Lepton flavor conserving $Z \to l^+ l^-$ decays in the general two Higgs doublet model

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We calculate the new physics effects to the branching ratios of the lepton flavor conserving decays $Z \to l^+ l^-$ in the framework of the general two Higgs doublet model. We predict the upper limits for the couplings $|\bar{\xi}^D_{N,\mu\tau}|$ and $|\bar{\xi}^D_{N,\tau\tau}|$ as 3×10^2 GeV and 1×10^2 GeV, respectively.

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I. INTRODUCTION

In the standard model (SM) of electroweak interactions lepton flavor is conserved. This conservation can be broken with the extension of the SM, such as the standard model, enlarged with massive, mixing neutrinos (νSM), permitting the existence of the massive neutrinos and the lepton mixing mechanism [1], νSM, extended with one heavy ordinary Dirac neutrino or two heavy right-handed singlet Majorana neutrinos [2], the Zee model [3], and the general two Higgs doublet model, which contains off diagonal Yukawa couplings in the lepton sector [4].

Leptonic Z decays are among the most interesting lepton flavor conserving (LFC) and lepton flavor violating (LFV) interactions and they reached great interest since the related experimental measurements are improved at present. With the Giga-Z option of the OESY TeV Energy Superconducting Linear Accelerator (TESLA) project, there is a possibility to increase Z bosons at resonance [5].

The processes $Z\to l^- l^+$ with $l=e,\mu,\tau$ are the among the LFC decays and they exist in the SM even at the tree level. The experimental predictions for the branching ratios of these decays are [6]

$$\begin{aligned} \mathrm{BR}(Z\to e^- e^+) &= 3.366\pm 0.0081\,\%, \\ \mathrm{BR}(Z\to \mu^- \mu^+) &= 3.367\pm 0.013\,\%, \\ \mathrm{BR}(Z\to \tau^- \tau^+) &= 3.360\pm 0.015\,\%, \end{aligned} \tag{1}$$

and the tree level SM predictions are

$$\begin{aligned} \mathrm{BR}(Z\to e^- e^+) &= 3.331\%, \\ \mathrm{BR}(Z\to \mu^- \mu^+) &= 3.331\%, \\ \mathrm{BR}(Z\to \tau^- \tau^+) &= 3.328\%. \end{aligned} \tag{2}$$

Comparison of these experimental and theoretical results shows that the main contribution comes from the SM in the tree level and the loop contributions, even the ones beyond the SM, should lie almost in the uncertainity of the measurements of these decays.

In the literature, there are various experimental and theoretical studies [7–15]. A method to determine the weak electric dipole moment was developed in Ref. [9]. The vector and axial vector coupling constants v_f and a_f in Z decays have been measured at the CERN e^+e^- collider LEP [11]. Furthermore, the measurements of the weak electric dipole moments of fermions have been performed [12]. In Ref. [13], various additional types of interactions have been studied and a way to measure these contributions in the process $Z\to\tau^-\tau^+$ was described. In Ref. [15], a new method to measure the electroweak mixing angle in Z decays to tau leptons has been proposed.

In the present work we study the LFC $Z\to l^- l^+$ decays, where $l=e,\mu,\tau$, in the model III version of 2HDM, which is the minimal extension of the SM. Since the SM prediction for these decays, even in the tree level, is almost the same as the experimental one, the contributions beyond the SM, which can exist at least in the loop level, should be small enough not to exceed the experimental results. This discussion stimulates us to study the BR's of these processes with the addition of the contributions beyond the SM and try to predict upper limits for the new couplings $\xi^D_{N,ij}$, $i,j=e,\mu,\tau$ existing in the model used. The restrictions of these couplings have been studied in the literature extensively. In Ref. [16], the coupling $|\bar{\xi}^D_{N,\mu\tau}|$ was predicted in the range 10^2-10^3 GeV, by using the constraint coming from the experimental limits of the μ lepton electric dipole moment EDM [17]. Furthermore, the coupling $|\bar{\xi}^D_{N,e\tau}|$ could be obtained as 10^{-4} GeV using the upper limit for the process $\mu\to e\gamma$, since this process fixes the combination $\bar{\xi}^D_{N,\mu\tau}\bar{\xi}^D_{N,e\tau}$. However, the upper limit for the coupling $\bar{\xi}^D_{N,\tau\tau}$ could not be predicted by using above processes (see Ref. [16] for details). Recently, the LFV $Z\to l_1^- l_2^+$ decays have been studied in Ref. [4] and it was shown that it is possible to reach the experimental upper limits of $\mathrm{BR}(Z\to i^\pm j^\pm)$ where $i,j=e,\mu;e,\tau;\mu,\tau$. Since these experimental data need improvement, more strict constraints for the couplings under consideration cannot be obtained in this case. However, the SM contribution to the BR's of the LFC $Z\to l^- l^+$ decays is large enough to describe the experimental results within experimental errors and, therefore, an analysis can be done for more strict upper limits of the new couplings appearing in model III.

