Supersymmetric 3-3-1 model

J. C. Montero, V. Pleitez, and M. C. Rodriguez

Instituto de Física Teórica, Universidade Estadual Paulista, Rua Pamplona, 145, 01405-900 São Paulo, SP, Brazil
(Received 15 December 2000; published 10 January 2002)

We build a complete supersymmetric version of a 3-3-1 gauge model using the superfield formalism. We point out that a discrete symmetry, similar to R symmetry in the minimal supersymmetric standard model, is possible to be defined in this model. Hence we have both R-conserving and R-violating possibilities. Analysis of the mass spectrum of the neutral real scalar fields show that in this model the lightest scalar Higgs boson has a mass upper limit, and at the tree level it is 124.5 GeV for a given illustrative set of parameters.

I. INTRODUCTION

Although the standard model (SM), based on the gauge symmetry $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, describes the observed properties of charged leptons and quarks, it is not the ultimate theory. However, the necessity to go beyond it, from the experimental point of view, comes at the moment only from neutrino data [1]. If neutrinos are massive then new physics beyond the SM is needed. From the theoretical point of view, the SM cannot be a fundamental theory since it has so many parameters and some important questions such as that of the number of families do not have an answer in its context. On the other hand, it is not clear what the physics beyond the SM should be. An interesting possibility is that at the TeV scale physics would be described by models which share some of the flaws of the SM but give some insight concerning some questions which remain open in the SM context.

One of these possibilities is that, at energies of a few TeVs, the gauge symmetry may be $SU(3)_c \otimes SU(3)_L \otimes U(1)_N$ (3-3-1 for shortness) instead of that of the SM [2,3]. In fact, this may be the last symmetry involving the lightest elementary particles: leptons. The lepton sector is exactly the same as in the SM but now there is a symmetry, at large energies among, say e^-, ν_e, and e^+. Once this symmetry is imposed on the lightest generation and extended to the other leptonic generations it follows that the quark sector must be enlarged by considering exotic charged quarks. It means that some gauge bosons carry lepton and baryon quantum number. Although this model coincides at low energies with the SM it explains some fundamental questions that are accommodated, but not explained, in the SM. These questions are the following.

(i) The family number must be a multiple of three in order to cancel anomalies [2,3]. This result comes from the fact that the model is anomaly free only if we have equal number of triplets and antitriplets, counting the $SU(3)_c$ colors, and furthermore requiring the sum of all fermion charges to vanish. However, each generation is anomalous, the anomaly cancellation occurs for the three, or multiple of three, together and not generation by generation like in the SM. This may provide a first step towards answering the flavor question.

(ii) Why $\sin^2 \theta_W < \frac{1}{4}$ is observed. This point comes from the fact that in the model of Ref. [2] we have that the $U(1)_N$

and $SU(3)_L$ coupling constants, g' and g, respectively, are related by

$$t^2 \equiv \frac{g'^2}{g^2} = \frac{\sin^2 \theta_W}{1 - 4 \sin^2 \theta_W}. \tag{1}$$

Hence, this 3-3-1 model predicts that there exists an energy scale, say μ, at which the model loses its perturbative character. The value of μ can be found through the condition $\sin^2 \theta_W(\mu) = 1/4$. However, it is not clear at all what is the value of μ; in fact, it has been argued that the upper limit on the vector bilepton masses is 3.5 TeV [4] instead of the 600 GeV given in Ref. [5]. Hence, if we want to implement supersymmetry in this model, and to remain in a perturbative regime, it is natural that supersymmetry is broken at the TeV scale. This is very important because one of the motivations for supersymmetry is that it can help to understand the hierarchy problem: if it is broken at the TeV scale. Notwithstanding, in the context of the SM it is necessary to assume that the breakdown of supersymmetry happens at the TeV scale. However, other 3-3-1 models, i.e., with different representation content, have a different upper limit for the maximal energy scale [6].

(iii) The quantization of the electric charge is possible even in the SM context. This is because of the classical (hypercharge invariance of the Yukawa interactions) and quantum constraints (anomalies) [7]. However, this occurs only family by family and if there are no right-handed neutrinos (neutrinos if massive must be Majorana fields); or, when the three families are considered together the quantization of the electric charge is possible only if right-handed neutrinos with Majorana mass term are introduced [8] or another Higgs doublet [9] or some neutral fermions [10] are introduced. On the other hand, in the 3-3-1 model [2,3,6] the charge quantization in the three families case does not depend on if neutrinos are massless or massive particles [11].

(iv) In the context of the SM with only one generation, as in the previous item, both classical and quantum constraints imply that the quantization of the charge and the vectorial nature of the electromagnetic interactions arise together. When right-handed neutrinos are added there is no charge quantization but the vectorial nature of electromagnetic interactions survives. Both of them are restored if neutrinos are Majorana particles [7]. In the three generation case neutrinos

ought to be also Majorana particles in order to retain both features of the electromagnetic interactions [8,12]. On the other hand, in all sorts of 3-3-1 models the quantization of the charge and the vectorial nature of the electromagnetic interactions are related one to another and are also independent of the nature of the neutrinos [13].

Last but not least, (v) if we accept the criterion that particle symmetries are determined by the known leptonic sector, and if each generation is treated separately, then $SU(3)_L$ is the largest chiral symmetry group to be considered among $(\nu,e,e^c)_L$. The lepton family quantum number is gauged; only the total lepton number, L, remains a global quantum number (or equivalently we can define $\mathcal{F}=B+L$ as the global conserved quantum number where B is the baryonic number [14]). On the other hand, if right-handed neutrinos do exist, as it appears to be the case [1], the symmetry among $(\nu_e,e,\nu_e^c,e^c)_L$ would be $SU(4)_L \otimes U(1)_N$ [15]. This is possibly the last symmetry among leptons. There is no room for $SU(5)_L \otimes U(1)_N$ if we restrict ourselves to the case of leptons with electric charges $\pm 1,0$ [16]. Hence, in this case all versions of the 3-3-1 model, for instance the one in Refs. [2,3,17], and the one in Refs. [6,18,19], are different $SU(3)_L$-projections of the larger $SU(4)_L$ symmetry [15].

