Spacetime supersymmetry in a nontrivial Neveu-Schwarz–Neveu-Schwarz superstring background

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In this paper we consider superstring propagation in a nontrivial Neveu-Schwarz–Neveu-Schwarz background. We deform the world sheet stress tensor and supercurrent with an infinitesimal $B_{\mu\nu}$ field. We construct the gauge-covariant super-Poincaré generators in this background and show that the $B_{\mu\nu}$ field spontaneously breaks spacetime supersymmetry. We find that the gauge-covariant spacetime momenta cease to commute with each other and with the spacetime supercharges. We construct a set of "magnetic" super-Poincaré generators that are conserved for constant field strength $H_{\mu\nu\lambda}$, and show that these generators obey a magnetic extension of the ordinary supersymmetry algebra.

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I. INTRODUCTION

Two-dimensional superconformal field theories are solutions to the classical superstring equations of motion. Their infinitesimal deformations [1] can be used to study superstring propagation in nontrivial backgrounds [2] and to elucidate the symmetry structure of string theory itself [3].

In this paper we describe superstring propagation in a nontrivial Neveu-Schwarz–Neveu-Schwarz (NS-NS) background. We start in Sec. II by deriving the infinitesimal deformations that preserve the superconformal structure. We show that they also preserve the nilpotency of the Becchi-Rouet-Stora-Tyutin (BRST) operators. We then construct the deformation that describes superstring propagation in a nontrivial two-form NS-NS background.

In Secs. III and IV we use this formalism to study superstring propagation in the two-form background. We construct the gauge-covariant super-Poincaré generators and compute the spacetime supersymmetry algebra in the presence of the two-form field. We find that the supersymmetry is spontaneously broken, and that the gauge-covariant spacetime momenta cease to commute with each other and with the spacetime supercharges.

In Sec. V we restrict our attention to the case of a constant three-form field strength. We construct a set of conserved "magnetic" super-Poincaré generators that give rise to a "magnetic" extension of the supersymmetry algebra. The magnetic supersymmetry is a generalization of the magnetic translational symmetry associated with point particles in a constant magnetic field [4].

We conclude with an Appendix in which we derive the most general two-form deformation that preserves the superconformal structure.

II. SUPERCONFORMAL DEFORMATIONS

In this paper we work in a Hamiltonian formalism in which the two-dimensional world sheet is parametrized by variables σ and τ. We define our superconformal field theory by three elements: (i) an algebra of operators, $\mathcal{A}$, (ii) a representation of that algebra, and (iii) two distinguished elements of $\mathcal{A}$, the holomorphic and antiholomorphic stress energy superfields $\mathcal{T}(\sigma,\theta)=T_F(\sigma)+\theta T(\sigma)$ and $\overline{\mathcal{T}}(\sigma,\theta)=\overline{T}_F(\sigma)+\overline{\theta T(\sigma)}$. The holomorphic components satisfy one copy of the super-Virasoro algebra,

$$
\begin{aligned}
[T(\sigma),T(\sigma')] &= -\frac{\mathrm{i}c}{24\pi}\delta'''(\sigma-\sigma')+2\mathrm{i}\,T(\sigma')\,\delta'(\sigma-\sigma') \\
&\quad -\mathrm{i}\,T'(\sigma')\,\delta(\sigma-\sigma') \\
\{T_F(\sigma),T_F(\sigma')\} &= -\frac{1}{2\sqrt{2}}T(\sigma')\,\delta(\sigma-\sigma') \\
&\quad +\frac{c}{24\sqrt{2}\,\pi}\delta''(\sigma-\sigma') \qquad (1)\\
[T(\sigma),T_F(\sigma')] &= \frac{3\mathrm{i}}{2}T_F(\sigma')\,\delta'(\sigma-\sigma') \\
&\quad -\mathrm{i}T_F'(\sigma')\,\delta(\sigma-\sigma');
\end{aligned}
$$

the antiholomorphic components satisfy another. The operators $T(\sigma)$ and $\overline{T}(\sigma)$ are the bosonic stress energy tensors, while $T_F(\sigma)$ and $\overline{T}_F(\sigma)$ are their supersymmetric partners.

The algebra $\mathcal{A}$ includes superfields $\Phi(\sigma)$ with bosonic and fermionic components, $\Phi(\sigma)=\Phi_B(\sigma)+\theta\Phi_F(\sigma)$. It also includes spin fields $S^\alpha(\sigma)$ whose presence renders $\mathcal{A}$ nonlocal. The states of the theory span representations of the super-Virasoro algebra. The highest weight states are created by superprimary fields, defined to be superfields $\Phi(\sigma)$ whose components satisfy

$$
\begin{aligned}
[T(\sigma),\Phi_F(\sigma')] &= \mathrm{i}\,d\,\Phi_F(\sigma')\,\delta'(\sigma-\sigma') \\
&\quad -\mathrm{i}\,\Phi_F'(\sigma')\,\delta(\sigma-\sigma') \qquad (2)
\end{aligned}
$$

$$[T(\sigma),\Phi_B(\sigma')] = \mathrm{i}\left(d+\frac{1}{2}\right)\Phi_B(\sigma')\,\delta'(\sigma-\sigma') - \mathrm{i}\,\Phi_B'(\sigma')\,\delta(\sigma-\sigma')$$

$$\{T_F(\sigma),\Phi_F(\sigma')\} = -\frac{1}{2\sqrt{2}}\Phi_B(\sigma')\,\delta(\sigma-\sigma')$$

$$[T_F(\sigma),\Phi_B(\sigma')] = \mathrm{i}\,d\,\Phi_F(\sigma')\,\delta'(\sigma-\sigma') - \frac{\mathrm{i}}{2}\Phi_F'(\sigma')\,\delta(\sigma-\sigma'),$$

and likewise for $\overline{T}(\sigma)$ and $\overline{T}_F(\sigma)$.

In what follows we study superconformal deformations, that is, variations of the stress energy superfields:

$$\begin{aligned} &T(\sigma)\to T(\sigma)+\delta T(\sigma), \quad T_F(\sigma)\to T_F(\sigma)+\delta T_F(\sigma)\\ &\overline{T}(\sigma)\to \overline{T}(\sigma)+\delta\overline{T}(\sigma), \quad \overline{T}_F(\sigma)\to \overline{T}_F(\sigma)+\delta\overline{T}_F(\sigma), \end{aligned} \tag{3}$$

consistent with the super-Virasoro algebra. This requires

$$\begin{aligned} &[\delta T(\sigma),T(\sigma')]+[T(\sigma),\delta T(\sigma')]\\ &\quad = 2\mathrm{i}\,\delta T(\sigma')\,\delta'(\sigma-\sigma') - \mathrm{i}\,\delta T'(\sigma')\,\delta(\sigma-\sigma')\\ &\{\delta T_F(\sigma),T_F(\sigma')\}+\{T_F(\sigma),\delta T_F(\sigma')\}\\ &\quad = -\frac{1}{2\sqrt{2}}\,\delta T(\sigma')\,\delta(\sigma-\sigma')\\ &[\delta T(\sigma),T_F(\sigma')]+[T(\sigma),\delta T_F(\sigma')]\\ &\quad = \frac{3\mathrm{i}}{2}\,\delta T_F(\sigma')\,\delta'(\sigma-\sigma') - \mathrm{i}\,\delta T_F'(\sigma')\,\delta(\sigma-\sigma')\\ &[\delta T(\sigma),\overline{T}(\sigma')]+[T(\sigma),\delta\overline{T}(\sigma')]=0\\ &[\delta T(\sigma),\overline{T}_F(\sigma')]+[T(\sigma),\delta\overline{T}_F(\sigma')]=0\\ &\{\delta T_F(\sigma),\overline{T}_F(\sigma')\}+\{T_F(\sigma),\delta\overline{T}_F(\sigma')\}=0, \end{aligned} \tag{4}$$

as well as analogous conditions from the antiholomorphic part of the algebra.