In model III, the neutral Higgs bosons h^0 and A^0 play the important role for the physics beyond the SM, in the calculation of the BR's of the LFC $Z\to l^- l^+$ decays. This analysis shows that the predictions of the upper limits for the couplings $|\bar{\xi}^D_{N,\mu\tau}|$ and $|\bar{\xi}^D_{N,\tau\tau}|$ are $\sim 3\times 10^2$ GeV and $\sim 1\times 10^2$ GeV, respectively, however, an upper limit for the

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coupling $|\overline{\xi}^{D}_{N,e\tau}|$ cannot be found due to the small contribution of new physics effects to the BR of the $Z\to e^- e^+$ decay. Here the predicted upper limit for $|\overline{\xi}^{D}_{N,\mu\tau}|$ is more strict compared to the previous studies. Furthermore, the upper limit for $|\overline{\xi}^{D}_{N,\tau\tau}|$ can be obtained. Here, the coupling $\overline{\xi}^{D}_{N,li}$ is defined as $\xi^{D}_{N,li}=\sqrt{4G_F/\sqrt{2}}\,\overline{\xi}^{D}_{N,li}$ and the definition of the coupling $\xi^{D}_{N,li}$ is given in Sec. II.

The paper is organized as follows. In Sec. II, we present the explicit expressions for the branching ratios of $Z\to l^- l^+$ in the framework of the model III. Section III is devoted to discussion and our conclusions.

II. $Z\to l^- l^+$ DECAY IN THE GENERAL TWO HIGGS DOUBLET MODEL

In the SM, lepton flavor is conserved since the matter content forbids the lepton flavor violation. However, most theories beyond the SM may bring flavor changing neutral currents (FCNC) at the tree level, unless some discrete *ad hoc* symmetries are imposed to eliminate them. The model I and II versions of the two Higgs doublet model (2HDM) [18] are the examples of the theories beyond where FCNC at the tree level is forbidden. In model I, the up-type and down-type quarks are coupled to the same scalar doublets, however, in model II, they are coupled to two different scalar ones. In the model III version of 2HDM, the *ad hoc* discrete symmetry in the Yukawa Lagrangian is not considered and the FCNC interactions at the tree level are allowed. This makes the LFV interactions possible. Furthermore, the existence of FCNC at the tree level brings new contributions to LFC decays. The most general Yukawa interaction for the leptonic sector in model III is

$$\mathcal{L}_Y=\eta^{D}_{ij}\overline{l}_{iL}\phi_1 E_{jr}+\xi^{d}_{ij}\overline{l}_{iL}\phi_2 E_{jR}+\text{H.c.}, \tag{3}$$

where i,j are family indices of leptons, L and R denote chiral projections $L(R)=1/2(1\mp\gamma_5)$, ϕ_i for $i=1,2$, are the two scalar doublets, l_{iL} and E_{jR} are lepton doublets and singlets respectively. The choice of ϕ_1 and ϕ_2,

$$\Phi_1=\frac{1}{\sqrt{2}}\left[\begin{pmatrix}0\\ v+H^0\end{pmatrix}+\begin{pmatrix}\sqrt{2}\chi^+\\ i\chi^0\end{pmatrix}\right],\quad \phi_2=\frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2}H^+\\ H_1+iH_2\end{pmatrix}, \tag{4}$$

with the vacuum expectation values

$$\langle\phi_1\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix},\quad \langle\phi_2\rangle=0, \tag{5}$$

helps us to decompose the SM particles and beyond in the tree level. In this case, the first doublet carries the SM particles and the other one is responsible for the particles beyond the SM. Therefore, we take H_1 and H_2 [see Eq. (4)] as the mass eigenstates h^0 and A^0 respectively, since no mixing between CP-even neutral Higgs bosons h^0 and the SM one H^0 occurs at the tree level. In Eq. (3) ξ^{D}_{ij} are the Yukawa matrices and they have in general complex entries. Notice that in the following we replace ξ^D with ξ^D_N where "N" denotes "neutral."

