Back reaction of quantum fields in an Einstein universe

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I study the back-reaction effect of the finite-temperature massless scalar field and the photon field in the background of the static Einstein universe. In each case I find a relation between the temperature of the universe and its radius. This relation exhibits a minimum radius below which no self-consistent solution for the Einstein field equation can be found. A maximum temperature marks the transition from the vacuum dominated era to the radiation dominated era. An interpretation of this behavior in terms of Bose-Einstein condensation in the case of the scalar field is given.

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I. INTRODUCTION

Many authors have investigated the behavior of quantum fields in curved spacetimes (for a thorough in-depth review see Ref. [1]). These investigations came in an endeavor to understand the origin of the universe and the creation of matter, presumably, out of an arbitrary state of nothing (the vacuum). The subject was initiated by the discovery of Penzias and Wilson [2] of the microwave background radiation, where it was observed that the galaxies swim in a global cold bath at about 2.73 K. The source of this radiation was found to be cosmic; therefore, it was called the cosmic microwave background (CMB) radiation. This radiation was found to be isotropic over a large angular scale of observation, and it has a Planck spectrum for a radiating blackbody at about 2.73 K.

The discovery of the CMB revived the theory of the hot origin of the universe (the big-bang model) which was worked out in the late 1940s by Gamow and his collaborators. The most refined analysis along this line predicted a cosmic background radiation at a temperature of about 5 K (for a concise recent review of the subject see Ref. [3]). Therefore the Penzias-Wilson discovery was considered a good verification of what was called the big bang model. However, since the Gamow model started with the universe at the times when the temperature was about 10^{12} K, the new interest in the origin of the universe sought much earlier times at much higher temperatures. The new interest arose in studying the state of the universe in the period from near the Planck time ($\sim 10^{-44}$ s) to the grand unification time ($\sim 10^{-34}$ s). This is the era when quantum effects played a decisive role in the subsequent developments of the universe, and it is also the era when particle processes could have left permanent imprints on the content of the universe.

The works dealing with this question started by the mid 1970s when matter fields were brought into connection with spacetime curvature through the calculation of the vacuum expectation value of the energy-momentum tensor $\langle 0|T_{\mu\nu}|0\rangle$ [4–8]. The motivations for studying this quantity stems from the fact that $T_{\mu\nu}$ is a local quantity that can be defined at a specific spacetime point, contrary to the particle concept which is global. The energy-momentum tensor also acts as a source of gravity in the Einstein field equations, therefore $\langle 0|T_{\mu\nu}|0\rangle$ plays an important role in any attempt to model a self-consistent dynamics involving the classical gravitational field coupled to the quantized matter fields. So, once $\langle 0|T_{\mu\nu}|0\rangle$ is calculated in a specified background geometry, we can substitute it on the right-hand side (RHS) of the Einstein field equation and demand self-consistency, i.e.,

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R=-8\pi\langle 0|T_{\mu\nu}|0\rangle, \tag{1}$$

where $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ is the metric tensor, and R is the scalar curvature.

The solution of Eq. (1) will determine the development of the spacetime in presence of the given matter field, for which $|0\rangle$ can be unambiguously defined. This is known as the "back-reaction problem." It is interesting to perform the calculation of $\langle 0|T_{\mu\nu}|0\rangle$ in Friedmann-Robertson-Walker (FRW) models because the real universe is, more or less, a sophisticated form of the Friedmann models. However, the time dependence of the spacetime metric generally creates unsolvable fundamental problems. One such problem was the definition of vacuum in a time-dependent background [9]; a time-dependent background is eligible for producing particles continuously, therefore, pure vacuum states in the Minkowskian sense do not exist. Also an investigation into the thermodynamics of a time-dependent system lacks the proper definition of thermal equilibrium, which is a basic necessity for studying finite-temperature field theory in curved backgrounds [10].

Of all the available solutions of the Einstein field equations, the static Einstein universe stands above the two fundamental challenges. First, being static, the Einstein universe leaves no ambiguity in defining the vacuum both locally and globally [1]. The same feature also allows for thermal equilibrium to be defined unambiguously. Furthermore, the Einstein static metric is conformal to all Robertson-Walker metrics, and it was shown by Kennedy [10] that thermal Green's functions for the static Einstein universe and the time-dependent Robertson-Walker universe are conformally related, hence deducing a (one-to-one) correspondence between the vacuum and the many particle states of both

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universes. Therefore, under the equilibrium condition, the thermodynamics of quantum fields in an Einstein universe of radius a is equivalent to that of an instantaneously static Friedmann-Roberson-Walker (FRW) universe of equal radius [4,7,11]. This means that the results obtained in the FRW universe would be qualitatively the same as those obtained in an Einstein universe.

