Nonperturbative continuity in graviton mass versus perturbative discontinuity

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We address the question of whether a graviton could have a small nonzero mass. The issue is subtle for two reasons: there is a discontinuity in the mass in the lowest tree-level approximation, and, moreover, the nonlinear four-dimensional theory of a massive graviton is not defined unambiguously. First, we reiterate the old argument that for vanishing graviton mass the lowest tree-level approximation breaks down since the higher order corrections are singular in the graviton mass. However, there can exist nonperturbative solutions which correspond to the summation of the singular terms, and these solutions are continuous in the graviton mass. Furthermore, we study a completely nonlinear and generally covariant five-dimensional model which mimics the properties of the four-dimensional theory of massive gravity. We show that the exact solutions of the model are continuous in the mass, yet the perturbative expansion exhibits a discontinuity in the leading order and singularities in higher orders as in the four-dimensional case. Based on exact cosmological solutions of the model we argue that the helicity-zero graviton state responsible for the perturbative discontinuity decouples from the matter in the limit of vanishing graviton mass in the full classical theory.

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I. INTRODUCTION

Could a graviton be massive? The naive answer to this question seems to be positive. Indeed, if the graviton Compton wavelength $\lambda_g = m_g^{-1}$ is large enough, let us say of the present Hubble size, we should not be able to tell the massive graviton from a massless one. In fact, astrophysical bounds are even milder, $\lambda_g > 10^{24}$ cm [1] (see also Refs. [2]). However, in general relativity (GR) the issue turns out to be more subtle. A dramatic observation was made in Refs. [3–5] according to which the predictions of massless GR, such as light bending by the Sun and the precession of the Mercury perihelion, differ by numerical factors from the predictions of the theory with a massive graviton, no matter how small the graviton mass is. This discontinuity, if true, would unambiguously prove that the graviton is strictly massless in Nature.

The arguments of Refs. [3–5] were based on the lowest tree-level approximation to interactions between sources. In this approximation the discontinuity has a clear physical interpretation. Indeed, a massive graviton in four dimensions has *five* physical degrees of freedom (helicities ± 2, ± 1, 0) while the massless graviton has only two (helicities ± 2). The exchange by the three extra degrees of freedom can be interpreted in the limit $m_g \to 0$ as an additional contribution due to one massless vector particle with two degrees of freedom (‘‘graviphoton’’ with helicities ± 1) plus one real scalar (‘‘graviscalar’’ with the helicity 0). The graviphotons do not contribute to the one-particle exchange—their derivative coupling to the conserved energy-momentum tensor vanishes. The graviscalar, on the other hand, is coupled to the trace of the energy-momentum tensor and its contribution is generically nonzero. It is what causes the discontinuity between the predictions of massless and massive theories in the lowest tree-level approximation.

However, as was argued in Ref. [6], this discontinuity does not persist in the full classical theory. It was shown that the lowest tree-level approximation to the calculation of interactions between two sources breaks down when the graviton mass is small. The next-to-leading terms in the corresponding expansion are huge since they are inversely proportional to powers of m_g. Thus, the truncation of the perturbative series does not make much sense and all higher order terms in the solution of the classical equations for the graviton field should be summed up. The summation leads to a nonperturbative solution that is continuous when $m_g \to 0$. The perturbative discontinuity shows up only at large distances where higher order terms are small; these distances are growing when $m_g \to 0$. In other words, the continuity is not perturbative and not uniform as a function of distance.

A simple reason why one could expect the violation of the lowest tree-level approximation is that it does not take into account the characteristic physical scale of the problem; while the nonperturbative calculation of the Schwarzschild solution does account for this effect. In the nonperturbative solution the coupling of the extra scalar mode to the matter is suppressed by the ratio of the graviton mass to the physical scale of the problem. Hence, the predictions of the massive theory could be made infinitely close to the predictions of the massless theory by taking small m_g.

The argument can be conveniently presented by considering the gravitational amplitude of scattering of a probe particle in the background gravitational field produced by a heavy static source. This amplitude has the following generic structure [note that we use the flat metric $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$]:

$$\tilde{h}^{\mu\nu}(q) t'_{\mu\nu} \propto \frac{a(q^2) t^{\mu\nu} t'_{\mu\nu} - b(q^2) t^{\mu}_{\mu} t'^{\nu}_{\nu}}{q^2 + m_g^2 - i\epsilon}, \qquad (1)$$

where $t_{\mu\nu} = p_\mu p_\nu$ and $t'_{\mu\nu} = p'_\mu p'_\nu$ refer to the heavy particle with four-momentum $p_\mu = (M, \vec{0})$ and to the light particle

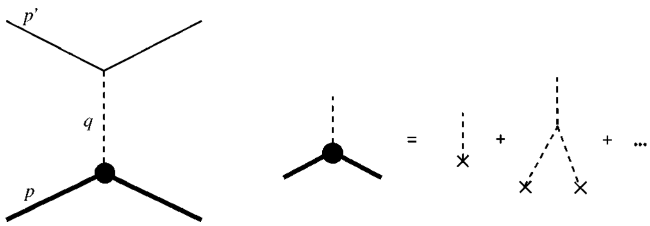

FIG. 1. Scattering of the probe particle at the gravitational field of the heavy source. The bold circle accounts for summation of the higher order iterations over the nonlinearities in the classical equations.

with momentum p'_μ, respectively (see Fig. 1).[1] The form factors $a(q^2)$ and $b(q^2)$ are functions of the momentum transfer q^2 and are defined by two parameters: the graviton mass m_g and the Schwarzschild radius $r_M=2G_NM$ of the heavy particle with mass M.

In the lowest tree-level approximation of the massive theory the form factors a and b are just constants and the unitarity (sum over five helicities) fixes their ratio, $a=3b$, while the same unitarity with two graviton states (helicities ± 2) in the massless theory gives $a=2b$. Therefore, a discontinuity [3–5] appears. However, this is only valid for small momenta $q \ll m_g(m_g r_M)^{-1/5}$, for which the higher order corrections are small [6]. In coordinate space this means that the linear approximation becomes valid only at the distance

$$r \gg r_m, \quad r_m \equiv \frac{(m_g r_M)^{1/5}}{m_g} = \frac{(2G_N M m_g)^{1/5}}{m_g}, \tag{2}$$

which for the Sun is bigger than the solar system size (see the discussion in the next section).

On the other hand, at $q \gg m_g(m_g r_M)^{-1/5}$, i.e., at shorter distances $r \ll r_m$, we expect that the summation of higher orders [6] returns the relation $a=2b$ of the massless theory. In other words, nonperturbative summation should lead to the decoupling of the graviscalar from the heavy source for distances $r \ll r_m$.