We restrict our attention to deformations that can be written in terms of superprimary fields. (We relax this condition in the Appendix.) Therefore we make the ansatz

$$\delta T_F = \Phi_F, \quad \delta\overline{T}_F = \widetilde{\Phi}_F, \tag{5}$$

where Φ_F and $\widetilde{\Phi}_F$ have dimension $(d,\overline{d})$ and $(d',\overline{d}')$, respectively. For ease of notation, we suppress their dependence on the coordinate σ. Substituting Eq. (5) into Eq. (4) and using Eq. (2), we see that δT_F and $\delta\overline{T}_F$ satisfy the deformation equations provided $\overline{d}=d'=1$, $d=\overline{d}'=\frac{1}{2}$, and

$$\delta T = \delta\overline{T} = 2\Phi_B, \tag{6}$$

where Φ_B is a (1,1) primary field. These solutions are the supersymmetric generalizations of the canonical deformations defined in Ref. [5].

This formalism can be used to study string propagation in a weak but nontrivial NS-NS background. We start with an undeformed theory that describes a closed superstring in flat Minkowski space. The corresponding superconformal field theory is defined by the following stress energy superfields:

$$\begin{aligned} T &= \frac{1}{2}\eta_{\mu\nu}\partial X^\mu \partial X^\nu - \frac{1}{2}\eta_{\mu\nu}\psi^\mu\partial\psi^\nu\\ \overline{T} &= \frac{1}{2}\eta_{\mu\nu}\overline{\partial} X^\mu \overline{\partial} X^\nu - \frac{1}{2}\eta_{\mu\nu}\widetilde{\psi}^\mu\overline{\partial}\widetilde{\psi}^\nu\\ T_F &= \frac{1}{2}\eta_{\mu\nu}\psi^\mu\partial X^\nu\\ \overline{T}_F &= \frac{1}{2}\eta_{\mu\nu}\widetilde{\psi}^\mu\overline{\partial} X^\nu, \end{aligned} \tag{7}$$

where X^μ, ψ^μ and $\widetilde{\psi}^\mu$ are world-sheet scalars and spinors, respectively. The algebra $\mathcal{A}$ includes composite operators constructed out of the matter fields $X^\mu, \psi^\mu, \widetilde{\psi}^\mu$, together with the spin fields $S^\alpha, \widetilde{S}^\alpha$ and the ghost fields b, c, β, γ, $\widetilde{b}$, $\widetilde{c}$, $\widetilde{\beta}$ and $\widetilde{\gamma}$. All operators are understood to be normal ordered.

We take the deformation to be

$$\begin{aligned} \delta T = \delta\overline{T} &= 2\Phi_B\\ &= B_{\mu\nu}(X)\overline{\partial}X^\nu\partial X^\mu + \partial_\lambda B_{\mu\nu}(X)\overline{\partial}X^\nu\psi^\lambda\psi^\mu\\ &\quad + \partial_\lambda B_{\mu\nu}(X)\partial X^\mu\widetilde{\psi}^\lambda\widetilde{\psi}^\nu + \partial_\rho\partial_\lambda B_{\mu\nu}(X)\psi^\lambda\psi^\mu\widetilde{\psi}^\rho\widetilde{\psi}^\nu, \end{aligned} \tag{8}$$

where Φ_B is the vertex operator for an infinitesimal NS-NS gauge field $B_{\mu\nu}$. The deformation is a (1,1) primary field if

$$\Box B_{\mu\nu}(X)=0, \quad \partial^\mu B_{\mu\nu}(X)=0. \tag{9}$$

The first of these expressions is an equation of motion for $B_{\mu\nu}$; the second is a gauge condition. In the Appendix, we present the deformations that give rise to gauge-covariant equations of motion for these fields.

The superpartners of δT and $\delta\overline{T}$ can be found by calculating the commutators of Φ_B with T_F and $\overline{T}_F$ and demanding that they satisfy Eqs. (4). This gives

$$\delta T_F = \frac{1}{2}[B_{\mu\nu}(X)\overline{\partial}X^\nu\psi^\mu + \partial_\lambda B_{\mu\nu}(X)\widetilde{\psi}^\lambda\widetilde{\psi}^\nu\psi^\mu] \tag{10}$$

$$\delta\overline{T}_F=\frac{1}{2}[B_{\mu\nu}(X)\partial X^\mu\tilde{\psi}^\nu+\partial_\lambda B_{\mu\nu}(X)\psi^\lambda\psi^\mu\tilde{\psi}^\nu].$$

It is tedious but straightforward to check that δT, $\delta\overline{T}$, δT_F and $\delta\overline{T}_F$ satisfy the superconformal deformation equations. These deformations are the same as in [6].

Instead of deforming the stress energy superfield, we could have deformed the BRST charges Q and $\overline{Q}$. Nilpotency then requires

$$\{Q,\delta Q\}=0,\quad \{\overline{Q},\delta\overline{Q}\}=0,\quad \{Q,\delta\overline{Q}\}+\{\overline{Q},\delta Q\}=0 \tag{11}$$

under the infinitesimal deformations

$$Q\rightarrow Q+\delta Q,\quad \overline{Q}\rightarrow\overline{Q}+\delta\overline{Q}. \tag{12}$$

The two approaches are equivalent on the local subalgebra defined by the Gliozzi-Scherk-Olive (GSO) projection. In fact, given a deformed BRST charge, the components of the deformed stress energy superfield can be extracted by calculating the commutator or anticommutator of Q with the ghost field b or β. Nilpotency of the BRST charge implies that the deformed T and T_F obey the super-Virasoro algebra. Conversely, given a deformed stress energy superfield, the BRST deformations are simply

$$\begin{aligned}\delta Q&=\int d\sigma\left[c\,\delta T-\frac{1}{2}\gamma\delta T_F\right]\\ \delta\overline{Q}&=\int d\sigma\left[\tilde{c}\,\delta\overline{T}-\frac{1}{2}\tilde{\gamma}\delta\overline{T}_F\right].\end{aligned} \tag{13}$$

It is straightforward to verify that the deformations (13) satisfy (11) when

$$\begin{aligned}Q=\int d\sigma\Bigg\{&c\left(T^{(X,\psi)}+\frac{1}{2}T^{(b,c,\beta,\gamma)}\right)\\ &-\frac{1}{2}\gamma\left[T_F^{(X,\psi)}+\frac{1}{2}T_F^{(b,c,\beta,\gamma)}\right]\Bigg\},\end{aligned} \tag{14}$$

and likewise for $\overline{Q}$.