Dowker and Critchley [12] considered the finite temperature corrections for the massless scalar field in an Einstein universe using the technique of finite-temperature Green's functions. Later Altaie and Dowker [13] calculated the finite temperature corrections to the massless scalar field, the neutrino field, and the photon field in the background of an Einstein universe. The results of the calculation for the photon field were used to deduce a self-consistent solution for the Einstein field equation, i.e., a back-reaction problem, from which a relation between the temperature and the radius of the Einstein universe was deduced. However, this relation was not fully exploited at that time and therefore some of the thermodynamical aspects were kept unexposed. Hu [11] considered the effects of finite-temperature conformally coupled massless scalar field in a closed Robertson-Walker universe using the results of Altaie and Dowker [13] and assuming that the thermal equilibrium is established for the scalar particles throughout the history of the universe. In the high-temperature limit Hu found that the universe expands linearly in cosmic time near the singularity. In the low-temperature limit, it reduces to the Starobinsky–de Sitter type solution where the singularity is avoided in an exponential expansion, concluding that the finite-temperature formalism provides a unifying framework for the description of the interplay of vacuum and radiation energy and their combined effect on the state of the early universe.

Recently Plunien *et al.* [14] considered the dynamical Casimir effect at finite temperature. They reported that finite temperatures can enhance the pure vacuum effect by several orders of magnitude. Although the relevance of this result was addressed in the context of an effort aiming at the experimental verification of the Casimir effect, it does have a useful implication in respect to the theoretical understanding of the finite temperature corrections to the vacuum energy density in closed spacetimes.

In this paper we will reconsider the calculation of the back-reaction effect of the conformally coupled massless scalar field and the photon field in the background of the Einstein static universe. The aim is to expose the thermal behavior of the system, analyze and interpret details that may have been overlooked in previous studies, and investigate the possibility of assigning any practical applicability of the results.

II. THE VACUUM ENERGY DENSITY AND BACK REACTION

The metric of the Einstein static universe is given by

$$ds^2=dt^2-a^2[d\chi^2+\sin^2\chi(d\theta^2+\sin^2\theta d\phi^2)], \tag{2}$$

where a is the radius of the spatial part of the universe S^3 and $0\leqslant\chi\leqslant\pi$, $0\leqslant\theta\leqslant\pi$, and $0\leqslant\phi\leqslant 2\pi$.

We consider an Einstein static universe being filled with a massless boson gas in thermal equilibrium at temperature T. The total energy density of the system can be written as

$$\langle T_{00}\rangle_{\rm tot}=\langle T_{00}\rangle_T+\langle T_{00}\rangle_0, \tag{3}$$

where $\langle T_{00}\rangle_0$ is the zero-temperature vacuum energy density and $\langle T_{00}\rangle_T$ is the corrections for finite temperatures, i.e.,

$$\langle T_{00}\rangle_T=\frac{1}{V}\sum_n \frac{d_n\epsilon_n}{\exp\beta\epsilon_n-1}, \tag{4}$$

where ϵ_n and d_n are the eigenenergies and degeneracies of the nth state, and $V=2\pi^2a^3$ is the volume of the spatial section of the Einstein universe.

To investigate the back-reaction effect of finite-temperature quantum fields on the behavior of the spacetime we should substitute for $\langle T_{00}\rangle_{\rm tot}$ on the RHS of the Einstein field, but this time with the cosmological constant λ, i.e.,

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R+g_{\mu\nu}\lambda=-8\pi\langle T_{\mu\nu}\rangle_{\rm tot}. \tag{5}$$

Indeed all the Einstein field equations for the system are satisfied due to the symmetry of the Einstein universe which is topologically described by $T\otimes S^3$ and due to the structure of $\langle T_{\mu\nu}\rangle$ in this geometry which comes to be diagonal and is given by (see [1], p. 186)

$$\langle T^{\nu}_{\mu}\rangle=\frac{p(s)}{2\pi^2a^4}\,\mathrm{diag}(1,-1/3,-1/3,-1/3), \tag{6}$$

where $p(s)$ is a spin-dependent coefficient which takes the values $p(0)=1/240$, $p(1/2)=17/960$, and $p(1)=11/120$.

