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Current constraints on the dark energy equation of state

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We combine complementary datasets from cosmic microwave background (CMB) anisotropy measurements, high redshift supernovae (SNIa) observations and data from local cluster abundances and galaxy clustering (LSS) to constrain the dark energy equation of state parametrized by a constant pressure-to-density ratio w_Q. Under the assumption of flatness, we find $w_Q < -0.85$ at 68% C.L., providing no significant evidence for quintessential behavior different from that of a cosmological constant. We then generalize our result to show that the constraints placed on a constant w_Q can be safely extended to dynamical theories. We consider a variety of quintessential dynamical models based on inverse power law, exponential and oscillatory scaling potentials. We find that SNIa observations are ''numbed'' to dynamical shifts in the equation of state, making the prospect of reconstructing $w(z)$ a challenging one indeed.

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I. INTRODUCTION

The discovery that the universe's evolution may be dominated by an effective cosmological constant [1] is one of the most remarkable cosmological findings of recent years. An exceptional opportunity is now opening up to decipher the nature of dark matter [2], to test the veracity of theories and reconstruct the dark matter's profile using a wide variety of observations over a broad redshift range.

One matter candidate that could possibly explain the observations is a dynamical scalar ''quintessence'' field. One of the strong aspects of quintessence theories is that they go some way to explaining the fine-tuning problem, why the energy density producing the acceleration is $\sim 10^{-120} M_{pl}^4$. A vast range of ''tracker'' (see for example [3,4]) and ''scaling'' (for example [5–8]) quintessence models exist which approach attractor solutions, giving the required energy density, independent of initial conditions. The common characteristic of quintessence models is that their equations of state, $w_Q = p/\rho$, vary with time whilst a cosmological constant remains fixed at $w_{Q=\Lambda} = -1$. Observationally distinguishing a time variation in the equation of state or finding w_Q different from -1 will therefore be a success for the quintessential scenario.

In this paper we will combine the latest observations of the cosmic microwave background (CMB) anisotropies provided by the Boomerang [9], DASI [10] and Maxima [11] experiments and the information from large scale structure (LSS) with the luminosity distance of high redshift supernovae (SNIa) to put constraints on the dark energy equation of state parametrized by a redshift independent quintessence-field pressure-to-density ratio w_Q. We will also make use of the Hubble Space Telescope (HST) constraint on the Hubble parameter $h = 0.72 \pm 0.08$ [12]. We will then also consider whether one can feasibly extract information about the time variation of w from observations.

The importance of combining different data sets in order to obtain reliable constraints on w_Q has been stressed by many authors (see e.g. [13,15,16]), since each dataset suffers from degeneracies between the various cosmological parameters and w_Q. Even if one restricts consideration to flat universes and to a value of w_Q constant with time the SNIa luminosity distance and position of the first CMB peak are highly degenerate in w_Q and Ω_Q, the energy density in quintessence.

The paper is therefore structured as follows. In Secs. II and III we will present the CMB, SNIa and LSS data used in the analysis. In Sec. IV we will present the results of our analysis. We will consider the implications for a dynamical w_Q in Sec. V. Section VI will be devoted to the discussion of the result and the conclusions.

II. CONSTRAINTS FROM CMB

The effects of quintessence on the angular power spectrum of the CMB anisotropies are several [15,17,18]. In the class of models we are considering, however, with a negligible value of Ω_Q in the early universe in order to satisfy the big bang nucleosynthesis (BBN) bound [19], the effects can be reduced to just two.

First, since the inclusion of quintessence changes the overall content of matter and energy, the angular diameter distance of the acoustic horizon size at recombination will be altered. In flat models (i.e. where the energy density in matter is equal to $\Omega_M = 1 - \Omega_Q$), this creates a shift in the peaks positions of the angular spectrum as

$$\mathcal{R} = \sqrt{(1-\Omega_Q)}\, y,$$

$$y = \int_0^{z_{dec}} [(1-\Omega_Q)(1+z)^3 + \Omega_Q (1+z)^{3(1+w_Q)}]^{-1/2} dz. \quad (1)$$

It is important to note that the effect is completely degenerate in the interplay between w_Q and Ω_Q. Furthermore, it does