What was not verified in Ref. [6] is a matching of the nonperturbative solution at $r \ll r_m$ with the exponentially decreasing linear solution at $r \gg r_m$. It might happen indeed that the solution matches an exponentially increasing function instead.[2] Boulware and Deser in [7] expressed their doubts about the existence of large distance matching. Moreover, they argued that there is no consistent interacting theory of the massive spin-2 field in $3+1$ dimensions. One of the arguments in Ref. [7] was that at quantum level the theory contained a sixth polarization in addition to the standard five polarizations. Furthermore, the mass term in the action is not uniquely defined beyond quadratic order in the fields.

These legitimate concerns can be addressed by embedding the 4D theory of a massless graviton into a five-dimensional theory—a route we take in the present paper. Indeed, gravity in five dimensions is well defined as a classical gauge theory; a massless graviton has exactly five states. For the matter fields which are confined to the four-dimensional brane the theory mimics a massive spin-2 particle with the fifth component of the momentum playing the role of the mass.[3]

The model which we discuss is that of Ref. [8]. In this model matter is localized on a brane. The brane worldvolume theory contains the induced 4D Einstein-Hilbert term due to which a five-dimensional graviton mimics the massive four-dimensional spin-2 state on the brane. In contrast with the four-dimensional massive theory, in this case the full nonlinear action can be written. The two-body problem for sources on the brane is now well defined. The amplitude has the same generic form (1) with substitution of $q^2+m_g^2$ by $q^2+m_c q$, where m_c is the counterpart of m_g in the model. We present the arguments in favor of the aforementioned behavior of the form factors $a(q^2)$ and $b(q^2)$. However, we did not manage to obtain an exact solution of the Schwarzschild problem in this case either.

Instead, we derive a number of pieces of evidence supporting the conjectured behavior from the exact cosmological solutions [9,10] of the model. We show that the lowest tree-level perturbative result is off by a factor of 4/3 as compared with the exact result and explain why the corresponding perturbation theory breaks down. Based on this, we expect that the perturbative discontinuity is indeed absent on the nonperturbative level in the full classical theory.

Recently, the problem of the vanishing graviton mass was studied in a different setup. It was shown in Refs. [11] and [12] that there is no mass discontinuity even in the lowest tree-level exchange on de Sitter (dS) [11,13] or anti–de Sitter (AdS) [11,12] backgrounds.[4] This fits well with the discussion presented above. Indeed, in the case of the (A)dS background, even the lowest tree-level approximation does take into account the presence of a mass scale of the problem, which in that case is given by the cosmological constant Λ. It was shown in [12] that the coupling of the graviscalar is proportional to m_g^2/Λ when $m_g \to 0$, and deviations from the massless model vanish in this limit. Since the cosmological constant in our world is restricted to $\Lambda \leq 10^{-84}\,\mathrm{GeV}^2$, the allowed graviton mass is in the range $m_g \ll 10^{-42}\,\mathrm{GeV}$—i.e., the graviton Compton wavelength is bigger than our horizon size. The existence of such a tiny graviton mass is immaterial

[1] To avoid confusion, note that we use $t_{\mu\nu}$ only as a kinematical structure of the vertices, not implying that it is the energy-momentum tensor.

[2] Such a solution can still be acceptable as long as the exponential growth of the solution takes over at distances much larger than the observable size of the Universe. This will take place if the graviton Compton wavelength $\lambda_g \gg 10^{28}$ cm.

[3] Note the analogy with the supersymmetric states whose mass is given by a central charge. This charge also can be viewed as an extra component of the momentum in the dimensionally enlarged space.

[4] The consideration for dS space is somewhat subtle since for $m_g^2 < 2\Lambda/3$ (Λ being the cosmological constant) unitarity is violated in the theory [13].

for all astrophysical and cosmological observations [11] (see also an interesting discussion of the continuity issue in the recent work [14]). Note that the nonperturbative continuity allows for a much wider range for the graviton mass, $m_g \ll (r_M/r^5)^{1/4}$. Here r is the maximal distance from the Sun where the data are obtained; see Sec. II for the numerics.

In Ref. [15] it was argued that in the (A)dS background the perturbative discontinuity reappears at the one-loop quantum level—a phenomenon very similar to the one-loop discontinuity for massive non-Abelian vector fields discussed in [4]. This is certainly true since the loops are sensitive to the number of particles running in the loop diagrams. From the practical point of view, however, the comparison of the theory with the experimental data on light bending by the Sun and the precession of the Mercury perihelion is not affected by the small quantum loop corrections. Indeed, while the graviscalar decouples from the classical source it is still coupled to the graviton and does contribute to the quantum loops. However, such effects of quantum gravity are suppressed and most likely cannot be disentangled in solar system measurements. For these reasons, in what follows we focus on the (dis)continuity in the classical theory only.

The paper is organized as follows. In Sec. II we recall the essence of the graviton mass discontinuity found in Refs. [3–5] and discuss the results of Ref. [6] where it was shown that there is in fact a continuity in the graviton mass in the full classical theory. In Sec. III we introduce a five-dimensional nonlinear model that mimics the properties of a four-dimensional massive gravitational theory. We show that the perturbative discontinuity that is present in the lowest tree-level approximation disappears in the exact solution of the model. In Sec. IV we discuss another exact solution of the nonlinear model, which interpolates between the four- and five-dimensional regimes. We conclude in Sec. V.

II. PRELIMINARIES: MASSIVE GRAVITON IN 4D

We will consider the following action for a massive graviton on a flat 4D background:

$$S_m = M_{\rm Pl}^2 \int d^4x \sqrt{|g|}\left(R + \frac{m_g^2}{4}[h_{\mu\nu}^2 - (h_\mu^\mu)^2]\right), \tag{3}$$

where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and the Planck mass $M_{\rm Pl}$ is related to the Newton constant G_N as $M_{\rm Pl}^2 = 1/(16\pi G_N)$. The mass term has the Pauli-Fierz form [16]; it is the only form that does not introduce ghosts [17]. We imply that indices in the mass term are raised and lowered by the tensor $\eta_{\mu\nu}$. If it were $g_{\mu\nu}$ instead the difference would appear only in terms cubic and higher in $h_{\mu\nu}$, which are not fixed anyway; higher powers of $h_{\mu\nu}$ could be arbitrarily added to the mass term.