Since we are interested in the energy density, we will consider the T_{00} only. In order to eliminate λ from Eq. (5) we multiply both sides with $g_{\mu\nu}$ and sum over μ and ν, then using the fact that $T^{\mu}_{\mu}=0$ for massless fields, and for the Einstein universe $R_{00}=0$, $g_{00}=1$, and $R=6/a^2$, we get

$$\frac{6}{a^2}=32\pi\langle T_{00}\rangle_{\rm tot}. \tag{7}$$

Note that in the general case conformal anomalies do appear in the expression for $\langle T^{\mu}_{\mu}\rangle$, but because of the high symmetry enjoyed by the Einstein universe these anomalies do not appear and $\langle T^{\mu}_{\mu}\rangle$ is found to be traceless for massless particles.

A. Scalar field

For a conformally coupled massless scalar field the zero-temperature vacuum energy density in an Einstein universe is [4,6]

$$\langle T_{00}\rangle_0=\frac{1}{480\pi^2a^4}. \tag{8}$$

The eigenenergies and degeneracies are $\epsilon_n=n/a$ and $d_n=n^2$, respectively, so that Eq. (3) gives

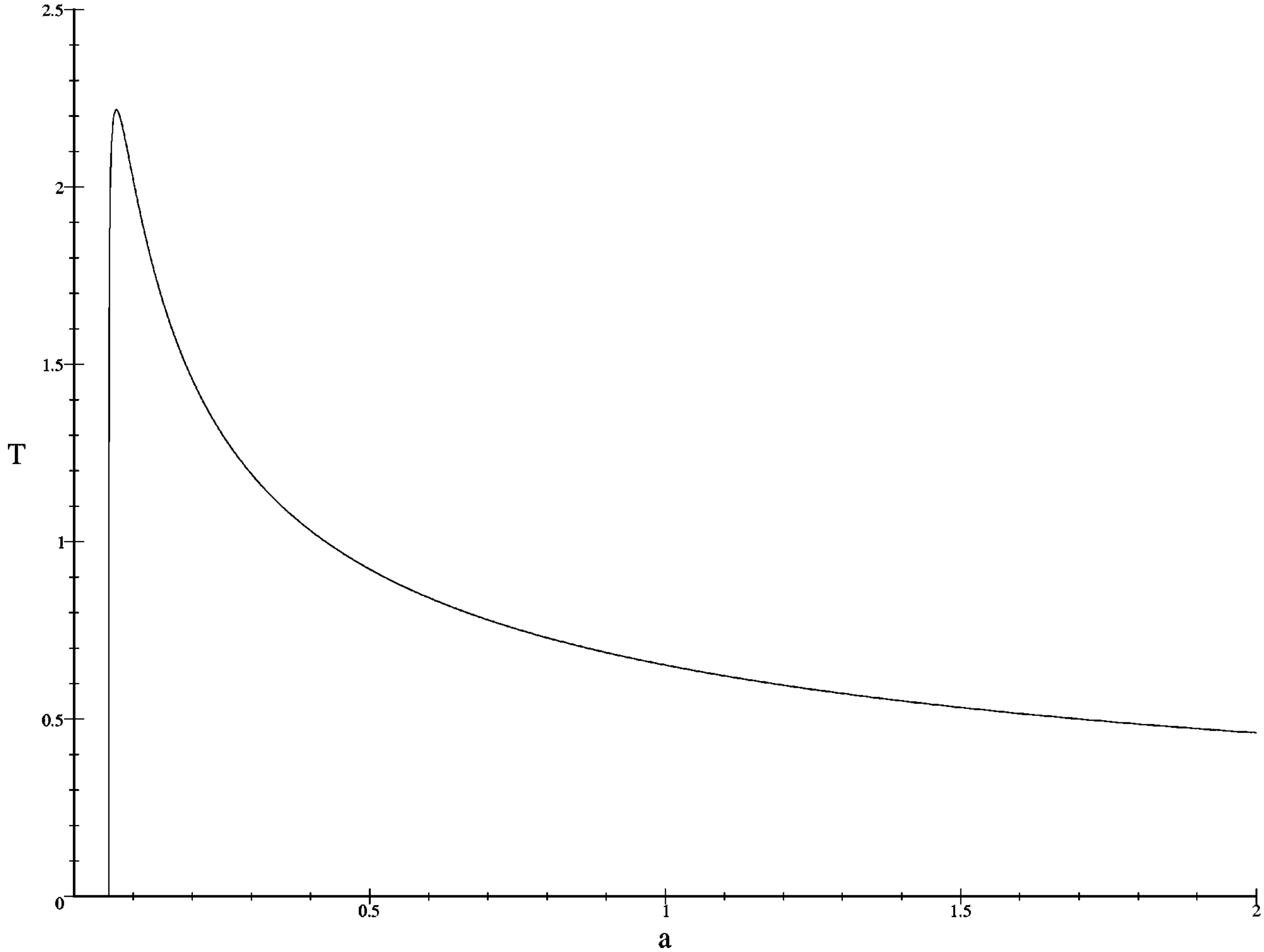

FIG. 1. The temperature-radius relationship for the massless scalar field in an Einstein universe.

$$\langle T_{00}\rangle_{\rm tot}=\frac{1}{2\pi^2a^4}\sum_{n=1}^{\infty}\frac{n^3}{\exp(n/Ta)-1}+\frac{1}{480\pi^2a^4}. \tag{9}$$

Using this mode-sum expression, Altaie and Dowker [13] calculated the finite temperature corrections for the vacuum energy density of the conformally coupled massless scalar field in the Einstein universe. The results which are functions of a single parameter $\xi(=Ta)$, were then subjected to the high and low-temperature limits. It was found that in the low-temperature (or small radius) limit the zero-temperature vacuum energy density is recovered, i.e.,

$$\lim_{\xi\to 0}\langle T_{00}\rangle_{\rm tot}=\frac{1}{480\pi^2a^4}, \tag{10}$$

and in the high temperature (or large radius) limit the behavior of the system is totally Planckian,

$$\lim_{\xi\to\infty}\langle T_{00}\rangle=\frac{\pi^2}{30}T^4. \tag{11}$$

In order to investigate the back-reaction effect of the field we substitute for $\langle T_{00}\rangle_{\rm tot}$ from Eq. (9) in Eq. (7) and request a self-consistent solution, we get