The general effective vertex for the interaction of on-shell Z boson with a fermionic current is given by

$$\Gamma_\mu=\gamma_\mu(f_V-f_A\gamma_5)+\frac{1}{m_W}(f_M+f_E\gamma_5)\sigma_{\mu\nu}q^\nu, \tag{6}$$

where q is the momentum transfer, $q^2=(p-p')^2$, f_V (f_A) is vector (axial-vector) coupling, $f_M(f_E)$ is proportional to the weak magnetic (electric dipole) moment of the fermion. Here $p(-p')$ is the four momentum vector of lepton (antilepton). For the $Z\to l^- l^+$ decay, the couplings f_V and f_A have contributions from the SM, f_V^{SM} and f_A^{SM}, even at the tree level, and all the couplings have contributions beyond the SM, f_I^{beyond}, where $I=V,A,M,E$. The explicit expressions for these couplings are

$$f_V^{\text{SM}}=\frac{ig}{\cos\theta_W}c_V,$$

$$f_A^{\text{SM}}=\frac{ig}{\cos\theta_W}c_A, \tag{7}$$

and

$$\begin{aligned}
f_V^{\text{beyond}}=\frac{-ig}{32\cos\theta_W\pi^2}\Bigg(&\int_0^1 dx\,c_V\eta_i^V(-1+x)\left(\ln\frac{L^{\text{self}}_{h^0}}{\mu^2}\frac{L^{\text{self}}_{A^0}}{\mu^2}\right)+\int_0^1 dx\int_0^{1-x}dy\Bigg\{\frac{1}{2}(-1+x+y)m_i m_{l^-}\eta_i^-\left(\frac{1}{L^{\text{ver}}_{h^0A_0}}-\frac{1}{L^{\text{ver}}_{A^0h^0}}\right)\\
&+c_V\Bigg[[-\eta_i^V m_i^2+(-1+x+y)\eta_i^+ m_i m_{l^-}]\frac{1}{L^{\text{ver}}_{h^0}}-[\eta_i^V m_i^2+(-1+x+y)\eta_i^+ m_i m_{l^-}]\frac{1}{L^{\text{ver}}_{A^0}}\\
&+\eta_i^V[2-(q^2xy+m_{l^-}^2(-1+x+y)^2]\left(\frac{1}{L^{\text{ver}}_{h^0}}+\frac{1}{L^{\text{ver}}_{A^0}}\right)+\ln\left(\frac{L^{\text{ver}}_{h^0}}{\mu^2}\frac{L^{\text{ver}}_{A^0}}{\mu^2}\right)\Bigg]\Bigg\}\Bigg),
\end{aligned}$$

$$
\begin{aligned}
f_A^{\text{beyond}}=&\frac{ig}{32\cos\theta_W\pi^2}\Bigg(\int_0^1 dx\, c_A\eta_i^V(-1+x)\left(\ln\frac{L_{h^0}^{\text{self}}}{\mu^2}\frac{L_{A^0}^{\text{self}}}{\mu^2}\right)-\int_0^1 dx\int_0^{1-x}dy\Bigg\{-2c_A\eta_i^V\ln\frac{L_{h^0A^0}^{\text{ver}}}{\mu^2}\frac{L_{A^0h^0}^{\text{ver}}}{\mu^2}\\
&+c_A\Bigg[[\eta_i^V m_i^2-(-1+x+y)\eta_i^+ m_i m_{l^-}]\frac{1}{L_{h^0}^{\text{ver}}}+[\eta_i^V m_i^2+(-1+x+y)\eta_i^+ m_i m_{l^-}]\frac{1}{L_{A^0}^{\text{ver}}}\\
&+\eta_i^V[2-(q^2xy-m_{l^-}^2(-1+x+y)^2]\left(\frac{1}{L_{h^0}^{\text{ver}}}+\frac{1}{L_{A^0}^{\text{ver}}}\right)+\ln\left(\frac{L_{h_0}^{\text{ver}}}{\mu^2}-\frac{L_{A^0}^{\text{ver}}}{\mu^2}\right)\Bigg]\Bigg\}\Bigg),\\
f_M^{\text{beyond}}=&\frac{gm_W}{256\cos\theta_W\pi^2}\int_0^1 dx\int_0^{1-x}dy\Bigg\{2\eta_i^- m_i(-1+x+y)\left(\frac{1}{L_{h^0A^0}^{\text{ver}}}-\frac{1}{L_{A^0h^0}^{\text{ver}}}\right)+\frac{1}{L_{h^0}^{\text{ver}}}(\eta_i^- m_i(y-x)-4c_V(x+y)\\
&\times[-\eta_i^+ m_i+2(-1+x+y)\eta_i^V m_{l^-}])+\frac{1}{L_{A^0}^{\text{ver}}}(\eta_i^- m_i(x-y)-4c_V(x+y)[\eta_i^+ m_i+2(-1+x+y)\eta_i^V m_{l^-}])\Bigg\},\\
f_E^{\text{beyond}}=&\frac{gm_W}{256\cos\theta_W\pi^2}\int_0^1 dx\int_0^{1-x}dy\Bigg\{2(1-x-y)\times[\eta_i^+ m_i+2(x-y)\eta_i^V m_{l^-}]\frac{1}{L_{h^0A^0}^{\text{ver}}}\\
&-[\eta_i^+ m_i+2(y-x)\eta_i^V m_{l^-}]\frac{1}{L_{A^0h^0}^{\text{ver}}}+\{m_i[\eta_i^+(y-x)+4c_V\eta_i^-(x+y)]+2(y-x)(-1+x+y)\eta_i^V m_{l^-}\}\frac{1}{L_{A^0}^{\text{ver}}}\\
&+\{m_i[\eta_i^+(x-y)+4c_V\eta_i^-(x+y)]-2(x-y)(-1+x+y)\eta_i^V m_{l^-}\}\frac{1}{L_{h^0}^{\text{ver}}}\Bigg\},
\end{aligned}
\tag{8}
$$