III. SPACETIME SYMMETRIES

In string theory, the stress energy superfields $\mathcal{T}_\Phi=T_{F(\Phi)}+\theta T_\Phi$ and $\overline{\mathcal{T}}_\Phi=\overline{T}_{F(\Phi)}+\overline{\theta}\overline{T}_\Phi$ depend on the spacetime fields Φ. Spacetime symmetries are superconformal deformations that induce changes in the spacetime fields:

$$\begin{aligned}\delta T&=\mathrm{i}[h,T_\Phi]=T_{\Phi+\delta\Phi}-T_\Phi\\ \delta T_F&=\mathrm{i}[h,T_{F(\Phi)}]=T_{F(\Phi+\delta\Phi)}-T_{F(\Phi)}\\ \delta\overline{T}&=\mathrm{i}[h,\overline{T}_\Phi]=\overline{T}_{\Phi+\delta\Phi}-\overline{T}_\Phi\\ \delta\overline{T}_F&=\mathrm{i}[h,\overline{T}_{F(\Phi)}]=\overline{T}_{F(\Phi+\delta\Phi)}-\overline{T}_{F(\Phi)}.\end{aligned} \tag{15}$$

The operator h is the generator of the spacetime symmetry; it is the zero mode of a sum of dimension (1,0) and (0,1) currents [7].

Any spacetime symmetry can be described in this way, including the gauge symmetry of a two-form field. The generator of two-form gauge symmetry is

$$\begin{aligned}h&=\int d\sigma\,d\theta d\overline{\theta}[\xi_\mu(\chi)D\chi^\mu-\xi_\mu(\tilde{\chi})\overline{D}\tilde{\chi}^\mu]\\ &=\int d\sigma[\xi_\mu(X)\partial X^\mu-\xi_\mu(X)\overline{\partial}X^\mu+\partial_\mu\xi_\nu(X)\psi^\mu\psi^\nu\\ &\quad-\partial_\mu\xi_\nu(X)\tilde{\psi}^\mu\tilde{\psi}^\nu],\end{aligned} \tag{16}$$

where $\chi^\mu=\psi^\mu+\theta X^\mu$, $\tilde{\chi}^\mu=\tilde{\psi}^\mu+\overline{\theta}X^\mu$ and D and $\overline{D}$ are superspace covariant derivatives. The integrand is a sum of terms of the correct dimensions provided

$$\Box\xi^\mu(X)=0,\quad \partial_\mu\xi^\mu(X)=0. \tag{17}$$

Let us check these assertions by computing the variations of the stress energy superfields:

$$\begin{aligned}\mathrm{i}[h,T]&=\partial_\mu\xi_\nu\overline{\partial}X^\mu\partial X^\nu+\partial_\lambda\partial_\mu\xi_\nu\overline{\partial}X^\mu\psi^\lambda\psi^\nu-\partial_\mu\xi_\nu\overline{\partial}X^\nu\partial X^\mu\\ &\quad-\partial_\lambda\partial_\mu\xi_\nu\partial X^\mu\tilde{\psi}^\lambda\tilde{\psi}^\nu\\ \mathrm{i}[h,T_F]&=\frac{1}{2}\partial_\mu\xi_\nu\overline{\partial}X^\mu\psi^\nu-\frac{1}{2}\partial_\mu\xi_\nu\overline{\partial}X^\nu\psi^\mu\\ &\quad-\frac{1}{2}\partial_\mu\partial_\lambda\xi_\nu\psi^\mu\tilde{\psi}^\lambda\tilde{\psi}^\nu+\frac{1}{2}\partial_\mu\partial_\lambda\xi_\nu\psi^\mu\psi^\lambda\psi^\nu\\ \mathrm{i}[h,\overline{T}]&=\partial_\mu\xi_\nu\overline{\partial}X^\mu\partial X^\nu+\partial_\lambda\partial_\mu\xi_\nu\overline{\partial}X^\mu\psi^\lambda\psi^\nu-\partial_\mu\xi_\nu\overline{\partial}X^\nu\partial X^\mu\\ &\quad-\partial_\lambda\partial_\mu\xi_\nu\partial X^\mu\tilde{\psi}^\lambda\tilde{\psi}^\nu\\ \mathrm{i}[h,\overline{T}_F]&=\frac{1}{2}\partial_\mu\xi_\nu\partial X^\nu\tilde{\psi}^\mu-\frac{1}{2}\partial_\mu\xi_\nu\partial X^\mu\tilde{\psi}^\nu\\ &\quad-\frac{1}{2}\partial_\mu\partial_\lambda\xi_\nu\tilde{\psi}^\mu\tilde{\psi}^\lambda\tilde{\psi}^\nu+\frac{1}{2}\partial_\mu\partial_\lambda\xi_\nu\tilde{\psi}^\mu\psi^\lambda\psi^\nu,\end{aligned} \tag{18}$$

where, in the interest of space, we suppress the arguments (X). From Eqs. (8) and (10) we see that the variations can indeed be described by a $B_{\mu\nu}$ spacetime field,

$$B_{\mu\nu}(X)=\partial_\nu\xi_\mu(X)-\partial_\mu\xi_\nu(X). \tag{19}$$

The deformations (18) induce a pure-gauge background for $B_{\mu\nu}$. The background preserves the gauge (9).

This construction can be readily generalized to an infinite class of infinitesimal gauge symmetries [9] and to finite symmetry transformations (T-duality) [10]. These higher symmetries are generated by operators which classically have higher dimension, such as

$$h=\int d\sigma\omega_{\mu\cdots\nu\cdots\rho\cdots\lambda}(X)\partial^w X^\mu\cdots\bar{\partial}^u X^\nu\cdots\psi^\rho\cdots\tilde{\psi}^\lambda. \tag{20}$$

The integrand is of dimension one if the functions $\omega_{\mu\cdots\nu\cdots\rho\cdots\lambda}$ satisfy differential constraints which can be viewed as gauge conditions. The transformation describes a spontaneously broken spacetime symmetry because it mixes massive and massless spacetime fields [11].