Besides the characteristic features given above, which we can consider predictions of the model, the model has some interesting phenomenological consequences. (a) An extended version of some 3-3-1 models may solve the strong CP problem. It was shown by Pal [20] that in 3-3-1 models [2,3,6,19] the more general Yukawa couplings admit a Peccei-Quinn symmetry [21] and that symmetry can be extended to the Higgs potential and, therefore, making it a symmetry of the entire Lagrangian. This is obtained by introducing extra Higgs scalar multiplets transforming under the 3-3-1 symmetry as $\Delta \sim (\mathbf{1},\mathbf{10},-3)$, for the model of Refs. [2,3], or $\Delta \sim (\mathbf{1},\mathbf{10},-1)$, for the model of Refs. [6,19]. In this case the resulting axion can be made invisible. The interesting thing is that in those sort of models the Peccei-Quinn symmetry is an automatic symmetry, in the sense that it does not have to be imposed separately on the Lagrangian but it is a consequence of the gauge symmetry and a discrete symmetry. (b) There exist new contributions to the neutrinoless double beta decay in models with three scalar triplets [14] or in the model with the sextet [22]. If the model is extended with a neutral scalar singlet it is possible to have a safe Majoron-like Goldstone boson and there are also contributions to that decay with Majoron emission [22]. (c) It is the simplest model that includes bileptons of both types: scalar and vector ones. In fact, although there are several models which include doubly charged scalar fields, not many of them incorporate doubly charged vector bosons: this is a particularity of the 3-3-1 model of Refs. [2,3]. (d) The model has several sources of CP violation. In the 3-3-1 model [2,3] we can implement the violation of the CP symmetry, spontaneously [23,24] or explicitly [25]. In models with exotic leptons it is possible to implement soft CP violation [26]. (e) The extra neutral vector boson Z' conserves flavor in the leptonic but not in the quark sector. The couplings to the leptons are leptophobic because of the suppression factor $(1-4s_W^2)^{1/2}$ but with some quarks there are enhancements

because of the factor $(1-4s_W^2)^{-1/2}$ [27]. (f) Although the minimal scalar sector of the model is rather complicated, with at least three triplets, we would like to stress that it contains all extensions of the electroweak standard model with extra scalar fields: two or more doublets [28], neutral gauge singlet [29], or doubly charged scalar fields [30], or a combination of all that. However, some couplings which are allowed in the multi-Higgs extensions are not in the present model when we consider an $SU(2) \otimes U(1)$ subgroup. Inversely, there are some interactions that are allowed in the present models that are not in the multi-Higgs extensions of the SM, for instance, trilinear couplings among the doublets which have no analog in the SM, or even in the minimal supersymmetric standard model (MSSM). It means that the model preserves the memory of the 3-3-1 original symmetry. Hence, in our opinion, the large Higgs sector is not an intrinsic trouble of this model. (g) Even if we restrict ourselves to leptons of charge $0,\pm 1$ we can have exotic neutral [18] or charged heavy leptons [17]. (h) Neutrinos can gain Majorana masses if we allow one of the neutral components of the scalar sextet to gain a nonzero vacuum expectation value [31], or if we introduce right-handed neutrinos [32], or if we add either terms in the scalar potential that break the total lepton number or an extra charged lepton transforming as a singlet under the 3-3-1 symmetry [33].

Of course, some of the benefits of this type of model, such as items (i), (ii), and (iv) above, can be considered only as a hint to the final resolution of those problems: they depend on the representation content and we can always ask ourselves what is the main principle behind the representation content. Anyway, we think that the 3-3-1 models have interesting features by themselves and that it is well motivated to generalize them by introducing supersymmetry. In the present paper we built exhaustively the supersymmetric version of the 3-3-1 model of Refs. [2,3]. A first supersymmetric 3-3-1 model, without the introduction of the scalar sextet, was considered some years ago by Duong and Ma [34].

The outline of the paper is as follows. In Sec. II we present the representation content of the supersymmetric 3-3-1 model. We build the Lagrangian in Sec. III. In Sec. IV we analyze the scalar potential; in particular, we found the mass spectrum of the neutral scalar and have shown that the lightest scalar field has an upper limit of 124.5 GeV. Our conclusion and a comparison with the supersymmetric model of Ref. [34] are in the last section. In the Appendixes we show the scalar mass matrices and the constraint equations for completeness.

II. THE SUPERSYMMETRIC MODEL

The fact that in the 3-3-1 model of Refs. [2,3] we have the constraint at tree level $\sin^2 \theta_W < 1/4$ means, as we said before, that the model predicts that there exists an energy scale, say μ, at which the model loses its perturbative character. This characteristic remains valid when supersymmetry is introduced and hence the breaking of the supersymmetry occurs in a natural way at the TeV scale in this supersymmetric 3-3-1 model. Some aspects of the supersymmetric 3-3-1 model (3-3-1s for short) have been already considered in

Refs. [34,35] and we will comment on these later.

However, let us first consider the particle content of the model without supersymmetry [2,3]. We have the leptons transforming as

$$L_l = \begin{pmatrix} \nu_l \\ l \\ l^c \end{pmatrix}_L \sim (\mathbf{1,3},0), \quad l = e, \mu, \tau. \tag{2}$$

In parentheses it appears the transformation properties under the respective factors $[SU(3)_C, SU(3)_L, U(1)_N]$. We have not introduced right-handed neutrinos and for the moment we assume here that the neutrinos are massless; however, see [31–33].

In the quark sector, one quark family is also put in the triplet representation

$$Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ J \end{pmatrix}_L \sim \left(\mathbf{3,3}, \frac{2}{3}\right), \tag{3}$$

and the respective singlets are given by

$$u_{1L}^c \sim \left(\mathbf{3^*,1}, -\frac{2}{3}\right), \quad d_{1L}^c \sim \left(\mathbf{3^*,1}, \frac{1}{3}\right), \quad J_L^c \sim \left(\mathbf{3^*,1}, -\frac{5}{3}\right), \tag{4}$$

writing all the fields as left handed.

The other two quark generations we put in the antitriplet representation

$$Q_{2L} = \begin{pmatrix} d_2 \\ u_2 \\ j_1 \end{pmatrix}_L, \quad Q_{3L} = \begin{pmatrix} d_3 \\ u_3 \\ j_2 \end{pmatrix}_L \sim \left(\mathbf{3,3^*}, -\frac{1}{3}\right), \tag{5}$$

and also with the respective singlets,

$$u_{2L}^c, u_{3L}^c \sim \left(\mathbf{3^*,1}, -\frac{2}{3}\right), \quad d_{2L}^c, d_{3L}^c \sim \left(\mathbf{3^*,1}, \frac{1}{3}\right),$$

$$j_{1L}^c, j_{2L}^c \sim \left(\mathbf{3^*,1}, \frac{4}{3}\right). \tag{6}$$

On the other hand, the scalars which are necessary to generate the fermion masses are

$$\eta = \begin{pmatrix} \eta^0 \\ \eta_1^- \\ \eta_2^+ \end{pmatrix} \sim (\mathbf{1,3},0), \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (\mathbf{1,3},+1),$$

$$\chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} \sim (\mathbf{1,3},-1), \tag{7}$$

and one way to obtain an arbitrary mass matrix for the leptons is to introduce the following symmetric antisextet:

$$S = \begin{pmatrix} \sigma_1^0 & \dfrac{h_2^+}{\sqrt{2}} & \dfrac{h_1^-}{\sqrt{2}} \\ \dfrac{h_2^+}{\sqrt{2}} & H_1^{++} & \dfrac{\sigma_2^0}{\sqrt{2}} \\ \dfrac{h_1^-}{\sqrt{2}} & \dfrac{\sigma_2^0}{\sqrt{2}} & H_2^{--} \end{pmatrix} \sim (\mathbf{1,6^*},0). \tag{8}$$