$$a^2=\frac{8}{3\pi}\sum_{n=1}^{\infty}\frac{n^3}{\exp(n/Ta)-1}+\frac{1}{90\pi}. \tag{12}$$

This equation determines a relation between the temperature T and the radius a of the Einstein universe in the presence of the conformally coupled massless scalar field. The solutions of this equation are shown in Fig. 1. Two regimes are recognized: one corresponding to small values of ξ where the temperature rises sharply reaching a maximum at $T_{\rm max}\approx 2.218T_p=3.15\times 10^{32}$ K at a radius $a_t\approx 0.072l_p=1.16\times 10^{-34}$ cm. Since this regime is controlled by the vacuum energy (the Casimir energy), we therefore prefer to call it the "Casimir regime." The second regime is what we call the "Planck regime," which corresponds to large values of ξ, and in which the temperature asymptotically approaches zero for very large values of a. This behavior was overlooked by Hu [11].

From Eq. (12) it is clear that at $T=0$ the radius of an Einstein universe has a minimum value a_0, below which no consistent solution of the Einstein field equation exist. This is given by

$$a_0=\left(\frac{1}{90\pi}\right)^{1/2}l_p. \tag{13}$$

Note that a_0 here is less than one Planckian length l_p, this goes beyond the range of validity of the quasi-classical approximation adopted in the present work. But fortunately, the region of validity of the approach can be extended if one takes the number of fields large enough (see, for instance, Ref. [15]).

From Eqs. (7) and (11) we can calculate the background (Tolman) temperature of the universe in the limit of large radius. This is given by

$$T_b=\left(\frac{45}{8\pi^3 a^2}\right)^{1/4}, \tag{14}$$

for example, at $a=1.38\times10^{28}$ cm we obtain $T=31.556$ K.

Conversely if we demand that the background temperature have the same value as the present equivalent temperature of the CMB radiation, i.e., 2.73 K, then the radius of the Einstein universe should be 1.294×10^{30} cm. This is about two orders of magnitude larger than the estimated Hubble length of 1.38×10^{28} cm.