In order to see the presence of the discontinuity in the lowest tree-level approximation let us compare free graviton propagators in the massless and massive theories. For the massless graviton we find

$$D^0_{\mu\nu;\alpha\beta}(q) = \left(\frac{1}{2}\eta_{\mu\alpha}\eta_{\nu\beta} + \frac{1}{2}\eta_{\mu\beta}\eta_{\nu\alpha} - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}\right)\frac{1}{q^2 - i\epsilon}, \tag{4}$$

where only the momentum independent parts of the tensor structure are kept. By a gauge choice the momentum dependent structures can be taken to be zero. On the other hand, there is no gauge freedom for the massive gravity given by the action (3), and the propagator takes the following form:

$$D^m_{\mu\nu;\alpha\beta}(q) = \left(\frac{1}{2}\tilde\eta_{\mu\alpha}\tilde\eta_{\nu\beta} + \frac{1}{2}\tilde\eta_{\mu\beta}\tilde\eta_{\nu\alpha} - \frac{1}{3}\tilde\eta_{\mu\nu}\tilde\eta_{\alpha\beta}\right)\frac{1}{q^2 + m_g^2 - i\epsilon}, \tag{5}$$

where

$$\tilde\eta_{\mu\nu} = \eta_{\mu\nu} + \frac{q_\mu q_\nu}{m_g^2}. \tag{6}$$

Note the $1/m_g^4$ and $1/m_g^2$ singularities of the propagator.

The difference in the numerical coefficients for the $\eta_{\mu\nu}\eta_{\alpha\beta}$ structure in the massless and massive propagators (1/2 versus 1/3) is what leads to the perturbative discontinuity [3–5]. No matter how small the graviton mass is, the predictions are substantially different in the two cases. The structure (5) gives rise to contradictions with observations.

To see how this comes about let us calculate the amplitude of the lowest tree-level exchange by a graviton between two sources with energy-momentum tensors $T_{\mu\nu}$ and $T'_{\alpha\beta}$ (the tilde denotes the quantities that are Fourier transformed to momentum space):

$$\mathcal{A}_0 \equiv -8\pi G_N \tilde T_{\mu\nu} D_0^{\mu\nu;\alpha\beta}\tilde T'_{\alpha\beta} = -\frac{8\pi G_N}{q^2}\left(\tilde T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\tilde T_\beta^\beta\right)\tilde T'^{\mu\nu}. \tag{7}$$

In the massive case this amplitude takes the form

$$\mathcal{A}_m \equiv -8\pi G_N \tilde T_{\mu\nu} D_m^{\mu\nu;\alpha\beta}\tilde T'_{\alpha\beta} = -\frac{8\pi G_N}{q^2 + m_g^2}\left(\tilde T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}\tilde T_\beta^\beta\right)\tilde T'^{\mu\nu}. \tag{8}$$

In the relativistic normalization we are using $\tilde T_{\mu\nu} = \langle p|T_{\mu\nu}|p\rangle = 2p_\mu p_\nu$ at zero momentum transfer, $q=0$. Suppose we take two probe massive static sources with masses M_1 and M_2. Then only $\tilde T_{00}$ and $\tilde T'_{00}$ are nonvanishing and the lowest tree-level graviton exchange determines the Newtonian interaction,

$$V_0(r) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec q\vec r}\frac{\mathcal{A}_0}{4M_1M_2} = -\frac{G_N M_1 M_2}{r},$$

$$V_m(r)=\int \frac{d^3q}{(2\pi)^3}\, e^{\,i\vec{q}\vec{r}}\frac{\mathcal{A}_m}{4M_1M_2} = -\frac{4}{3}\,\frac{G_N M_1 M_2}{r}\,e^{-m_g r}. \tag{9}$$

Expressions (7) and (8) give different results for the Newtonian attraction even in the range $r \ll \lambda_g$ where one can neglect the exponential decrease. This difference can be eliminated by redefining the Newton coupling for the massive theory as follows:

$$\tilde{G}_N = \tfrac{4}{3}\, G_N, \tag{10}$$

where G_N is the Newton constant of the massless theory. For nonrelativistic problems the predictions of the massive theory with the coupling rescaled by a factor 3/4 at $m_g \to 0$ are identical to those of the massless theory with the coupling G_N.

However, this is not enough to warrant the viability of the massive model. The relativistic predictions in the two cases are different [4,5]. For instance, the predictions for light bending by the Sun are in conflict. At the classical level the trace of the energy-momentum tensor for light is zero. Therefore, the second term on the right-hand side of Eqs. (7) and (8) is not operative for light. Hence, the amplitudes $\mathcal{A}_0$ and $\mathcal{A}_m$ are identical in this case. However, we have established above that calculations in the massive theory should be performed with the rescaled Newton constant. Taking this fact into account, the prediction for light bending in the massive theory is off by 25% [3–5].

We could certainly take an opposite point of view, namely, do not rescale the Newton constant of the massive theory. In this case the predictions for light bending in the massive and massless models would be identical. However, the Newton force between static sources would differ by a factor of 4/3.

The above considerations are based on the lowest perturbative approximation. The question is whether these results hold in the full classical theory. Normally, one would expect that for solar system distances the lowest approximation is well justified. However, it was argued in Ref. [6] that the approximation breaks down in the massive theory for relatively short distances. Since this breaking manifests itself in a rather interesting way we will briefly summarize the results of Ref. [6] below.

To see the inconsistency of the perturbative expansion in G_N let us look (following [6]) at the Schwarzschild solution of Eq. (3). We parametrize the interval for a massive spherically symmetric body as follows:

$$ds^2 = -e^{\nu(\rho)}dt^2 + e^{\sigma(\rho)}d\rho^2 + e^{\mu(\rho)}\rho^2(d\theta^2+\sin^2\theta\, d\phi^2). \tag{11}$$

In the massless theory the function μ is redundant due to the reparametrization invariance of the theory; it can be put equal to zero. However, in the massive case this gauge symmetry is broken and μ is nonzero. Therefore, in order to compare the results in the massive and massless cases one has to do the substitution

$$r \equiv \rho \exp\left(\frac{\mu}{2}\right), \quad \exp(\lambda) \equiv \left(1+\frac{\rho}{2}\frac{d\mu}{d\rho}\right)^{-2}\exp(\sigma-\mu). \tag{12}$$

The standard Schwarzschild solution of the massless theory takes the following form:

$$\nu^{\rm Schw}(r) = -\lambda^{\rm Schw}(r) = \ln\left(1-\frac{r_M}{r}\right) = -\frac{r_M}{r} - \frac{1}{2}\left(\frac{r_M}{r}\right)^2 + \cdots,$$
$$\mu^{\rm Schw}(r)=0. \tag{13}$$

Here $r_M \equiv 2G_N M$ is the gravitational radius of the source of mass M.