Now, we introduce the minimal set of particles in order to implement the supersymmetry [36]. We have the sleptons corresponding to the leptons in Eq. (2), squarks related to the quarks in Eqs. (4)–(6), and the Higgsinos related to the scalars given in Eqs. (7) and (8). Besides, in order to cancel chiral anomalies generated by the superpartners of the scalars, we have to add the following Higgsinos in the respective antimultiplets:

$$\tilde{\eta}' = \begin{pmatrix} \tilde{\eta}'^0 \\ \tilde{\eta}_1'^+ \\ \tilde{\eta}_2'^- \end{pmatrix}_L \sim (\mathbf{1,3^*},0), \quad \tilde{\rho}' = \begin{pmatrix} \tilde{\rho}'^- \\ \tilde{\rho}'^0 \\ \tilde{\rho}'^{--} \end{pmatrix}_L \sim (\mathbf{1,3^*}, -1),$$

$$\tilde{\chi}' = \begin{pmatrix} \tilde{\chi}'^+ \\ \tilde{\chi}'^{++} \\ \tilde{\chi}'^0 \end{pmatrix}_L \sim (\mathbf{1,3^*},+1), \tag{9a}$$

$$\tilde{S}' = \begin{pmatrix} \tilde{\sigma}_1'^0 & \dfrac{\tilde{h}_2'^-}{\sqrt{2}} & \dfrac{\tilde{h}_1'^+}{\sqrt{2}} \\ \dfrac{\tilde{h}_2'^-}{\sqrt{2}} & \tilde{H}_1'^{--} & \dfrac{\tilde{\sigma}_2'^0}{\sqrt{2}} \\ \dfrac{\tilde{h}_1'^+}{\sqrt{2}} & \dfrac{\tilde{\sigma}_2'^0}{\sqrt{2}} & \tilde{H}_2'^{++} \end{pmatrix}_L \sim (\mathbf{1,6},0). \tag{9b}$$

There are also the scalar partners of the Higgsinos defined in Eq. (9) and we will denote them by η', ρ', χ', S'. This is the particle content which we will consider as the minimal 3-3-1s model if the charged lepton masses are generated by the antisextet S.

Summarizing, we have in the 3-3-1s model the following superfields: $\hat{L}_{e,\mu,\tau}$, $\hat{Q}_{1,2,3}$, $\hat{\eta}$, $\hat{\rho}$, $\hat{\chi}$, $\hat{S}$; $\hat{\eta}'$, $\hat{\rho}'$, $\hat{\chi}'$, $\hat{S}'$; $\hat{u}_{1,2,3}^c$, $\hat{d}_{1,2,3}^c$, $\hat{J}^c$, and $\hat{j}_{1,2}^c$, i.e., 23 chiral superfields, and 17 vector superfields: $\hat{V}^a$, $\hat{V}^\alpha$, and $\hat{V}'$. In the MSSM there are 14 chiral superfields and 12 vector superfields.

III. THE LAGRANGIAN

With the superfields introduced in the last section we can build a supersymmetric invariant Lagrangian. It has the following form:

$$\mathcal{L}_{331} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}. \tag{10}$$

Here $\mathcal{L}_{SUSY}$ is the supersymmetric piece, while $\mathcal{L}_{soft}$ explicitly breaks SUSY. Below we will write each of these Lagrangians in terms of the respective superfields.

A. The supersymmetric term

The supersymmetric term can be divided as follows:

$$\mathcal{L}_{SUSY} = \mathcal{L}_{Lepton} + \mathcal{L}_{Quarks} + \mathcal{L}_{Gauge} + \mathcal{L}_{Scalar}, \quad (11)$$

where each term is given by

$$\mathcal{L}_{Lepton} = \int d^4\theta [\hat{\bar{L}} e^{2g\hat{V}} \hat{L}], \quad (12)$$

$$\begin{aligned}
\mathcal{L}_{Quarks} = \int d^4\theta [\, & \hat{\bar{Q}}_1 \exp\{[2g(\hat{V}_c + \hat{V}) + (2g'/3)\hat{V}']\} \hat{Q}_1 \\
& + \hat{\bar{Q}}_\alpha \exp\{[2g(\hat{V}_c + \hat{\bar{V}}) - (g'/3)\hat{V}']\} \hat{Q}_\alpha \\
& + \hat{\bar{u}}_i^c \exp\{[2g\hat{\bar{V}}_c - (2g'/3)\hat{V}']\} \hat{u}_i^c \\
& + \hat{\bar{d}}_i^c \exp\{[2g\hat{\bar{V}}_c + (g'/3)\hat{V}']\} \hat{d}_i^c \\
& + \hat{\bar{J}}^c \exp\{[2g\hat{\bar{V}}_c - (5g'/3)\hat{V}']\} \hat{J}^c \\
& + \hat{\bar{j}}_\beta^c \exp\{[2g\hat{\bar{V}}_c + (4g'/3)\hat{V}']\} \hat{j}_\beta^c] \quad (13)
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{L}_{Gauge} = \frac{1}{4} \Big[& \int d^2\theta \, [W_c^a W_c^a + W^a W^a + W' W'] \\
& + \int d^2\bar{\theta} \, [\bar{W}_c^a \bar{W}_c^a + \bar{W}^a \bar{W}^a + \bar{W}' \bar{W}'] \Big], \quad (14)
\end{aligned}$$

where $\hat{V}_c = T^a \hat{V}_c^a$, $\hat{\bar{V}}_c = \bar{T}^a \hat{V}_c^a$, $\hat{V} = T^a \hat{V}^a$, $\hat{\bar{V}} = \bar{T}^a \hat{V}^a$, and $T^a = \lambda^a/2$ $(\bar{T}^a = -\lambda^{*a}/2)$ are the generators of triplets (antitriplets) of $SU(3)$, i.e., $a = 1, \ldots, 8$, and g and g' are the gauge couplings of $SU(3)$ and $U(1)$, respectively. W_c^a, W^a, and W' are the strength fields, and they are given by

