U(1), and thus the information related to the c.m. can be gained simply by the Tr operation, relation (2.16). So the internal degrees of freedom of the bound state, which consist of the relative positions of N D0-branes together with the dynamics of strings stretched between the D0-branes, are described by the SU(N) sector of the matrix coordinates. It is easy to see that the commutator potential in the action has some flat directions, along which the eigenvalues can take arbitrarily large values. But it is understood that, by considering the quantum effects and in the case that we expect formation of the bound state, we should expect suppression of the large values of the internal degrees of freedom [28]. Consequently, it is expected that the SU(N) sector of the matrix coordinates will take mean values like $\langle X_a^i\rangle\sim\ell$ ($a=1,\ldots,N^2-1$, not $a=0$ as c.m.), with

ℓ as the bound state size scale mentioned above.[5] We should mention that, although the c.m. is represented by the U(1) sector, its dynamics is affected by the interaction of the ingredients of the bound state with the SU(N) sector of external fields, similar to the situation we imagine in the case of the van der Waals force.

The important question about the interaction of a bound state (as an extended object) with an external field is about the regime in which the substructure of the bound state is probed. As we mentioned in the Introduction, in our case the quanta of RR fields are the representatives of the external field. The quanta are coming from a source and so, as it makes things easier, we ignore their dynamics. The source is introduced into our problem by the gauge field $A_\mu(x,t)$. These fields appear in the action through their functional dependence on the matrix coordinates X. In fact, this is the key to probing the substructure of the bound state. According to the non-Abelian Fourier expansion we mentioned above, we have

$$A_\mu(X,t)=\int d^dk\,\overline{A}_\mu(k,t)e^{ik_iX^i}, \tag{3.3}$$

in which $\overline{A}_\mu(k,t)$ are the Fourier components of the fields $A_\mu(x,t)$ (i.e., fields in ordinary coordinates). One can imagine scattering processes that are designed to probe inside the bound state. As in every other scattering process, the two limits of the momentum modes, corresponding to long and short wavelengths, behave differently.

In the limit $\ell|k|\to 0$ (long wavelength regime), the field A_μ is not involved in the X matrices mainly. This means that the fields appear to be nearly constant inside the bound state, and in an estimation we have

$$e^{ik_iX^i}\sim e^{ik_iX^i_{\mathrm{c.m.}}}. \tag{3.4}$$

So in this limit we expect that the substructure and consequently noncommutativity will not be seen [Fig. 1(a)]. As a consequence, after interaction with a long wavelength mode, it is not expected that the bound state will jump to another energy level different from the first one. It should be noted that the c.m. dynamics can be affected as well in this case.

In the limit $\ell|k|=$finite (short wavelength regime), the fields depend on the coordinates X inside the bound state, and so the substructure responsible for noncommutativity should be probed [Fig. 1(b)]. In fact, we know that the noncommutativity of D0-brane coordinates is a consequence of the strings which are stretched between D0-branes. So, by these kinds of scattering processes, one should be able to probe both D0-branes (as pointlike objects), and the strings stretched between them. In this case, it is completely to be expected that the energy levels of the incoming and outgoing bound states will be different, since the ingredients of bound state substructure can absorb quanta of energy from the incident wave. In this case the c.m. dynamics can be affected in a novel way by the interaction of the substructure with the external fields (the van der Waals effect).

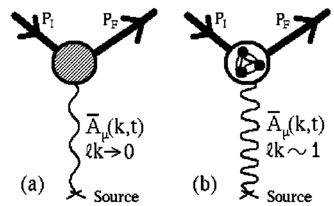

FIG. 1. Substructure is not experienced by the long wavelength modes (a). Because of their functional dependence on the matrix coordinates, the short wavelength modes can probe inside the bound state (b). ℓ and $\overline{A}_\mu(k,t)$ represent the size of the bound state and the Fourier modes, respectively.

In the general case, one can gain more information about the substructure of a bound state by analyzing the recoil effect on the source. To do this, one should be able to include the dynamics of the source in the formulation. Considering the dynamics of the source, in terms of quantized field theory, means that we consider the processes in which the source and the target exchange one quantum of gauge field with definite wavelength and frequency, although off shell, as $A_\mu(x,t)\sim\epsilon_\mu e^{ik_ix^i-i\omega t}$. This kind of process is shown in Fig. 2.

B. Path integral quantization

In this subsection we consider the quantization of D0-brane dynamics, using the path integral method. The theory on the world line has gauge symmetry, defined by the transformations (2.21). We should fix this symmetry, and here we use simply the temporal gauge, defined by the condition $a_t(t)\equiv 0$. So after the Wick rotation $t\to -it$ and $A_0\to -iA_0$, we have the following expression for the path integral of our system:

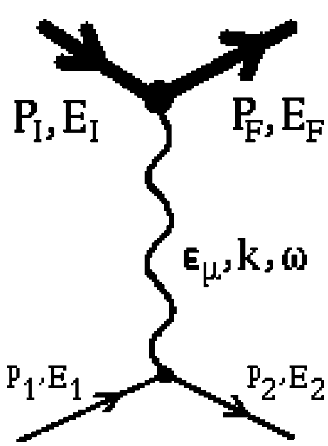

FIG. 2. Exchange of one photon between a D0-brane bound state (thick lines) and another source (thin lines).