where

$$
\begin{aligned}
&L_{h^0}^{\text{self}}=m_{h^0}^2(1-x)+[m_i^2-m_{l^-}^2(1-x)]x,\quad L_{A^0}^{\text{self}}=L_{1,h^0}^{\text{self}}(m_{h^0}\to m_{A^0}),L_{h^0}^{\text{ver}}=m_{h^0}^2(1-x-y)+m_i^2(x+y)-q^2xy,\\
&L_{h^0A^0}^{\text{ver}}=m_{h^0}^2x+m_i^2(1-x-y)+(m_{A^0}^2-q^2x)y,\quad L_{A^0}^{\text{ver}}=L_{h^0}^{\text{ver}}(m_{h^0}\to m_{A^0}),\quad L_{A^0h^0}^{\text{ver}}=L_{h^0A^0}^{\text{ver}}(m_{h_0}\to m_{A^0}),
\end{aligned}
\tag{9}
$$

and

$$
\eta_i^V=\xi_{N,il}^{D*}\xi_{N,li}^D,\quad \eta_i^+=\xi_{N,il}^{D*}\xi_{N,il}^{D*}+\xi_{N,li}^D\xi_{N,li}^D,\quad \eta_i^-=\xi_{N,il}^{D*}\xi_{N,il}^{D*}-\xi_{N,li}^D\xi_{N,li}^D. \tag{10}
$$

The parameters c_V and c_A are $c_A=-1/4$ and $c_V=1/4-\sin^2\theta_W$. In Eq. (10) the flavor changing couplings $\xi_{N,li}^D$ represent the effective interaction between the internal lepton i, ($i=e,\mu,\tau$) and outgoing (incoming) $l^-(l^+)$ one.

In general the Yukawa couplings $\xi_{N,li}^D$ are complex and they can be parametrized as

$$
\xi_{N,il}^D=|\xi_{N,il}^D|e^{i\theta_{il}}, \tag{11}
$$

with lepton flavors i, l and CP violating parameters θ_{il}. Notice that parameters θ_{il} are the sources of the lepton EDM.

Finally the BR for the LFC process $Z\to l^-l^+$, for the vanishing external lepton masses, can be written as

$$
\text{BR}(Z\to l^-l^+)=\frac{m_Z}{12\pi\Gamma_Z}\left\{\left|f_V\right|^2+\left|f_A\right|^2+\frac{1}{2\cos^2\theta_W}(|f_M|^2+|f_E|^2)\right\}, \tag{12}
$$

where $f_V=f_V^{\text{SM}}+f_V^{\text{beyond}}$, $f_A=f_A^{\text{SM}}+f_A^{\text{beyond}}$, and $f_M=f_M^{\text{beyond}}$, $f_E=f_E^{\text{beyond}}$. Here Γ_Z is the total decay width of Z boson, namely $\Gamma_Z=2.490\pm0.007$ GeV.