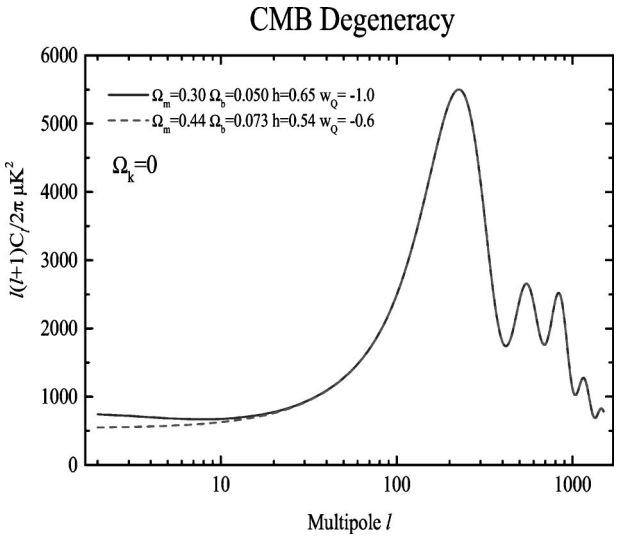

FIG. 1. CMB power spectra and the angular diameter distance degeneracy. The models are computed assuming flatness, $\Omega_k=1-\Omega_M-\Omega_Q=0$). The integrated Sachs-Wolfe effect on large angular scale slightly breaks the degeneracy. The degeneracy can be broken with a strong prior on h, in this paper we use the results from the HST.

not qualitatively add any new features additional to those produced by the presence of a cosmological constant [21] and it is not highly sensitive to further time dependencies of w_Q.

Secondly, the time-varying Newtonian potential after decoupling will produce anisotropies at large angular scales through the integrated Sachs-Wolfe (ISW) effect. The curve in the CMB angular spectrum on large angular scales depends not only on the value of w_Q but also its variation with redshift. However, this effect will be difficult to disentangle from the same effect generated by a cosmological constant, especially in view of the affect of cosmic variance and/or gravity waves on the large scale anisotropies.

In order to emphasize the importance of degeneracies between all these parameters while analyzing the CMB data, we plot in Fig. 1 some degenerate spectra, obtained keeping the physical density in matter $\Omega_M h^2$, the physical density in baryons $\Omega_b h^2$ and $\mathcal{R}$ fixed. As we can see from the plot, models degenerate in w_Q can be constructed. However, as we will utilize in the next sections, the combination of the different datasets can break the mentioned degeneracies.

To constrain w_Q from CMB, we perform a likelihood analysis comparing the recent CMB observations with a set of models with cosmological parameters sampled as follows: $0.1<\Omega_m<1.0$, $-1.0\leqslant w_Q\leqslant -0.55$, $0.015<\Omega_b<0.20$; $0<\Omega_Q<0.9$ and $0.45<h<0.95$. We vary the spectral index of the primordial density perturbations within the range $n_s=0.60,\ldots,1.40$, we allow for a possible reionization of the intergalactic medium by varying the CMB photons optical depth in the range $0.0<\tau_C<0.4$ and we re-scale the fluctuation amplitude by a pre-factor C_{10}, in units of C_{10}^{COBE}. We also restrict our analysis to *flat* models, $\Omega_{tot}=1$, and we add a conservative external prior on the age of the universe $t_0>10$ Gyrs (see e.g. [24]).

In order to speed-up the computation time of the theoretical models for different w_Q we make use of a k-splitting technique [20]. Basically a $w_Q=-1$ and $w_Q>-1$ model are calculated in two different ways. For low l the 2 models are computed in the ordinary way by solving the Boltzmann equation, in order to properly take into account the ISW effect. For the high l just a flat, $w_Q=-1$ model is calculated. This $w_Q=-1$ model is then shifted using the expression for the angular diameter distance in Eq. (1) to obtain the $w_Q>-1$ models.

The theoretical models are computed using a modified version of the publicly available CMBFAST program [22] and are compared with the recent BOOMERanG-98, DASI and MAXIMA-1 results. The power spectra from these experiments were estimated in 19, 9 and 13 bins respectively, spanning the range $25\leqslant l\leqslant 1100$. We approximate the experimental signal C_B^{ex} inside the bin to be a Gaussian variable, and we compute the corresponding theoretical value C_B^{th} by convolving the spectra computed by CMBFAST with the respective window functions. When the window functions are not available, as in the case of Boomerang-98, we use top-hat window functions. The likelihood for a given cosmological model is then defined by $-2\ln\mathcal{L}=(C_B^{th}-C_B^{ex})M_{BB'}(C_{B'}^{th}-C_{B'}^{ex})$ where C_B^{th} (C_B^{ex}) is the theoretical (experimental) band power and $M_{BB'}$ is the Gaussian curvature of the likelihood matrix at the peak. We consider 10%, 4% and 4% Gaussian distributed calibration errors (in μK) for the BOOMERanG-98, DASI and MAXIMA-1 experiments respectively and we take into account for the beam error in BOOMERanG-98 by analytic marginalization as in [23]. We also include the Cosmic Background Explorer (COBE) data using Knox's RADPACK packages.

