Active-sterile neutrino oscillations in the early Universe: Asymmetry generation at low $|\delta m^2|$ and the Landau-Zener approximation

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It is well established that active-sterile neutrino oscillations generate large neutrino asymmetries for very small mixing angles ($\sin^2 2\theta_0 \lesssim 10^{-4}$), negative values of δm^2, provided that $|\delta m^2| \gtrsim 10^{-4}$ eV^2. By numerically solving the quantum kinetic equations, we show that the generation still occurs at much lower values of $|\delta m^2|$. We also describe the borders of the generation at small mixing angles and show how our numerical results can be analytically understood within the framework of the Landau-Zener approximation thereby extending previous work based on the adiabatic limit. This approximate approach leads to a fair description of the MSW dominated regime of the neutrino asymmetry evolution and is also able to correctly reproduce its final value. We also briefly discuss the impact that neutrino asymmetry generation could have on big bang nucleosynthesis, CMBR and relic neutrinos.

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I. INTRODUCTION

Previous work has established that $\nu_\alpha - \nu_{\rm sterile}$ ($\alpha = e,\mu,\tau$) neutrino oscillations typically generate large [$\mathcal{O}(0.1)$] neutrino asymmetries for *negative values* of δm^2 [1] and *small mixing angles* ($\sin^2 2\theta_0 \ll 1$) [1–4]. Such a large value for the neutrino asymmetry can have several interesting consequences and applications including a suppression of the sterile neutrino production [5,2,6], a modification of the standard big bang nucleosynthesis (BBN) predictions in different ways [2,7,3,8–10] [2] and, perhaps not emphasized enough, it implies a relic α neutrino background in which mainly neutrinos or antineutrinos (according to the sign of the generated asymmetry) are present.

We define the *asymmetry* of the particle species X as *the net number of X* particles at temperature T per number of photons at some fixed initial temperature $T_{\rm in}$, such that $m_\mu \gg T_{\rm in} \gg m_{\rm el}$, in a comoving volume R^3:

$$L_X \equiv \frac{N_X - N_{\bar{X}}}{N_\gamma^{\rm in}}, \tag{1}$$

where $N_X \equiv n_X R^3$ is the number of X-particles in R^3, while n_x is the X-particle density.[3]

The exact *borders* of the generation in the space of mixing parameters have not yet been fully determined. Using the so called "static approximation" in [2] it was found that, to have a final neutrino asymmetry larger than 10^{-5}, the following rough *lower limit on the mixing angle* (for a fixed δm^2) holds:

$$\sin^2 2\theta_0 \gtrsim 6(5) \times 10^{-10} \left(\frac{|\delta m^2|}{\rm eV^2} \right)^{-1/6}, \quad \alpha = e(\mu,\tau). \tag{4}$$

An *upper limit on the mixing angle* for which the neutrino generation occurs is still unknown as numerical solutions are made quite difficult (i.e. CPU time consuming) by the presence of a sterile neutrino production prior the onset of neutrino asymmetry generation that is strongly coupled to the neutrino asymmetry generation. Moreover, at large angles ($\sin^2 2\theta_0 \gtrsim 10^{-6}$), the phenomenon of rapid oscillations of the asymmetry at the onset of the generation appears [12],

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[1] We define $\delta m^2 \equiv m_2^2 - m_1^2$, with m_1 and m_2 eigenvalues of the mass eigenstates such that in the limit of $\theta_0 \to 0$ they coincide with the α and sterile weak eigenstates respectively. With this definition in mind, one can say that *negative values of δm^2 imply a sterile neutrino lighter than the α*.

[2] A particular interesting case is the possibility that a neutrino asymmetry generation can solve the claimed discrepancy between the value of baryon density from standard BBN with that one inferred from recent cosmic microwave background radiation (CMBR) anisotropies observations [11].

[3] If one defines

$$T_\nu \equiv T_{\rm in} \frac{R_{\rm in}}{R} \tag{2}$$

it is simple to check that the asymmetry L_X is connected to the *asymmetry abundance* (relative to photons) $\eta_X \equiv (n_X - n_{\bar{X}})/n_\gamma$ by the simple relation

$$\eta_X = h\, L_X, \tag{3}$$

where $h \equiv (T_\nu/T)^3 \le 1$ is the *dilution factor* that takes into account the photon production during the electron-positron annihilations around $m_e/2 \simeq 0.25$ MeV.

making things even more complicated.[4] Requiring that the final effective number of extra-neutrino species (in energy density) does not violate the BBN bound, $\Delta N_\nu^\rho < \Delta N_\nu^{\max} < 1$, and with the definition $f(\Delta N_\nu^{\max}) \equiv \ln(1-\Delta N_\nu^{\max})/\ln(0.4)$, one finds, for $\alpha = e(\mu,\tau)$, the constraint [2,6]

$$\sin^2 2\theta_0 \lesssim 2.1(4.1)\times 10^{-5}\left(\frac{\mathrm{eV}^2}{|\delta m^2|}\right)^{1/2} f^2(\Delta N_\nu^{\max}), \quad (5)$$

valid for $|\delta m^2| \gtrsim 10^{-4}$ eV^2 (with $\delta m^2 < 0$) and $0.9 \gtrsim \Delta N_\nu^{\max} \gtrsim 0.3$. This constraint is however applicable only in the case of two flavor $\nu_\alpha - \nu_s$ (with $\alpha = \mu,\tau$) neutrino oscillations. In the general case of multiflavor oscillations the neutrino asymmetry produced by the oscillations with the largest $|\delta m^2|$ can suppress the sterile neutrino production from the other oscillations [5,2,6]. Therefore Eq. (5) can only possibly apply for oscillations with the largest $|\delta m^2|$, with the constraints on other two flavor oscillations being much weaker. Also, if $\alpha = e$, or if the muon or tauon neutrino is also mixed to the electron neutrino then an electron neutrino asymmetry can be generated, the constraint $\Delta N_\nu^\rho < \Delta N_\nu^{\max}$ has to be replaced by a more general constraint $\Delta N_\nu^\rho + \Delta N_\nu^{f_{\nu_e}} < \Delta N_\nu^{\max}$, in which the quantity $\Delta N_\nu^{f_{\nu_e}}$ takes into account the BBN effect of the deviation of the electron neutrino distribution from the standard case, in which a Fermi-Dirac distribution with zero chemical potential is assumed.[5] If a positive electron neutrino asymmetry is generated before the freezing of the neutron to proton abundance ratio, then $\Delta N_\nu^{f_{\nu_e}}$ is negative [1,7,3,9,11]. In this case the upper limit on the mixing angle can be strongly relaxed and is related to the limit for the neutrino asymmetry generation, that however, as we said, has not been determined. Moreover one has conservatively to keep in mind that the current observational situation on the values of primordial nuclear abundances cannot strictly exclude a value $\Delta N_\nu^{\max} \gtrsim 1$, for which no BBN constraints on active-sterile neutrino oscillations would apply at all if $\Delta N_\nu^{f_{\nu_e}} = 0$.[6] It has also to be mentioned that the region of the mixing parameter space in which a positive electron neutrino asymmetry is generated such that $\Delta N_\nu^{f_{\nu_e}}$ is negative and with a mixing angle large enough that $\Delta N_\nu^\rho \sim 1$, would be very interesting as in this case the standard BBN would be modified such that the primordial nuclear abundances predictions would still agree with the measured values and at the same time a higher value for the baryon density would be allowed, in agreement with that one inferred from CMBR acoustic peaks [14,11].

For

$$|\delta m^2| \gtrsim 10^{-4} \ \ \mathrm{eV}^2, \quad (6)$$

the numerical calculations performed with the quantum kinetic equations (QKE) [15] found that the final value of the neutrino asymmetry is in the range 0.23–0.35 approximately independently of $\sin 2\theta_0$ for most of the range of values set by Eqs. (4),(5) [3]. These calculations also showed that the final values can be well reproduced within the *adiabatic approximation*. Let us shortly recall the main features of the neutrino asymmetry generation effect and of what the adiabatic approximation consists.[7] Neutrino asymmetry evolution in the region of mixing parameters set by Eqs. (4), (5) and (6), is characterized by the presence of a *critical temperature* T_c that is approximately given by the following expression:

$$T_c \simeq T^0_{\mathrm{res}}(y_c) \equiv 15.0(18.6)\left(\frac{|\delta m^2|}{\mathrm{eV}^2}\right)^{1/6}\left(\frac{2}{y_c}\right)^{1/3} \mathrm{MeV}, \quad (7)$$

where $y_c \equiv p_c/T$ is the *critical (dimensionless) resonant momentum* and its value is $\simeq 2$.[8] The *total effective asymmetry* $L^{(\alpha)}$ is defined as[9]

$$L^{(\alpha)} \equiv 2\, L_{\nu_\alpha} + \tilde{L}^{(\alpha)} \quad (8)$$

where $\tilde{L}$ contains the asymmetry of the other neutrino flavors $(\beta \neq \alpha)$ and also a contribution coming from electrons and baryons. We assume that $\tilde{L}$ is of the same order of magnitude as the baryon asymmetry $\sim 5\times 10^{-10}$, a value that we adopted in our calculations.[10] The evolution of the total asymmetry experiences different stages. At high $T \gtrsim T_c$, oscillations drive $|L| \ll |\tilde{L}|$. When T approaches T_c a decrease of $|L|$ to arbitrarily small values is prevented by the presence of chemical potentials in neutrino equilibrium distributions and $|L|$ starts to grow slowly. At $T = T_c$, by definition, L

[4]Actually the appearance of neutrino asymmetry sign oscillations, in the region where the sterile neutrino production is not negligible, may not be a coincidence. It is likely that the strong coupling between a copious sterile neutrino production and the neutrino asymmetry evolution is responsible for the rapid sign changes [13].