Let us compare this with the perturbative in G_N solution of the massive theory obtained in Ref. [6]. In the leading plus next-to-leading approximation in G_N the solution reads

$$\nu \simeq -\frac{r_M}{r}\left[1+\frac{7}{32}\frac{r_M}{m_g^4 r^5}\right],$$
$$\lambda \simeq \frac{1}{2}\frac{r_M}{r}\left[1-\frac{21}{8}\frac{r_M}{m_g^4 r^5}\right],$$
$$\mu \simeq \frac{1}{2}\frac{r_M}{m_g^2 r^3}\left[1+\frac{21}{4}\frac{r_M}{m_g^4 r^5}\right]. \tag{14}$$

We note the following peculiarities of the results (14).

(1) In the leading order there is a finite discontinuity in the expression for λ: the result of the massless theory in Eq. (13) differs from the result of the massive model by a factor 1/2. This is precisely the discontinuity that is seen in the lowest approximation.

(2) The next-to-leading corrections in Eq. (14) are governed by the ratio $r_M/m_g^4 r^5$ and are singular in m_g.

(3) For any given distance r there is a value of m_g below which the perturbative expansion in G_N breaks down.

These results are in correspondence with the perturbative series for the scattering amplitude described by the Feynman graphs in Fig. 1. The leading terms in the expansions (14) are given by the diagram of first order in the source, i.e., the diagram with one cross. The terms singular in m_g in the propagator (5) do not contribute in this order. In the next order (the diagram with two crosses in Fig. 1) we have two extra propagators which could provide a singularity in m_g up to $1/m_g^8$. The two leading terms $1/m_g^8$ and $1/m_g^6$ do not contribute again so the result contains only the $1/m_g^4$ singularity as in Eq. (14).

To demonstrate how badly the expansion in powers of G_N breaks down let us take the largest allowed value for the graviton mass, $m_g = (10^{25}\,\text{cm})^{-1}$ [1,2] and calculate the correction to the leading result in the gravitational field of the Sun. We will find that at distances of the order of the solar system size, i.e., at $r \sim 10^{15}$ cm, the next-to-leading corrections in Eq. (14) are about 10^{32} times bigger than the leading terms. Therefore, this expansion is unacceptable.

For a light enough graviton, however, a consistent perturbative expansion could be organized in powers of m_g. In this case one finds [6]

$$\nu(r) = -\frac{r_M}{r} + \mathcal{O}\Big(m_g^2\sqrt{r_M r^3}\Big),$$

$$\lambda(r) = \frac{r_M}{r} + \mathcal{O}(m_g^2\sqrt{r_M r^3}),$$

$$\mu(r) = \sqrt{\frac{8r_M}{13r}} + \mathcal{O}(m_g^2 r^2), \tag{15}$$

where only the leading terms in r_M/r are retained. These expressions are valid in the following interval:

$$r_M \ll r \ll r_m, \qquad r_M = 2G_N M, \qquad r_m = \frac{(m_g r_M)^{1/5}}{m_g}. \tag{16}$$

For the gravitational field of the Sun this would correspond to the interval

$$3\times 10^5\ \text{cm} \ll r \ll 10^{21}\ \text{cm}, \tag{17}$$

where the lower bound is less than the radius of the Sun and the upper bound is of the order of a galaxy scale. Thus, for practical calculations within the solar system this expansion is well suited.

As we see, the expressions for ν and λ in the leading approximation coincide with those of the massless theory (13). Thus, there is *no mass discontinuity*. Moreover, the expressions (15) explicitly show nonanalyticity in G_N, that is $\mu \propto \sqrt{G_N}$, while in ν and λ nonanalytic terms are proportional to m_g^2.

We discussed in the Introduction subtle issues concerning the validity of the results discussed above arising even on the classical level: the nonlinear theory of massive gravity is not uniquely defined and it is complicated to make sure that the solutions which have no discontinuity do indeed satisfy the boundary conditions at infinity, i.e., that for $r \gg 1/m_g$ the solution matches the exponentially decreasing function.

As we already noted even the exponentially growing solution can be acceptable when the graviton Compton wavelength becomes larger than the observable size of the Universe. The Yukawa factors due to the graviton mass, $\exp(\pm m_g r)$, can be made to be arbitrarily close to unity by decreasing the graviton mass. However, as we discussed above, this does not warrant the continuity of the $m_g \to 0$ limit since the coefficients in front of the perturbative potentials in the massive and massless theories (9) are different and m_g independent. Therefore, the question of whether the graviton could have a nonzero mass effectively reduces to the question of whether the graviton could have five polarizations. Indeed, these extra polarizations are responsible for the m_g independent discontinuity in the coefficients in the potentials (9). Therefore, in what follows below we will address the question: *Can the graviton which describes the data in our observable Universe have five degrees of freedom*?

In the next section we present a model based on five dimensions where the massless graviton naturally has five degrees of freedom. The model is free of all the problems of the 4D massive gravity discussed above. We perform our analysis within this completely nonlinear theory in which exact solutions can be found. These solutions are compared with the perturbative results. We find that the picture outlined in the work [6] (and discussed above) holds.

III. BRANE MODEL OF MASSIVE GRAVITONS

The 5D model we will discuss was introduced in [8]. The gravitational part of the action takes the form

$$S = M_*^3 \int d^4x\, dy \sqrt{|G|}\mathcal{R} + M_{\rm Pl}^2 \int d^4x \sqrt{|g|} R(x), \tag{18}$$

where M_* is a parameter of the theory and $M_{\rm Pl} = 1.7 \times 10^{18}\ \text{GeV} \gg M_*$. Furthermore, G_{AB} is a 5D metric tensor, $A = \{\mu,5\} = \{0,1,2,3,5\}$, $\mathcal{R}$ is the five-dimensional Ricci scalar, and $g_{\mu\nu}$ denotes the induced metric on the brane, which we take as

$$g_{\mu\nu}(x) \equiv G_{\mu\nu}(x, y=0), \qquad \mu,\nu = 0,1,2,3, \tag{19}$$

neglecting the brane fluctuations.

We assume that our observable 4D world (4D matter) is confined to a tensionless brane (a tensionless hyperplane in this case) which is fixed at the point $y=0$ in the extra fifth dimension.[5] In other words, we assume that the energy-momentum tensor of 4D matter has the factorized form $T_{\mu\nu}(x)\,\delta(y)$. We also imply the presence of the Gibbons-Hawking boundary term on the brane; this provides the correct Einstein equations in the bulk. These simplifications help to keep the presentation clear and do not affect our main results. The brane world aspects of the model (18) were studied in detail in Refs. [8,18–20].