III. CONSTRAINTS FROM SUPERNOVAE

Evidence that the universe's expansion rate was accelerating was first provided by two groups, the SCP and High-Z Search Team [1] using type Ia supernovae (SNIa) to probe the nearby expansion dynamics. SNIa are good standard candles, as they exhibit a strong phenomenological correlation between the decline rate and peak magnitude of the luminosity. The observed apparent bolometric luminosity is related to the luminosity distance, measured in Mpc, by $m_B=M+5\log d_L(z)+25$, where M is the absolute bolometric magnitude. The luminosity distance is sensitive to the cosmological evolution through an integral dependence on the Hubble factor $d_l=(1+z)\int_0^z[dz'/H(z',\Omega_Q,w_q)]$ and therefore can be used to constrain the scalar equation of state. We evaluate the dark energy Ω/w likelihoods assuming a constant equation of state, such that $H(z)=\rho_0\Sigma_i\Omega_i(1+z)^{(3+3w_i)}$. The predicted m_{eff} is then calculated by calibration with low-z supernovae observations [25] where the Hubble relation $d_l\approx H_0 cz$ is obeyed. We calculate the likelihood, $\mathcal{L}$, using the relation $\mathcal{L}=\mathcal{L}_0\exp(-\chi^2(\Omega,w_Q)/2)$ where $\mathcal{L}_0$ is an arbitrary normalization and χ^2 is evaluated using the observations of the SCP group, marginalizing over H_0. As can be seen in Fig. 2 there is an inherent degeneracy in the luminosity distance in the Ω_M/w_Q plane; one can see

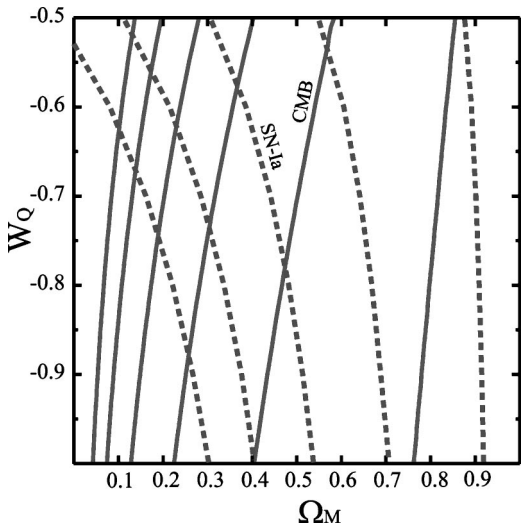

FIG. 2. Contours of constant R (CMB) and SNIa luminosity distance in the w_Q-Ω_M plane. The degeneracy between the two distance measures can be broken by combining the two sets of complementary information. The luminosity distance is chosen to be equal to d_l at $z=1$ for a fiducial model with $\Omega_\Lambda=0.7$, $\Omega_M=0.3$, $h=0.65$. (We note that as $\Omega_Q=1-\Omega_M$ goes to zero the dependence of R and d_L upon w_Q also become zero, as there is no dark energy present.)

that little can be found out about the equation of state from luminosity distance data alone. However, the degeneracies of CMB and SNIa data complement one another so that together they offer a more powerful approach for constraining w_Q.

IV. RESULTS

Table I shows the 1σ constraints on w_Q for different combinations of priors, obtained after marginalizing over all remaining *nuisance* parameters. The analysis is restricted to *flat* universes. One can see that w_Q is poorly constrained from CMB data alone, even when the HST strong prior on the Hubble parameter, $h=0.72\pm0.08$, is assumed. Adding a big bang nucleosynthesis prior, $\Omega_b h^2=0.020\pm0.005$, has small effect on the CMB+HST result. Adding SNIa breaks

TABLE I. Constraints on w_Q and $\Omega_M=1-\Omega_Q$ using different priors and datasets. We always assume flatness and $t_0>10$ Gyr. The 1σ limits are found from the 16% and 84% integrals of the marginalized likelihood. The HST prior is $h=0.72\pm0.08$ while for the BBN prior we use the conservative bound $\Omega_b h^2=0.020\pm0.005$.