The previous discussion can also be carried through in terms of the BRST formalism. Let us suppose that Q_Φ and $\bar{Q}_\Phi$ are nilpotent BRST charges, functions of the spacetime fields, and h is the zero mode of a sum of dimension (1,0) and (0,1) currents. Then $\Phi\to\Phi+\delta\Phi$ is a spacetime symmetry if

$$\begin{aligned}\delta Q_\Phi&=\mathrm{i}[h,Q_\Phi]=Q_{\Phi+\delta\Phi}-Q_\Phi\\ \delta\bar{Q}_\Phi&=\mathrm{i}[h,\bar{Q}_\Phi]=\bar{Q}_{\Phi+\delta\Phi}-\bar{Q}_\Phi\,.\end{aligned}\tag{21}$$

For the case at hand, the BRST operator Q is given by

$$\begin{aligned}Q=\int d\sigma c\bigg(&\frac{1}{2}\eta_{\mu\nu}\partial X^\mu\partial X^\nu-\frac{1}{2}\eta_{\mu\nu}\psi^\mu\partial\psi^\nu\\ &-\frac{3}{2}\beta\partial\gamma-\frac{1}{2}\partial\beta\gamma\bigg)\\ &+\int d\sigma\bigg(bc\partial c+\frac{1}{2}\gamma\eta_{\mu\nu}\psi^\mu\partial X^\nu-\frac{1}{4}b\gamma^2\bigg),\end{aligned}\tag{22}$$

and h is given in Eq. (16). We compute the commutator and find

$$\begin{aligned}\delta Q=\mathrm{i}[h,Q]=\int d\sigma c[&\partial_\mu\xi_\nu\bar{\partial}X^\mu\partial X^\nu+\partial_\lambda\partial_\mu\xi_\nu\bar{\partial}X^\mu\psi^\lambda\psi^\nu\\ &-\partial_\mu\xi_\nu\bar{\partial}X^\nu\partial X^\mu-\partial_\lambda\partial_\mu\xi_\nu\partial X^\mu\tilde{\psi}^\lambda\tilde{\psi}^\nu]\\ &-\frac{1}{4}\int d\sigma\gamma[\partial_\mu\xi_\nu\bar{\partial}X^\mu\psi^\nu-\partial_\mu\xi_\nu\bar{\partial}X^\nu\psi^\mu\\ &-\partial_\mu\partial_\lambda\xi_\nu\psi^\mu\tilde{\psi}^\lambda\tilde{\psi}^\nu+\partial_\mu\partial_\lambda\xi_\nu\psi^\mu\psi^\lambda\psi^\nu].\end{aligned}\tag{23}$$

Comparing with Eqs. (14) and (18), we see that this deformation can be absorbed in the two-form gauge potential (19).

IV. SUPERSYMMETRY ALGEBRA

We are now ready to compute the supersymmetry algebra in the two-form gauge field background. We start with the undeformed super-Poincaré generators,

$$P^{\mu(0)}=\int d\sigma(\partial X^\mu+\bar{\partial}X^\mu),\quad Z^{\mu(0)}=\int d\sigma(\partial X^\mu-\bar{\partial}X^\mu)\tag{24}$$

$$Q_\alpha^{(-1/2)}=\int d\sigma J_\alpha^{(-1/2)},\quad \tilde{Q}_\alpha^{(-1/2)}=\int d\sigma\tilde{J}_\alpha^{(-1/2)}$$

where $P^{\mu(0)}$ and $Z^{\mu(0)}$ are the spacetime translation and winding number generators, $Q_\alpha^{(-1/2)}$ and $\tilde{Q}_\alpha^{(-1/2)}$ are the spacetime supercharges, and the supersymmetry currents are given by

$$J_\alpha^{(-1/2)}=S_\alpha e^{-\phi/2},\quad \tilde{J}_\alpha^{(-1/2)}=\tilde{S}_\alpha e^{-\tilde{\phi}/2}.\tag{25}$$

The operators are in the canonical picture; the superscripts indicate the ghost charges of the operators. It is a small calculation to show that the generators obey the following commutation relations:

$$\begin{aligned}&[P^{\mu(0)},P^{\nu(0)}]=0,\quad [P^{\mu(0)},Q_\alpha^{(-1/2)}]=0,\\ &[P^{\mu(0)},\tilde{Q}_\alpha^{(-1/2)}]=0\\ &[Z^{\mu(0)},Z^{\nu(0)}]=0,\quad [Z^{\mu(0)},Q_\alpha^{(-1/2)}]=0,\\ &[Z^{\mu(0)},\tilde{Q}_\alpha^{(-1/2)}]=0\\ &\{Q_\alpha^{(-1/2)},Q_\beta^{(-1/2)}\}=(\gamma_\mu)_{\alpha\beta}\int d\sigma\psi^\mu e^{-\phi},\\ &\{\tilde{Q}_\alpha^{(-1/2)},\tilde{Q}_\beta^{(-1/2)}\}=(\gamma_\mu)_{\alpha\beta}\int d\sigma\tilde{\psi}^\mu e^{-\tilde{\phi}}.\end{aligned}\tag{26}$$

To interpret this algebra, we recall the picture changing operation that is an essential ingredient of superstring theory [12]. Picture changing maps a BRST-invariant operator $O^{(q)}$ of ghost charge q to an equivalent operator of charge $q+1$ via the commutator

$$O^{(q+1)}=[Q,2\xi O^{(q)}],\tag{27}$$

where ξ is defined through the bosonization of the superconformal ghosts,

$$\beta=e^{-\phi}\partial\xi,\quad \gamma=e^{\phi}\eta.\tag{28}$$

It is straightforward to show that under the picture changing,

$$\begin{aligned}&[Q,2\xi\psi^\mu e^{-\phi}]+[\bar{Q},\tilde{\xi}\tilde{\psi}^\mu e^{-\tilde{\phi}}]=\partial X^\mu+\bar{\partial}X^\mu\\ &[Q,2\xi\psi^\mu e^{-\phi}]-[\bar{Q},\tilde{\xi}\tilde{\psi}^\mu e^{-\tilde{\phi}}]=\partial X^\mu-\bar{\partial}X^\mu.\end{aligned}\tag{29}$$

This implies that

$$\begin{aligned}P^{\mu(-1)}&=\int d\sigma(\psi^\mu e^{-\phi}+\tilde{\psi}^\mu e^{-\tilde{\phi}}),\\ Z^{\mu(-1)}&=\int d\sigma(\psi^\mu e^{-\phi}-\tilde{\psi}^\mu e^{-\tilde{\phi}})\end{aligned}\tag{30}$$

are the momentum and winding number generators in the (-1) picture. Using these relations, we can write the last line of Eq. (26) in a familiar form,

$$\{Q_{i\alpha}^{(-1/2)},Q_{j\beta}^{(-1/2)}\}=(\gamma^\mu)_{\alpha\beta}(\delta_{ij}P_\mu^{(-1)}+\epsilon_{ij}Z_\mu^{(-1)}),\tag{31}$$

where $Q_{1\alpha}^{(-1/2)} \equiv Q_\alpha^{(-1/2)} + \tilde{Q}_\alpha^{(-1/2)}$ and $Q_{2\alpha}^{(-1/2)} \equiv Q_\alpha^{(-1/2)} - \tilde{Q}_\alpha^{(-1/2)}$. The spacetime supercharges close into the usual $N=2$ super-Poincaré algebra, modulo a change of picture.