[5]A more detailed discussion on the definition of ΔN_ν^ρ and $\Delta N_\nu^{f_{\nu_e}}$ can be found in [11].

[6]More generally no BBN constraint applies if $\Delta N_\nu^{\max} \gtrsim 1 + \max(\Delta N_\nu^{f_{\nu_e}})$.

[7]See also [16] for a more detailed review.

[8]This is true until the constraint (5) on the mixing angle is adopted, otherwise y_c can grow to much higher values [2,6] as an effect of sterile neutrino production. The asymmetry generation should go off when y_c becomes so high to be in the tail of the distribution ($y_c > y_c^{\mathrm{off}} \gtrsim 10$), but this has not yet been studied by numerical calculations and the exact value of y_c^{off} has not been determined.

[9]When not necessary we will drop the subscript in $L^{(\alpha)}, \tilde{L}^{(\alpha)}$ and just indicate them with $L, \tilde{L}$.

[10]In any case the final value of L is not dependent on a particular choice for $\tilde{L}$. Moreover, note that in the realistic case of multiflavor oscillations a mixing $\nu_{\beta\neq\alpha} \leftrightarrow \nu_s$ could create an $|L_{\nu_\beta}| \gtrsim 5\times 10^{-10}$ prior the occurrence of $\nu_\alpha \leftrightarrow \nu_s$ oscillations. Thus, in the case of multiflavor mixing, our assumption is valid if either the $\nu_\alpha \leftrightarrow \nu_s$ oscillations have the largest $|\delta m^2|$ or the oscillations $\nu_\beta \leftrightarrow \nu_s$ do not have the right parameters (negative δm^2, large enough mixing angle, ...) to create a large $|L_{\nu_\beta}|$.

$=\tilde{L}$ and $L_{\nu_\alpha}=0$.[11] Until this momentum, the resonant momentum $y_{\rm res}=p_{\rm res}/T$ of neutrinos and antineutrinos is practically the same and evolving as $\propto T^{-3}$. At $T\lesssim T_c$ the asymmetry growth becomes exponential and the neutrino and antineutrino resonances quickly become separated: for $L>0$ the antineutrino resonant momentum gets smaller than the critical momentum y_c, while the neutrino resonant momentum increases rapidly moving into the tail of the distribution where the number of neutrinos becomes negligibly small (and vice-versa for negative L). When $|L^{(\alpha)}|$ reaches a threshold value $L_t^{(\alpha)}\simeq 1.0(0.4)\times 10^{-6}(y_c|\delta m^2|/\mathrm{eV}^2)^{1/3}$ for $\alpha=e,(\mu,\tau)$, nonlinear effects of the medium start to act and the exponential growth turns into a power law (with $L\propto T^{-4}$ initially), while (for L positive) anti-neutrino resonant momentum gets a minimum value $y_{\rm res}^{\rm min}$ and then starts to grow again. A relation between $y_{\rm res}$ and L can be obtained from the resonance condition and is given by the following approximate expression:

$$y_{\rm res}\simeq \frac{|\delta m^2|/\mathrm{eV}^2}{a_0 L (T/\mathrm{MeV})^4}, \tag{9}$$

with $a_0\equiv(4\sqrt{2}\zeta(3)/\pi^2)\times 10^{12} G_F\ \mathrm{MeV}^2\simeq 8$. When $y_{\rm res}\sim 10$ the asymmetry stops its growth and gets frozen to its final value $L_{\rm f}\sim 0.3$. It is simple, from the Eq. (9), to see that this happens at a temperature given by

$$T_f\approx 0.5\ \mathrm{MeV}\left(\frac{|\delta m^2|}{\mathrm{eV}^2}\right)^{1/4}. \tag{10}$$

It is quite interesting that this complicated behavior of the neutrino evolution can be regarded as the effect of two different physical regimes. The relevant quantity is the ratio $r\equiv(l_{\rm int}/l_{\rm osc})_{\rm res}$ between the interaction length and the oscillation length for the resonant neutrinos. If $r\lesssim 1$ collisions are able to inhibit the Mikheyev-Smirnov-Wolfenstein (MSW) effect and the regime is *collision dominated* and can be described by a simplified set of kinetic equations obtainable within the static approximation procedure [2,17]. On the other hand if $r\gtrsim 1$, the MSW effect can take place and the regime is MSW *dominated*. In [16] it has been clarified that the transition between the two regimes depends on the value of the asymmetry and not just on the temperature. It is in fact possible to show that the following asymptotic expressions for r holds:

$$r\simeq\frac{\sin 2\theta_0}{0.8(2.0)\times 10^{-2}}(|L|\ll L_t) \tag{11}$$

$$r\simeq\frac{\sin 2\theta_0}{0.8(2.0)\times 10^{-2}}\left(\frac{|L|}{L_t}\right)^{3/2}(|L|\gg L_t). \tag{12}$$

First of all note that a collisional regime can exist only if $\sin 2\theta_0\lesssim 10^{-2}$. Thus $\sin 2\theta_0\simeq 10^{-2}$ is a border between two very different regions of mixing parameters. In this paper we will only be interested to the case of *very small mixing angles* ($\sin 2\theta_0\ll 10^{-2}$) in which collisions are important at low values of the asymmetry ($|L|<L_t$), independently on the temperature. In this case only when $|L|\gtrsim(10^{-2}/\sin 2\theta_0)^{2/3}L_t$ the MSW regime can begin. It is then crucial that, during the collisional regime, a fast growth of the asymmetry occurs for the MSW effect to start to be effective and for the resonance of neutrinos and antineutrinos to become separated. If in fact now one assumes (for $L>0$) that the expansion is slow enough that the resonant momentum of antineutrinos, after the minimum, increases so slowly that antineutrinos adiabatically cross the resonance, then all α-antineutrinos are converted into antisterile neutrinos. This easily explains why the final values of the α-neutrino asymmetry are in a range of values slightly lower than the value $N_{\bar{\nu}_\alpha}(T_{\rm in})/N_\gamma(T_{\rm in})=0.375$, corresponding to an extreme situation in which all anti-neutrinos are converted. This simple picture is the *adiabatic approximation* first described in [3].

As we said, the numerical solution of the QKE's have shown that for $|\delta m^2|\gg 10^{-4}\ \mathrm{eV}^2$ and for the mixing angle within the limits (4) and (5) (without however a systematic investigation at the borders) this physical approximated two-regime picture works very well in the description of the neutrino asymmetry evolution and of its final value.[12]

What about for $|\delta m^2|<10^{-4}\ \mathrm{eV}^2$? Does neutrino asymmetry generation still occur? Does the static-adiabatic approximation picture still work?

The authors of Ref. [18], with an analytical procedure based on the mean momentum approximation,[13] were finding that, even though an exponential growth occurs for $|\delta m^2|\gtrsim 10^{-7}\ \mathrm{eV}^2$, soon the nonlinear term in the effective potential is able stop the growth and concluded that no neutrino asymmetry (larger than 10^{-7}) can be generated in any point of the parameter space. This result seemed to be confirmed by numerical results [19]. These were obtained also employing the mean momentum approximation for $\delta m^2=-10^{-5}\ \mathrm{eV}^2$ and $\theta_0=0.1$. At some critical temperature an oscillatory behavior of the neutrino asymmetry, with an amplitude exponentially growing, was found, but the amplitude was not growing higher than 10^{-7}, although this may simply be because the evolution was stopped at $T\simeq 0.3\ \mathrm{MeV}$, below the freezing temperature of neutron to proton abundance ratio or simply an artifact of not including the momentum

[11]Within the static approximation a definition of T_c is more rigorous [1,2]. This is a sort of numerical definition.

[12]Excluding the phenomenon of rapid oscillations at the onset of the exponential growth for $\sin^2 2\theta_0\gtrsim 10^{-6}$ [12] that requires some further physical picture still not completely understood.