Let us study the gravitational potential between two static bodies located on the brane. This can be calculated from the action (18). The corresponding Green function is conveniently represented by working in momentum space in the four world-volume directions and in position space with respect to the transverse coordinate y. For the time being we can neglect the tensorial structure of the propagator (to be discussed below) and calculate the scalar part of the Green function. This can be done by calculating the corresponding propagator in a theory with scalars only, which have bulk and brane kinetic terms similar to Eq. (18). The result of the calculation reads as follows [8]:

$$\tilde{G}(q, y=0) = \frac{1}{M_{\rm Pl}^2}\frac{1}{q^2 + m_c\sqrt{q^2}}, \tag{20}$$

where we introduce the parameter

[5] A simplest possibility is to consider a brane at a fixed point of the $\mathbf{R}/\mathbf{Z}_2$ orbifold.

$$m_c \equiv \frac{1}{r_c} \equiv \frac{2M_*^3}{M_{\rm Pl}^2}. \tag{21}$$

The Green function (20) has unusual features. It has a tachyonic pole at $q^2 = -q_0^2 + q^{-2} = m_c^2$ which corresponds to decay into the continuous tower of Kaluza-Klein states (which arise from the reduction of the 5D graviton). Although the five-dimensional graviton is well defined, from the 4D perspective it looks like an unstable particle with width m_c. Nevertheless, the rules of integration for the propagator (20) in the complex energy plane can be defined consistently.

In particular, using Eq. (20) we can find the static potential $\phi(r)$. The result can be written in terms of special functions and has different asymptotic behavior for small and large distances (see Ref. [18]). The "crossover scale" between these two regimes is defined by r_c given in Eq. (21). At short distances, i.e., when $r \ll r_c$,

$$\phi(r) = -\frac{1}{8\pi^2 M_{\rm Pl}^2}\frac{1}{r}\left\{\frac{\pi}{2} + \left[-1+\gamma-\ln\left(\frac{r_c}{r}\right)\right]\left(\frac{r}{r_c}\right) + \mathcal{O}(r^2)\right\}. \tag{22}$$

Here $\gamma \simeq 0.577$ is the Euler constant. The leading term in this expression has the familiar $1/r$ scaling of the four-dimensional Newton law with the correct numerical coefficient. The leading correction is given by the logarithmic *repulsion* term in Eq. (22).

Let us turn now to the large distance behavior. For $r \gg r_c$ one finds

$$\phi(r) = -\frac{1}{16\pi^2 M_*^3}\frac{1}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right). \tag{23}$$

The long distance potential scales as $1/r^2$ in accordance with the 5D Newton law. Thus, the crossover scale (21) should be sufficiently large to avoid conflict with astronomical observations. In [8] it was estimated that for $M_* \sim 1$ TeV the crossover scale r_c is around 10^{15} cm, which is roughly the size of the solar system. This is too low to be consistent with data. Therefore, the scale M_* should be taken to be at least a couple of orders smaller than 1 TeV. This is in no conflict with any gravitational or Standard Model measurements (see the discussions in Refs. [19,20]). We take $r_c \geq 10^{25}$ cm, which corresponds to $M_* \leq 1$ GeV.

The parameter m_c plays a role in this model which in many respects is similar to that of the graviton mass m_g in Eq. (3). Indeed, as $m_c \to 0$, gravity on a brane becomes 4D Newtonian at larger and larger distances. Moreover, the four-dimensional interaction in the model with the action (18) can be interpreted as an exchange of a four-dimensional state with width equal to m_c [8]. In the next section we will find even closer similarities between m_c and m_g.

A. Perturbative discontinuity

To see that the model (18) exhibits a discontinuity in the one-graviton tree-level approximation let us calculate, following [8], the tensorial structure of the one-graviton exchange. To this end we will solve the Einstein equations in the linear approximation in h_{AB} which is the deviation from the flat 5D metric,

$$G_{AB} = \eta_{AB} + h_{AB}. \tag{24}$$

We choose the *harmonic gauge* in the bulk:

$$\partial^A h_{AB} = \tfrac{1}{2}\,\partial_B h_C^C. \tag{25}$$

In this gauge from the $\{\mu 5\}$ and $\{55\}$ components of the sourceless equations of motion it follows that

$$h_{\mu 5} = 0, \qquad h_5^5 = h_\mu^\mu. \tag{26}$$

Let us turn to the $\{\mu\nu\}$ components of the Einstein equations. After some simplifications they take the form

$$\begin{aligned}&[M_*^3 \partial_A \partial^A + M_{\rm Pl}^2 \delta(y)\partial_\alpha \partial^\alpha] h_{\mu\nu} \\ &\quad = -\left\{T_{\mu\nu} - \tfrac{1}{3}\eta_{\mu\nu}T_\alpha^\alpha\right\}\delta(y) + M_{\rm Pl}^2 \delta(y)\partial_\mu \partial_\nu h_\alpha^\alpha.\end{aligned} \tag{27}$$

There are two terms on the right-hand side of this equation. The first one has a structure which is identical to that of a massive 4D graviton (or, equivalently, of a massless 5D graviton). The second term on the right-hand side, which contains derivatives $\partial_\mu \partial_\nu$, is not important at the moment since it vanishes when it is contracted with the conserved energy-momentum tensor. As a result, the amplitude of interaction of two test sources takes the form

$$\tilde{h}_{\mu\nu}(q,y=0)\tilde{T}'^{\mu\nu}(q) \propto \frac{\tilde{T}^{\mu\nu}\tilde{T}'_{\mu\nu} - \frac{1}{3}\tilde{T}_\mu^\mu \tilde{T}'^\nu_\nu}{q^2 + m_c q}, \tag{28}$$

where $q \equiv \sqrt{q^2}$. We see that the tensor structure is the same as in the case of the massive 4D theory [see Eq. (8)].

In analogy with the discussions in the previous section we might expect that the lowest tree-level approximation will break down in the next iterations in the classical source. A further indication of this is the existence of the terms singular in m_c in the expression for the gravitational field $h_{\mu\nu}$ produced by a static source. We write the energy-momentum tensor for the source as follows:

$$T_{\mu\nu}(x) = -M\,\delta_{\mu 0}\delta_{\nu 0}\delta^{(3)}(\vec{x}), \tag{29}$$

where M is its rest mass. As before, let us make a Fourier transform with respect to four world-volume coordinates. Then the solution is as follows:

$$\tilde{h}_{00}(q,y) = c\,\frac{1}{2}\tilde{G}_N M \frac{1}{q^2 + m_c q}\exp(-q|y|), \tag{30}$$

$$\begin{aligned}\tilde{h}_{ij}(q,y) &= c\,\frac{1}{4}\tilde{G}_N M \frac{\delta_{ij}}{q^2 + m_c q}\exp(-q|y|) + c\tilde{G}_N M \\ &\quad \times \frac{q_i q_j}{m_c q}\frac{1}{q^2 + m_c q}\exp(-q|y|),\end{aligned} \tag{31}$$

where $c=-16\pi$. These expressions, taken at $y=0$, should be contrasted with the lowest order expressions for the Schwarzschild solution in 4D theory with a massless graviton:

$$\tilde{h}_{00}^{\rm Schw}(q)=c\,\frac{1}{2}\,G_N M\,\frac{1}{q^2}, \tag{32}$$

$$\tilde{h}_{ij}^{\rm Schw}(q)=c\,\frac{1}{2}\,G_N M\,\frac{\delta_{ij}}{q^2}. \tag{33}$$

Comparing the expressions (30), (31) to those in (32), (33) we draw the following three conclusions.