We now extend this analysis to the $B_{\mu\nu}$ background. We first need to find gauge-covariant versions of the supersymmetry generators. We begin by computing the two-form gauge transformations of Eq. (24),

$$\begin{aligned}
\mathrm{i}[h,\partial X_\mu] &= \frac{1}{2}[\partial_\mu \xi_\nu(X) - \partial_\nu \xi_\mu(X)](\partial X^\nu - \bar{\partial} X^\nu) \\
&\quad + \frac{1}{2}\partial_\mu \partial_\lambda \xi_\rho(X)(\psi^\lambda \psi^\rho - \tilde{\psi}^\lambda \tilde{\psi}^\rho) \\
\mathrm{i}[h,\bar{\partial} X_\mu] &= \frac{1}{2}[\partial_\mu \xi_\nu(X) - \partial_\nu \xi_\mu(X)](\partial X^\nu - \bar{\partial} X^\nu) \\
&\quad + \frac{1}{2}\partial_\mu \partial_\lambda \xi_\rho(X)(\psi^\lambda \psi^\rho - \tilde{\psi}^\lambda \tilde{\psi}^\rho)
\end{aligned} \tag{32}$$

$$\mathrm{i}[h,S_\alpha e^{-\phi/2}] = \frac{1}{2}(\gamma^{\rho\lambda})_\alpha^\beta \partial_\rho \xi_\lambda(X) S_\beta e^{-\phi/2}$$

$$\mathrm{i}[h,\tilde{S}_\alpha e^{-\tilde{\phi}/2}] = -\frac{1}{2}(\gamma^{\rho\lambda})_\alpha^\beta \partial_\rho \xi_\lambda(X) \tilde{S}_\beta e^{-\tilde{\phi}/2},$$

where h is given by Eq. (16). These expressions suggest that we take the following operators to be the gauge-covariant super-Poincaré generators in the canonical picture and the (infinitesimal) $B_{\mu\nu}$ background:

$$\hat{P}^{\mu(0)} = \int d\sigma(\hat{\partial} X^\mu + \hat{\bar{\partial}} X^\mu), \quad \hat{Z}^{\mu(0)} = \int d\sigma(\hat{\partial} X^\mu - \hat{\bar{\partial}} X^\mu) \tag{33}$$

$$\hat{Q}_\alpha^{(-1/2)} = \int d\sigma \hat{J}_\alpha^{(-1/2)}, \quad \hat{\tilde{Q}}_\alpha^{(-1/2)} = \int d\sigma \hat{\tilde{J}}_\alpha^{(-1/2)},$$

where

$$\begin{aligned}
\hat{\partial} X_\mu &= \partial X_\mu + \frac{1}{2}B_{\mu\nu}(X)(\partial X^\nu - \bar{\partial} X^\nu) + \frac{1}{2}\partial_\nu B_{\mu\lambda}(X) \\
&\quad \times (\psi^\nu \psi^\lambda - \tilde{\psi}^\nu \tilde{\psi}^\lambda) \\
\hat{\bar{\partial}} X_\mu &= \bar{\partial} X_\mu + \frac{1}{2}B_{\mu\nu}(X)(\partial X^\nu - \bar{\partial} X^\nu) + \frac{1}{2}\partial_\nu B_{\mu\lambda}(X) \\
&\quad \times (\psi^\nu \psi^\lambda - \tilde{\psi}^\nu \tilde{\psi}^\lambda)
\end{aligned} \tag{34}$$

$$\hat{J}_\alpha^{(-1/2)} = S_\alpha e^{-\phi/2} + \frac{1}{4}(\gamma^{\rho\lambda})_\alpha^\beta B_{\rho\lambda}(X) S_\beta e^{-\phi/2}$$

$$\hat{\tilde{J}}_\alpha^{(-1/2)} = \tilde{S}_\alpha e^{-\tilde{\phi}/2} - \frac{1}{4}(\gamma^{\rho\lambda})_\alpha^\beta B_{\rho\lambda}(X) \tilde{S}_\beta e^{-\tilde{\phi}/2}.$$

It is not hard to check that the generators are indeed covariant under two-form gauge transformations.

A nontrivial $B_{\mu\nu}$ field spontaneously breaks the translational symmetry of Minkowski space [11]. This can be seen from the commutator of the deformed stress energy tensor with the gauge-covariant translation current $\hat{\partial} X_\mu$,

$$\begin{aligned}
&[T(\sigma) + \delta T(\sigma), \hat{\partial} X_\mu(\sigma')] \\
&\quad = \mathrm{i}\,\hat{\partial} X_\mu(\sigma')\,\delta'(\sigma-\sigma') - \mathrm{i}\,\hat{\partial} X'_\mu(\sigma')\,\delta(\sigma-\sigma') \\
&\qquad - \mathrm{i}\,H_{\mu\nu\lambda}(X)\partial X^\nu \bar{\partial} X^\lambda\,\delta(\sigma-\sigma') \\
&\qquad - 2\mathrm{i}\,\partial_\nu H_{\mu\rho\sigma}(X)\psi^\nu \psi^\rho \bar{\partial} X^\sigma\,\delta(\sigma-\sigma') \\
&\qquad - 2\mathrm{i}\,\partial_\nu H_{\mu\rho\sigma}(X)\tilde{\psi}^\nu \tilde{\psi}^\rho \partial X^\sigma\,\delta(\sigma-\sigma') \\
&\qquad - \mathrm{i}\,\partial_\mu \partial_\lambda H_{\nu\rho\sigma}(X)\psi^\nu \psi^\rho \tilde{\psi}^\lambda \tilde{\psi}^\sigma\,\delta(\sigma-\sigma'),
\end{aligned} \tag{35}$$

where we work to first order in the $B_{\mu\nu}$ field. The symmetry is conserved if $\hat{\partial} X_\mu$ is primary and of dimension one. This requires that the field strength $H_{\mu\nu\lambda}=0$. For nonzero $H_{\mu\nu\lambda}$, the gauge-covariant translations are spontaneously broken, just as they are for the point particle in a constant magnetic field.

The supersymmetry currents $\hat{J}_\alpha^{(-1/2)}$ and $\hat{\tilde{J}}_\alpha^{(-1/2)}$ are also spontaneously broken for nonvanishing $H_{\mu\nu\lambda}$. This follows from the commutator

$$\begin{aligned}
&[T(\sigma) + \delta T(\sigma), \hat{J}_\alpha^{(-1/2)}(\sigma')] \\
&\quad = \mathrm{i}\,\hat{J}_\alpha^{(-1/2)}(\sigma')\,\delta'(\sigma-\sigma') - \mathrm{i}\,\hat{J}_\alpha^{(-1/2)\prime}(\sigma')\,\delta(\sigma-\sigma') \\
&\qquad + \frac{\mathrm{i}}{2}H_{\mu\nu\lambda}(X)(\gamma^{\mu\nu})_\alpha^\beta S_\beta e^{-\phi/2}\bar{\partial} X^\lambda\,\delta(\sigma-\sigma') \\
&\qquad + \mathrm{i}(\gamma^{\mu\nu})_\alpha^\beta \partial_\rho H_{\mu\nu\lambda} S_\beta e^{-\phi/2}\tilde{\psi}^\rho \tilde{\psi}^\lambda\,\delta(\sigma-\sigma').
\end{aligned} \tag{36}$$

A nonzero field strength spontaneously breaks spacetime supersymmetry, as expected from supergravity.