$$W_{\alpha c}^a = -\frac{1}{8g} \bar{D}\bar{D} e^{-2g\hat{V}_c} D_\alpha e^{2g\hat{V}_c},$$

$$W_\alpha^a = -\frac{1}{8g} \bar{D}\bar{D} e^{-2g\hat{V}} D_\alpha e^{2g\hat{V}},$$

$$W_\alpha' = -\frac{1}{4} \bar{D}\bar{D} D_\alpha \hat{V}'. \quad (15)$$

Finally

$$\begin{aligned}
\mathcal{L}_{Scalar} = \int d^4\theta [\, & \hat{\bar{\eta}} e^{2g\hat{V}} \hat{\eta} + \hat{\bar{\rho}} e^{2g\hat{V} + g'\hat{V}'} \hat{\rho} + \hat{\bar{\chi}} e^{2g\hat{V} - g'\hat{V}'} \hat{\chi} \\
& + \hat{\bar{S}} e^{2g\hat{V}} \hat{S} + \hat{\bar{\eta}}' e^{2g\hat{\bar{V}}} \hat{\eta}' + \hat{\bar{\rho}}' e^{2g\hat{\bar{V}} - g'\hat{V}'} \hat{\rho}' \\
& + \hat{\bar{\chi}}' e^{2g\hat{\bar{V}} + g'\hat{V}'} \hat{\chi}' + \hat{\bar{S}}' e^{2g\hat{\bar{V}}} \hat{S}'] + \int d^2\theta \, W \\
& + \int d^2\bar{\theta} \, \bar{W}, \quad (16)
\end{aligned}$$

where W is the superpotential, which we discuss in the next subsection.

B. Superpotential

The superpotential of our model is given by

$$W = \frac{W_2}{2} + \frac{W_3}{3}, \quad (17)$$

with W_2 having only two chiral superfields and the terms permitted by our symmetry are

$$W_2 = \mu_0 \hat{L} \hat{\eta}' + \mu_\eta \hat{\eta} \hat{\eta}' + \mu_\rho \hat{\rho} \hat{\rho}' + \mu_\chi \hat{\chi} \hat{\chi}' + \mu_S \hat{S} \hat{S}', \quad (18)$$

and in the case of three chiral superfields the terms are

$$\begin{aligned}
W_3 = & \lambda_1 \epsilon \hat{L}\hat{L}\hat{L} + \lambda_2 \epsilon \hat{L}\hat{L}\hat{\eta} + \lambda_3 \hat{L}\hat{L}\hat{S} + \lambda_4 \epsilon \hat{L}\hat{\chi}\hat{\rho} + f_1 \epsilon \hat{\rho}\hat{\chi}\hat{\eta} \\
& + f_2 \hat{\eta}\hat{\eta}\hat{S} + f_3 \hat{\chi}\hat{\rho}\hat{S} + f_1' \epsilon \hat{\rho}'\hat{\chi}'\hat{\eta}' + f_2' \hat{\eta}'\hat{\eta}'\hat{S}' \\
& + f_3' \hat{\chi}'\hat{\rho}'\hat{S}' + \sum_i [\kappa_{1i} \hat{Q}_1 \hat{\eta}' \hat{u}_i^c + \kappa_{2i} \hat{Q}_1 \hat{\rho}' \hat{d}_i^c] \\
& + \kappa_3 \hat{Q}_1 \hat{\chi}' \hat{J}^c + \sum_{\alpha i} [\kappa_{4\alpha i} \hat{Q}_\alpha \hat{\eta} \hat{d}_i^c + \kappa_{5\alpha i} \hat{Q}_\alpha \hat{\rho} \hat{u}_i^c] \\
& + \sum_{\alpha\beta} \kappa_{6\alpha\beta} \hat{Q}_\alpha \hat{\chi} \hat{j}_\beta^c + \sum_{\alpha ij} \kappa_{7\alpha ij} \hat{Q}_\alpha \hat{L}_i \hat{d}_j^c \\
& + \sum_{i,j,k} \xi_{1ijk} \hat{d}_i^c \hat{d}_j^c \hat{u}_k^c + \sum_{ij\beta} \xi_{2ij\beta} \hat{u}_i^c \hat{u}_j^c \hat{j}_\beta^c + \sum_{i\beta} \xi_{3i\beta} \hat{d}_i^c \hat{J}^c \hat{j}_\beta^c, \quad (19)
\end{aligned}$$

with $i, j, k = 1, 2, 3$, $\alpha = 2, 3$, and $\beta = 1, 2$.

All the seven neutral scalar components $\eta^0, \rho^0, \chi^0, \sigma_2^0, \eta'^0, \rho'^0$, and χ'^0 gain nonzero vacuum expectation values. This arises from the mass matrices for quarks. In fact, defining $\langle \eta^0 \rangle = v_\eta/\sqrt{2}$, $\langle \eta^{0'} \rangle = v_\eta'/\sqrt{2}$, etc, from the superpotential in Eq. (19) the following mass matrices arise:

$$\Gamma^u = \frac{1}{\sqrt{2}} \begin{pmatrix} \kappa_{11} v_\eta' & \kappa_{12} v_\eta' & \kappa_{13} v_\eta' \\ \kappa_{521} v_\rho & \kappa_{522} v_\rho & \kappa_{523} v_\rho \\ \kappa_{531} v_\rho & \kappa_{532} v_\rho & \kappa_{533} v_\rho \end{pmatrix}, \quad (20)$$

for the u quarks, and

$$\Gamma^d = \frac{1}{\sqrt{2}} \begin{pmatrix} \kappa_{21} v'_\rho & \kappa_{22} v'_\rho & \kappa_{23} v'_\rho \\ \kappa_{421} v_\eta & \kappa_{422} v_\eta & \kappa_{423} v_\eta \\ \kappa_{431} v_\eta & \kappa_{432} v_\eta & \kappa_{4333} v_\eta \end{pmatrix}, \qquad (21)$$

for the d quarks, and for the exotic quarks, J and $j_{1,2}$, we have $M_J = \kappa_3 v'_\chi$ and

$$\Gamma^j = \frac{v_\chi}{\sqrt{2}} \begin{pmatrix} \kappa_{621} & \kappa_{622} \\ \kappa_{631} & \kappa_{632} \end{pmatrix}, \qquad (22)$$

respectively.