(i) Upon the substitution $\tilde{G}_N \to G_N$ the $\{00\}$ components coincide for large momenta, or, equivalently, for $r \ll r_c$.

(ii) The $\{ij\}$ component of the 5D theory consists of two terms. The first term, after the substitution $\tilde{G}_N \to G_N$, is twice as small as the corresponding term on the right hand side of the Schwarzschild solution (33). This is what gives rise to the discontinuity.

(iii) There is an additional term in the expression for $\tilde{h}_{ij}(q,y=0)$ which is proportional to

$$\frac{q_i q_j}{m_c q}.$$

This term does not contribute to the one-graviton exchange in leading order because of conservation of the energy-momentum tensors (the diagram with a single cross in Fig. 1). However, it does contribute to higher order diagrams (the ones with two and more cross in Fig. 1). This term is singular in m_c and the perturbation theory in G_N breaks down when $m_c \to 0$.

Given these arguments, we conclude that for a consistent calculation of the interaction between two sources on a brane we should find the Schwarzschild solution that sums up all the orders of the Born expansion for the classical equations. Unfortunately, we could not manage to find the analytic solution. However, implying the existence of a smooth limit $m_c \to 0$, one could perform the expansion in m_c in analogy with the 4D massive case [6].

The $\{\mu\nu\}$ component of the Einstein equation for the action (18) can be integrated with respect to y in the interval $-\epsilon \le y \le \epsilon$ with $\epsilon \to 0$. The resulting equation takes the form

$$\overline{\mathcal{G}_{\mu\nu}}(x)+m_c\int_{-\epsilon}^{+\epsilon}\mathcal{G}_{\mu\nu}(x,y)\,dy=-\frac{M}{2M_{\rm Pl}^2}\,\delta_{\mu 0}\,\delta_{\nu 0}\,\delta^{(3)}(x), \tag{34}$$

where $\overline{\mathcal{G}_{\mu\nu}}$ and $\mathcal{G}_{\mu\nu}(x,y)$ denote the Einstein tensor of the world-volume and bulk theories, respectively. Since the extrinsic curvature has a finite jump across the brane, the second term on the left-hand side of Eq. (34) is nonzero even in the limit $\epsilon \to 0$. This term is proportional to the parameter m_c with respect to which the expansion is performed (we imply that the metric is nonsingular in m_c; this seems to be a reasonable requirement for a physically meaningful solution).

Then it is clear from Eq. (34) that in the lowest approximation in m_c one recovers the usual 4D Schwarzschild solution of the massless theory (13). For the calculation of the subdominant corrections in m_c and for matching conditions at infinity, however, numerical simulations are needed. Note that in this case the solution should be matched at infinity to a well known 5D Schwarzschild solution which decreases as $(r_M/r)^2$ at infinity. This is an easier task compared to the 4D massive case where the power-law solution at short distances should be matched with the Yukawa potential at infinity.[6]

Does this mean that we cannot analytically compare the perturbative and nonperturbative results in the model (18)? Not at all. Instead of finding the exact Schwarzschild solution we perform a similar analysis for other solutions which can be obtained explicitly. In the next section we discuss an exact nonperturbative cosmological solution of the model (18) found in Refs. [9,10] which differs from the perturbative result by 4/3.

B. Nonperturbative continuity

In this section we study the cosmological solution in the model (18) found in Refs. [9] and [10]. It was already noticed in [9] that the cosmological evolution in Eq. (18) is governed by a Newton constant that differs from the "Newton" constant of perturbation theory by 4/3. We will discuss this discrepancy in detail.

Our goal is as follows. We consider the solution of the model (18) that describes the expansion of the matter dominated Universe. We will perform two distinct calculations for this. First we find the solution based on the Newtonian approximation. This calculation makes use of the lowest order potential between objects on the brane. As a second step we find the corresponding exact nonperturbative cosmological solution of the Einstein equations. In the domain where the Newtonian approximation is legitimate, the perturbative result for the cosmological solution would coincide under the normal circumstances with the nonperturbative one, as happens in a 4D world with a massless graviton. However, we find a discrepancy of a factor of 4/3 in these two methods.

Let us start with the perturbative approach. As we established in the previous subsection the one-graviton exchange in the lowest approximation gives rise to the following expression for the potential of a massive source at short distances $r \ll r_c$:

$$\phi(r)=-\tilde{G}_N\frac{M}{r}. \tag{35}$$

The appearance of the constant $\tilde{G}_N$ instead of G_N in this expression is related to the fact that we used the lowest tree-level approximation.

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[6] In Ref. [21] the asymptotic form of the Schwarzschild solution for $m_c \to 0$ was also discussed and, moreover, certain generalizations of cosmological solutions of the model (18) were obtained.

Let us now use the standard consideration of Newtonian cosmology.[7] Consider a spherical ball with some uniform matter density in it. We assume that the radius of the ball R is much smaller then r_c and that we are in a regime where the Newtonian approximation is valid. In this case the potential of the ball on its surface takes the form

$$\phi_{\rm ball}(r=R)=-\tilde{G}_N\frac{M}{R}. \tag{36}$$

Let us consider a pointlike probe particle of mass m_0 which is located right on the surface of the ball. We neglect the back reaction of this probe particle on the ball. The energy conservation condition for the system of the ball and probe takes the form

$$\frac{m_0\dot{R}^2}{2}-\tilde{G}_N\frac{Mm_0}{R}=\frac{km_0}{2}, \tag{37}$$

where the overdot denotes the time derivative and k is some constant. We would like to calculate the time evolution of the radius R. In the regime under consideration this is equivalent to the time evolution of the scale factor in Friedmann-Lemaitre-Robertson-Walker cosmology. In what follows we consider the solution that corresponds to the expansion of a flat, i.e., $k=0$, matter dominated Universe. For $k=0$ we rewrite Eq. (37) as follows:

$$\left(\frac{\dot{R}}{R}\right)^2=\frac{8\pi}{3}\tilde{G}_N\rho, \tag{38}$$

where the density ρ for the matter dominated Universe is related to the scale factor R as follows:

$$\rho=\frac{u}{R^3}, \tag{39}$$

where u is some constant.