B. Photon field

The vacuum energy density of this field at zero temperature is given by [6]

$$\langle T_{00}\rangle_0=\frac{11}{240\pi^2 a^4}. \tag{15}$$

The total energy density of the system in terms of the mode-sum can be written as

$$\langle T_{00}\rangle_{\rm tot}=\frac{1}{\pi^2 a^4}\sum_{n=2}^{\infty}\frac{n(n^2-1)}{\exp(n/Ta)-1}+\frac{11}{240\pi^2 a^4}. \tag{16}$$

In the low-temperature limit the result reduces to [13]

$$\lim_{\xi\to0}\langle T_{00}\rangle_{\rm tot}=\frac{11}{240\pi^2 a^4}. \tag{17}$$

Substituting this into Eq. (7) we get

$$a_{0\gamma}=\left(\frac{11}{45\pi}\right)^{1/2} l_p. \tag{18}$$

This is the minimum radius for an Einstein static universe filled with photons at finite temperatures.

In the high-temperature (or large radius) limit the result is

$$\lim_{\xi\to\infty}\langle T_{00}\rangle_{\rm tot}=\frac{\pi^2}{15}T^4-\frac{1}{6}\frac{T^2}{a^2}. \tag{19}$$

The back reaction of the field can be studied if we substitute Eq. (16) into Eq. (7) where this time we obtain

$$a^2=\frac{16}{3\pi}\sum_{n=2}^{\infty}\frac{n(n^2-1)}{\exp(n/Ta)-1}+\frac{11}{45\pi}. \tag{20}$$

The solutions to this equation are depicted in Fig. 2, where we see that the behavior is qualitatively the same as that encountered in the conformally coupled scalar field case. The minimum radius permissible for a self-consistent solution to exist in the presence of the photon field is $a_0=0.279 l_p$, and the maximum temperature $T_{\rm max}=1.015 T_p=1.44\times10^{32}$ K at $a_t=0.34 l_p=5.5\times10^{-34}$ cm.

The background (Tolman) temperature of the photon field is

$$T_{b\gamma}=\left(\frac{45}{16\pi^3 a^2}\right)^{1/4}. \tag{21}$$

At the radius of 1.38×10^{28} cm we obtain a background temperature of 30.267 K, and if we require that the background temperature have the same value as the average measured value of 2.73 K, the radius of the Einstein universe has to be 1.83×10^{30} cm. Again more than two orders of magnitude larger than the estimated value of Hubble length.

III. BOSE-EINSTEIN CONDENSATION

In an earlier work [16], we studied the Bose-Einstein condensation (BEC) of nonrelativistic spin 0 and spin 1 particles in an Einstein universe. We found that the finiteness of the system resulted in smoothing-out the singularities of the thermodynamic functions which are normally found in infinite systems, so that the phase transitions in curved space become noncritical. We also remarked about the enhancement of the condensate fraction and the displacement of the specific-heat maximum toward higher temperatures. Singh and Pathria [17] considered the BEC of a relativistic conformally coupled massive scalar field. Their results confirmed our earlier findings of the nonrelativistic case. Recently we considered the BEC of the relativistic massive spin-1 field in an Einstein universe [18]. Again the results confirmed our earlier findings concerning the general features of the BEC in closed spacetimes. So the above-mentioned features became established general features of the BEC of quantum fields in curved spaces.

Parker and Zhang [19] considered the ultrarelativistic BEC of the minimally coupled massive scalar field in an Einstein universe in the limit of high temperatures. They showed, among other things, that an ultrarelativistic BEC can occur at very high temperatures and densities in the Einstein universe, and by implication in the early stages of a dynamically changing universe. Parker and Zhang [20], also showed that the Bose-Einstein condensate can act as a source for inflation leading to a de Sitter type universe. However, Parker and Zhang gave no specific value for the condensation temperature of the system.

Here we are going to use the ready result obtained for the condensation temperature T_c of the conformally coupled massless scalar field in order to explain the change in behavior of the system from the Casimir regime to the Planck regime. This change is taking place at a well defined maximum temperature which we called the transition temperature T_t.

