Reply to "Comment on 'Modification of Z boson properties in the quark-gluon plasma' and 'Two-loop contribution to high mass dilepton production by a quark-gluon plasma'"

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We identify the main difference between our work and earlier work on the imaginary part of two-loop massive vector self-energies at finite temperature.

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In their Comment, Aurenche *et al.* address two issues: citation of previous work on dilepton production in quark-gluon plasma, and a discrepancy between that work and ours. We address both of those issues in turn.

In [1] we computed the decay of the Z boson in a quark-gluon plasma at temperature $T \ll m_Z$. To our knowledge this was the first publication on that topic. Afterwards we realized that the mathematical results could be applied to heavy dilepton production ($T \ll M$) in a quark-gluon plasma too [2]. We are grateful to Aurenche *et al.* for pointing out earlier work on heavy dilepton production. However, of the 31 references cited in their Comment, only 5 of them are directly relevant [3–7], and their results differ from ours.

Contrary to the claims of Aurenche *et al.*, we have given sufficient details in [1,2] and [8] for the interested reader to reproduce our results. In addition, we have written a more pedagogical paper [9] that displays in great detail how two-loop self-energies are computed at finite temperature and how the results are related to multiple scattering in the medium. We have checked our notes, *and* we have recomputed by independent means all the results described in [1] and [2].

The results may be expressed either as the imaginary part of the Z-boson self-energy or as its decay rate. In an obvious notation $\Gamma = -\mathrm{Im}\,\Gamma/m_Z$. (Note that only the limit where the 3-momentum of the massive vector meson is zero is considered.) These results may be expressed as various contributions (limits $m_f \ll T \ll m_Z$ assumed throughout). The notation follows that of [1]. Here F denotes the fusion reaction $g + Z \rightarrow q + \overline{q}$, C denotes the Compton-like reaction $q + Z \rightarrow q + g$ or the related reaction with the incoming quark replaced by an antiquark, D denotes the three-body decay $Z \rightarrow g + q + \overline{q}$, V denotes a vertex correction, and S denotes a quark self-energy correction:

$$\mathrm{Im}\,\Pi = -\frac{2}{3}\frac{\alpha_s}{\pi^2}\sum_f \left[g_A^2(f) + g_V^2(f)\right]\int_0^\infty d\omega\{n_{\mathrm{BE}}(\omega)[F + D_g + V_g + S_g] + n_{\mathrm{FD}}(\omega)[C + D_q + V_q + S_q]\}. \tag{1}$$

The individual terms are

$$F = -2\omega + \left[2\omega + 2m_Z + \frac{m_Z^2}{\omega}\right]\ln\left(\frac{2m_Z\omega}{k_c^2}\right),$$

$$D_g = -2\omega + \left[2\omega - 2m_Z + \frac{m_Z^2}{\omega}\right]\ln\left(\frac{2m_Z\omega}{k_c^2}\right),$$

$$V_g = 4\omega - 2\frac{m_Z^2}{\omega}\ln\left(\frac{2m_Z\omega}{k_c^2}\right),$$

$$S_g = 4\omega + 4\omega\ln\left(\frac{2m_Z\omega}{k_c^2}\right), \tag{2}$$

and

$$C = 4\omega + [-2\omega + m_Z]\ln\left(\frac{2m_Z\omega}{k_c^2}\right),$$

$$D_q = 4\omega + [-2\omega - m_Z]\ln\left(\frac{2m_Z\omega}{k_c^2}\right),$$

$$V_q = -8\omega + 8\omega\ln\left(\frac{2m_Z\omega}{k_c^2}\right),$$

$$S_q = 4\omega + 4\omega\ln\left(\frac{2m_Z\omega}{k_c^2}\right). \tag{3}$$

The expression for V_q given here is twice as big as given originally [1]. The origin of that 2 is a combinatoric factor coming from the two internal quark lines of the vertex. The expression for D_q contains the term 4ω which was originally given as 2ω. It turns out that the nonlog terms are sensitive to the explicit implementation of the cutoff. We had used two implementations: an invariant cutoff k_c described in more detail later, and a small quark mass. The same cutoff must be used consistently for all terms, and unfortunately we originally quoted D_q using the mass cutoff.

The above results are readily reproduced independently of the method used in [1] and [8]. The Compton and fusion cross sections are well-known; kinetic theory is used to compute these reaction rates with a Bose-Einstein distribution $n_{\mathrm{BE}}(\omega)$ for an incoming gluon and a Fermi-Dirac distribution $n_{\mathrm{FD}}(\omega)$ for an incoming quark or antiquark. (Under the conditions quoted the Pauli suppression and Bose enhancement corrections in the final state are of order $e^{-m_Z/2T}$ and totally ignorable.) Both of these reactions have a single exchanged quark. The square of the 4-momentum transfer, or virtuality, requires a cutoff $t \leqslant -k_c^2$. This removes a small piece of phase space that must be treated with resummation techniques [10] as successfully used in real photon production [11].