[5]There is another way to justify this expectation. It is known that diagonal SU(N) matrices represent the relative positions of D0-branes, which are expected to be of the order of ℓ in a bound state. But due to the symmetry transformation we introduced in the previous section, the diagonal and nondiagonal elements in the matrices can mix with each other, representing the same mechanical system. So the size scale associated with the diagonal elements should be valid for the nondiagonal elements also.

$$\langle X_F,t_F|X_I,t_I\rangle \sim \int [DX][Da_t]\,\delta(a_t)\det\left|\frac{\delta a_t}{\delta\Lambda}\right| e^{-S_{\mathrm{D0}}[X,a_t]}, \tag{3.5}$$

in which $\delta(a_t)$ supports the gauge fixing condition, $\det|\delta a_t/\delta\Lambda|$ is the determinant that arises by variation of the gauge fixing condition, and finally $S_{\mathrm{D0}}[X,a_t]$ is the action (2.7) evaluated between (X_F,t_F) and (X_I,t_I), as "final" and "initial" conditions. The variation of the gauge fixing condition can be calculated easily in our case, and it is found to be [for $U(X,t)=\exp(-i\Lambda)$]

$$a_t=0\rightarrow a_t'=\delta a_t=-iU^\dagger\partial_t U=-\partial_t\Lambda(X,t)+O(\Lambda^2), \tag{3.6}$$

and consequently we have $\delta a_t(t)/\delta\Lambda(t')=-\partial_t\delta(t-t')$. So we see that the determinant and consequently the corresponding ghosts are decoupled from our dynamical fields X.[6] So, up to a normalization factor, we have for the above expression of the path integral:

$$\langle X_F,t_F|X_I,t_I\rangle \sim \int [DX]e^{-S_{\mathrm{D0}}[X,a_t\equiv 0]}. \tag{3.7}$$

To calculate the path integral in a general background we have to use a perturbation expansion in powers of the charge q; this expansion is also valid for weak external fields (A_0,A_i). So we have

$$\begin{aligned}\langle X_F,t_F|X_I,t_I\rangle &\\ \sim \int [DX]&\exp\left[-\int_{t_I}^{t_F}dt\,\mathrm{Tr}\left(\frac{1}{2}m\dot{X}_i\dot{X}^i + m\frac{[X^i,X^j]^2}{4(2\pi l^2)^2}\right)\right]\\ &\times\sum_{n=0}^{\infty}\frac{q^n}{n!}\left\{i\int_{t_I}^{t_F}dt\,\mathrm{Tr}[\dot{X}^iA_i(X,t)+A_0(X,t)]\right\}^n.\end{aligned} \tag{3.8}$$

As mentioned before, from the point of view of D0-brane dynamics, the commutator potential $[X^i,X^j]^2$ is responsible for the formation of D0-brane bound states [27]. Although the problem of finding the full set of eigenenergies and eigenvectors of the corresponding Hamiltonian is very difficult, we assume that this full set is at hand. It is logical to separate the c.m. variables from the internal ones; we show those of the c.m. by the momenta $P_{\mathrm{c.m.}}$ and $|P_{\mathrm{c.m.}}\rangle$, and the internal ones by the energy $E_{\{n\}}$ and $|\{n\}\rangle$, in which $\{n\}$ represents all the quantum numbers associated with the internal dynamics. We recall that the c.m. is free in the case $q=0$. It is worth recalling that, in general, we expect the eigenenergies to have the general form $E_{\{n\}}=g(\{n\})\ell^{-1}$, with $g(\{n\})$ as a function of the quantum numbers $\{n\}$, and also the condition

$$\langle X|\{n\}\rangle\rightarrow 0 \quad \text{for } |X|\gg\ell \tag{3.9}$$

for the wave functions, with $\ell\sim m^{-1/3}l^{2/3}$ the size scale of the bound state as we mentioned before. As in any other quantum mechanical system, for the case $q=0$ the general expression of the propagator can be used:

$$\langle X_2,t_2|X_1,t_1\rangle_{q=0}=\sum_{P_{\mathrm{c.m.}}}\sum_{\{n\}}\langle X_2|P_{\mathrm{c.m.}},\{n\}\rangle\langle P_{\mathrm{c.m.}},\{n\}|X_1\rangle \times e^{-i(P^2_{\mathrm{c.m.}}/2Nm+E_{\{n\}})(t_2-t_1)}, \tag{3.10}$$

with the definition $|P_{\mathrm{c.m.}},\{n\}\rangle\equiv|P_{\mathrm{c.m.}}\rangle\otimes|\{n\}\rangle$. We can now insert the propagator above in the expression (3.8), noting that the perturbation expansion has terms involving the velocity $\dot{X}$. Based on the standard representation of "slicing" used for path integrals, finally the following expression for the first order of perturbation is found (see [30]):

$$\begin{aligned}\langle X_F,t_F|X_I,t_I\rangle\sim&\langle X_F,t_F|X_I,t_I\rangle_{q=0}\\ &+i\mathcal{N}\lim_{\Delta t\to 0}\sum_{k=1}^{n}\int d^dX_{k-1}d^dX_kd^dX_{k+1}\\ &\times\langle X_F,t_F|X_{k+1},t_{k+1}\rangle_{q=0}\\ &\times 2\Delta t\cdot\mathrm{Tr}\left(q\frac{X^i_{k+1}-X^i_{k-1}}{2\Delta t}A_i(X_k,t) + qA_0(X_k,t)\right)\\ &\times\langle X_{k-1},t_{k-1}|X_I,t_I\rangle_{q=0}\\ &\times e^{-S_{q=0}[k,k-1;\Delta t]}e^{-S_{q=0}[k+1,k;\Delta t]}+O(q^2),\end{aligned} \tag{3.11}$$