The condensation temperature of the conformally coupled massive scalar field is given by [21]

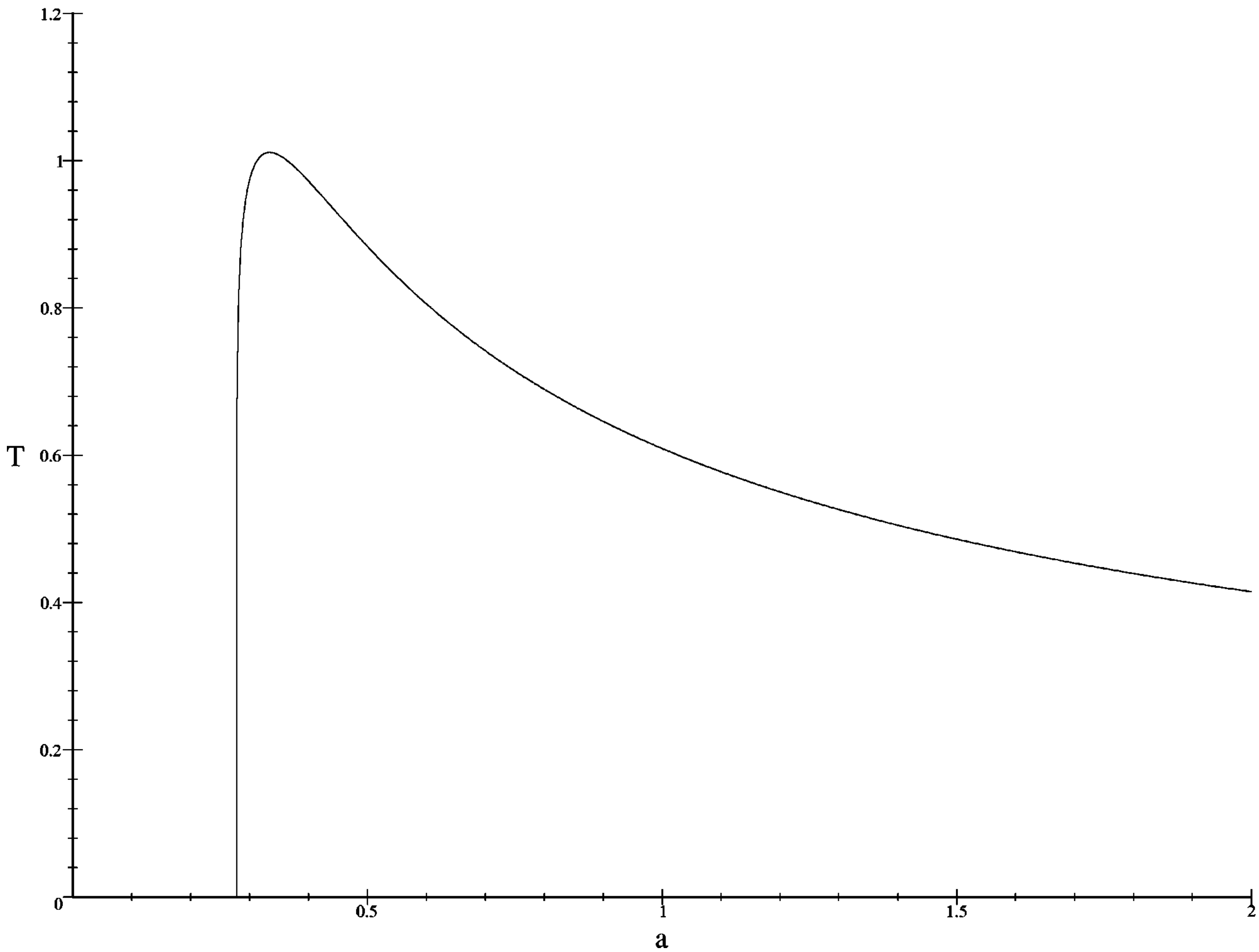

FIG. 2. The temperature-radius relationship for the photon field in an Einstein universe.

$$T_c=\left(\frac{3q}{m}\right)^{1/2}\left(1+\frac{1}{m^2a^2}\right)^{-1/4}. \tag{22}$$

In the massless limit this gives

$$T_c=\sqrt{3qa}, \tag{23}$$

where q is the number density of the particles.