The book by Field [12] devotes chapter 2 to a detailed analysis of the decay of a massive virtual photon in vacuum from which one can obtain the results for the Z boson. The three-body decay involves Pauli suppression $1-n_{\rm FD}$ for the quark and antiquark and Bose enhancement $1+n_{\rm BE}$ for the gluon. With t denoting the invariant mass squared of the virtual quark that decays into the final state quark and gluon, we impose a cutoff $t \geqslant k_c^2$. The vertex and self-energy corrections described by Field can most simply be extended to finite temperature by modifying the propagator in the loops [13]:

$$\frac{i}{p^2+i\epsilon} \rightarrow \frac{i}{p^2+i\epsilon} + \frac{2\pi}{\exp(|p_0|/T)-1}\,\delta(p^2). \tag{4}$$

There is a corresponding expression for quarks. These corrections involve an interference between amplitudes of zero and first order in α_s. Again, one places a cutoff k_c^2 on the invariant mass of any quark-gluon pair.

Aurenche *et al.* claim that all log terms cancel. This would happen if the sign of S_g and S_q given above were reversed. Indeed, upon investigation those terms are given the opposite sign in [3–7]. For example, combining Eqs. (3.23), (3.18), and (3.9) of [4], and explicitly evaluating their expression (3.9) with the same invariant cutoff k_c as we have used, we find that their S_g and S_q have logs with the opposite sign. (The nonlog terms are not as easy to extract from their expressions.) Since they all follow the same method of calculation it is no surprise that they all obtain the same answer. Their method uses the quark wave-function renormalization Z_2 at finite temperature. To regulate the divergence they add a finite temperature counterterm to the Lagrangian. (In contrast, we have not added any finite temperature counterterms.) Since it is crucial to get the sign right (the relative sign between S_g and S_q is fixed and we are in agreement on that) we used the analysis of Field [12], section 2.4, and simply replaced the vacuum propagator with the finite temperature one as discussed above. We get the signs as given in Eqs. (2) and (3) above. Even if the signs were to be reversed, there would still remain finite temperature corrections of relative order T^2/m_Z^2 which Aurenche *et al.* deny. In fact, the width of the Z boson in medium would be *less* than in vacuum, a very difficult situation to understand physically. (This correction is not due to Pauli blocking).

The cutoff must be chosen in the parametric range $gT \ll k_c \ll T$. The hole cut out of phase space $|t|<k_c^2$ should be filled in by resummation analogous to how it was done for real photons [11]. Although we have not done a complete resummation analysis for the Z boson, we have computed the single quark loop diagram with one line dressed in the manner of [11]. The result is an additional contribution of $4\omega \ln(k_c^2/m_T^2)$ to both Eqs. (2) and (3), where $m_T = g_s T/\sqrt{3}$. (This same result can be inferred from the analysis of Thoma and Traxler [14], which has not been cited by Aurenche *et al.*) This may be viewed as eliminating the cutoff dependence of the self-energy terms. In effect k_c is replaced by the mass of a quark propagating through the plasma with a typical thermal momentum: the dispersion relation of such a quark is $E=\sqrt{p^2+m_T^2}$. This quark is actually a collective excitation of the plasma. Even though most of the lines in the Feynman graphs are hard in the sense that they are of order T or higher, they still need to be dressed on account of their proximity to the light cone. The rest of the $\ln(k_c)$ dependence should be cancelled in a similar way, probably by dressing the vertices. The latter was not necessary for real photons, but there is a difference between a vector particle with zero mass and high momentum and one with high mass and zero momentum. On the other hand, if it could be argued that the self-energy contributions are actually zero, then *all* k_c dependence in Eqs. (2) and (3) would cancel, leaving a term of order $\ln(m_Z/\alpha_s T)$. This is an open question.

We have not included vacuum corrections. For the Z boson these are subsumed in the experimentally measured width. As pointed out by Aurenche *et al.* the rate for $q+\overline{q} \rightarrow l^+ + l^-$ in plasma does get modified by vacuum vertex and self-energy corrections; see chapter 2 of Field where they are explicitly evaluated. We should have listed the two-loop vacuum correction to the (virtual) photon self-energy in [2]. Thus

$$\mathrm{Im}\,\Pi_{\rm vacuum} = -\frac{e_q^2}{4\pi} M^2 \left(1+\frac{\alpha_s}{\pi}\right). \tag{5}$$

In summary, we are grateful to Aurenche *et al.* for pointing out the inconsistency of our results with those of [3–7]. We have identified the primary source of disagreement, but have been unable to resolve it. Unfortunately the thermal effects on Z-boson decay are far too small to be measured at accelerator energies, making this more of an academic exercise.

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