Even though the super-Poincaré symmetries are spontaneously broken, one can still compute the supersymmetry algebra in the $B_{\mu\nu}$ background. It is a small exercise to show that the gauge-covariant supercharges obey the following anticommutation relations:

$$\{\hat{Q}_\alpha^{(-1/2)}, \hat{Q}_\beta^{(-1/2)}\} = (\gamma^\mu)_{\alpha\beta}\int d\sigma \times \left[\psi_\mu e^{-\phi} + \frac{1}{2}B_{\mu\nu}(X)\psi^\nu e^{-\phi}\right] \tag{37}$$

$$\{\hat{\tilde{Q}}_\alpha^{(-1/2)}, \hat{\tilde{Q}}_\beta^{(-1/2)}\} = (\gamma^\mu)_{\alpha\beta}\int d\sigma \times \left[\tilde{\psi}_\mu e^{-\tilde{\phi}} - \frac{1}{2}B_{\mu\nu}(X)\tilde{\psi}^\nu e^{-\tilde{\phi}}\right]$$

$$\{\hat{Q}_\alpha^{(-1/2)}, \hat{\tilde{Q}}_\beta^{(-1/2)}\} = 0,$$

to first order in the $B_{\mu\nu}$ field. Rewriting these expressions in terms of $\hat{Q}^{(-1/2)}_{1\alpha}$ and $\hat{Q}^{(-1/2)}_{2\alpha}$, we find

$$\{\hat{Q}^{(-1/2)}_{i\alpha},\hat{Q}^{(-1/2)}_{j\beta}\}=\delta_{ij}(\gamma^{\mu})_{\alpha\beta}\int d\sigma\Big[\psi_{\mu}e^{-\phi}+\tilde{\psi}_{\mu}e^{-\tilde{\phi}}+\frac{1}{2}B_{\mu\nu}(X)(\psi^{\nu}e^{-\phi}-\tilde{\psi}^{\rho}e^{-\tilde{\phi}})\Big]+\epsilon_{ij}(\gamma^{\mu})_{\alpha\beta}\int d\sigma\Big[\psi_{\mu}e^{-\phi}-\tilde{\psi}_{\mu}e^{-\tilde{\phi}}+\frac{1}{2}B_{\mu\nu}(X)(\psi^{\nu}e^{-\phi}+\tilde{\psi}^{\rho}e^{-\tilde{\phi}})\Big]. \quad (38)$$

To interpret this expression, we must define the picture changing operation in the gauge field background. From Eq. (27) we find

$$\delta O^{(q+1)}=[\delta Q,2\xi O^{(q)}]+[Q,2\delta\xi O^{(q)}]+[Q,2\xi\delta O^{(q)}], \quad (39)$$

where $\delta O^{(p)}$ is the deformation in the p picture, $\delta\xi$ is the deformation of the ghost ξ, and δQ is the deformation of the BRST charge. Using this relation, it is not hard to show that the first term on the right-hand side of Eq. (38) is the gauge-covariant momentum generator in the -1 picture,

$$\int d\sigma\Big\{\Big[Q+\delta Q,2\xi\Big(\psi_{\mu}e^{-\phi}+\frac{1}{2}B_{\mu\nu}(X)\psi^{\nu}e^{-\phi}\Big)\Big]+\Big[\bar{Q}+\delta\bar{Q},2\tilde{\xi}\Big(\tilde{\psi}_{\mu}e^{-\tilde{\phi}}+\frac{1}{2}B_{\mu\nu}(X)\tilde{\psi}^{\nu}e^{-\tilde{\phi}}\Big)\Big]\Big\}=\int d\sigma(\hat{\partial}X_{\mu}+\hat{\bar{\partial}}X_{\mu}). \quad (40)$$

The second term is the gauge-covariant winding number generator in the same picture. Combining these results, we find

$$\{\hat{Q}^{(-1/2)}_{i\alpha},\hat{Q}^{(-1/2)}_{j\beta}\}=(\gamma^{\mu})_{\alpha\beta}(\delta_{ij}\hat{P}^{(-1)}_{\mu}+\epsilon_{ij}\hat{Z}^{(-1)}_{\mu}). \quad (41)$$

We can use similar techniques to compute the remaining parts of the supersymmetry algebra:

$$[\hat{P}^{(0)}_{\mu},\hat{P}^{(0)}_{\nu}]=-\int d\sigma[H_{\mu\nu\lambda}(X)(\partial X^{\lambda}-\bar{\partial}X^{\lambda})+\partial_{\lambda}H_{\mu\nu\rho}(X)\times(\psi^{\lambda}\psi^{\rho}-\tilde{\psi}^{\lambda}\tilde{\psi}^{\rho})]$$

$$[\hat{Q}^{(-1/2)}_{\alpha},\hat{P}^{(0)}_{\mu}]=(\gamma^{\rho\lambda})_{\alpha}^{\beta}\int d\sigma H_{\mu\rho\lambda}(X)S_{\beta}e^{-\phi/2} \quad (42)$$

$$[\hat{\bar{Q}}^{(-1/2)}_{\alpha},\hat{P}^{(0)}_{\mu}]=-(\gamma^{\rho\lambda})_{\alpha}^{\beta}\int d\sigma H_{\mu\rho\lambda}(X)\tilde{S}_{\beta}e^{-\tilde{\phi}/2}.$$

We see that the gauge-covariant momenta and supersymmetry charges cease to commute in the presence of a nontrivial NS-NS background field. For the case of constant $H_{\mu\nu\lambda}$, however, the commutators (42) simplify considerably. We find

$$[\hat{P}^{(0)}_{\mu},\hat{P}^{(0)}_{\nu}]=-H_{\mu\nu\lambda}Z^{\lambda(0)}$$

$$[\hat{Q}^{(-1/2)}_{\alpha},\hat{P}^{(0)}_{\mu}]=(\gamma^{\rho\lambda})_{\alpha}^{\beta}H_{\mu\rho\lambda}\hat{Q}^{(-1/2)}_{\beta} \quad (43)$$

$$[\hat{\bar{Q}}^{(-1/2)}_{\alpha},\hat{P}^{(0)}_{\mu}]=-(\gamma^{\rho\lambda})_{\alpha}^{\beta}H_{\mu\rho\lambda}\hat{\bar{Q}}^{(-1/2)}_{\beta}.$$

This algebra is similar to that of the supersymmetric point particle in a constant electromagnetic background.

We have checked our results by verifying that the Jacobi identity still holds. For example, we compute

$$[\hat{Q}^{(-1/2)}_{\alpha},[\hat{P}^{(0)}_{\mu},\hat{P}^{(0)}_{\nu}]]+[\hat{P}^{(0)}_{\mu},[\hat{P}^{(0)}_{\nu},\hat{Q}^{(-1/2)}_{\alpha}]]+[\hat{P}^{(0)}_{\nu},[\hat{Q}^{(-1/2)}_{\alpha},\hat{P}^{(0)}_{\mu}]]=(\gamma^{\lambda\rho})_{\alpha\beta}\int d\sigma(\partial_{\lambda}H_{\mu\nu\rho}-\partial_{\rho}H_{\lambda\mu\nu}-\partial_{\mu}H_{\nu\rho\lambda}+\partial_{\nu}H_{\rho\lambda\mu}), \quad (44)$$

which vanishes because of the Bianchi identity.