[13]This approximation works very well in the description of the sterile neutrino production [24] for reasons explained in [6]. However in the case of the neutrino asymmetry generation, especially in the MSW dominated regime, the momentum dependence is such a crucial feature that the mean momentum approximation is a very drastic one and in our opinion it should be regarded more as a toy model.

degree of freedom. In any case the authors of Ref. [19] concluded that neutrino oscillations cannot generate a large neutrino asymmetry in the early Universe, in apparent agreement with [18]. In [1,2,7,3] it was shown that a large neutrino asymmetry can be generated due to the crucial role of the collisions, thus disproving the general conclusions of [18,19], although this result is still not necessarily in disagreement with the conclusions of [18,19] for $|\delta m^2|\lesssim 10^{-4}$ and this point has still remained unclear.

It has also to be said that the main attention was put at higher values of $|\delta m^2|$ because only in this case the asymmetry generation occurs early enough to modify the standard BBN predictions for the nuclear abundances. Moreover, there are also qualitative reasons to suspect that at low $|\delta m^2|$ the static approximation is not valid. In [1,2,8] it has been in fact pointed out that for $|\delta m^2|\lesssim 10^{-4}$ eV^2, oscillations between collisions can give a nonnegligible contribution to the rate of the neutrino asymmetry, suggesting a possible deviation from the static approximation. This statement is justified observing that, if one requires that the change of the mixing angle in matter is negligible between collisions on average,[14] then the critical temperature has to be higher than approximately 3 MeV and from the Eq. (7) this implies just $|\delta m^2|\gtrsim 10^{-4}$ eV^2. On the other hand we already noted that the role of collisions in inhibiting the MSW effect, for low values of the asymmetry, is effective at all temperatures (for $\sin 2\theta_0 \lesssim 10^{-2}$).

Which kind of deviations from the static approximation are then expected at small $|\delta m^2|$ and, above all, does the asymmetry generation still occur? Are oscillations in the neutrino asymmetry expected to appear as the mean momentum calculations of [19] suggest?

The purpose of this paper is to answer these questions, mainly understanding the exact border of neutrino asymmetry generation at low values of $|\delta m^2|$. We will however also give some results concerning the presence of neutrino asymmetry oscillations. Moreover we will study the border of the generation at small mixing angles using the QKE's and show that the limit (4), derived in the static approximation, has actually to be replaced with a less stringent one (Sec. II). We will show that the small mixing angle border can be explained within an analysis on the limits of validity of the adiabaticity approximation in the description of the MSW dominated regime. This analysis will also involve extending the adiabatic approximation in the border region and outside by means of the Landau-Zener approximation, that is able to describe how the neutrino asymmetry generation dies at small mixing angles (Sec. III). In Sec. IV we will study the cosmological consequences of the neutrino asymmetry generation in the light of the new results. If neutrino asymmetry generation occurs also at low values of $|\delta m^2|$, even though this implies that it is generated at temperatures too low to produce remarkable changes in standard BBN, one can still consider which kind of cosmological effects (and maybe observations) can result from it. In particular we will correct some wrong claims, which have appeared recently in the literature, on the role that the neutrino asymmetry generated in active-sterile neutrino oscillations would have on CMBR. We will consider the implications on the possibilities of a detection of relic neutrino background by the light of recent studies [20]. In Sec. V we will draw the conclusions of this work.

[14]This is equivalent to imposing that the derivative of the effective potential, with negligible neutrino asymmetry, is much less than the total interaction rate.

II. NUMERICAL RESULTS

Set of equations and numerical procedure

We have numerically solved the QKE's for the density matrix distribution in the momentum space, describing the statistical properties of the ν_α,ν_s states, including their mixing. Since we are interested in extending previous studies to values of $|\delta m^2|/\mathrm{eV}^2 < 10^{-4}$ and since from Eq. (10), obtained within the adiabatic approximation, one would have that the neutrino asymmetry is generated down to temperatures below $T\sim 0.05$ MeV, we have to take into account the occurrence of electron-positron annihilations that will make $T_\nu \neq T$. The first modification is that the re-scaled momentum $p\,R/R_{\rm in} = p\,T_{\rm in}/T_\nu$ and thus now all neutrino distributions are more conveniently expressed in terms of $y\equiv p/T_\nu$.

The QKE's describing neutrino propagation are given by the following set of equations for 8 distributions in momentum space [15], $\vec{P}(y,t), P_0(y,t), \bar{\vec{P}}(y,t), \bar{P}_0(y,t)$ (for notational simplicity we drop the dependence on momentum and on time):

$$\frac{dP_x}{dt} = -\lambda\, P_y - D\, P_x \tag{13}$$

$$\frac{dP_y}{dt} = \lambda\, P_x - \beta\, P_z - D\, P_y \tag{14}$$

$$\frac{dP_z}{dt} = \beta\, P_y + R \tag{15}$$

$$\frac{dP_0}{dt} = R. \tag{16}$$

The function $R(y,t)$ takes into account the *(thermal) redistribution* of neutrinos at each quantum state due to the effect of collisions, that in a linear approximation is given by [4,17]

$$R \simeq \Gamma\left[\frac{f_\xi^{\rm eq}}{f_0} - \frac{1}{2}(P_0 + P_z)\right] \tag{17}$$

where $f_\xi^{\rm eq}(y,t) \equiv [1+e^{y-\xi(t)}]^{-1}$ is the Fermi-Dirac distribution, $\xi(t)$ is the *(dimensionless) chemical potential* and $f_0 \equiv f^{\rm eq}_{\xi=0}$. The variables P_0 and P_z can be replaced with the variables $z_s = (P_0 - P_z)/2$ and $z_\alpha = (P_0 + P_z)/2$. These variables are the diagonal entries of the density matrix and are related to the α and sterile neutrino distributions simply by $z_{\alpha,s} \equiv f_{\nu_\alpha,s}/f_0$. In this case Eqs. (15) and (16) are replaced by the following ones:

$$\frac{dz_s}{dt} = -\frac{1}{2}\beta P_y \tag{18}$$

$$\frac{dz_\alpha}{dt} = -\frac{dz_s}{dt} + R \tag{19}$$

while the repopulation function R now can be recasted as $R = \Gamma(z_\alpha^{\rm eq} - z_\alpha)$.

The anti-neutrinos barred distributions $\bar{\vec{P}}, \bar{P}_0$ obey the same equations with the same *total collision rate* Γ, the same *decoherence parameter D*, and the same function β.[15] The function λ, containing the effective potential of neutrinos, depends explicitly also on L and for anti-neutrinos it changes such that $\bar{\gamma}(L) = \gamma(-L)$. Thus the presence of a *total asymmetry L* couples together the neutrino and antineutrino sets of equations. They are coupled also by the presence of the chemical potentials in the equilibrium active neutrino distributions. These are in fact different if a *neutrino asymmetry* is present, considering that

$$L_{\nu_\alpha} = \frac{1}{4\zeta(3)} \int_0^\infty dy\, y^2 \left[\frac{1}{1+e^{y-\xi}} - \frac{1}{1+e^{y-\bar{\xi}}} \right]. \tag{20}$$

Note that if $\tilde{L} = 0$, then one would have simply $L = 2\, L_{\nu_\alpha}$ and $L = L_{\nu_\alpha} = 0$ would be an exact solution of the set of equations. Above the critical temperature this would be a stable solution and L would get so small that the statistical fluctuations on very small scales would determine the sign of the asymmetry below the critical temperature when the solution becomes unstable. This sign would be different on distances as small as the scale of the dominant statistical fluctuations that is smaller than the interaction length [12]. On these scales neutrino free-steam and the free-streaming is faster than the neutrino asymmetry generation such that immediately regions with different sign would destroy each other and no asymmetry generation could take place. This picture is consistent with the idea that the generation of a neutrino asymmetry can occur only if CP is violated at some level, in this case due to the presence of a CP asymmetric medium ($\tilde{L} \neq 0$) that can push the neutrino asymmetry toward some direction in any point of the space and strongly enough to dominate on statistical fluctuations. This surely happens in a large region of mixing parameters where the presence of rapid oscillations can be excluded [12].

Note also that the integral equation (20) is coupled to the set of differential equations for the $\vec{P}, P_0, \bar{\vec{P}}, \bar{P}_0$, as the value of the neutrino asymmetry appears in the expression for λ. However, as already discussed in previous works [2,3,9,12],

[15]We neglect a small term, proportional to the asymmetry L, that makes the collision rate Γ for neutrinos and $\bar{\Gamma}$ for anti-neutrinos different [8]. Within the static approximation this term proves not to give any difference. It has to be investigated whether in the region of rapid oscillations this term can play or not any role.