CMB+HST	$w_Q<-0.62$
	$0.15<\Omega_M<0.45$
CMB+HST+BBN	$-0.95<w_Q<-0.62$
	$0.15<\Omega_M<0.42$
CMB+HST+SNIa	$-0.94<w_Q<-0.74$
	$0.16<\Omega_M<0.34$
CMB+HST+SNIa+LSS	$w_Q<-0.85$
	$0.28<\Omega_M<0.43$

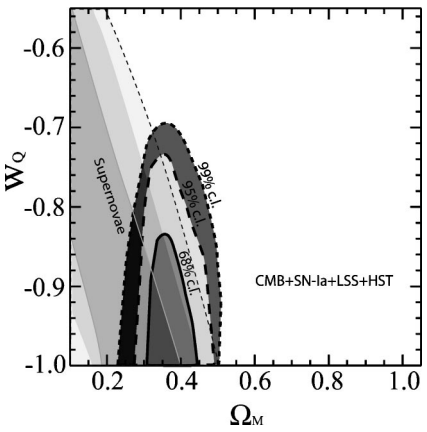

FIG. 3. The likelihood contours in the (Ω_M, w_Q) plane, with the remaining parameters taking their best-fitting values for the joint CMB+SNIa+LSS analysis described in the text. The contours correspond to 0.32, 0.05 and 0.01 of the peak value of the likelihood, which are the 68%, 95% and 99% confidence levels respectively.

the CMB Ω_Q-w_Q degeneracy and improves the upper limit on w_Q, with $w_Q<-0.74$. Finally, including information from local cluster abundances through $\sigma_8=(0.55\pm0.1)\Omega_M^{-0.5}$, where σ_8 is the rms mass fluctuation in spheres of $8h^{-1}$ Mpc, further breaks the quintessential-degeneracy, giving $w_Q<-0.85$ at 1σ. Also reported in Table I are the constraints on Ω_M. As we can see, the combined data suggests the presence of dark energy with high significance, even in the case CMB+HST. It is interesting to project our likelihood in the Ω_Q-w_Q plane. Proceeding as in [26], we

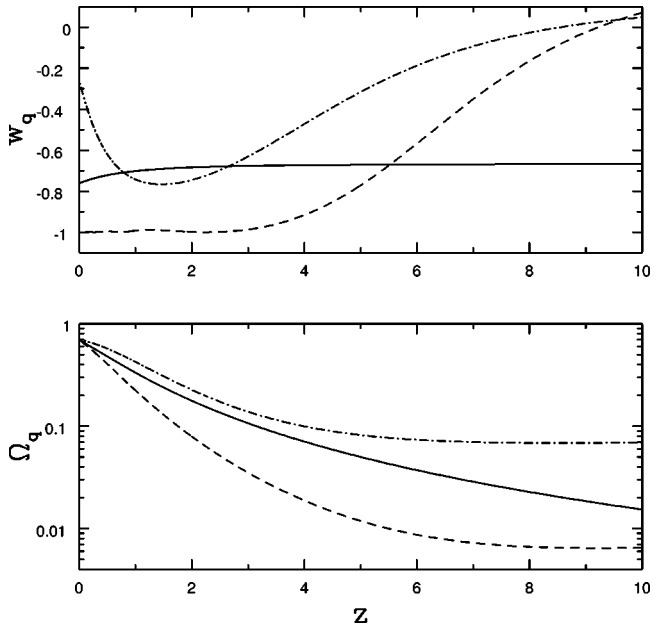

FIG. 4. The variation of Ω_Q and w_Q with redshift for the three models described in Sec. V. The power law potential (full line) w_Q shows a steady small variation, the exponential feature potential, (short dashed line) acts remarkably like a cosmological constant at late times, after a deviation from behaving like normal matter with $w_Q\sim0$, whilst the oscillating potential (dot-dash) shows a continual variation in w_Q.