For the Einstein universe specifically we have shown that [see Eq. (7)]

$$\langle T_{00}\rangle_{\rm tot}=\frac{3}{16\pi a^2}. \tag{24}$$

This means that, if the back reaction of the field is to be taken into consideration, the number density of the particles in the system q will be inversely proportional to a^2, this enables us to write

$$qa^2=\text{const}, \tag{25}$$

for any values of q and a. This means that

$$q_b=q_0\left(\frac{a_0}{a_p}\right)^2. \tag{26}$$

Substituting this into Eq. (23) we get the expression for the condensation temperature of the conformally coupled massless scalar field in the Einstein universe as

$$T_c=\sqrt{\frac{3q_pa_p^2}{a_0}}. \tag{27}$$

The estimated upper bound on the net average particle number density of the universe at present is $q_p<10^{-24}\ \text{cm}^{-3}$ (see Dolgov and Zeldovich [22]). If this upper bound is adopted, then using Eq. (27) we can calculate the condensation temperature of the conformally coupled field at any specified radius. If we substitute for the radius of the Einstein universe the estimated value of Hubble length, i.e., $a_p=1.8\times10^{28}$ cm, then we can write

$$T_c=\frac{1.268}{\sqrt{a_0}}. \tag{28}$$

We have already found that the transition from the Casimir regime to the Planck regime takes place at $a_0=0.072l_p$. Substituting this into Eq. (28) we get

$$T_c=4.725T_p. \tag{29}$$

Clearly we obtain a condensation temperature which is the same order of magnitude as the transition temperature obtained earlier for the conformally coupled massless scalar particles.

This strongly suggests that the transition from the Casimir regime into the Planck regime is taking place as a result of Bose-Einstein condensation of the vacuum energy so that as the condensate is formed, a free absorption and emission of massless quanta by the condensate is expected to take place and the system will start behaving according to Planck's law.

IV. CONCLUSIONS

There are numerous publications which present quantum field theoretic calculations performed in the Einstein universe; these studies have contributed to achieving a better understanding of the interplay of spacetime curvature and of quantum field theoretic effects. Furthermore the fact that the Einstein universe is being conformally related to the time-dependent Robertson-Walker universe encourages us to do calculations in the Einstein universe.

In conclusion I can say that the present study exhibited some features of the thermodynamical behavior of the Einstein static universe. In presenting the results of this investigation I stress the fact that due to the static nature of the Einstein universe, the following results are specific to the case considered and should not be taken to imply an evolving cosmological state.

The main findings are as follows.

(1) The thermal development of the universe is a direct consequence of the state of its global curvature.

(2) The universe avoids the singularity at $T=0$ through quantum effects (the Casimir effect) because of the nonzero value of $\langle T_{00}\rangle_0$. A nonzero expectation value of the vacuum energy density always implies a symmetry breaking event.

(3) During the Casimir regime the universe is totally controlled by vacuum. The energy content of the universe is a function of its radius. Using the conformal relation between the static Einstein universe and the closed FRW universe [10], this result indicates that in a FRW model there would be a continuous creation of energy out of vacuum as long as the universe is expanding, a result which was confirmed by Parker long ago [23]. The steep, nearly vertical line in Figs. 1 and 2 suggests that the real universe started violently and had to relax later.

(4) At high temperatures new quantum-thermal effects do interfere causing a phase transition at about $T_{\max}=2.218T_p=3.15\times10^{32}$ K for the massless scalar field and at $T_{\max}=1.015T_p=1.44\times10^{32}$ K for the photon field. The calculations show that a Bose-Einstein condensation of massless quanta (at least in the scalar field case) may be responsible for the transition. The values of these peaks agrees with the expectations of particle physics in respect to the era of total unification of forces.

It should be emphasized too that the Einstein static universe is unstable; being dependent on the value of the cosmological constant, the solutions will surely reflect this instability. However, although not a realistic cosmological model, the Einstein universe provides a useful theoretical model to achieve better understanding of the interplay of spacetime curvature and of quantum field theoretic effects. In this respect we note that the recent findings of Plunien *et al.* [14] that finite temperatures can enhance the pure vacuum effect by several orders of magnitude can be used to explain the behavior of our system during the Casimir (vacuum) regime, since this means that the finite temperature corrections will surely enhance the positive vacuum energy density of our closed system causing the system to behave, thermodynamically, as being controlled by the vacuum energy. So, one can confidently assume that the original massless particles that existed during the Casimir regime are basically those which were born out of vacuum through the mechanism of the Casimir effect plus the finite temperature enhancement deduced by Plunien *et al.* Indeed, a similar behavior to the case of dynamical Casimir effect inside a resonantly vibrating cavity presented by Plunien *et al.* is observed here where the number of particles increases all the time. This interpretation, i.e., the finite-temperature enhancement of the Casimir energy, explains, physically, the behavior of quantum fields at finite temperature during the Casimir regime.

ACKNOWLEDGMENT

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