in which $t_j-t_I=j\cdot\Delta t$ and $t_F-t_I=(n+1)\Delta t$. In the above, $S_{q=0}[j,j+1;\Delta t]$ is the value of the action in the exponential of Eq. (3.8) evaluated between the points (X_j,t_j) and (X_{j+1},t_{j+1}) $(1\leqslant j\leqslant n)$ in the limit $\Delta t\to 0$. The normalization constant $\mathcal{N}$ contains sufficient powers of Δt to make the final result finite and independent of Δt. The sum Σ_k comes from slicing the potential term $\int dt\,\mathrm{Tr}(\dot{X}\cdot A+A_0)$ in the path integral (3.8), and it will eventually change to the time integral $\int dt$ over the intermediate times in which the interaction occurs. It is worth recalling that spatial integrals like $\int d^dX$ are in fact $\int\prod_{a=0}^{N^2-1}d^dX_a$. We mention that for the velocity independent term $A_0(X,t)$ the integrals of $d^dX_{k\pm1}$ can be performed to get the new propagators, and after the change $X_k\rightarrow X$ we simply find an expression like

[6]This case is similar to the so-called axial gauge in the extreme limit $\lambda\rightarrow\infty$ (p. 196 of [29]).

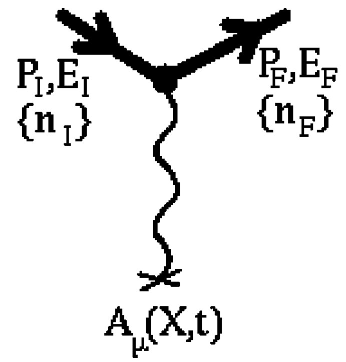

FIG. 3. The graph for the transition amplitude between states with definite c.m. momenta and energies (P,E), and internal energy specified by the quantum numbers $\{n\}$.

$$\sim i\int_{t_I}^{t_F}dt\int d^dX\langle X_F,t_F|X,t\rangle_{q=0}\mathrm{Tr}[qA_0(X,t)]\times\langle X,t|X_I,t_I\rangle_{q=0}+O(q^2),\tag{3.12}$$

which is the familiar expression for velocity independent interactions.

For many practical aims, we should find the S-matrix elements between states with definite momenta and energies (Fig. 3). This can be done by the proper transformations of the amplitudes $\langle X_F,t_F|X_I,t_I\rangle$ in coordinate space.

Because there is less knowledge about the propagator (3.10), expression (3.11) can still not be used for actual calculations. As mentioned before, we expect that the spatial integrations $\int d^dX$ will get their main contribution from the volume of the bound state $V\sim\ell^d$. So as an approximation, and to know a little more about the result, we may ignore the commutator potential, and do the integrations in the finite volume $V\sim\ell^d$, or simply put $\int d^dX_a\sim\ell^d$, for $a\neq 0=$c.m. By doing this, we can verify the general aspects of probing the substructure of the bound state discussed in the previous subsection.

C. Effective interaction vertex of photon and free D0-branes

In the considerations of the previous subsection, the background $(A_0(x,t),A_i(x,t))$ was taken to be arbitrary. Here we take an example in which the D0-branes interact with a monotonic incident wave, defined by the condition $\overline{A}_\mu(k',\omega')=\epsilon_\mu\delta^d(k'-k)\,\delta(\omega'-\omega)$, with ϵ_μ as the polarization vector, and the following definition for the Fourier modes:

$$\overline{A}_\mu(k',\omega')\equiv\frac{1}{(2\pi)^{d+1}}\int d^dx\,dt\,A_\mu(x,t)e^{-ik_i'x^i+i\omega't}.\tag{3.13}$$

So the corresponding gauge field is $A_\mu(X,t)\sim\epsilon_\mu\exp(ik_iX^i-i\omega t)$. In addition, here we ignore the commutator potential, and consequently it is assumed that all of the N^2 degrees of freedom, including those N that describe the position of D0-branes, are free for $q=0$. So we have the following expression for the path integral:

$$\langle X_F,t_F|X_I,t_I\rangle\sim\int[DX]\exp\left[-\int_{t_I}^{t_F}dt\,\mathrm{Tr}\left(\frac{1}{2}m\dot{X}_i\dot{X}^i\right)\right]\times\left\{1+iq\int_{t_I}^{t_F}dt\,\mathrm{Tr}[\dot{X}^iA_i(X,t)+A_0(X,t)]+O(q^2)\right\}.\tag{3.14}$$