A straightforward calculation of the commutator of the string coordinate X^{μ} with the generator of two-form gauge transformations shows that the string coordinate does not change with a $B_{\mu\nu}$ background field. This is in contrast to the open string, in which case the string coordinate deforms and becomes non-commutative [13].

V. MAGNETIC SUPERSYMMETRY

In the previous section we saw that a $B_{\mu\nu}$ field spontaneously breaks spacetime supersymmetry. It is interesting to ask whether any deformations of the spacetime symmetries remain conserved in this background. In this section we shall see that there are indeed such generators for constant $H_{\mu\nu\lambda}$. We call them ‘‘magnetic’’ super-Poincaré generators in analogy to the magnetic translation operators that can be constructed for point particles in a constant magnetic field [4].

The basic approach is as before. We start by deforming the gauge-covariant super-Poincaré currents (34) by a sum of (0,1) and (1,0) operators. We then compute the conditions that follow from the requirement that the new currents be primary and dimension one with respect to the deformed stress tensor. We find that these conditions require $H_{\mu\nu\lambda}$ to be constant, and furthermore, that the deformed currents be of the following form:

$$\partial X^M_\mu = \hat{\partial} X_\mu - H_{\mu\nu\lambda} X^\lambda \partial X^\nu - H_{\mu\nu\lambda} \psi^\nu \psi^\lambda \tag{45}$$

$$\bar{\partial} X^M_\mu = \hat{\bar{\partial}} X_\mu + H_{\mu\nu\lambda} X^\lambda \bar{\partial} X^\nu + H_{\mu\nu\lambda} \tilde{\psi}^\nu \tilde{\psi}^\lambda$$

$$J^{(-1/2)M}_\alpha = \hat{J}^{(-1/2)}_\alpha - \frac{1}{2} (\gamma^{\mu\nu})_\alpha^\beta H_{\mu\nu\lambda} X^\lambda S_\beta e^{-\phi/2}$$

$$\tilde{J}^{(-1/2)M}_\alpha = \hat{\tilde{J}}^{(-1/2)}_\alpha + \frac{1}{2} (\gamma^{\mu\nu})_\alpha^\beta H_{\mu\nu\lambda} X^\lambda S_\beta e^{-\phi/2}.$$

The index M indicates that these are conserved, gauge-covariant, ''magnetic'' super-Poincaré currents in a constant $H_{\mu\nu\lambda}$ background.

Once we have the magnetic currents, it is a simple exercise to compute the magnetic supersymmetry algebra. We find

$$\{Q^{(-1/2)M}_{i\alpha}, Q^{(-1/2)M}_{j\beta}\} = (\gamma^\mu)_{\alpha\beta} (\delta_{ij} P^{(-1)M}_\mu + \epsilon_{ij} Z^{(-1)M}_\mu), \tag{46}$$

and

$$[P^{(0)M}_\mu, P^{(0)M}_\nu] = 2 H_{\mu\nu\lambda} Z^{\lambda(0)M}$$

$$[Q^{(-1/2)M}_\alpha, P^{(0)M}_\mu] = -(\gamma^{\rho\lambda})_\alpha^\beta H_{\mu\rho\lambda} Q^{(-1/2)M}_\beta \tag{47}$$

$$[\tilde{Q}^{(-1/2)M}_\alpha, P^{(0)M}_\mu] = (\gamma^{\rho\lambda})_\alpha^\beta H_{\mu\rho\lambda} \tilde{Q}^{(-1/2)M}_\beta .$$

In these expressions,

$$P^{(0)M}_\mu = \int d\sigma (\partial X^M_\mu + \bar{\partial} X^M_\mu)$$
$$\tag{48}$$
$$Z^{(0)M}_\mu = \int d\sigma (\partial X^M_\mu - \bar{\partial} X^M_\mu)$$

and

$$P^{(-1)M}_\mu = \hat{P}^{(-1)}_\mu - H_{\mu\nu\lambda} \int d\sigma X^\lambda (\psi^\nu e^{-\phi} - \tilde{\psi}^\nu e^{-\tilde{\phi}})$$
$$\tag{49}$$
$$Z^{(-1)M}_\mu = \hat{Z}^{(-1)}_\mu - H_{\mu\nu\lambda} \int d\sigma X^\lambda (\psi^\nu e^{-\phi} + \tilde{\psi}^\nu e^{-\tilde{\phi}})$$

are the magnetic translation and winding generators in the 0 and -1 pictures. This is the magnetic supersymmetry algebra that holds in a constant $H_{\mu\nu\lambda}$ background.

VI. CONCLUSIONS

In this paper we discussed deformations of the fermionic string. We showed how to deform the stress tensor, the supercurrent and the BRST charges in a way consistent with superconformal invariance. We used the technique to study superstring propagation in a nontrivial two-form NS-NS background.

Our main result was the construction of the gauge-covariant super-Poincaré generators in the presence of a $B_{\mu\nu}$ field. We found that the $B_{\mu\nu}$ field generically breaks spacetime supersymmetry. For the case of constant field strength $H_{\mu\nu\lambda}$, we found ''magnetic'' extensions of the spacetime super-Poincaré generators. The magnetic generators are conserved and gauge covariant; they are generalizations of the magnetic translation operators that can be constructed for point particles in a constant magnetic field. For the case at hand, the magnetic super-Poincaré generators close into a magnetic extension of the spacetime supersymmetry algebra.

The techniques presented here can be readily extended to the case of a weak Ramond-Ramond background. Work along these lines is currently in progress.

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APPENDIX

In this appendix we show how to describe a NS-NS two-form potential in an arbitrary gauge [8]. We do this by first performing an arbitrary gauge transformation about flat spacetime. We then replace the transformation parameters by the $B_{\mu\nu}$ field.

We start by computing the commutator of h, the generator of a symmetry transformation, with the stress tensor T and the supercurrent T_F. We assume that h has the form (16), but we do not impose the differential constraints on $\xi_\mu(X)$. This gives