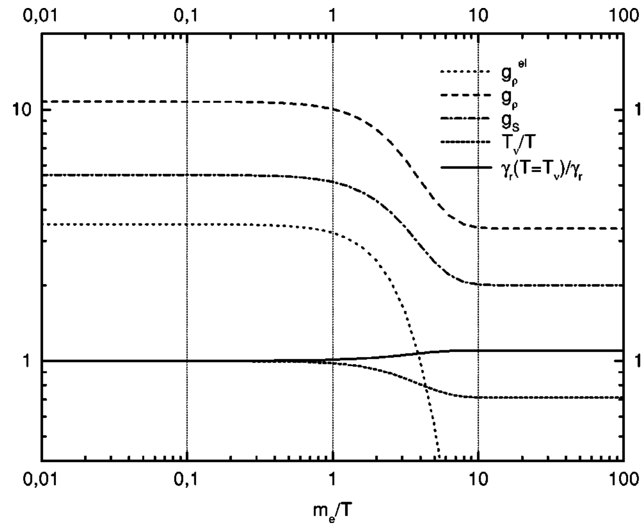

FIG. 1. Effect of electron-positron annihilations on the degrees of freedom, on neutrino temperature and on the adiabaticity parameter.

it is numerically more convenient to calculate the neutrino asymmetry not from Eq. (20) but through the following integro-differential equation:

$$\frac{dL_{\nu_\alpha}}{dt} = \frac{1}{8\zeta(3)} \int \beta(y)[P_y(y) - \bar{P}_y(y)] f_0(y) y^2 dy. \tag{21}$$

The set of equations (13)–(17),(21) is still not ''closed'' as one still needs some equations describing the evolution of the chemical potentials ξ and $\bar{\xi}$ in the equilibrium active neutrino distributions. We use the instantaneous chemical decoupling procedure developed in Ref. [8] with kinetic decoupling temperatures, $T_{\rm dec}^\alpha = 3.5$ MeV for $\alpha = \mu, \tau$ and 2.5 MeV for $\alpha = e$. The approximate validity of this procedure has been checked in Ref. [9].

The last step is to replace the time with temperature. It is well known that the neutrino temperature T_ν can be expressed as a function of the photon temperature T, simply imposing entropy conservation during electron-positron annihilations:

$$T_\nu = T \left[\frac{g_S(m_e/T)}{g_S(0)} \right]^{1/3} \tag{22}$$

with the effective number of (entropy) degrees of freedom of photons and electron-positrons given by

$$g_S(x) = 2 + \frac{45}{\pi^4} \int_0^\infty dy\, y^2 \frac{\sqrt{y^2+x^2} + \frac{1}{3}\frac{y^2}{\sqrt{y^2+x^2}}}{e^{\sqrt{y^2+x^2}} + 1} \tag{23}$$

and $g_S(0) = 11/2$ (in Fig. 1 both g_s and T_ν/T are shown for illustrative purposes). One can choose either T or T_ν as the independent variable for the numerical calculations (we used both choices in two different codes). For our discussion it is simpler to choose T_ν and thus we have to express all quantities as a function of T_ν. The quantity β can be written as

$$\beta(y,T_\nu)=\frac{\delta m^2}{2\,y\,T_\nu}\sin 2\theta_0. \tag{24}$$

Since we treat neutrinos as fully decoupled during the period of electron-positron annihilations, interactions will be effective only until $T\simeq T_\nu$ and thus we can simply write

$$\Gamma(y,T_\nu)\simeq 1.27(0.92)G_F^2T_\nu^5 y, \quad \alpha=e(\mu,\tau). \tag{25}$$

The quantity λ can be written as

$$\lambda(y,T_\nu,L)=-\frac{\delta m^2}{2\,y\,T_\nu}[\cos 2\theta_0-b(y,T_\nu)\pm a(y,T_\nu,L)] \tag{26}$$

where the $+$ $(-)$ sign holds for neutrinos (anti-neutrinos). The term $b(y,T_\nu)$ is the finite temperature contribution to the (dimensionless) effective potential in the early Universe [21]. This term is important when $|L|\ll L_t$, but it becomes negligible after the exponential growth, when $|L|\gg L_t$, for $T\lesssim T_c$. We will mainly consider $T_c\gtrsim 0.25$ MeV for which electron-positron number densities are still approximately equal to the ultra-relativistic limit. Thus we can just replace T with T_ν and write[16]

$$b(y,T_\nu)=-\frac{\mathrm{eV}^2}{\delta m^2}\left(\frac{T_\nu}{T_\alpha}\right)^6 y^2 \tag{27}$$

where $T_\alpha\simeq 18.9(23.4)$ MeV for $\alpha=e(\mu,\tau)$. The term a is proportional to the total asymmetry L. Electron-positron annihilations do not change electron asymmetry in $\tilde{L}$ and as usually is done, we also neglect a small change in $\tilde{L}$ arising from neutron-proton conversions. Thus we can write the term a in the usual way, where now however T has to be replaced with T_ν because of the definition (1) of particle asymmetry that we are using:

$$a\simeq -a_0\frac{\mathrm{eV}^2}{\delta m^2}L\,T_\nu^4 y. \tag{28}$$

So far, the electron positron-annihilations have not caused any change in the equations when these are written in T_ν. This because we could approximately assume that α- neutrinos, during the electron positron annihilations, are not sensitive to all other particle species, either because they do not interact with them any more or because the contribution of the other particle species in the effective potentials can be neglected when $T\lesssim 0.25$ MeV and $|L|\gg L_t$. There is however an indirect effect due to the expansion rate. In replacing the time t with the neutrino temperature T_ν, one needs to calculate the quantity

$$\frac{dT_\nu}{dt}\simeq -\frac{1.66\sqrt{g_\rho}T_\nu^3}{M_{\rm Pl}}\left(\frac{T}{T_\nu}\right)^2, \tag{29}$$

where $M_{\rm Pl}$ is the Planck mass and the number of (energy density) degrees of freedom is given by[17]

$$g_\rho(T_\nu)=2+\frac{21}{4}\left(\frac{T_\nu}{T}\right)^4+\frac{60}{\pi^4}\int_0^\infty dy\,\frac{y^2\sqrt{y^2+x^2}}{1+e^{\sqrt{y^2+x^2}}} \tag{30}$$

with $x\equiv m_e/T$ and where T has to be regarded as a function of T_ν (in Fig. 1 the function g_ρ is also plotted). Thus electron-positron annihilations cause a change in the expansion rate. In Sec. IV we will obtain physical insight into this effect.

In Figs. 2–4 we show the evolution of the absolute value of the total asymmetry with the temperature T. In Fig. 2 we show solutions for many different values of $-\delta m^2$ but for a fixed value of the mixing angle $\sin^2 2\theta_0=10^{-8}$ and for $\alpha=e$. The solid lines are the solutions obtained solving the set of QKE equations. The dotted lines are curves obtained integrating the QKE's only until $r<3$ [see the Eq. (12)]. From that moment we started to use the *adiabatic approximation* as described in [3,9]. It can be seen how the asymmetry generation occurs at values of $-\delta m^2$ much below 10^{-4} eV^2

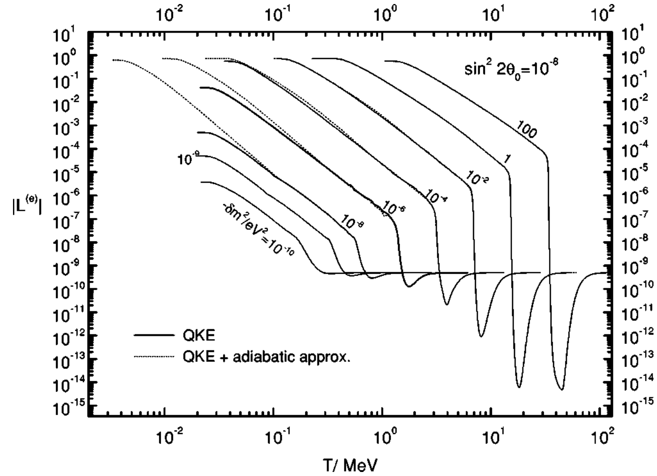

FIG. 2. Evolution of $L^{(e)}=2L_{\nu_e}+\tilde{L}$ with (photon) temperature for $\sin^2 2\theta_0=10^{-8}$ and values of $-\delta m^2$ as indicated. The solid lines are the QKE's solutions while the dotted lines employ an adiabatic description for the growth in the MSW dominated regime. The initial value is $L^{(e)}=\tilde{L}=5\times 10^{-10}$.

[16] Actually we will even consider values of $-\delta m^2/\mathrm{eV}^2\lesssim 10^{-11}$ for which $T_c\lesssim 0.25$ MeV. In this case one should replace the constant T_α with a function of temperature taking into account that electron and positrons number densities decrease during annihilations. We neglected this modification and used the standard Notzold-Raffelt expression for the finite temperature term.

[17] We neglect any contribution to g_ρ from the sterile neutrino production or from the modification of α neutrino distribution, due to the generation of the asymmetry. This because we are primarily interested in small values of mixing angles satisfying the Eq. (5) and to values of $|\delta m^2|/\mathrm{eV}^2\ll 100$ for which most of the neutrino asymmetry is generated below $T_{\rm cdec}^\alpha$ and thus $\Delta N_\nu^\rho\simeq 0$.