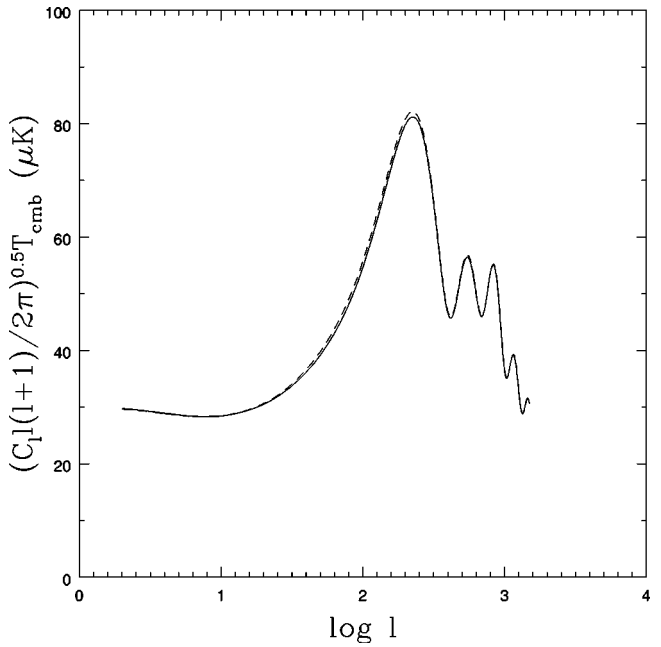

FIG. 5. Comparison of CMB temperature power spectra for the dynamical quintessence model with an exponential potential with a feature described in Sec. V, and a model with constant $w_Q = w_{eff}$ for the dynamical model. In both cases with $\Omega_Q = 0.7$ and $H_0 = 65$. One can see that the constant model is a remarkably good approximation to the dynamical model, despite the equation of state of the dynamical model varying significantly from the effective value from recombination until nowadays.

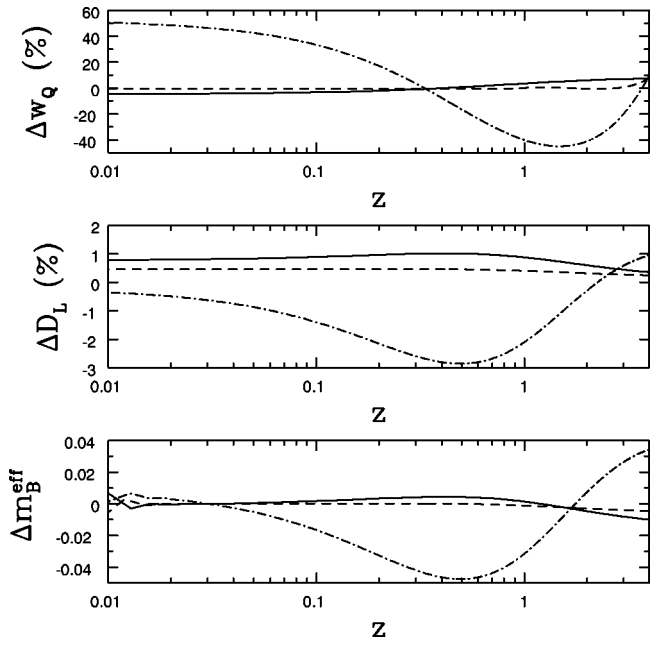

FIG. 6. The implication of a deviation $\Delta w_Q = 1 - w_{eff}/w_Q$ on $\Delta D_L = 1 - D_L(w_{eff})/D_L(w_Q)$ and $\Delta m_B = m_B(w_Q) - m_B(w_{eff})$ for the three models in Sec. V. The tracker potential (full line) has $w_{eff} = -0.727$, the exponential potential with feature (dashed line) has $w_{eff} = -0.993$ and the oscillatory potential (dot-dash) has $w_{eff} = -0.529$. Notice that D_L and m_B are in turn both desensitized to any variation in w_Q.

attribute a likelihood to a point in the (Ω_M, w_Q) plane by finding the remaining parameters that maximize it. We then define our 68%, 95% and 99% contours to be where the likelihood falls to 0.32, 0.05 and 0.01 of its peak value, as would be the case for a two dimensional multivariate Gaussian. In Fig. 3 we plot likelihood contours in the (Ω_M, w_Q) plane for the joint analyses of CMB+SNIa+HST+LSS data together with the contours from the SNIa dataset only. As we can see, the combination of the data breaks the luminosity distance degeneracy.