$$
\begin{aligned}
i[h,T]=&\partial_\mu\xi_\nu\bar\partial X^\mu\partial X^\nu+\partial_\lambda\partial_\mu\xi_\nu\bar\partial X^\mu\psi^\lambda\psi^\nu-\frac{1}{2}\Box\partial_\mu\xi_\nu\bar\partial X^\mu\partial X^\nu-\frac{1}{2}\Box\partial_\lambda\partial_\mu\xi_\nu\bar\partial X^\mu\psi^\lambda\psi^\nu+\partial_\nu\partial_\lambda\partial_\mu\xi^\mu\partial X^\nu\bar\partial X^\lambda\\
&-\frac{1}{2}\partial_\nu\partial_\lambda\partial_\mu\xi^\mu\partial X^\nu\partial X^\lambda-\frac{1}{2}\partial_\nu\partial_\mu\xi^\mu\partial^2X^\nu-\frac{1}{2}\partial_\nu\partial_\lambda\partial_\mu\xi^\mu\bar\partial X^\nu\bar\partial X^\lambda-\frac{1}{2}\partial_\nu\partial_\mu\xi^\mu\bar\partial^2X^\nu+\frac{1}{2}\Box\partial_\mu\xi_\nu\partial X^\mu\partial X^\nu+\frac{1}{2}\Box\xi_\mu\partial^2X^\mu\\
&+\frac{1}{2}\Box\partial_\nu\partial_\mu\xi_\lambda\bar\partial X^\nu\psi^\mu\psi^\lambda-\frac{1}{2}\Box\partial_\nu\partial_\mu\xi_\lambda\partial X^\nu\psi^\mu\psi^\lambda+\frac{1}{2}\Box\partial_\mu\xi_\nu\partial\psi^\mu\psi^\nu+\frac{1}{2}\Box\partial_\mu\xi_\nu\psi^\mu\partial\psi^\nu-\frac{1}{2}\Box\partial_\nu\xi_\mu\bar\partial X^\mu\partial X^\nu\\
&+\frac{1}{2}\Box\partial_\nu\xi_\mu\bar\partial X^\mu\bar\partial X^\nu-\partial_\nu\xi_\mu\bar\partial X^\mu\partial X^\nu-\frac{1}{2}\Box\partial_\nu\partial_\mu\xi_\lambda\partial X^\nu\tilde\psi^\mu\tilde\psi^\lambda-\partial_\lambda\partial_\mu\xi_\nu\partial X^\mu\tilde\psi^\lambda\tilde\psi^\nu+\frac{1}{2}\Box\partial_\nu\partial_\mu\xi_\lambda\bar\partial X^\nu\tilde\psi^\mu\tilde\psi^\lambda\\
&+\frac{1}{2}\Box\xi_\mu\bar\partial^2X^\mu-\frac{1}{2}\Box\partial_\mu\xi_\nu\tilde\psi^\mu\bar\partial\tilde\psi^\nu-\frac{1}{2}\Box\partial_\mu\xi_\nu\bar\partial\tilde\psi^\mu\tilde\psi^\nu
\end{aligned}\tag{A1}
$$

and

$$
\begin{aligned}
i[h,T_F]=&\frac{1}{2}\partial_\mu\xi_\nu\bar\partial X^\mu\psi^\nu+\frac{1}{2}\Box\partial_\mu\xi_\nu\partial X^\mu\psi^\nu-\frac{1}{2}\Box\partial_\mu\xi_\nu\bar\partial X^\mu\psi^\nu+\frac{1}{2}\Box\xi_\mu\partial\psi^\mu-\frac{1}{2}\partial_\mu\partial_\nu\partial_\lambda\xi^\mu\partial X^\nu\psi^\lambda+\frac{1}{2}\partial_\mu\partial_\nu\partial_\lambda\xi^\mu\bar\partial X^\nu\psi^\lambda\\
&-\frac{1}{2}\partial_\mu\partial_\nu\xi^\mu\partial\psi^\nu+\frac{1}{2}\partial_\mu\partial_\lambda\xi_\nu\psi^\mu\psi^\lambda\psi^\nu-\frac{1}{2}\partial_\mu\xi_\nu\bar\partial X^\nu\psi^\mu-\frac{1}{2}\partial_\mu\partial_\lambda\xi_\nu\psi^\mu\tilde\psi^\lambda\tilde\psi^\nu.
\end{aligned}\tag{A2}
$$

Substituting $B_{\mu\nu}=\partial_\mu\xi_\nu-\partial_\nu\xi_\mu$ into these equations, we find for the deformations that describe two-form propagation in flat spacetime,

$$
\begin{aligned}
\delta T=&\left(B_{\mu\nu}(X)-\frac{1}{2}\Box B_{\mu\nu}(X)-\partial_\nu\partial^\lambda B_{\lambda\mu}(X)\right)\bar\partial X^\mu\partial X^\nu-\left(\frac{1}{2}\Box B_{\mu\nu}-\frac{1}{2}\partial_\mu\partial^\lambda B_{\lambda\nu}(X)\right)\partial X^\mu\partial X^\nu+\frac{1}{2}\partial_\mu\partial^\lambda B_{\lambda\nu}(X)\bar\partial X^\mu\bar\partial X^\nu\\
&+\frac{1}{2}\partial^\mu B_{\mu\nu}(X)\partial^2X^\nu+\frac{1}{2}\partial^\mu B_{\mu\nu}(X)\bar\partial^2X^\nu+\frac{1}{2}\Box\partial_\lambda B_{\nu\mu}(X)\partial X^\mu\psi^\lambda\psi^\nu-\left(\partial_\lambda B_{\nu\mu}(X)-\frac{1}{2}\Box\partial_\lambda B_{\nu\mu}(X)\right)\bar\partial X^\mu\psi^\lambda\psi^\nu\\
&-\left(\partial_\lambda B_{\mu\nu}(X)+\frac{1}{2}\Box\partial_\lambda B_{\mu\nu}(X)\right)\partial X^\mu\tilde\psi^\lambda\tilde\psi^\nu+\frac{1}{2}\Box\partial_\lambda B_{\mu\nu}(X)\bar\partial X^\mu\tilde\psi^\lambda\tilde\psi^\nu-\frac{1}{2}(\partial_\lambda\partial^\mu B_{\nu\mu}(X)-\partial_\nu\partial^\mu B_{\lambda\mu}(X))\partial\psi^\lambda\psi^\nu\\
&-\partial_\rho\partial_\lambda B_{\mu\nu}(X)\psi^\lambda\psi^\mu\tilde\psi^\rho\tilde\psi^\nu-\frac{1}{2}(\partial_\lambda\partial^\mu B_{\mu\nu}(X)-\partial_\nu\partial^\mu B_{\mu\lambda}(X))\bar\partial\tilde\psi^\lambda\tilde\psi^\nu
\end{aligned}\tag{A3}
$$

and

$$
\begin{aligned}
\delta T_F=&\left(\frac{1}{2}B_{\mu\nu}(X)-\frac{1}{2}\Box B_{\mu\nu}(X)-\frac{1}{2}\partial_\nu\partial^\lambda B_{\lambda\mu}(X)\right)\bar\partial X^\mu\psi^\nu-\left(\frac{1}{2}\Box B_{\mu\nu}(X)-\frac{1}{2}\partial_\mu\partial^\lambda B_{\lambda\nu}(X)\right)\partial X^\nu\psi^\mu-\frac{1}{2}\partial^\mu B_{\nu\mu}(X)\partial\psi^\nu\\
&-\frac{1}{2}\partial_\lambda B_{\mu\nu}(X)\psi^\mu\tilde\psi^\nu\tilde\psi^\lambda.
\end{aligned}\tag{A4}
$$

One can check that the deformations δT and δT_F satisfy the super-Virasoro algebra provided

$$\Box B_{\mu\nu}-\partial_\mu\partial^\lambda B_{\lambda\nu}-\partial_\nu\partial^\lambda B_{\mu\lambda}=0.\tag{A5}$$

This is nothing but the gauge-covariant equation of motion for $B_{\mu\nu}$. Imposing the gauge condition $\partial^\mu B_{\mu\nu}=0$, we recover the gauge-fixed expressions discussed in the previous sections.

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