From Eqs. (20), (21), and (22) we see that all the vacuum expectation values (VEVs) have to be different from zero in order to give mass to all quarks. Notice also that the u-like and d-like mass matrices have no common VEVs. On the other hand, the charged lepton mass matrix is already given by $M^l_{ij} = v_{\sigma_2} \lambda_{3ij}/\sqrt{2}$, where v_{σ_2} is the VEV of the $\langle \sigma^0_2 \rangle$ component of the antisextet S in Eq. (8). However, $v'_{\sigma_{1,2}}$ can both be zero since the sextet S' does not couple to leptons at all.

The terms which are proportional to the following constants: μ_0 in Eq. (18) and λ_1, λ_4, f_2, f'_2, κ_7, and $\xi_{1,2,3}$ in Eq. (19) violate the conservation of the $\mathcal{F} = B + L$ quantum

number. For instance, if we allow the ξ_1 term it implies in proton decay [35]. However, if we assume the global $U(1)_\mathcal{F}$ symmetry, it allows us to introduce the R-conserving symmetry [37], defined as $R = (-1)^{3\mathcal{F} + 2S}$. The $\mathcal{F}$ number attribution is

$$\mathcal{F}(U^{--}) = \mathcal{F}(V^-) = -\mathcal{F}(J_1) = \mathcal{F}(J_{2,3}) = \mathcal{F}(\rho^{--})$$

$$= \mathcal{F}(\chi^{--}) = \mathcal{F}(\chi^-) = \mathcal{F}(\eta^-_2) = \mathcal{F}(\sigma^0_1) = 2, \qquad (23)$$

with $\mathcal{F} = 0$ for the other Higgs scalar, while for leptons and the known quarks $\mathcal{F}$ coincides with the total lepton and baryon numbers, respectively. As in the MSSM this definition implies that all known standard model's particles have even R parity while their supersymmetric partners have odd R parity. The terms ξ_2, λ_4 were not considered in Ref. [35]. However, the term with ξ_2 involves an exotic quark (heavier than the proton) so the analysis in that reference is still valid.

As usual, the supersymmetry breaking is accomplished by including the most general renormalizable soft-supersymmetry breaking terms but now, they must be also consistent with the 3-3-1 gauge symmetry. We will also include terms which explicitly violate the R-like symmetry. These soft terms are given by

$$\mathcal{L}_{soft} = -\frac{1}{2}\left[m_{\lambda_c} \sum_a \lambda^a_c \lambda^a_c + m_\lambda \sum_a (\lambda^a \lambda_a) + m' \lambda \lambda + \text{H.c.} \right] - m^2_L \tilde{L}^\dagger \tilde{L} - m^2_{Q_1} \tilde{Q}^\dagger_1 \tilde{Q}_1 + \sum_i [\tilde{u}^{c\dagger}_i m^2_{u_i} \tilde{u}^c_i - \tilde{d}^{c\dagger}_i m^2_{d_i} \tilde{d}^c_i]$$

$$- m^2_J \tilde{J}^{c\dagger} \tilde{J}^c - \sum_\beta \tilde{j}^{c\dagger}_\beta m^2_{j_\beta} \tilde{j}^c_\beta - \sum_\alpha m^2_{Q_\alpha} \tilde{Q}^\dagger_\alpha \tilde{Q}_\alpha - m^2_\eta \eta^\dagger \eta - m^2_\chi \chi^\dagger \chi - m^2_\rho \rho^\dagger \rho - m^2_s \text{Tr}(S^\dagger S) - m^2_{\eta'} \eta'^\dagger \eta'$$

$$- m^2_{\rho'} \rho'^\dagger \rho' - m^2_{\chi'} \chi'^\dagger \chi' - m^2_{s'} \text{Tr}(S'^\dagger S') - \left[k_1 \epsilon \rho \chi \eta + k_2 \eta \eta S^\dagger + k_3 \chi \rho S^\dagger + k'_1 \epsilon \rho' \chi' \eta' + k'_2 \eta' \eta' S'^\dagger + k'_3 \chi' \rho' S'^\dagger \right.$$

$$+ M^2 \tilde{L} \eta^\dagger + \varepsilon_0 \sum_{ijk} \epsilon_{ijk} \tilde{L}_i \tilde{L}_j \tilde{L}_k + \varepsilon_1 \sum_{ijk} \epsilon_{ijk} \tilde{L}_i \tilde{L}_j \eta_k + \varepsilon_2 \sum_{ij} \tilde{L}_i \tilde{L}_j S_{ij} + \varepsilon_3 \sum_{ijk} \epsilon_{ijk} \tilde{L}_i \chi_j \rho_k$$

$$+ \sum_i \tilde{Q}_1 (\zeta_{1i} \eta' \tilde{u}^c_i + \zeta_{2i} \rho' \tilde{d}^c_i + \zeta_{3J} \chi' \tilde{J}^c) + \sum_\alpha \tilde{Q}_\alpha \left[\sum_i \left(\omega_{1\alpha i} \eta \tilde{d}^c_i + \omega_{2\alpha i} \rho \tilde{u}^c_i + \sum_j \omega_{3\alpha ij} \tilde{L}_i \tilde{d}^c_j \right) + \sum_\beta \omega_{4\alpha\beta} \chi \tilde{j}^c_\beta \right]$$

$$+ \sum_i \left(\sum_{jk} \mathsf{s}_{1ijk} \tilde{d}^c_i \tilde{d}^c_j \tilde{u}^c_k + \sum_\alpha \left[\mathsf{s}_{2i\alpha} \tilde{d}^c_i \tilde{J}^c \tilde{j}^c_\alpha + \sum_j \mathsf{s}_{3ij\alpha} \tilde{u}^c_i \tilde{u}^c_j \tilde{j}^c_\alpha \right] \right) + \text{H.c.} \right]. \qquad (24)$$

The terms proportional to k_2, k'_2, M^2, ε_0, ε_3, ω_3, and $\mathsf{s}_{1,2,3}$ violates the R-parity symmetry.

The $SU(3)$ invariance tell us that

$$m_L = \begin{pmatrix} m_{\tilde{\nu}} & 0 & 0 \\ 0 & m_{\tilde{l}} & 0 \\ 0 & 0 & m_{\tilde{l}^c} \end{pmatrix}, \qquad (25)$$

and we need $m_{\tilde{l}^c} = m_{\tilde{l}} = m_{\tilde{\nu}}$ and the same for the other mass

parameters. More details of the Lagrangian are given elsewhere [38].