III. DISCUSSION

Flavor conserving $Z\to l^+l^-$ decays are possible at the tree level in the SM model and the contribution of one loop corrections to the tree level result is small. Our aim is to determine the new physics effects to the BR of these decays and to predict the restrictions for the free parameters of the model used. The model we study is the model III version of 2HDM, which may bring considerable contribution to the BR ($Z\to l^+l^-$) beyond the SM. However, in model III, there are a large number of free parameters, namely, the masses of charged and neutral Higgs bosons, the Yukawa couplings that can be complex in general. The dimensionfull Yukawa couplings in the lepton sector are $\bar{\xi}_{N,ij}^D$, $i,\ j=e,\ \mu,\ \tau$ and it is necessary to restrict them using the present and forthcoming experiments.

The couplings $\bar{\xi}_{N,ij}^D$, $i,\ j=e,\ \mu$ can be neglected compared to $\bar{\xi}_{N,\tau i}^D$ $i=e,\ \mu,\ \tau$ with the assumption that the strength of these couplings are related with the masses of leptons denoted by the indices of them. Furthermore, we

TABLE I. The values of the input parameters used in the numerical calculations.

Parameter	Value
m_e	0 (GeV)
m_μ	0.106 (GeV)
m_τ	1.78 (GeV)
m_W	80.26 (GeV)
m_Z	91.19 (GeV)
G_F	1.16637×10^{-5} (GeV^{-2})
Γ_Z	2.490 (GeV)
$\sin\theta_W$	$\sqrt{0.2325}$

assume that $\bar{\xi}^D_{N,ij}$ is symmetric with respect to the indices i and j. Therefore, the Yukawa couplings $\bar{\xi}^D_{N,\tau e}$, $\bar{\xi}^D_{N,\tau,\mu}$ and $\bar{\xi}^D_{N,\tau\tau}$ play the main role in our lepton conserving decays, $Z\to l^+l^-$.

For $\bar{\xi}^D_{N,\mu\tau}$, the constraint coming from the experimental limits of μ lepton EDM [17],

$$0.3\times 10^{-19}\, e-cm< d_\mu <7.1\times 10^{-19} e \text{ cm} \tag{13}$$

(see Ref. [16] for details) or the deviation of the anomalous magnetic moment (AMM) of muon over its SM prediction [19] due to the recent experimental result of muon AMM by the $g-2$ Collaboration [20], can be used. The coupling $\bar{\xi}^D_{N,e\tau}$ is restricted using the experimental upper limit of the BR of the process $\mu\to e\gamma$ and the above constraint for $\bar{\xi}^D_{N,\mu\tau}$, since $\mu\to e\gamma$ decay can be used to fix the Yukawa combination $\bar{\xi}^D_{N,\mu\tau}\,\bar{\xi}^D_{N,e\tau}$. Using the the experimental bounds of μ lepton EDM and the upper limit of the BR of the process $\mu\to e\gamma$, $|\bar{\xi}^D_{N,\mu\tau}|$ ($|\bar{\xi}^D_{N,e\tau}|$) has been predicted at the order of the magnitude of 10^2-10^3 ($10^{-5}-10^{-3}$) GeV (see Ref. [16]). For $|\bar{\xi}^D_{N,\tau\tau}|$ no prediction has been done yet.

The present work is devoted to study on the lepton flavor conserving decays $Z\to l^-l^+$, where $l=e,\ \mu,\ \tau$. The main contribution to the BR's of these processes come from the SM in the tree level. In the calculations, we take into account the one loop contributions including the neutral Higgs bosons h^0 and A^0 beyond the SM and we neglect the one loop diagrams including the charged $H^\pm$ bosons beyond the SM, since $m_{H^\pm}$ is large [21] compared to the masess of neutral Higgs masses, namely, $m_{H^\pm}>5m_{h^0,A^0}$. We see that, with this additional part, the BR's of the $Z\to l^-l^+$ decays can be enhanced, by adjusting the Yukawa couplings and the upper limits of these Yukawa couplings can be restricted using the existing experimental measurements. Notice that in the theoretical calculations, we take the Yukawa couplings complex, however we use their magnitudes in the numerical analysis, since the BR's of the processes under consideration are not sensitive to the *CP* violating part of these couplings. Throughout our calculations we use the input values given in Table I.