V. CONSTRAINING DYNAMICAL MODELS

In the previous section we obtained bounds on the equation of state parameter w_Q by assuming it is a constant, independent of the redshift. However, in quintessential models the equation of state can vary with time. It is therefore useful to discuss how well our constraints on w_Q apply to less trivial models. There are a wide variety of quintessential models; we illustrate our analysis using representatives of three of the most general classes of model: the inverse power law, $V(\phi) = V_0/\phi^p$ [3] with $p = 1$, an exponential scaling potential with a feature, $V(\phi) = V_0 e^{-\lambda\phi}[A + (\phi - \phi_0)^2]$ [18], with $\lambda = 10$, $A = 0.008$, $\phi_0 = 25.8$, and an oscillatory scaling potential, $V(\phi) = V_0 e^{-\lambda\phi}[1 + A\sin(\nu\phi)]$ [27] with $\lambda = 4$, $A = 0.98$, $\nu = 0.51$. In each case V_0 is chosen so that $\Omega_Q = 0.7$ and H_0 is 65. The particular time dependent characteristics of each are shown in Fig. 4.

If we are to constrain dynamical models, we need to understand how well the effect of a time varying w_ϕ can be modeled by a constant w_{eff}. In [28] it was shown that in models in which the dark energy component is negligible at last scattering, the CMB and matter power spectra are well approximated by $w_{eff} = \int da \Omega_Q(a) w_Q(a)$. This is demonstrated in Fig. 5 in the case of the exponential potential with a feature, using $w_{eff} = -0.993$. In [19] we showed that if the dark energy component *is* a significant proportion, as can be seen in scaling quintessence models, the dark energy component can be modelled as an additional contribution to the effective number of relativistic degrees of freedom. We restrict ourselves to former case in which Ω_ϕ(MeV) is negligible. In [13] it is noted that although w_{eff} is a good measure for modelling CMB spectra it may not be such a good measure when considering d_l. We investigate whether this is actually the case using the three models outlined above, a power law, exponential scaling and scaling oscillatory potential, as test cases. In Fig. 6 we show that even a substantial deviation from w_{eff} at late times produces a small change in ρ_Q. The effect on m_B is doubly ''numbed,'' first because of the smoothing by the integral relation with H in d_l, see [14] for a previous discussion of this, and secondly because $m_B \sim \log d_l$. As a result the bolometric magnitude taken from the SNIa data is highly insensitive to variations in the equation of state. This does not bode well if we are to try and reconstruct the time varying equation of state from observations. It looks more likely that w_{eff} will be a more tangible observable.

VI. CONCLUSIONS

In this paper we have provided new constraints on the dark energy equation of state parameter w_Q by combining different cosmological data. The new CMB results provided by Boomerang and DASI improve the constraints from previous and similar analysis (see e.g., [13,28,29]), with $w_Q < -0.85$ at 68% C.L. ($w_Q < -0.76$ at 95% C.L.). We have also demonstrated how the combination of CMB data with other datasets is crucial in order to break the $\Omega_Q - w_Q$ degeneracy. The constraints from each single datasets are, as expected, quite broad but compatible between each other, providing an important consistency test. When comparison is possible (i.e. restricting to similar priors and datasets), our analysis is compatible with other recent analysis on w_Q [31]. Our final result is perfectly in agreement with the $w_Q = -1$ cosmological constant case and gives no support to a quintessential field scenario with $w_Q > -1$. A frustrated network of domain walls or a purely exponential scaling field are excluded at high significance. In addition a number of quintessential models are highly disfavored, power law potentials with $p \geqslant 1$ and the oscillatory potential discussed in this paper, to name a few.

It will be the duty of higher redshift datasets, for example from clustering observations [30] to point to a variation in w that might place quintessence in a more favorable light.

The result obtained here, however, could be plagued by some of the theoretical assumptions we made. The CMB and LSS constraints can be weakened by the inclusion of a background of gravity waves or of isocurvature perturbations or by adding features in the primordial perturbation spectra. These modifications are not expected in the most basic and simplified inflationary scenario but they are still compatible with the present data. The SNIa result has been obtained under the assumption of a constant-with-time w_Q. We have shown that in general w_{eff} is a rather good approximation for dynamical quintessential models since the luminosity distance depends on w_Q through a multiple integral that smears its redshift dependence. As we show in the previous section, our result is therefore valid for a wide class of quintessential models. This ''numbing'' of sensitivity to w_Q implies that maybe an effective equation of state is the most tangible parameter able to be extracted from supernovae. However with the promise of large data sets from Planck and SNAP satellites, opportunities may yet still be open to reconstruct a time varying equation of state [16].

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