IV. THE SCALAR POTENTIAL

The scalar potential is written as

$$V_{331} = V_D + V_F + V_{\text{soft}}, \qquad (26a)$$

where

$$V_D = -\mathcal{L}_D = \frac{1}{2}(D^a D^a + DD)\frac{g'^2}{2}(\rho^\dagger \rho - \rho'^\dagger \rho' - \chi^\dagger \chi + \chi'^\dagger \chi')^2 + \frac{g^2}{8}\sum_{i,j}(\eta_i^\dagger \lambda_{ij}^a \eta_j + \rho_i^\dagger \lambda_{ij}^a \rho_j + \chi_i^\dagger \lambda_{ij}^a \chi_j$$

$$+ S_{ij}^\dagger \lambda_{jk}^a S_{kl} - \eta_i'^\dagger \lambda_{ij}^{*a} \eta_j' - \rho_i'^\dagger \lambda_{ij}^{*a} \rho_j' - \chi_i'^\dagger \lambda_{ij}^{*a} \chi_j' - S_{ij}'^\dagger \lambda_{jk}^{*a} S_{kl}')^2, \tag{26b}$$

$$V_F = -\mathcal{L}_F = \sum_m F_m^* F_m = \sum_{i,j,k}\left[\left|\frac{\mu_\eta}{2}\eta_i' + \frac{f_1}{3}\epsilon_{ijk}\rho_j \chi_k + \frac{2f_2}{3}\eta_i S_{ij}\right|^2 + \left|\frac{\mu_\rho}{2}\rho_i' + \frac{f_1}{3}\epsilon_{ijk}\chi_j \eta_k + \frac{f_3}{3}\chi_i S_{ij}\right|^2\right.$$

$$+ \left|\frac{\mu_\chi}{2}\chi_i' + \frac{f_1}{3}\epsilon_{ijk}\rho_j \eta_k + \frac{f_3}{3}\rho_i S_{ij}\right|^2 + \left|\frac{\mu_s}{2}S_{ij}' + \frac{f_2}{3}\eta_i \eta_j + \frac{f_3}{3}\chi_i \rho_j\right|^2 + \left|\frac{\mu_\eta}{2}\eta_i + \frac{f_1'}{3}\epsilon_{ijk}\rho_j' \chi_k' + \frac{2f_2'}{3}\eta_i' S_{ij}'\right|^2 + \left|\frac{\mu_\rho}{2}\rho_i\right.$$

$$\left. + \frac{f_1'}{3}\epsilon_{ijk}\chi_j' \eta_k' + \frac{f_3'}{3}\chi_i' S_{ij}'\right|^2 + \left|\frac{\mu_\chi}{2}\chi_i + \frac{f_1'}{3}\epsilon_{ijk}\rho_j' \eta_k' + \frac{f_3'}{3}\rho_i' S_{ij}'\right|^2 + \left|\frac{\mu_s}{2}S_{ij} + \frac{f_2'}{3}\eta_i' \eta_j' + \frac{f_3'}{3}\chi_i' \rho_j'\right|^2\right], \tag{26c}$$

$$V_{soft} = -\mathcal{L}_{soft}^{scalar} = m_\eta^2 \eta^\dagger \eta + m_\rho^2 \rho^\dagger \rho + m_\chi^2 \chi^\dagger \chi + m_s^2 \,\mathrm{Tr}(S^\dagger S) + m_{\eta'}^2 \eta'^\dagger \eta' + m_{\rho'}^2 \rho'^\dagger \rho' + m_{\chi'}^2 \chi'^\dagger \chi' + m_{s'}^2 \,\mathrm{Tr}(S'^\dagger S')$$

$$+ [k_1 \epsilon\, \rho\chi\eta + k_2 \eta S^\dagger \eta + k_3 \chi^T S^\dagger \rho + k_1' \epsilon\, \rho'\chi'\eta' + k_2' \eta' S'^\dagger \eta' + k_3' \chi'^T S'^\dagger \rho' + \mathrm{H.c.}]. \tag{26d}$$

Note that $k_{1,2,3}, k'_{1,2,3}$ has dimension of mass and that the terms which are proportional to k_2, k_2' and f_2, f_2' violate the R parity.

It is instructive to rewrite Eqs. (26b) as follows:

$$V_D = \frac{g'^2}{2}(\rho^\dagger \rho - \chi^\dagger \chi - \rho'^\dagger \rho' + \chi'^\dagger \chi')^2 + \frac{g^2}{8}\left[\frac{4}{3}\sum_i (X_i^\dagger X_i)^2\right.$$

$$+ 2\sum_{i,j}(X_i^\dagger X_j)(X_j^\dagger X_i) + 2\sum_i (X_i^\dagger S)(S^\dagger X_i)$$

$$- \frac{4}{3}\sum_{i,j}(X_i^\dagger X_i)(X_j^\dagger X_j) - \frac{2}{3}\mathrm{Tr}(S^\dagger S)\sum_i (X_i^\dagger X_i)$$

$$+ 2\,\mathrm{Tr}[(S^\dagger S)^2] - \frac{2}{3}(\mathrm{Tr}\, S^\dagger S)^2 + \mathrm{pt}\right], \tag{27}$$

where "pt" in the expression above denotes the replacements $X_i' \leftrightarrow X_i$ but not in g' which is always the coupling constant of the $U(1)_N$ factor. In the same way we rewrite Eq. (26c) as

$$V_F = \sum_i \frac{\mu_{X_i}^2}{4}(X_i^\dagger X_i + \mathrm{pt}) + \frac{|\mu_S^2|}{4}[\mathrm{Tr}(S^\dagger S) + \mathrm{pt}] + \frac{1}{6}\sum_{i\neq j\neq k}\left\{\frac{\mu_{X_i}^*}{6}[f_1 \epsilon_{ijk}X_i'^\dagger X_j X_k + f_1' \epsilon_{ijk}X_i^\dagger X_j' X_k']\right.$$

$$+ \frac{f_2}{3}\left[\mu_\eta^* \eta'^\dagger S\eta + \frac{\mu_S^*}{2}\eta^\dagger S'\eta\right] + \mathrm{pt} + \frac{f_3}{6}[\mu_\rho^* \rho'^\dagger S\chi + \mu_\chi^* \chi'^\dagger S\rho + \mu_S^* \chi^\dagger S'\rho] + \mathrm{pt} + \mathrm{H.c.}\Big\}$$

$$+ |f_1|^2\left[\sum_{i\neq j}(X_i^\dagger X_i)(X_j^\dagger X_j) - (X_j^\dagger X_i)(X_i^\dagger X_j)\right] + \mathrm{pt} + \frac{4|f_2|^2}{9}[(\eta S)^\dagger(\eta S) + (\eta^\dagger \eta)^2]$$