A similar theory for a charged particle in ordinary space is considered in Appendix A, to extract the field theory vertex function of the coupling of a photon to incoming or outgoing charged particles. So the result of the path integral above can be considered as the matrix coordinate version of the example of Appendix A. We continue with an expression like that of Eq. (3.11), as

$$\begin{aligned}\langle X_F,t_F|X_I,t_I\rangle\sim&\langle X_F,t_F|X_I,t_I\rangle_{\text{f.p.}}+i\mathcal{N}\lim_{\Delta t\to0}\sum_{k=1}^{n}\int d^dX_{k-1}d^dX_kd^dX_{k+1}\\&\times\langle X_F,t_F|X_{k+1},t_{k+1}\rangle_{\text{f.p.}}\\&\times2\Delta t\cdot\mathrm{Tr}\left(q\frac{X_{k+1}^i-X_{k-1}^i}{2\Delta t}A_i(X_k,t)+qA_0(X_k,t)\right)\\&\times\langle X_{k-1},t_{k-1}|X_I,t_I\rangle_{\text{f.p.}}\\&\times e^{-S_{\text{f.p.}}[k,k-1;\Delta t]}e^{-S_{\text{f.p.}}[k+1,k;\Delta t]}+O(q^2),\end{aligned}\tag{3.15}$$

in which $S_{\text{f.p.}}$ and $\langle\cdots\rangle_{\text{f.p.}}$ are the action and the propagator of free particles, respectively; see Appendix B for the explicit expressions. The integrations $d^dX_{k\pm1}$ can be done to get new propagators, and after the change $X_k\to X$ we find

$$\begin{aligned}\langle X_F,t_F|X_I,t_I\rangle\sim&\langle X_F,t_F|X_I,t_I\rangle_{\text{f.p.}}+i\mathcal{N}'\int_{t_I}^{t_F}dt\int d^dX\langle X_F,t_F|X,t\rangle_{\text{f.p.}}\\&\times\mathrm{Tr}\left\{q\left(\frac{X_F^i-X^i}{t_F-t}+\frac{X^i-X_I^i}{t-t_I}\right)A_i(X,t)+qA_0(X,t)\right\}\langle X,t|X_I,t_I\rangle_{\text{f.p.}}+O(q^2).\end{aligned}\tag{3.16}$$

Up to now the gauge field can be in any arbitrary form. Also, since in this case we have ignored the commutator potential and so the degrees of freedom are free for $q=0$, we can easily use the momentum basis for the incoming and outgoing states; see Fig. 4. So for the S-matrix element in the momentum-energy basis, we have the expression

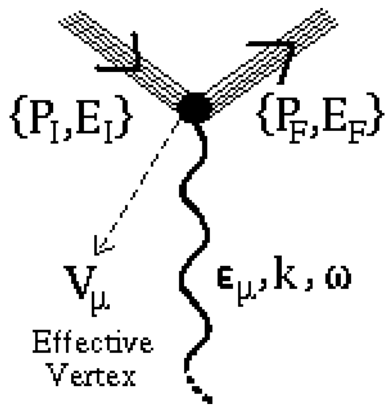

FIG. 4. The graph for the transition amplitude between states with definite momenta and energies, specified by the set $\{P,E\}$ for all N^2 degrees of freedom. Here we use thin lines as incoming and outgoing states, to emphasize that these states are free before and after the vertex of interaction.

$$S_{FI}\sim \prod_{a=0}^{N^2-1} \delta^d(P_{Fa}-P_{Ia})\,\delta(E_{Fa}-E_{Ia}) + (\cdots)\int_{t_I}^{t_F} dt \int d^dX \int d^dX_I d^dX_F \times \prod_{a=0}^{N^2-1} \left(e^{i(E_{Fa}t_F-E_{Ia}t_I)}e^{-i(P_{Fa}\cdot X_{Fa}-P_{Ia}\cdot X_{Ia})}\right) \times\langle X_F,t_F|X,t\rangle_{f.p.} \times \mathrm{Tr}\Bigg[iq\epsilon\cdot\left(\frac{X_F-X}{t_F-t}+\frac{X-X_I}{t-t_I}\right) \times e^{ik\cdot X-i\omega t}+iq\epsilon_0 e^{ik\cdot X-i\omega t}\Bigg] \times\langle X,t|X_I,t_I\rangle_{\mathrm{f.p.}}+O(q^2), \tag{3.17}$$

in which $E_a=P_a^2/(2Nm)$ for both I and F states [by convention $\mathrm{Tr}(T^aT^b)=N\delta^{ab}$], and the symbol $A\cdot B$ is for the inner product A_iB^i. We recall that the subscripts a and b count the N^2 independent degrees of freedom associated with the $N\times N$ Hermitian matrices X and P. Some of the integrations above can be done (see Appendix B), and the resulting expression is found to be

$$S_{FI}\sim \prod_{a=0}^{N^2-1} \delta^d(P_{Fa}-P_{Ia})\,\delta(E_{Fa}-E_{Ia})+(\cdots) \times\delta^d(P_{F\mathrm{c.m.}}-P_{I\mathrm{c.m.}}-k)\,\delta\left(\sum_{a=0}^{N^2-1}(E_{Fa}-E_{Ia})-\omega\right) \times\int\prod_{b=1}^{N^2-1} d^d\hat{X}_b e^{i(P_{Ib}-P_{Fb})\cdot\hat{X}_b}\mathrm{Tr}\{iq[\epsilon\cdot(P_F+P_I) +\epsilon_0]e^{ik\cdot\hat{X}}\}+O(q^2), \tag{3.18}$$