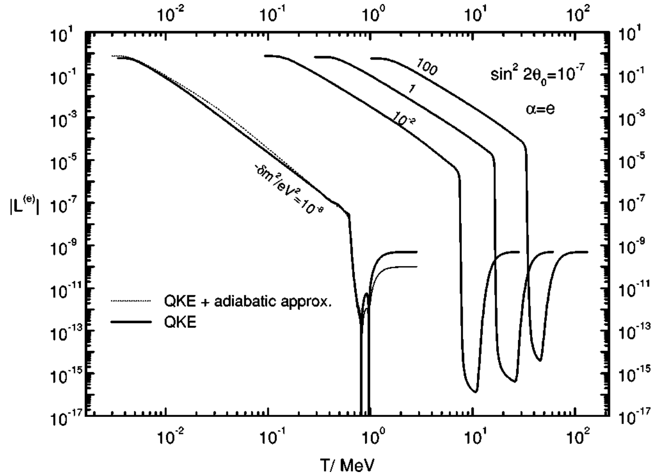

FIG. 3. The same as in Fig. 2 but for $\sin^2 2\theta_0=10^{-7}$ and a different choice of values for $-\delta m^2$. For $-\delta m^2/\mathrm{eV}^2=10^{-8}$ we also show an example for a different choice of the initial asymmetry.

and goes off only around $-\delta m^2 \simeq 10^{-10}$ eV^2. In particular for $-\delta m^2=10^{-10}$ eV^2 the final value of neutrino asymmetry is still at the level of 10^{-5}.

The numerical convergence of the solutions has been obtained integrating around the resonance [9,12] but the interval of the integration highly increases for low $|\delta m^2|$ and covers up to two orders of magnitude in momentum space. The behavior at low values of the asymmetry is well reproduced by the static approximation at any value of $|\delta m^2|$. In particular note that no sign changes are present.

In Fig. 3 solutions for $\sin^2 2\theta_0=10^{-7}$, $-\delta m^2/\mathrm{eV}^2 =100,1,10^{-2},10^{-8}$ and $\alpha=e$ are shown. While for the first two cases there are no sign changes, which was already known [12], in the case $-\delta m^2=10^{-8}$ two sign changes appear. Note that, changing the initial neutrino asymmetry, the solutions converge to the same values at some stage. The final sign is still the same of $\tilde{L}$, like the static approximation predicts. For the numerical convergence of this solution a momentum integration over an interval of 5 orders of magnitude has been necessary, much larger than for $\sin^2 2\theta_0 =10^{-8}$. In Fig. 4 another example is shown with the mixing parameters $\sin^2 2\theta_0=10^{-7}$, $\delta m^2/\mathrm{eV}^2=-10^{-7}$ for the $\alpha =\mu$ case. In this case three sign changes are present and the final sign is therefore opposite to that of $\tilde{L}$. We tested this solution with both codes, obtaining the same behavior. Thus we can say that the numerical calculations suggest the appearance of *slow oscillations* in the neutrino asymmetry. These seem to have a different nature than the rapid oscillations observed in [12] for $\sin^2 2\theta_0 \gtrsim 10^{-6}$ and $10^{-2} \lesssim |\delta m^2|/\mathrm{eV}^2 \lesssim 500$. They are "slower" and with just a few sign changes and do not occur during the exponential growth but before, above the critical temperature. Moreover, even though the integration has to be very accurate, a numerical convergence can be obtained and thus any character of chaoticity seems to be excluded. This is also supported by the fact that starting with two slightly different initial conditions the final sign is the same. We can thus exclude the possibility that statistical fluctuations, estimated in [12], can influence the solutions. However, it would be desirable to have some analytical insight supporting the presence of sign changes found in the numerical results. In any case, if confirmed, they would suggest a deviation from the static approximation at low $|\delta m^2|$ and sufficiently high mixing angles ($\sin^2 2\theta_0 \sim 10^{-7}$) due to the presence of sign changes that are not predicted by the static approximation. We did not investigate whether at even higher mixing angles, rapid oscillations appear, as in the case of high $-\delta m^2$. This requires a dedicated analysis beyond the goals of this work.

In Fig. 5(a) we give the final value of the asymmetry for $\alpha=e$, five values of the mixing angle ($\sin^2 2\theta_0 =10^{-7}$, 10^{-8}, 10^{-9}, 4×10^{-10}, 10^{-10}) and for $-\delta m^2$ in the range $10^{-13} < -\delta m^2/\mathrm{eV}^2 < 10^4$.[18] In Fig. 5(b) the final value of the asymmetry is shown in the case $\alpha=\mu$ and for two values of the mixing angle ($\sin^2 2\theta_0=10^{-7}$ and 10^{-8}) and one can see that there is not much difference with the case $\alpha=e$. This because in the MSW regime, when the neutrino asymmetry gets its final value, the finite temperature term in the effective potential [the b-term, see Eq. (27)] is negligible and only the Wolfenstein term is relevant [the a-term; see Eq. (28)]. This term is the same for $\alpha=e$ and $\alpha=\mu$.

For values of the mixing angles $\sin^2 2\theta_0=10^{-7}$, 10^{-8} and 10^{-9}, the final value of the neutrino asymmetry can be quite easily understood as being due to MSW transitions as the resonance momentum passes through the neutrino distribution. Previous work focused on the adiabatic limit where complete conversion occurred. The solid curves describe the results obtained solving the QKE. The dotted lines correspond to the results obtained with the adiabatic approxima-

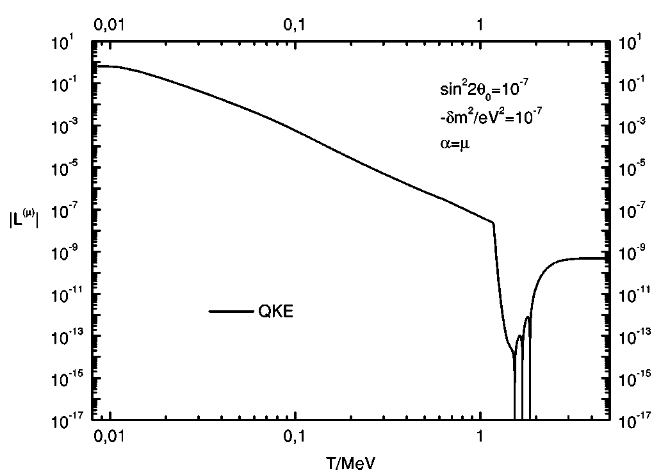

FIG. 4. An example of evolution with temperature for $L^{(\mu)}$. The mixing parameters are $\sin^2 2\theta_0=-\delta m^2/\mathrm{eV}^2=10^{-7}$.

[18]The chosen upper limit for $|\delta m^2|$ comes from the requirement to have the critical temperature $T_c \lesssim 100$ MeV considering that, for higher temperatures, the expression that we used for the effective potentials are not valid. However in the case $\alpha=e$, direct mass measurements from tritium β decay limit $-\delta m^2$ to be less than $\simeq 100$ eV^2.

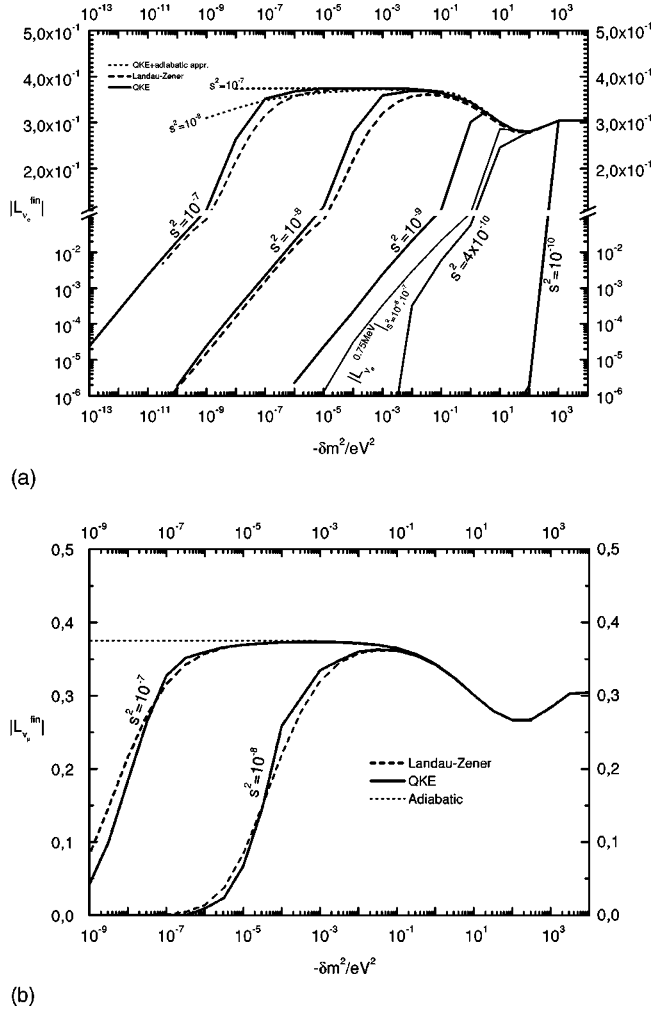

FIG. 5. Final values of the neutrino asymmetry in the case $\alpha = e$ (a) and $\alpha=\mu$ (b). Solid lines are solutions from the QKE's. Dashed lines are the solution with Landau-Zener approximation. Dotted lines are the adiabatic limit. (a) we also show with a thin solid line the value of neutrino asymmetry at $T=0.75$ MeV (for mixing angles $\gtrsim 10^{-8}$) that is approximately the freezing temperature of the neutron to proton ratio.