$$+ \mathrm{pt} + \frac{|f_3|^2}{9}[(\chi S)^\dagger(\chi S) + (\rho S)^\dagger(\rho S) + (\chi^\dagger \chi)^\dagger(\rho^\dagger \rho)] + \mathrm{pt} + \frac{2f_1^* f_2}{9}\epsilon(\rho\chi)^\dagger \eta S + \mathrm{pt}$$

$$+ \left[\frac{f_1^* f_3}{9}\epsilon(\chi\eta)^\dagger \chi S + \mathrm{pt} + \frac{f_2^* f_3}{9}(\eta^\dagger \chi)(\eta^\dagger \rho) + \mathrm{pt} + \mathrm{H.c.}\right], \tag{28}$$

where "pt" in the expression above denotes as before the replacements $X_i' \leftrightarrow X_i$, and $f'_{1,2,3} \leftrightarrow f_{1,2,3}$, and $k'_{1,2,3} \leftrightarrow k_{1,2,3}$. We have omitted $SU(3)$ indices since we have denoted the unprimed triplets wherever it is possible as $X_i = \eta, \rho, \chi$ and $X_i' = \eta', \rho', \chi'$ but in each term only an unprimed (primed) field appears.

We can now work out the mass spectra of the scalar and pseudoscalar fields by making a shift of the form $X \rightarrow (1/\sqrt{2})(v_X + H_X + iF_X)$ (similarly for the case of the primed fields) for all the neutral scalar fields of the multiplets X_i. Note that H_X and F_X are not mass eigenstates yet. We will denote H_i, $i=1,\ldots,8$ and A_i, $i=1,\ldots,6$ the respective massive fields; $G_{1,2}$ will denote the two neutral Goldstone bosons. The mass matrices appear in the Appendix A, for the real scalars, and in the Appendix B for the pseudoscalar case. The constraint equations are given in the Appendix C. We will use the following set of parameters in the scalar potential below:

$$f_1 = f_3 = 1, \quad f_1' = f_3' = 10^{-6} \quad \text{(dimensionless)}, \quad (29)$$

and

$$-k_1 = k_1' = 10, \quad k_3 = k_3' = -100,$$

$$-\mu_\eta = \mu_\rho = -\mu_s = \mu_\chi = 1000 \quad \text{(in GeV)}, \quad (30)$$

we also use the constraint $V_\eta^2 + V_\rho^2 + 2V_2^2 = (246 \text{ GeV})^2$ coming from M_W, where, we have defined $V_\eta^2 = v_\eta^2 + v_\eta'^2$, $V_\rho^2 = v_\rho^2 + v_\rho'^2$, and $V_2^2 = v_{\sigma_2}^2 + v_{\sigma_2'}^2$. Assuming that $v_\eta = 20$, $v_\chi = 1000$, $v_{\sigma_2} = 10$, and $v_\eta' = v_\rho' = v_{\sigma_2}' = v_\chi' = 1$ in GeV, the value of v_ρ is fixed by the constraint above.

With this set of values for the parameters the real mass eigenstates H_i are obtained by the diagonalization of the mass matrix given in the Appendix A. Besides the constraint equations (Appendix C) and imposing the positivity of the eigenvalues (mass square), and the values for the parameters given above, we obtain the following values for the masses of the scalar sector (in GeV): $M_{H_i} = 121.01$, 277.14,515.26,963.68,1218.8,1243.24,3797.86, and 4516.43, where $i = 1, \ldots, 8$ and $M_{H_j} > M_{H_i}$ with $j > i$. In the pseudoscalar sector we have verified analytically that the mass matrix in the Appendix B, has two Goldstone bosons as it should be. The other six physical pseudoscalars have the following masses, with the same parameters as before, in GeV, $M_{A_i} = 276.4,515.3,963.65,1243.24,3797.85$, and 4516.43.

The behavior of the lightest scalar (H_1) and pseudoscalar (A_1) as a function of v_χ is shown in Fig. 1 for a given choice of the parameters, we see that, at the tree level, there is an upper limit for the mass of the lightest scalar $M_{H_1} < 124.5$ GeV and that for these values of the parameters $M_{A_1} > M_Z$. Other values of the parameters give higher or lower values for the upper limit of M_{H_1}. Of course, radiative corrections have to be taken into account, however, this has to be done in the context of the supersymmetric 3-3-1 model which is not in the scope of the present work. Hence the mass square of the lightest real scalar boson has an upper bound (see Fig. 1)

$$M_{H_1}^2 \lesssim (124.5 + \epsilon)^2 \text{ GeV}^2, \quad (31)$$

where 124.5 GeV is the tree value ($\epsilon = 0$). We recall that in the MSSM if $M_{A_1} > M_Z$ the upper limit on the mass of the
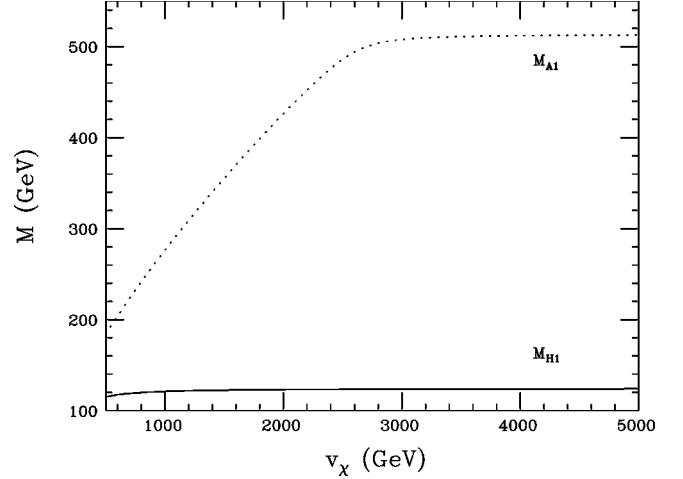

FIG. 1. The eigenvalues of the mass matrix given in Appendix A; M_{H_1} is the mass of the lightest scalar and M_A the mass of the lightest pseudoscalar.

lightest neutral scalar is M_Z at the tree level but radiative corrections raise it to 130 GeV [39].