in which the second series of δ functions have appeared as support for the total momentum and total energy conservation. The last expression contains the Tr and integrals over the matrix coordinates $\hat{X}[\mathrm{Tr}(\hat{X})=0]$, and although the improved forms in some special cases ($N=2$ or in the large-N limit) are accessible, the result in the general case is not known. We mention that such integrals for ordinary coordinates as that of Appendix A can be calculated exactly. We can present the general form of the result as

$$S_{FI}\sim \prod_{a=0}^{N^2-1} \delta^d(P_{Fa}-P_{Ia})\,\delta(E_{Fa}-E_{Ia})+(\cdots) \times\delta^d(P_{F\mathrm{c.m.}}-P_{I\mathrm{c.m.}}-k)\,\delta\left(\sum_{a=0}^{N^2-1}(E_{Fa}-E_{Ia})-\omega\right) \times[iq\epsilon\cdot V(P_{Ia,Fa},k)+iq\epsilon_0V_0(P_{Ia,Fa},k)]+O(q^2), \tag{3.19}$$

in which $V^\mu(P_{Ia,Fa},k)$, as the effective vertex function (see Fig. 4), has the general form

$$V^i=\mathrm{Tr}[(P_F^i+P_I^i)H(P_{Ia,Fa},k)],$$
$$V^0=\mathrm{Tr}[H(P_{Ia,Fa},k)], \tag{3.20}$$

with $H(P_{Ia,Fa},k)$ as a matrix depending on P_{Ia}, P_{Fa} ($a=1,\ldots,N^2-1,a\neq\mathrm{c.m.}$), and k. In the case of ordinary coordinates for covariant theory we find simply $V^\mu\sim(p_I+p_F)^\mu$; see Appendix A.

IV. CONCLUSION AND DISCUSSION

In this work we provide the basic elements of the interaction of D0-brane bound states and one-form RR photons, using the world-line formulation. At the classical level, we checked that the action is invariant under the gauge transformation of the gauge fields in the bulk theory. Also, because of the matrix nature of the coordinates, we see that new symmetry transformations exist, under which the gauge fields transform as gauge fields of a non-Abelian gauge theory. We interpret this observation as the case in which "the fields rotate due to rotation of coordinates." We derived the Lorentz-like equations of motion, and the covariance of the equations was checked under the symmetry transformations. It is seen that the c.m. is white or colorless with respect to the SU(N) sector of the background fields.

At the quantum level, we developed the perturbation theory of the interaction of D0-branes with the RR gauge fields. In particular, using the path integral method, we wrote down the expression for the propagator in the first order of perturbation, which can be converted to the amplitudes of the scattering processes by an arbitrary external source. We discussed how the functional dependence of the gauge fields provides the base for probing the substructure of the bound states.

One natural extension of the studies in this work is for the supersymmetric case. Particularly in the case of maximal supersymmetry ($d=9$), we have the D0-branes of the matrix model, coupled to the one-form RR background. As mentioned in the Introduction, in the matrix model picture D0-branes are assumed to be the supergravitons of 11-dimensional supergravity in the light-cone gauge, and in particular in this case they play the role of the ''photons'' of the one-form RR field in ten dimensions. The interaction of one D0-brane with a bound state of D0-branes is studied in the context of the matrix model, and according to the matrix model interpretation the commutator potential is responsible for the interaction of the single D0-brane (maybe viewed as one RR photon) and the bound state. The known results are those of different orders of loop calculations. It will be interesting to check whether the perturbation expansion in the charge q of this work can reproduce the loop expansion results of the matrix model.

Another extension of the studies of this work might be to include the gravitational effects, specifically by considering nonflat metrics. Comparison to the matrix model calculations can also be done in this case.

One interesting question is about the field theory that may correspond to the world-line theory of matrix coordinates in the presence of a one-form background. For the case of ordinary coordinates, by studies like those of [31], it has been understood that the quantized world-line theory of a charged particle in the presence of the gauge field $A_\mu(x)$ corresponds to a quantized field theory of interaction of charges and photons. As an example, in Appendix A we derived the field theory vertex function for the interaction of a photon with the current of incoming and outgoing particles. In the previous section, we showed how various amplitudes can be calculated in principle by the world-line theory, at least in the perturbative regime. As we saw, our knowledge of the exact values of the amplitudes is restricted, and hence the discussion here will be based on some qualitative considerations.

Probably one of the best guiding observations is the matrix nature of the gauge fields in the world-line formulation. The components of the gauge field in the matrix basis are defined simply by

$$A^\mu(X)=A^\mu_a(X_b)T^a,\quad A^\mu_a(X_b)\equiv\frac{1}{N}\mathrm{Tr}[A^\mu(X)T^a], \tag{4.1}$$

in which $A^\mu_a(X_b)$ are some functions (numbers) depending on the matrix coordinates. The most famous matrix gauge fields we know are those of non-Abelian gauge theories, and it is tempting to see what kind of relation between these two kinds of matrix gauge field can be verified; on one side the quantum theory of matrix gauge fields, and on the other side the quantum mechanics of matrix coordinates.