This is nothing but the Friedmann equation for the scale factor R for a flat matter dominated Universe. We find the solution for the scale factor:

$$R^3(t)=6\pi u\tilde{G}_N t^2. \tag{40}$$

This solution is consistent with the fact that we choose the time period when $R\ll r_c$ so that the brane world evolves in accordance with the laws of 4D theory. What is important in our solution is the numerical coefficient in the relation (40), which different from that in the 4D massless gravity case—it contains $\tilde{G}_N=(4/3)G_N$ instead of G_N. Below we will show that the exact solution matches the one in massless gravity in the limit $m_c\to 0$.

Before discussing the exact solution let us explain why the Newtonian approach outlined above does not produce a correct coefficient. It is due to the effects of nonlinear terms: similar to the Schwarzschild problem in 4D massive gravity discussed in Sec. II these corrections are defined by powers of the parameter

$$\frac{Gu}{m_c^2R^3}\sim\frac{1}{m_c^2t^2}. \tag{41}$$

It is clear that these corrections blow up at $m_c\to 0$ and we need to sum them up. The corrections seem to be small at the later time $t\gg 1/m_c$, but as we will see the 4D approach stops working at this epoch.

Let us now solve the same problem using the exact Einstein equations. We parametrize the 5D interval in the following form:

$$ds^2=-N^2(t,y)dt^2+A^2(t,y)dx_idx^i+B^2(t,y)dy^2. \tag{42}$$

The 4D scale factor is defined as follows:

$$R(t)\equiv A(t,y=0). \tag{43}$$

The solution was found in [9] and [10]:

$$N=1-|y|\frac{\ddot{R}}{\dot{R}},\quad A=R-|y|\dot{R},\quad B=1, \tag{44}$$

and the 4D scale factor obeys the following modified Friedmann equation:

$$\left(\frac{\dot{R}}{R}\right)^2=\frac{8\pi}{3}G_N\rho-m_c\frac{\dot{R}}{R}. \tag{45}$$

The $m_c\to 0$ limit of this equation is clearly incompatible with Eq. (38), which is based on the leading order approximation in the massive theory, but coincides with the result of massless gravity. This certainly implies that the Hubble parameter $\dot{R}/R$ is continuous in this limit—an assertion we verify below by presenting the exact solution of Eq. (45).

We can absorb the parameters m_c and G_N in Eq. (45) by rescaling:

$$t=\frac{\tau}{m_c},\quad \rho=\frac{3m_c^2}{32\pi G_N}\tilde{\rho},$$
$$\left(\frac{\tilde{\rho}'}{\tilde{\rho}}\right)^2=\frac{9}{4}\tilde{\rho}+3\frac{\tilde{\rho}'}{\tilde{\rho}},\quad \tilde{\rho}'=\frac{d\tilde{\rho}}{d\tau}. \tag{46}$$

After introducing the variable

$$x\equiv 1+\tilde{\rho}=1+\frac{32\pi G_N}{3m_c^2}\rho, \tag{47}$$

the exact solution can be written in terms of elementary functions for $\tau(x)$:

$$\frac{3}{2}m_ct=\frac{1}{\sqrt{x}-1}+\frac{1}{2}\log\frac{\sqrt{x}+1}{\sqrt{x}-1}. \tag{48}$$

[7]For a careful treatment and interpretation of Newtonian cosmology see, e.g., [22].

When $\tau = m_c t \gg 1$ we get for the scale factor

$$R^3 = \frac{8\pi G_N u}{m_c} t \left[1 - \frac{\log(3m_c t) + 1}{3m_c t} + \cdots \right]. \tag{49}$$

This unusual behavior [compare with the 4D Newtonian cosmology in Eq. (40)] is typical of a pure brane cosmology regime [23] where one has $H^2 \propto \rho^2$—indeed, $G_N/m_c = 1/(32\pi M_*^3)$ plays the role of G_N in the 5D world. It is only relevant to the late time cosmology, $t \gg 1/m_c$—the epoch where the Hubble parameter is small, $H \sim 1/t \ll m_c$, and the expansion enters the 5D regime, as analyzed in [9]. Therefore, 4D Newtonian cosmology is not applicable at this epoch.

For $\tau = m_c t \ll 1$,

$$R^3 = 6\pi G_N u t^2 \left[1 - \frac{3}{4} m_c t + \cdots \right]. \tag{50}$$

In correspondence with the difference of Eqs. (45) and (38), discussed above we see that R^3 at $m_c = 0$ is different from the expression in Eq. (40) that was obtained using the lowest tree-level approximation by the same factor 3/4—it contains G_N instead of $\tilde{G}_N$. Note that the exact expression for R^3 is linear in G_N—no higher orders are present.

The exact solution considered above gives an explicit demonstration of the nonperturbative continuity in the limit $m_c \to 0$. This continuity is not uniform—for the given value of t the parameter m_c should be much smaller than $1/t$. This is the strongest constraint on the graviton mass coming from cosmology, $m_c \ll H_0$, where $H_0 \sim 10^{-42}$ GeV is the present day Hubble parameter.

IV. INTERPOLATING SOLUTION

In this section we discuss a cosmological solution found in [10] and show that it interpolates between the regimes with 4D and 5D tensor structures.

Let us start with the brane action (18) and in addition introduce in the theory a negative cosmological constant on the brane Λ_b and the matter density $\rho \geq |\Lambda_b|$ (we put the pressure equal to zero for simplicity). The time evolution of such a 4D brane universe is interesting; it evolves asymptotically to a static Minkowski space on the brane without any fine-tuning [10]. The asymptotic form of the metric is as follows:

$$ds^2 = -(1 + b|y|)^2 dt^2 + dx^i dx_i + dy^2, \tag{51}$$

where the constant b is

$$b \equiv |\Lambda_b|/4M_*^3. \tag{52}$$

In fact, this is a solution to the equation

$$\mathcal{R}_{AB} - \frac{1}{2} G_{AB} \mathcal{R} = \frac{1}{2M_*^3} T_{AB}(x)\,\delta(y), \tag{53}$$

where the energy-momentum tensor on the brane is

$$T_{\mu\nu} = \mathrm{diag}(0, -\Lambda_b, -\Lambda_b, -\Lambda_b), \quad T_{5\mu} = T_{55} = 0, \tag{54}$$

i.e., $\rho + \Lambda_b \to 0$ in this limit. To warrant the 4D behavior, the induced 4D Ricci scalar on the brane was added in [10].

The important thing is that the early cosmology of this model is standard, with no discontinuity in the Newton constant. Indeed, the Friedmann equation is given in Eq. (45) where ρ should be substituted by $\rho + \Lambda_b$. The Newton constant on the right-hand side of this equation is the conventional 4D gravitational constant which reflects *no discontinuity*. This is true as far as the early cosmology is concerned.