tion, with a slight difference between Figs. 5(a) and 5(b). In Fig. 5(a) ($\alpha=e$), the results correspond to the solutions in Figs. 2 and 3 obtained combining QKE and the adiabatic approximation as already illustrated. In Fig. 5(b) the adiabatic approximation is started from $T\sim T_c/2$ with an initial asymmetry corresponding to a resonant momentum $y_{\rm res}^{\rm min}=0.3$ [see Eq. (9)]. In the first case one gets a compromise of a fast procedure (most of the CPU time is required in the MSW dominated regime) but still able to reproduce accurately the correct neutrino asymmetry evolution at any temperature. In the second case one has a super-fast procedure able to reproduce accurately the final stages ($|L|\gtrsim 10^{-2}$) of the evolution, and a fair description of the intermediate stages. At high values of $-\delta m^2$ the adiabatic approximation well describes the results obtained using the QKE's. Note also that, for $|\delta m^2|/\mathrm{eV}^2 \gg 10^{-2}$, not all anti-neutrinos are converted into anti-sterile neutrinos (we refer to positive L, otherwise neutrinos would be converted instead of anti-neutrinos) and $|L_{\nu_\alpha}^{\rm fin}|$ can be well below the value 0.375 corresponding to a full conversion and for $y_{\rm res}^{\rm min} \ll 1$. This is because at high values of $|\delta m^2|$ the MSW dominated regime occurs mostly at temperature above ~ 1 MeV and collisions are able to re-distribute thermally the asymmetry after it has been produced at $y=y_{\rm res}$. In particular this means that (on statistical average) some anti-neutrinos in quantum states with $y>y_{\rm res}$ will move to quantum states with $y<y_{\rm res}$. Since $y_{\rm res}$ increases with time, this means that these anti-neutrinos cannot be converted any more. In a pictorial way, we can say that these anti-neutrinos *evade* the resonance and manage not to be converted. This *evasion effect* can be described correctly only numerically.

For lower values of $-\delta m^2$ the asymmetry becomes large only for low temperatures $\lesssim 1$ MeV, where collisions have an approximately negligible effect and the evasion effect does not occur. The final value of the neutrino asymmetry gets very close to the upper limit of 0.375 around $-\delta m^2 \sim 10^{-2}$ eV^2. For lower values of $-\delta m^2$ the adiabatic approximation predicts that the final value remains approximately constant at 0.375.[19] However at low enough values of $-\delta m^2$, the adiabatic approximation clearly fails in reproducing the results obtained fully integrating the QKE's. These show that, for a fixed value of $\sin^2 2\theta_0$, there is a value of $-\delta m^2/\mathrm{eV}^2$ below which the final value of the asymmetry becomes rapidly negligible. The region of mixing parameters where the adiabatic approximation holds and the final asymmetry $|L_{\nu_\alpha}^{\rm fin}| \gtrsim 0.8 |L_{\nu_\alpha}^{\rm fin}|^{(\rm ad)}$, with the value of the adiabatic limit $|L_{\nu_\alpha}^{\rm fin}|^{(\rm ad)}$ in the range 0.26–0.375 (the exact value depending on $-\delta m^2$), is given approximately by

$$\sin^2 2\theta_0 \left(\frac{|\delta m^2|}{\mathrm{eV}^2}\right)^{1/4} \gtrsim 1.2\times 10^{-9}. \tag{31}$$

It is also simple to infer from Fig. 5(a)[20] the following equations describing the iso-$|L_{\nu_\alpha}^{\rm fin}|$ curves in the space of mixing parameters:

$$\sin^2 2\theta_0 \left(\frac{|\delta m^2|}{\mathrm{eV}^2}\right)^{1/4} \simeq 0.8\times 10^{-9} |L_{\nu_\alpha}^{\rm fin}|^{1/4}. \tag{32}$$

[19]More precisely, as it is shown Fig. 5(a), for $\sin^2 2\theta_0 = 10^{-8}$ in the adiabatic approximation, the final value gets again slowly lower than 0.375 for $-\delta m^2/\mathrm{eV}^2 \simeq 10^{-2}$. This because in that case we integrated the QKE's in the collision dominated regime and in this case the correct value for the initial $y_{\rm res}^{\rm min}$ at which the MSW conversions start is reproduced. This value gets as high as 1 at $-\delta m^2 = 10^{-8}$ eV^2 and thus not all-antineutrinos are converted as it happens in Fig. 5(b). On the other hand, for larger $\sin^2 2\theta_0$, the value of $y_{\rm res}^{\rm min} \ll 1$ and the two procedures, the fast one and the super-fast one, give the same answer.

[20]We concentrated, for values of the final asymmetry much less than 0.01, on the case $\alpha=e$. However in the case $\alpha=\mu$ one does not expect differences, as the Wolfenstein term in the effective potential, responsible for the final value of the asymmetry in the MSW regime, is the same.

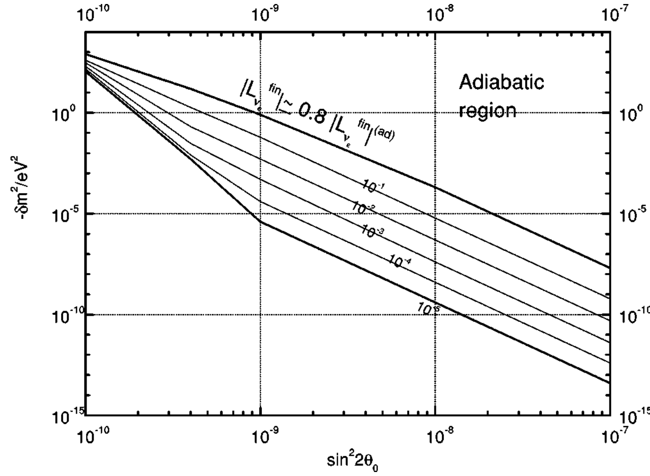

FIG. 6. Adiabatic region and iso-$|L^{\rm fin}_{\nu_e}|$ curves.

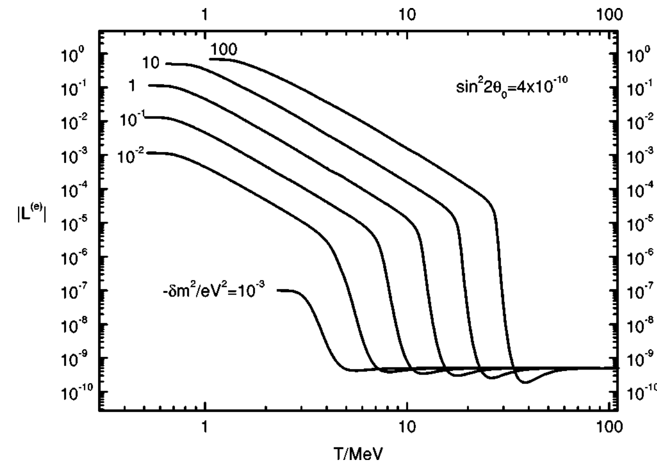

FIG. 7. The same as in Fig. 2 but for $\sin^2 2\theta_0 = 4\times10^{-10}$ and a different choice of values for $-\delta m^2$. For $-\delta m^2 = 10^{-3}$ eV^2 the MSW dominated regime never starts.

For $|L^{\rm fin}_{\nu_\alpha}| > 10^{-5}$ one finds a larger region compared to the condition (4) from the static approximation. This is because the MSW effect enhances the neutrino asymmetry generation compared to a collision dominated regime. The curves (32) are however valid, if one consider values of $|L^{\rm fin}_{\nu_\alpha}| \geqslant 10^{-5}$, only for $\sin^2 2\theta_0 \gtrsim 10^{-9}$. For smaller angles the validity of Eq. (32) breaks down for $10^{-5} \leqslant |L^{\rm fin}_{\nu_\alpha}| \lesssim L^{\star}_{\nu_\alpha}(\sin^2 2\theta_0)$. For example for $\sin^2 2\theta_0 = 4\times10^{-10}$ we found that $L^{\star}_{\nu_\alpha} \simeq 10^{-3}$ corresponding to $-\delta m^2/{\rm eV}^2 = 10^{-2}$ and this is clearly visible in Fig. 5(a). For lower values of $-\delta m^2$ and a fixed value of $\sin^2 2\theta_0$, there is a sharp cutoff in the generation.