Now we would like to parametrize the BR as

$$\mathrm{BR}=\mathrm{BR}^{\mathrm{SM}}+\mathrm{BR}^{\mathrm{beyond}}, \tag{14}$$

where BR$^{\mathrm{SM}}$ is coming from only the SM part. In Eq. (14), BR$^{\mathrm{beyond}}$ gets contributions from the combination of the SM and beyond, which we denote as BR$^{\mathrm{mixed}}$, and from only beyond the SM, which we denote as BR$^{\mathrm{pure\ beyond}}$.

Figure 1 represents $|\bar{\xi}^D_{N,e\tau}|$ dependence of the BR$^{\mathrm{beyond}}$ $(Z\to e^-e^+)$ for $m_{h^0}=80$ GeV and $m_{A^0}=90$ GeV. Here the coupling $|\bar{\xi}^D_{N,e\tau}|$ stands in the range $10^{-4}-10^{-3}$ GeV respecting the above restrictions and the BR$^{\mathrm{beyond}}$ can take the values at most at the order of the magnitude of 10^{-15}. This value is negligible compared to even the uncertainity in the

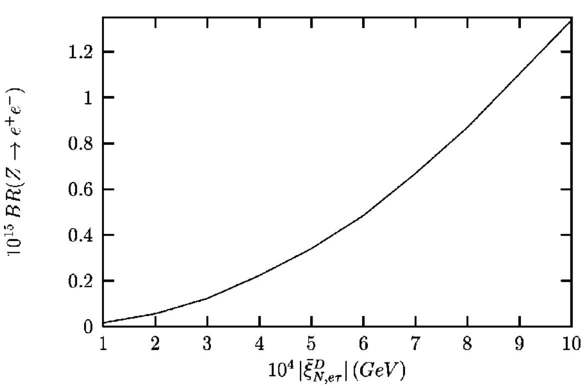

FIG. 1. $|\bar{\xi}^D_{N,e\tau}|$ dependence of the BR$^{\mathrm{beyond}}$ $(Z\to e^-e^+)$ for $m_{h^0}=80$ GeV and $m_{A^0}=90$ GeV.

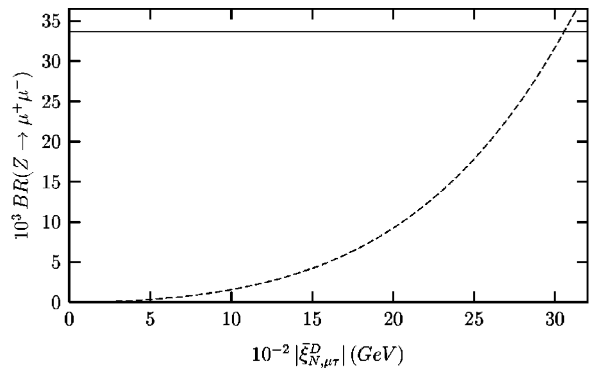

FIG. 2. $|\bar{\xi}^D_{N,\mu\tau}|$ dependence of the BR$^{\mathrm{beyond}}$ $(Z\to\mu^-\mu^+)$ for $m_{h^0}=80$ GeV and $m_{A^0}=90$ GeV. Here the solid (dashed) line represents the SM (beyond) contribution.

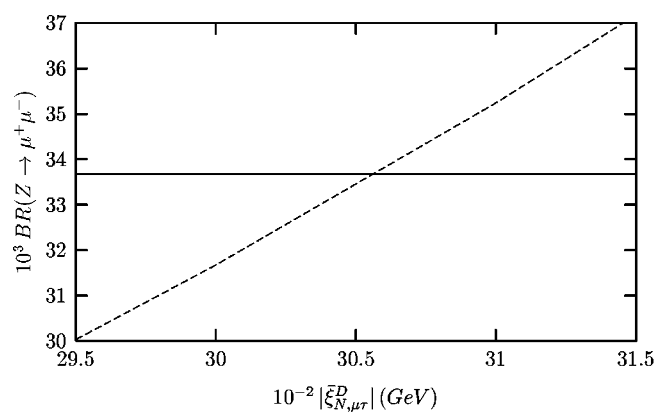

FIG. 3. The same as Fig. 2 but the enlarged one around the BR$(Z\to\mu^-\mu^+)\sim 3.3\%$.