The best base we found for the possible relation mentioned above was the suggested relation of [4], the map between field configurations of noncommutative and ordinary gauge theories. The suggested map preserves the gauge equivalence relation, and it is emphasized that, due to the different natures of the gauge groups, this map cannot be an isomorphism between the gauge groups. Since for the considerations below there is no essential difference between fermions and bosons, we take the example of the interaction of fermionic matter with the non-Abelian gauge field $\mathcal{A}_\mu(x)$, which is described by the action

$$\begin{aligned}S=\int d^{d+1}x[\bar\psi(i\gamma^\mu\partial_\mu-m)\psi-g\,\mathrm{Tr}(J^\mu\mathcal{A}_\mu\\ -\tfrac14\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu})],\\ \mathcal{A}^\mu(x)=\mathcal{A}^\mu_a(x)T^a,\quad \mathcal{F}^{\mu\nu}(x)=\mathcal{F}^{\mu\nu}_a(x)T^a,\\ \mathcal{F}_{\mu\nu}\equiv[\mathcal{D}_\mu,\mathcal{D}_\nu],\quad \mathcal{D}_\mu\equiv\partial_\mu-ig\mathcal{A}_\mu,\end{aligned} \tag{4.2}$$

in which the term $J^\mu\mathcal{A}_\mu$ is responsible for the interaction; it may be chosen as that of the minimal coupling $J^a_\mu=i\bar\psi\gamma_\mu T^a\psi$. Gauge invariance specifies the behavior of the current J_μ under the gauge transformations to be $J(x)\rightarrow J'(x)=U^\dagger J(x)U$.

On the side of the world-line theory of matrix coordinates, in contrast to the example of previous sections, here we consider a covariant theory, presented by an action like

$$S[X]=\int d\tau\,\mathrm{Tr}[\tfrac12 mD_\tau X^\mu D_\tau X^\mu-qD_\tau X^\mu A_\mu(X)+\cdots], \tag{4.3}$$

in which we have dropped any kind of potential, including the commutator potential of D0-branes. In the above, τ parameterizes the world line, and $D_\tau=\partial_\tau+i[a_\tau,\ ]$ is the covariant derivative along the world line with a_τ as the world-line gauge field.[7] The gauge field $A(X)$ depends on the symmetrized products of X's. In the same spirit as the transformations in the world-line theory of D0-branes, we take

$$\begin{aligned}X^\mu\rightarrow\tilde X^\mu=U^\dagger X^\mu U,\\ a_\tau\rightarrow\tilde a_\tau=U^\dagger a_\tau U-iU^\dagger\partial_\tau U,\\ A_\mu(X)\rightarrow\tilde A_\mu(\tilde X)=U^\dagger A_\mu(X)U-iU^\dagger\delta_\mu U\end{aligned} \tag{4.4}$$

as the gauge transformation in the covariant theory, with $U=\exp[-i\Lambda(X,\tau)]$. We mention that $D_\tau X^\mu$ transforms as $D_\tau X^\mu\rightarrow\tilde D_\tau\tilde X^\mu=U^\dagger D_\tau X^\mu U$ under the transformations. Following the relations (2.14) and (2.15), we can define the field strength as

$$F_{\mu\nu}(X)\equiv\delta_\mu A_\nu(X)-\delta_\nu A_\mu(X), \tag{4.5}$$

and so the field strength transforms as $F_{\mu\nu}\rightarrow\tilde F_{\mu\nu}=U^\dagger F_{\mu\nu}U$; see Eq. (2.23). Now, we want to sketch the map between the field theory in space-time and the world-line theory of a charged particle in a matrix space. It is natural to assume that the map should relate the objects in the two theories as shown in Table I.

We mention that (1) it is enough that the gauge fields are related up to a gauge transformation, (2) the objects on both sides are matrices, and (3) the field strengths and currents of

[7] See [12] for an example of these objects in a covariant theory.

TABLE I. The quantities that should be related by the map in field and world-line theories. Λ is the symbol for the gauge transformation parameter in the two theories.

Non-Abelian field theory	$\Leftrightarrow$	Gauge theory on matrix space
$\mathcal{A}^{\mu}(x)=\mathcal{A}^{\mu}_{a}(x)T^{a}$	$\sim$	$A^{\mu}(X)+$gauge trans. terms
$\mathcal{F}^{\mu\nu}(x)=\mathcal{F}^{\mu\nu}_{a}(x)T^{a}$	$\sim$	$F^{\mu\nu}(X)$
$J^{\mu}(x)=J^{\mu}_{a}(x)T^{a}$	$\sim$	$D_{\tau}X^{\mu}$
$\Lambda(x)=\Lambda_{a}(x)T^{a}$	$\sim$	$\Lambda(X)$

the two theories transform identically under the gauge transformations. Since in this case we have matrices of equal sizes on both sides, it may be considered as a case in which one is able to find a one-to-one map between the two theories. It remains for future studies to check the relation quantitatively, in particular by comparing the amplitudes as observable quantities.