In Fig. 6 we plotted the iso-$|L^{\rm fin}_{\nu_e}|$ curves for $-\log|L^{\rm fin}_{\nu_e}| = 1,2,3,4,5$. For $\sin^2 2\theta_0 \gtrsim 10^{-9}$ the curves (32) are found while for smaller mixing angles they do not hold and the generation goes off much faster for a decreasing $-\delta m^2$ and for a fixed value of $\sin^2 2\theta_0$.

One can easily explain this cutoff in the generation of neutrino asymmetry at low values of mixing angles ($\sin^2 2\theta_0 \lesssim 10^{-9}$) where Eq. (32) starts not to be valid for $|L^{\rm fin}_{\nu_e}| < L^{\star}$. This can be done in terms of current understanding of neutrino asymmetry generation outlined in the Introduction. In fact for too small mixing angles the collision dominated regime[21] is not efficient enough in generating a neutrino asymmetry $|L^{(e)}| \gtrsim (10^{-2}/\sin 2\theta_0)^{2/3} L_t$ and the MSW dominated regime cannot start at all [see Eq. (12)]. This can be seen in Fig. 7, where we plotted the evolution of the neutrino asymmetry with temperature for $\sin^2 2\theta_0 = 4\times10^{-10}$ and a few values of $-\delta m^2/{\rm eV}^2$. One can see how, in the case $-\delta m^2/{\rm eV}^2 = 10^{-3}$, the asymmetry gets frozen much before it can get the value $(10^{-2}/\sin 2\theta_0)^{2/3} L_t \simeq 10^{-4}$ and the MSW dominated regime never starts.

We are still lacking in a physical description of Eq. (32) for $\sin^2 2\theta_0 \gtrsim 10^{-9}$. In the next section, we will develop a simple physical picture providing such a description improving the adiabatic approximation in the MSW dominated regime.

III. THE LANDAU-ZENER APPROXIMATION

We will now develop a useful extension to the adiabatic approximation by taking into account non-adiabatic effects which inevitably occur for a choice of mixing parameters outside of the region (31). For definiteness let us refer to positive values of the asymmetry. In this case when the total asymmetry has reached values $L^{(\alpha)} \gg L_t$, the resonant momentum of anti-neutrinos is given by the expression (9). After it has reached its minimum value $y^{\rm min}_{\rm res}$, it starts to grow again eventually passing all of the anti-neutrino distribution at $y > y^{\rm min}_{\rm res}$.[22] In the adiabatic limit all anti-neutrinos at $y = y_{\rm res}$ are converted. It is then easy to find the following rate equation for the neutrino asymmetry [3]:

$$\left(\frac{dL_{\nu_\alpha}}{dt}\right)_{\rm ad} = \frac{y^2_{\rm res}}{4\zeta(3)}[f_{\bar\nu_\alpha}(y_{\rm res}) - f_{\bar\nu_s}(y_{\rm res})]\frac{dy_{\rm res}}{dt}. \qquad (33)$$

In the case of negative asymmetry one has to make the replacement $[f_{\bar\nu_\alpha} - f_{\bar\nu_s}] \rightarrow [f_{\nu_s} - f_{\nu_\alpha}]$.

In order to incorporate nonadiabatic effects, which can be important if $\sin^2 2\theta$ is small enough, we will use the standard Landau-Zener approximation [22]. In this case, due to the quantum level crossing between matter eigenstates, not all active anti-neutrinos will be converted into sterile anti-neutrinos. The number of active anti-neutrinos that undergo level crossing and are not converted is given by the well known "jumping probability" $P \equiv e^{-\pi\gamma_r/2}$ where $\gamma_r \equiv |2\dot{\bar\theta}_m \bar l_m|^{-1}_{\rm res}$ is the *adiabaticity parameter* at the resonance

[21]Note that the rate of neutrino asymmetry generation in the collision dominated regime, as deduced within the static approximation, is proportional to $\sin^2 2\theta_0$ [1,2].

[22]We recall that at this stage the resonant momentum of neutrinos is by far in the tail of the distribution and continues to increase further while the asymmetry grows and therefore neutrinos are completely out of the game.

and $\overline{\theta}_m$ and $\overline{l}_m$ are respectively the mixing angle and the oscillation length in matter given by

$$\sin 2\overline{\theta}_m = \frac{\sin 2\theta_0}{\sqrt{\sin^2 2\theta_0 + (\cos 2\theta_0 - b - a)^2}} \tag{34}$$

$$\overline{l}_m = \frac{2\,y\,T/|\delta m^2|}{\sqrt{\sin^2 2\theta_0 + (\cos 2\theta_0 - b - a)^2}}. \tag{35}$$

Since we are considering a regime of high values of the asymmetry ($L \gg L_t$), one can neglect the finite temperature term b in the effective potential and thus one can find easily the following expression for the adiabaticity parameter:

$$\gamma_r = \frac{|\delta m^2|}{2y_{\rm res}T_\nu}\frac{\sin^2 2\theta_0}{|da/dt|_{\rm res}}. \tag{36}$$

For computational purposes this can be then transformed with easy algebraic arrangements making use of the Eq. (9), defining $\alpha \equiv -d\ln L_{\nu_\alpha}/d\ln T_\nu > 0$, replacing time derivative with neutrino temperature derivative [using the expression (29)] and finding, in the end,

$$\gamma_{\rm res} \simeq 1.8\times 10^{10}\sqrt{\frac{10.75}{g_\rho(T_\nu)}}\left(\frac{T_\nu}{T}\right)^2 \frac{\sin^2 2\theta_0 L_{\nu_\alpha} T_\nu}{|4-\alpha|}. \tag{37}$$

For $\alpha = 4$ the adiabaticity parameter is infinite and this is perfectly understandable because when this happens, from Eq. (9), one can see that $y_{\rm res} = y_{\rm res}^{\rm min}$ and $dy_{\rm res}/dt = 0$.[23] The Landau-Zener approximation can be started from $y_{\rm res} \simeq y_{\rm res}^{\rm min}$ and in this case the rate equation for the asymmetry will be given by

$$\frac{dL_{\nu_\alpha}}{dt} = \left(\frac{dL_{\nu_\alpha}}{dt}\right)_{\rm ad}(1 - e^{-(\pi/2)\gamma_{\rm res}}). \tag{38}$$

This equation can be solved numerically and clearly is much less CPU time consuming then the QKE's. The results from the Landau-Zener approximation for the final value of the asymmetry are shown in Figs. 5(a) and 5(b) with dashed lines and we find a good agreement with the QKE's in both cases $\alpha = e$ and $\alpha = \mu, \tau$. The Landau-Zener approximation provides useful physical insight into the numerical results. First of all we can understand Eq. (31) for the adiabatic region looking at the expression (37) for the adiabaticity parameter. The jumping probability starts to be non-negligible for $\gamma_r \lesssim 2$. The adiabaticity decreases (on average) during neutrino asymmetry (MSW dominated) growth, since initially $\alpha = 4$ while when the growth stops at $T = T_{\rm f}$ one has $\alpha = 0$. The whole MSW regime will be then adiabatic and the final value of neutrino asymmetry will be close to 0.375 if $\gamma_{\rm res}(T_{\rm f}) \gtrsim 2$. Using the expression (10) for the final temperature one can immediately check that this condition reproduces the condition (31) for the adiabaticity region in the space of mixing parameters. We can also try to estimate the equation (32) for the iso-asymmetry curves. For $\gamma_r^{\rm fin} \ll 1$ the final neutrino asymmetry is given approximately by

$$L_{\nu_\alpha}^{\rm fin} \sim 0.375\frac{\pi}{2}\langle\gamma_r\rangle. \tag{39}$$

In this equation the quantity $\langle\gamma_r\rangle$ is the statistical average of the adiabaticity parameter done with the Fermi-Dirac distribution with zero chemical potential (assuming that all generation of the asymmetry occurs below the chemical decoupling temperature). If one neglects the annihilations and assumes $T_\nu = T$ and $g_\rho = 10.75 = $ const, then this quantity can be expressed through the expression (37) where the quantity α must be replaced with its averaged value $\langle\alpha\rangle$ and $\langle L_{\nu_\alpha}\rangle \simeq L_{\rm f}/2$. Again, from the expression (9) for the resonant momentum and using $y_{\rm res}^{\rm fin} \simeq 10$, obtains an expression for $T_{\rm f} \simeq (|\delta m^2|/{\rm eV}^2)^{1/4}(L_{\nu_\alpha}^{\rm fin})^{1/4}/3$, and the iso-asymmetry curves (32) are roughly reproduced when $\langle\alpha\rangle \simeq 3$, that is a quite reasonable value considering that α decreases from $\alpha = 4$ when $y = y_{\rm res}^{\rm min}$ to $\alpha = 0$ when $y_{\rm res} \gtrsim 10$ an the asymmetry gets frozen to its final value. Thus the Landau-Zener approximation provides the correct approach to get a physical insight into the generation of neutrino asymmetry at low $-\delta m^2$ and small mixing angles.