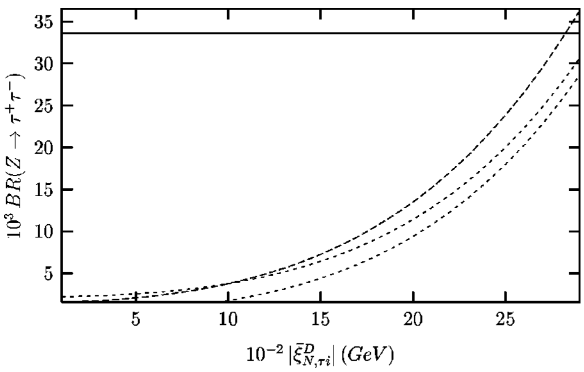

FIG. 4. $|\bar{\xi}^D_{N,\tau i}|$ $(i=\mu,\tau)$ dependence of the $\mathrm{BR}^{\mathrm{beyond}}$ $(Z \to \tau^- \tau^+)$ for $m_{h^0}=80$ GeV and $m_{A^0}=90$ GeV. Here the solid line is devoted to the SM, dashed line to the $|\bar{\xi}^D_{N,\tau\mu}|$ dependence for $|\bar{\xi}^D_{N,\tau\tau}|=10^3$ GeV, small dashed line to the $|\bar{\xi}^D_{N,\tau\tau}|$ dependence for $|\bar{\xi}^D_{N,\tau\mu}=10^3$ GeV and dotted line to the $|\bar{\xi}^D_{N,\tau\tau}|$ dependence for $|\bar{\xi}^D_{N,\tau\mu}|=3\times 10^2$ GeV.

experimental result of the BR $(Z\to e^- e^+)$, $\sim 0.008\%$. This concludes that the new physics effects in model III does not bring any contribution to the BR of the process $Z \to e^- e^+$ and a new constraint for the coupling $|\bar{\xi}^D_{N,e\tau}|$ cannot be found. Notice that the $\mathrm{BR}^{\mathrm{pure\ beyond}}$ $(Z\to e^- e^+)$ is at the order of the magnitude of 10^{-28}. We also study the Higgs boson h^0 mass m_{h^0} dependence of the $\mathrm{BR}^{\mathrm{beyond}}$ $(50 < m_{h^0} < 80\,\mathrm{GeV})$ and observe that, for $m_{h^0}=50$ GeV, there is an enhancement more than a factor of 2 larger than the result for $m_{h^0}=80$ GeV.

In Fig. 2 we present $|\bar{\xi}^D_{N,\mu\tau}|$ dependence of the $\mathrm{BR}^{\mathrm{beyond}}$ $(Z\to \mu^- \mu^+)$ for $m_{h^0}=80$ GeV and $m_{A^0}=90$ GeV. Here the solid (dashed) line represents the SM (beyond) contribution. It can seen that the $\mathrm{BR}^{\mathrm{beyond}}$ can reach the SM values for the large values of the coupling, $|\bar{\xi}^D_{N,\mu\tau}| \sim 3.1\times 10^3$ GeV, which lies in the above constraint region. Not to exceed the experimental result of the process $Z\to\mu^-\mu^+$, we need to predict an upper limit for this coupling. By taking into account the uncertainity of the experimental result, namely, 0.013% we get the upper limit of the coupling as $|\bar{\xi}^D_{N,\mu\tau}| \sim 3\times 10^2$ GeV. The $\mathrm{BR}^{\mathrm{pure\ beyond}}$ $(Z\to e^- e^+)$ can also reach the SM value for $|\bar{\xi}^D_{N,\mu\tau}| \sim 3.5\times 10^3$ GeV. Furthermore, the Higgs boson h^0 mass m_{h^0} dependence of the $\mathrm{BR}^{\mathrm{beyond}}$ $(50 < m_{h^0} < 80\,\mathrm{GeV})$ shows that for $m_{h^0}=50$ GeV, there is an enhancement more than a factor of 3 larger than the result for $m_{h^0}=80$ GeV. Figure 3 is the enlarged version of Fig. 2 around the BR $(Z\to\mu^-\mu^+) \sim 3.3\%$.

Figure 4 is devoted to the $|\bar{\xi}^D_{N,\tau i}|$ $(i=\mu,\tau)$ dependence of the $\mathrm{BR}^{\mathrm{beyond}}$ $(Z\to\tau^-\tau^+)$ for $m_{h^0}=80$ GeV and $m_{A^0}=90$ GeV. Here the solid (dashed, small, dotted) line represents the SM (beyond). The dashed line is devoted to the $|\bar{\xi}^D_{N,\tau\mu}|$ dependence for $|\bar{\xi}^D_{N,\tau\tau}|=10^3$ GeV, the small dashed line is to the $|\bar{\xi}^D_{N,\tau\tau}|$ dependence for $|\bar{\xi}^D_{N,\tau\mu}|=10^3$ GeV, and the dotted line is to the $|\bar{\xi}^D_{N,\tau\tau}|$ dependence for $|\bar{\xi}^D_{N,\tau\mu}|=3\times 10^2$ GeV, which is the upper limit of $|\bar{\xi}^D_{N,\tau\mu}|$, obtained using $\mathrm{BR}^{\mathrm{beyond}}$ $(Z\to\mu^-\mu^+)$. From this figure, it is observed