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APPENDIX A: PERTURBATION THEORY OF A CHARGED PARTICLE IN ORDINARY SPACE BY PATH INTEGRAL METHOD

As an exercise, and to complete the basics of the present paper, here we review the perturbation theory of a charged particle in an electromagnetic background. In particular, we extract the vertex function of the coupling of a photon to incoming and outgoing (bosonic) charged particles. In contrast to the nonrelativistic theory of the paper, here we consider a covariant example. A good reference for this discussion is [30]. The action we use, initially in Euclidean space-time, is simply

$$S=\int d\tau[\tfrac{1}{2}m\dot{x}^{2}-iq\dot{x}^{\mu}A_{\mu}(x)]. \tag{A1}$$

We begin with an expression similar to the formula (3.11) of the text:

$$\begin{aligned}\langle x_F,\tau_F|x_I,\tau_I\rangle\sim&\langle x_F,\tau_F|x_I,\tau_I\rangle_{\text{f.p.}}\\&+i\mathcal{N}\lim_{\Delta\tau\to 0}\sum_{k=1}^{n}\int d^{d+1}x_{k-1}d^{d+1}x_{k}\\&\times d^{d+1}x_{k+1}\langle x_F,\tau_F|x_{k+1},\tau_{k+1}\rangle_{\text{f.p.}}\\&\times 2\Delta\tau\cdot\left(q\frac{x^{\mu}_{k+1}-x^{\mu}_{k-1}}{2\Delta\tau}A_{\mu}(x_k,\tau)\right)\\&\times\langle x_{k-1},\tau_{k-1}|x_I,\tau_I\rangle_{\text{f.p.}}\\&\times e^{-S_{\text{f.p.}}[k,k-1;\Delta\tau]}e^{-S_{\text{f.p.}}[k+1,k;\Delta\tau]}+O(q^2),\end{aligned} \tag{A2}$$

in which the normalization constant $\mathcal{N}$ contains sufficient powers of $\Delta\tau$ to regulate the final result, and we have the following relations:

$$\begin{aligned}\langle x_2,\tau_2|x_1,\tau_1\rangle_{\text{f.p.}}\sim& e^{-m(x_2-x_1)^2/2(\tau_2-\tau_1)}\\&\sim\int d^{d+1}l\exp\left(il\cdot(x_2-x_1)\right.\\&\left.-i\frac{l^2}{2m}(\tau_2-\tau_1)\right),\end{aligned} \tag{A3}$$

$$S_{\text{f.p.}}[j+1,j;\Delta\tau]=\frac{m(x_{j+1}-x_j)^2}{2\Delta\tau}, \tag{A4}$$

with $A\cdot B=A^{\mu}B_{\mu}$. Performing the integrations $dx_{k\pm1}$ to replace the new propagators, and after the change $x_k\to x$, we find

$$\begin{aligned}\langle x_F,\tau_F|x_I,\tau_I\rangle\sim&\langle x_F,\tau_F|x_I,\tau_I\rangle_{\text{f.p.}}\\&+i\mathcal{N}'\int_{\tau_I}^{\tau_F}d\tau\int d^{d+1}x\langle x_F,\tau_F|x,\tau\rangle_{\text{f.p.}}\\&\times\left\{q\left(\frac{x_F-x}{\tau_F-\tau}+\frac{x-x_I}{\tau-\tau_I}\right)\cdot A(x)\right\}\\&\times\langle x,\tau|x_I,\tau_I\rangle_{\text{f.p.}}+O(q^2).\end{aligned} \tag{A5}$$

From now on we restrict the calculation to the plane wave $A_{\mu}(x)\sim\epsilon_{\mu}e^{ik_{\nu}x^{\nu}}$. To find the S-matrix elements, it is usual to go to momentum space, and we have the expression

$$\begin{aligned}S_{FI}\sim&\delta^{d}(p_F-p_I)\delta(E_F-E_I)+\frac{\mathcal{N}''e^{-im^2(\tau_F-\tau_I)/2}}{\tau_F-\tau_I}\\&\times\int_{\tau_I}^{\tau_F}d\tau\int d^{d+1}x\int d^{d+1}x_I\int d^{d+1}x_F\\&\times e^{-ip_F\cdot x_F}e^{ip_I\cdot x_I}\langle x_F,\tau_F|x,\tau\rangle_{\text{f.p.}}\left\{e^{ik\cdot x}\exp\left[iq\epsilon\cdot\left(\frac{x_F-x}{\tau_F-\tau}\right.\right.\right.\\&\left.\left.\left.+\frac{x-x_I}{\tau-\tau_I}\right)\right]\right\}_{\text{linear in }\epsilon}\langle x,\tau|x_I,\tau_I\rangle_{\text{f.p.}}+O(q^2),\end{aligned} \tag{A6}$$

in which $p_F^2 = p_I^2 = -m^2$, and to make the calculation easier we have exponentiated the ϵ_μ; so we should keep only the linear term in ϵ finally. By using the momentum representation of the propagator $\langle\cdots\rangle_{\text{f.p.}}$ we find

$$S_{FI} \sim \delta^d(p_F - p_I)\,\delta(E_F - E_I) + (\cdots)\,\delta^d(p_F - p_I - k) \times \delta(E_F - E_I - k_0)[iq\epsilon_\mu(p_F + p_I)^\mu] + O(q^2), \tag{A7}$$

in which we recognize the field theory result $\epsilon\cdot(p_F + p_I)$ for the vertex function (p. 548 of [29]).