The Landau-Zener approach is also useful to get a physical insight into the effect of the electron-positron annihilations. From the expression (37) for the adiabaticity parameter we can in fact study the following quantity:

$$f(T/m_e) \equiv \left[\frac{\gamma_r(T = T_\nu)}{\gamma_r}\right] = \sqrt{\frac{g_\rho}{10.75}\left(\frac{T}{T_\nu}\right)^4}. \tag{40}$$

One can plot this function (see Fig. 1) and discover that it monotonically increases from 1 to its asymptotical value $f(0) \simeq 1.1$. Thus the answer is that *the effect of electron-positron annihilations is to make the MSW conversions less adiabatic and thus to decrease the neutrino asymmetry generation.* This is an effect clearly negligible in the fully adiabatic region, since if $\gamma_r(T = T_\nu) \gg 1$ then also $\gamma_r \gg 1$ and the final value of the neutrino asymmetry does not change. While considering Eq. (39), one can see that for $|L_{\nu_\alpha}^{\rm fin}| \ll 1$ the annihilations decrease the final value of 10%, just a correcting effect.

IV. CAN THE GENERATION OF NEUTRINO ASYMMETRY BE OBSERVED?

A. Big bang nucleosynthesis

It remains to briefly discuss the impact of neutrino asymmetry generation on cosmology. The most straightforward

[23] Thus in this precise moment, and only in this moment $L \propto T^{-4}$ (exactly). After this moment a power law with $\alpha = 4$ cannot hold exactly, since if all neutrinos are adiabatically MSW converted, the resonant momentum has to start to grow to sustain the asymmetry growth. However while $L \ll 0.1$, $L \propto T^{-4}$ (approximately) since the resonance need only move very slowly to produce the required asymmetry. One could measure the average value of α fitting the numerical solutions if one is really interested in this quantity. See [23] for a detailed analysis on this (in our opinion academic) issue.

consequence is a modification of the predictions of standard big bang nucleosynthesis [3,8,9]. Standard BBN is modified by active-sterile neutrino oscillations in two ways. One is by changing the energy density, described by ΔN_ν^ρ, and the second is by modifying the standard electron neutrino distributions $[e^{p/T}+1]^{-1}$, described by $\Delta N_\nu^{f_{\nu_e}}$. This second change occurs both because an electron neutrino asymmetry can be generated and also because oscillations below the thermal decoupling temperature at ~ 1 MeV produce deviations from thermal equilibrium. In the case of $\alpha=\mu,\tau$, clearly in the idealized case of two neutrino mixing considered in this paper, only the first effect is present in first approximation.[24] However this effect is not significant unless a significant neutrino asymmetry is generated before the chemical decoupling temperature of about 3.5 MeV. This in turn requires relatively large values of $-\delta m^2 \gtrsim 100\ \mathrm{eV}^2$. Even if all the asymmetry is produced before the chemical decoupling (for $-\delta m^2 \gg 100\ \mathrm{eV}^2$), one gets at maximum a value of $\Delta N_\nu^\rho \simeq 0.4$ [9]. In the case $\alpha=e$ the neutrino asymmetry generation gives also a significant contribution to $\Delta N_\nu^{f_{\nu_e}}$ and in particular if the generated asymmetry is positive then negative values of $\Delta N_\nu^{f_{\nu_e}}$ are possible [3]. However, this effect is only important when a significant neutrino asymmetry is produced before the freezing of the neutron to proton ratio at $T\approx 0.75$ MeV. This requires a $-\delta m^2 \gtrsim 0.01\ \mathrm{eV}^2$ [9]. This can also be seen from Fig. 5(a) where the value of the asymmetry at $T=0.75$ MeV has been plotted for mixing angles large enough ($\sin^2 2\theta_0 \gtrsim 10^{-8}$) that the generation is adiabatic at $-\delta m^2/\mathrm{eV}^2 \gtrsim 10^{-2}$. Also in the multi-flavor case where a heavier ν_τ (and/or ν_μ) oscillates with the ν_s which generates a large L_{ν_τ} (and/or L_{ν_μ}) asymmetry, and subsequent oscillations of ν_τ (and/or ν_μ) with ν_e transfers some of this asymmetry to the ν_e sector (thereby generating L_{ν_e}) one similarly obtains that significant L_{ν_e} production requires a $-\delta m^2 \gtrsim 0.5\ \mathrm{eV}^2$ [3,8,9]. *Thus for* $|\delta m^2| \ll 10^{-2}\ \mathrm{eV}^2$, *the asymmetry generation effects are unlikely to produce observable deviations on the standard BBN predictions.*

B. Cosmic microwave background

It has been pointed out how a neutrino asymmetry generated above the chemical decoupling temperature would imply a ΔN_ν^ρ able to change the matter-radiation equivalence and thus modifying the height and position of CMBR acoustic peaks [25]. However in this case such an asymmetry cannot be generated from active-sterile neutrino oscillations because, as already pointed out discussing BBN, they generate the asymmetry generally below the chemical decoupling and thus produce a value of ΔN_ν^ρ not high enough to produce remarkable effects. However a ΔN_ν^ρ is also produced for mixing angles large enough [see Eq. (5)] that a significant sterile neutrino production occurs. This also modifies the matter-radiation equivalence time and gives effects on CMBR anisotropies [26]. In the case of negative $-\delta m^2$ the neutrino asymmetry generation can modify the sterile neutrino production, especially in multiflavor mixing. Thus this indirect effect of neutrino asymmetry generation has effects on CMBR. In other words a calculation of ΔN_ν^ρ including the account of asymmetry generation is needed to derive the correct effects on CMBR anisotropies.

[24] Actually a small effect on electron neutrino distribution, giving a nonzero $\Delta N_\nu^{f_{\nu_e}}$, is present also for $\alpha=\mu,\tau$ if one describes exactly the chemical decoupling. This is because a few electron neutrino-antineutrino annihilations will occur to restore μ (or τ) number densities depleting electron neutrino number densities [24,9].

C. Relic neutrinos

We have seen that the effects of neutrino asymmetry generation on BBN for low values of $-\delta m^2$ are not significant, while on CMBR they can be only indirect. However it is still remarkable that neutrino asymmetry generation would lead to a relic neutrino background in which mainly antineutrinos or neutrinos are present. At the moment nobody has been able to experimentally detect relic neutrinos. However it has been noted in [20] that the presence of an asymmetry in the relic neutrino background can have observable effects in future neutrino detectors. If very high energy α-neutrinos are produced from cosmological sources at high redshift, their vacuum probability oscillation can be changed by matter effects due to the presence of an asymmetry in the relic background. Thus, if one knows the vacuum probability and the fluxes and energy spectrum of produced neutrinos, it is possible in principle to observe a change in the oscillation probability induced by matter effects. These effects depend crucially on the value of $\eta_0^{(\alpha)} \equiv h_0 L^{(\alpha)}$, where $h_0=4/11$ is the dilution factor at the present. Thus the maximum value of $\eta_0^{(\alpha)}$ that can be generated from active-sterile neutrino oscillations is $\simeq 0.27$. According to the analysis presented in [20], such a value can give at maximum effects of about 0.5% in the change of vacuum probability which is probably unobservably small. Nevertheless, it shows how the possibilities of detecting relic neutrino background can be changed by the presence of an asymmetry produced by active-sterile neutrino oscillations and thus the issue of relic neutrino background detection should be reconsidered in light of this and deserves further investigations.

V. CONCLUSIONS

We have shown that the generation of large neutrino asymmetry via active-sterile neutrino oscillations occurs for much lower values of $|\delta m^2|$ than previously supposed. We have made a detailed study of the final asymmetry values in the $\sin^2 2\theta_0 \leqslant 10^{-7}$ parameter region. The evolution of neutrino asymmetry in this parameter region can be understood quite simply by means of the Landau-Zener approximation that made possible to derive analytical results in good agreement with the numerical ones. For very small $-\delta m^2 \ll 0.01\ \mathrm{eV}^2$, the oscillations generate the asymmetry too late to have significant effects on BBN. The account of neutrino asymmetry generation can modify the sterile neutrino production (this is especially true when considered within a multiflavor mixing scheme) and thus has to be taken into account when studying the effects on CMBR anisotropies. Properties of relic neutrino background are also modified by neutrino asymmetry generation and this observation should certainly encourage new investigations on relic neutrinos detection.

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