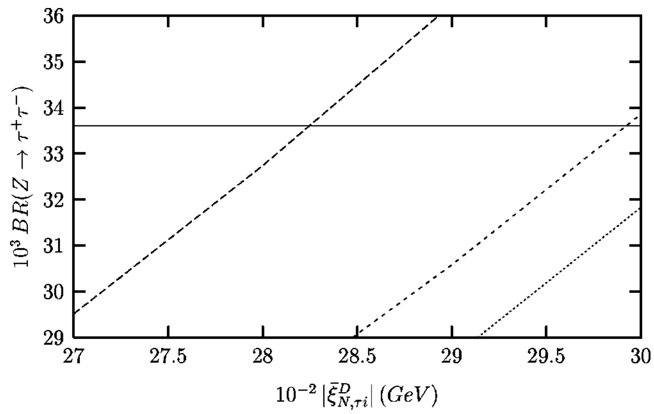

FIG. 5. The same as Fig. 4 but the enlarged one around the $\mathrm{BR}(Z\to\tau^-\tau^+)\sim 3.3\%$.

that $\mathrm{BR}^{\mathrm{beyond}}$ can reach the SM value for the large values of the couplings $|\bar{\xi}^D_{N,\mu\tau}| \sim 3.0\times 10^3$ GeV and $|\bar{\xi}^D_{N,\mu\tau}| \sim 3.0\times 10^3$ GeV. Our aim is to get a $\mathrm{BR}^{\mathrm{beyond}}$ so that the BR does not exceed its experimental value. To ensure this, first, we choose the upper limit of the coupling $|\bar{\xi}^D_{N,\mu\tau}|$ as 3×10^{-2} GeV, respecting the $\mathrm{BR}^{\mathrm{beyond}}$ $(Z\to\mu^-\mu^+)$ and then use the uncertainity of the experimental result for BR $(Z\to\tau^-\tau^+)$, namely, 0.015%. We predict the upper limit of the coupling as $|\bar{\xi}^D_{N,\mu\tau}| < 1\times 10^2$ GeV (see dotted line in the Fig. 4). Notice that $\mathrm{BR}^{\mathrm{pure\ beyond}}$ $(Z\to e^- e^+)$ can reach the SM value for $|\bar{\xi}^D_{N,\mu\tau}| \sim 3.5\times 10^3$ GeV and $|\bar{\xi}^D_{N,\tau\tau}| \sim 3\times 10^3$ GeV. For completeness we study the mass m_{h^0} dependence of the $\mathrm{BR}^{\mathrm{beyond}}$ $(50 < m_{h^0} < 80\,\mathrm{GeV})$ and observe that for $m_{h^0}=50$ GeV, there is an enhancement less than a factor of 2 larger than the result for $m_{h^0}=80$ GeV. Figure 5 is the enlarged version of Fig. 4 around the BR $(Z\to\tau^-\tau^+) \sim 3.3\%$, similar to Fig. 3.

As a summary, we study the $\mathrm{BR}^{\mathrm{beyond}}$'s of the lepton flavor conserving decays $Z\to l^- l^+ (l=e,\mu,\tau)$ in model III. We observe that the one loop diagrams due to neutral Higgs bosons h^0 and A^0 can give considerable contributions and it is possible even to reach the tree level SM result. This forces one to predict the upper limits for the Yukawa couplings in the lepton sector: An upper limit for the coupling $|\bar{\xi}^D_{N,e\tau}|$ cannot be found since the new physics effects to the BR$(Z\to e^- e^+)$ are extremely small compared to the SM result, we predict the upper limit of the coupling as $|\bar{\xi}^D_{N,\mu\tau}| < 3\times 10^2$ GeV; we predict the upper limit of the coupling as $|\bar{\xi}^D_{N,\tau\tau}| < 1\times 10^2$ GeV. Furthermore, $\mathrm{BR}^{\mathrm{beyond}}$ is not so much sensitive to the mass m_{h^0}.

In the future, with the more reliable experimental result of the BR's of above processes, it would be possible to predict the values of the free parameters, existing in the models beyond the SM, more accurately.

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