APPENDIX B: CALCULATION OF *S*-MATRIX ELEMENT FOR MATRIX COORDINATES IN MOMENTUM BASIS

Here we present the derivation of Eq. (3.18), starting with Eq. (3.17). By using the definitions

$$\langle X_2, t_2 | X_1, t_1\rangle_{\text{f.p.}} \sim \int \prod_{a=0}^{N^2-1} d^{d+1}L_a \exp\left(iL_a\cdot(X_{2a} - X_{1a}) - i\frac{L_a^2}{2Nm}(t_2 - t_1)\right), \tag{B1}$$

$$S_{\text{f.p.}}[j+1, j; \Delta t] = \sum_{a=0}^{N^2-1} \frac{Nm(X_{j+1,a} - X_{j,a})^2}{2\Delta t}, \tag{B2}$$

we find for Eq. (3.17)

$$\begin{aligned} S_{FI} \sim{}& \prod_{a=0}^{N^2-1} \delta^d(P_{Fa} - P_{Ia})\,\delta(E_{Fa} - E_{Ia}) + (\cdots)\int_{t_I}^{t_F} dt \int d^dX \int d^dX_I d^dX_F \prod_{a=0}^{N^2-1} \left(e^{i(E_{Fa}t_F - E_{Ia}t_I)} e^{-i(P_{Fa}\cdot X_{Fa} - P_{Ia}\cdot X_{Ia})}\right) \\ &\times \int \prod_{b=0}^{N^2-1} d^{d+1}Q_b \exp\left(iQ_b\cdot(X_{Fb} - X_b) - i\frac{Q_b^2}{2Nm}(t_F - t)\right)\left(\sum_{c=0}^{N^2-1}\left\{\exp\left[iq\epsilon\cdot\left(\frac{X_{Fc} - X_c}{t_F - t} + \frac{X_c - X_{Ic}}{t - t_I}\right)\right]\right\}_{\text{linear in }\epsilon}\right. \\ &\left.\times \mathrm{Tr}(T^c e^{ik\cdot X - i\omega t}) + \mathrm{Tr}(iq\epsilon_0 e^{ik\cdot X - i\omega t})\right) \times \int \prod_{e=0}^{N^2-1} d^{d+1}L_e \exp\left(iL_e\cdot(X_e - X_{Ie}) - i\frac{L_e^2}{2Nm}(t - t_I)\right) + O(q^2), \end{aligned} \tag{B3}$$

in which to make the calculation easier we have exponentiated the $\vec{\epsilon}$; so we should keep only the linear term in ϵ finally. In the above the symbol $A\cdot B$ is for the inner product A_iB^i. It is worth recalling that the spatial integrals like $\int d^dX$ are in fact $\int\prod_{a=0}^{N^2-1} d^dX_a$. Here we leave the term ϵ_0 for the reader to evaluate. After doing the integrations over $d^dX_{I,F}$, we have

$$\begin{aligned} S_{FI} \sim{}& \prod_{a=0}^{N^2-1} \delta^d(P_{Fa} - P_{Ia})\,\delta(E_{Fa} - E_{Ia}) + (\cdots) \\ &\times \int_{t_I}^{t_F} dt \int \prod_{a=0}^{N^2-1} d^dX_a e^{i(E_{Fa}t_F - E_{Ia}t_I)} \times \int \prod_{b=0}^{N^2-1} d^{d+1}Q_b d^{d+1}L_b e^{-i(Q_b^2/2Nm)(t_F - t)} e^{-i(L_b^2/2Nm)(t - t_I)} e^{i(L_b - Q_b)\cdot X_b} \\ &\times \left(\sum_{c=0}^{N^2-1}\prod_{e=0}^{N^2-1} \delta^d\left(Q_e - P_{Fe} + \frac{q\epsilon\delta_{ce}}{t_F - t}\right)\delta^d\left(L_e - P_{Ie} + \frac{q\epsilon\delta_{ce}}{t - t_I}\right)\right. \\ &\left.\times \left\{\exp\left[iq\epsilon\cdot X_c\left(\frac{-1}{t_F - t} + \frac{1}{t - t_I}\right)\right]\right\}_{\text{linear in }\epsilon} \mathrm{Tr}(T^c e^{ik\cdot X - i\omega t})\right) + O(q^2). \end{aligned} \tag{B4}$$

By using the δ functions we can simply perform the integrations over d^dQ and d^dL. Also, based on the fact that $\exp(ik\cdot X) = \exp(ik\cdot X_{\text{c.m.}}\mathbf{1}_N)\exp(ik\cdot\hat{X})$, with $\mathrm{Tr}(\hat{X}) = 0$, we can perform the integration over $d^dX_0 = d^dX_{\text{c.m.}}$. By recalling that $E_a = P_a^2/(2Nm)$ for I and F states [by convention $\mathrm{Tr}(T^aT^b) = N\delta^{ab}$], and in the limits $t_I \to -\infty$ and $t_F \to \infty$, we arrive at

$$\begin{aligned} S_{FI} \sim{}& \prod_{a=0}^{N^2-1} \delta^d(P_{Fa} - P_{Ia})\,\delta(E_{Fa} - E_{Ia}) + (\cdots)\,\delta^d(P_{F\text{c.m.}} - P_{I\text{c.m.}} - k)\,\delta\left(\sum_{a=0}^{N^2-1}(E_{Fa} - E_{Ia}) - \omega\right)\int \prod_{b=1}^{N^2-1} d^d\hat{X}_b e^{i(P_{Ib} - P_{Fb})\cdot\hat{X}_b} \\ &\times \sum_{c=0}^{N^2-1}\{e^{(i/m)q\epsilon\cdot(P_{Fc} + P_{Ic})}\}_{\text{linear in }\epsilon} \mathrm{Tr}(T^c e^{ik\cdot\hat{X}}) + O(q^2). \end{aligned} \tag{B5}$$

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