Let us now look at the late cosmology, or more precisely at the form of the metric (51) to which the solution asymptotes. The metric on the brane is Minkowskian and static everywhere with only dependence on y. For small values of y, which satisfy $b|y| \ll 1$, this metric can be obtained as a perturbation on the flat Minkowski space. Indeed, for small perturbations (24) in the harmonic gauge (25) we find Eq. (27) with the energy-momentum tensor defined in Eq. (54). This equation has the 5D tensor structure on the right hand side. Let us now note that the energy-momentum tensor (54) satisfies the relation

$$T_{ij} - \tfrac{1}{3} T \eta_{ij} = 0, \quad i,j = 1,2,3. \tag{55}$$

Therefore, the equation for h_{ij} is simplified. This is completely due to the 5D tensor structure; in fact if we had a 4D tensor structure this would not be so. Furthermore, the solution of Eq. (27) in the gauge (25) can be written in the following form:

$$h_{00} = -h_{55} = -\frac{|\Lambda_b|}{2M_*^3}|y|, \quad h_{ij} = 0, \quad h_{\mu 5} = 0. \tag{56}$$

One can indeed verify that this solution coincides to first order with the exact solution (51). For this we perform the following gauge transformation of the exact solution (the two different signs correspond to the two sides of the brane):

$$y = \mathrm{sgn}(z) \frac{1}{b} [(1 + 2b|z| + 2b^2 z^2)^{1/2} - 1]. \tag{57}$$

After this the metric takes the form

$$ds^2 = -(1 + 2b|z| + 2b^2 z^2) dt^2 + dx^i dx_i + \frac{(1 + 2b|z|)^2}{1 + 2b|z| + 2b^2 z^2} dz^2, \tag{58}$$

which in leading order coincides with the perturbative solution.

Therefore, we conclude that the cosmological solution of Ref. [10] does indeed provide an explicit example with both asymptotic regimes: at small distances (small Hubble radius) the behavior is four dimensional with the 4D tensor structure, whereas at large distances (large Hubble radius) the behavior has the 5D tensor structure. In this sense the solution discussed above captures the important features of a

Schwarzschild solution of 4D massive theory; this is not surprising since it is asymptotically (in time) Minkowski on the brane.

V. DISCUSSION AND CONCLUSIONS

We discussed a nonlinear five-dimensional generally covariant model which resembles many crucial properties of a massive graviton in four dimensions. The mass discontinuity is present in the lowest tree-level approximation; however, this approximation breaks down for vanishing graviton mass and all the tree-level graphs should be taken into account. The resulting expression for the nonperturbative classical calculation is continuous in the graviton mass. Thus, there is no mass discontinuity in the full classical theory.

There are three extra degrees of freedom in the massive (or five-dimensional) theory compared to the massless one. Among these degrees of freedom only the helicity-0 state (the graviscalar) has a nonzero coupling to 4D matter. However, this coupling tends to zero in the full classical theory as the graviton mass (or m_c in the 5D example) vanishes. Thus, all the extra degrees of freedom decouple in the massless limit.

The interesting issue that we did not discuss in the paper is the emission of a helicity-0 graviton. Based on our observations and using the unitarity arguments we expect that the nonperturbative amplitudes of the radiation of the helicity-0 state by 4D matter fields will also vanish with the graviton mass, while they are nonvanishing in the lowest tree-level approximation as was shown in Ref. [24].

In the small mass limit the extra degrees of freedom of a massive theory form an independent sector which decouples from our matter as the graviton mass goes to zero. These degrees of freedom do interact with each other; moreover, in perturbation theory these interactions are singular in the limit $m_g \rightarrow 0$. Certainly, on top of the classical effects there is the issue of quantum loops, which we did not discuss in the present work. However, the loop effects are suppressed and most likely they cannot be disentangled in existing measurements.

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[1] Particle Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
[2] A. S. Goldhaber and M. M. Nieto, Phys. Rev. D **9**, 1119 (1974); C. M. Will, *ibid.* **57**, 2061 (1998); J. Uzan and F. Bernardeau, *ibid.* **64**, 083004 (2001); P. Binetruy and J. Silk, Phys. Rev. Lett. **87**, 031102 (2001).
[3] Y. Iwasaki, Phys. Rev. D **2**, 2255 (1970).
[4] H. van Dam and M. Veltman, Nucl. Phys. **B22**, 397 (1970).
[5] V. I. Zakharov, JETP Lett. **12**, 312 (1970).
[6] A. I. Vainshtein, Phys. Lett. **39B**, 393 (1972).
[7] D. G. Boulware and S. Deser, Phys. Rev. D **6**, 3368 (1972).
[8] G. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B **485**, 208 (2000).
[9] C. Deffayet, Phys. Lett. B **502**, 199 (2001).
[10] C. Deffayet, G. Dvali, G. Gabadadze, and A. Lue, Phys. Rev. D **64**, 104002 (2001).
[11] I. I. Kogan, S. Mouslopoulos, and A. Papazoglou, Phys. Lett. B **503**, 173 (2001).
[12] M. Porrati, Phys. Lett. B **498**, 92 (2001).
[13] A. Higuchi, Nucl. Phys. **B282**, 397 (1987); **B325**, 745 (1989).
[14] I. I. Kogan, S. Mouslopoulos, A. Papazoglou, and L. Pilo, hep-th/0105255.
[15] F. A. Dilkes, M. J. Duff, J. T. Liu, and H. Sati, Phys. Rev. Lett. **87**, 041301 (2001); M. J. Duff, J. T. Liu, and H. Sati, Phys. Lett. B **516**, 156 (2001).
[16] M. Fierz, Helv. Phys. Acta **12**, 3 (1939); M. Fierz and W. Pauli, Proc. R. Soc. London **A173**, 211 (1939).
[17] P. van Nieuwenhuizen, Nucl. Phys. **B60**, 478 (1973).
[18] G. Dvali and G. Gabadadze, Phys. Rev. D **63**, 065007 (2001).
[19] G. Dvali, G. Gabadadze, M. Kolanovic, and F. Nitti, Phys. Rev. D **64**, 084004 (2001).
[20] C. Deffayet, G. Dvali, and G. Gabadadze, Phys. Rev. D (to be published), astro-ph/0105068.
[21] R. Dick, Class. Quantum Grav. **18**, R1 (2001).
[22] G. Dautcourt, gr-qc/9603001.
[23] P. Binetruy, C. Deffayet, and D. Langlois, Nucl. Phys. **B565**, 269 (2000).
[24] P. van Nieuwenhuizen, Phys. Rev. D **7**, 2300 (1973).