V. CONCLUSIONS

We have built the complete supersymmetric version of the 3-3-1 model of Refs. [2,3]. Another possibility in this 3-3-1 model which avoids the introduction of the scalar sextet, S, was considered some years ago by Duong and Ma, Ref. [34], who built the supersymmetric version of that model. The sextet was substituted by a single charged lepton singlet $E_L \sim (\mathbf{1},1)$ and $E_L^c \sim (\mathbf{1},-1)$. Here we would like to point out the differences between our version of the supersymmetric 3-3-1 model and that of Ref. [34]. (a) Duong and Ma assumed that the breaking of $SU(3)_L \otimes U(1)_N \rightarrow SU(2)_L \otimes U(1)_Y$ occurred before the breaking of supersymmetry and the resulting model is a supersymmetric $SU(2)_L \otimes U(1)_Y$ model. Even in this case the scalar potential involving doublets of the residual gauge symmetry does not coincide with the potential of the MSSM. In the present work, we have considered that the supersymmetry is broken at the same time as the 3-3-1 gauge symmetry. Hence, we have to consider the complete 3-3-1 scalar potential. It means that in the Duong and Ma supersymmetric model there are no doubly charged charginos and exotic charged squarks. (b) In Ref. [34] it was assumed that some of the VEVs have zero value, unlikely we have considered all (but $\sigma_2'^0$ and σ_1^0) of them different from zero. Hence we are able to obtain realistic quark and charged lepton masses, as can be seen from Eqs. (20), (21), and (22). In Ref. [34] some of these masses have to be generated by radiative corrections [40]. From (a) and (b) we see that the supersymmetric 3-3-1 model considered in this work has different phenomenological features from the supersymmetric 3-3-1 model of Duong and Ma.

On the other hand, we would like to recall that in the MSSM with soft-breaking terms, it has been shown that at the tree level the lightest scalar has a mass which is less than M_Z if the mass of the top quark has an upper bound of 200

GeV [41] and this upper limit does not depend on the scale of supersymmetry masses and on the scale of supersymmetry breaking and holds independently of the short distance or large mass behavior of the theory [42] and it does not depend on the number of the Higgs doublets [43]. Radiative corrections [42] and the addition of extra Higgs multiplets [42,44] shift the limit above depending on the top mass or on the type of extra scalars. However, in the 3-3-1s that we have built here the limit of Ref. [41] is avoided.

From the phenomenological point of view there are several possibilities. Since it is possible to define the R-parity symmetry, the phenomenology of this model with R parity conserved has similar features to that of the R-conserving MSSM: the supersymmetric particles are pair produced and the lightest neutralino is the lightest supersymmetric particle. The mass spectra of all particles in this model are considered in Ref. [38]. However, there are differences between this model and the MSSM with or without R-parity breaking: due to the fact that there are doubly charged scalar and vector fields. Hence, we have doubly charged charginos which are mixtures of the superpartners of the U-vector boson with the doubly charged Higgsinos. This implies new interactions that are not present in the MSSM, for instance $\tilde{\chi}^{--}\tilde{\chi}^0 U^{++}$, $\tilde{\chi}^-\tilde{\chi}^- U^{++}$, $\tilde{l}^-l^-\tilde{\chi}^{++}$, where $\tilde{\chi}^{++}$ denotes any doubly charged chargino. Moreover, in the chargino production, besides the usual mechanism, we have additional contributions coming from the U-bilepton in the s channel. Due to this fact we expect that there will be an enhancement in the cross section of production of these particles in e^-e^- colliders, such as the Next Linear Collider (NLC) [38]. We will also have the singly charged charginos and neutralinos, as in the MSSM, where there are processes like $\tilde{l}^-l^+\tilde{\chi}^0$, $\tilde{\nu}_l l^-\tilde{\chi}^+$, with $\tilde{l}$ denoting any slepton; $\tilde{\chi}^-$ denotes singly charged chargino and $\tilde{\nu}_L$ denotes any sneutrino. The only difference is that in the MSSM there are five neutralinos and in the 3-3-1s model there are eight neutralinos.

Finally, we would like to call attention that, whatever the energy scale μ at which $\sin^2\theta_W(\mu)=1/4$ is in the nonsupersymmetric 3-3-1 model, when supersymmetry is added it will result in a rather different value for μ. In conclusion, we can say that the present model has a rich phenomenology that deserves to be studied in more detail.

APPENDIX A: MASS MATRIX OF THE SCALAR NEUTRAL FIELDS

Here we write down the complete symmetric mass matrix in the scalar CP-even sector; the constraint equations given in the Appendix C have been already taken into account:

$$\mathbf{M}_{11}=\frac{g^2 v_\eta^2}{3}+\frac{1}{18\sqrt{2}v_\eta}(f_1 f_3 v_\rho^2 v_{\sigma_2}-18k_1 v_\rho v_\chi+3f_1 v'_\rho \mu_\rho v_\chi$$
$$+f_1 f_3 v_{\sigma_2}v_\chi^2-3f'_1 \mu_\eta v'_\rho v'_\chi+3f_1 \mu_\chi v_\rho v'_\chi),$$

$$\mathbf{M}_{12}=-\frac{g^2 v_\eta v_\rho}{6}+\frac{1}{9\sqrt{2}}\left(\sqrt{2}f_1^2 v_\eta v_\rho-f_1 f_3 v_\rho v_{\sigma_2}\right.$$
$$\left.+9k_1 v_\chi-\frac{3}{2}\mu_\chi v'_\chi\right),$$

$$\mathbf{M}_{13}=-\frac{g^2 v_\eta v_\rho}{6}+\frac{1}{9\sqrt{2}}\left(9k_1 v_\rho-\frac{3}{2}f_1 \mu_\rho v'_\rho\right.$$
$$\left.+\sqrt{2}f_1^2 v_\eta v_\chi-f_1 f_3 v_{\sigma_2}v_\chi\right),$$

$$\mathbf{M}_{14}=\frac{g^2 v_\eta v_{\sigma_2}}{6}-\frac{f_1 f_3}{18\sqrt{2}}(v_\rho^2+v_\chi^2),\quad \mathbf{M}_{15}=-\frac{g^2 v_\eta v'_\eta}{3},$$

$$\mathbf{M}_{16}=\frac{g^2 v_\eta v'_\rho}{6}-\frac{1}{6\sqrt{2}}(\mu_\rho v_\chi-\mu_\eta v'_\chi),$$

$$\mathbf{M}_{17}=\frac{g^2 v_\eta v'_\chi}{6}+\frac{1}{6\sqrt{2}}(f'_1 \mu_\eta v'_\rho-f_1 \mu_\chi v_\rho),$$

$$\mathbf{M}_{18}=\frac{g^2 v_\eta v_{\sigma_2}}{6},$$

$$\mathbf{M}_{22}=\left(\frac{g^2}{3}+g'^2\right)v_\rho^2-\frac{v_\chi}{12\sqrt{2}v_\rho}(12k_1 v_\eta+6\sqrt{2}k_3 v_{\sigma_2}$$
$$+2f_1 \mu_\eta v'_\eta+\sqrt{2}f_3 \mu_s v'_{\sigma_2})+\frac{v'_\chi}{12\sqrt{2}v_\rho}(2f'_1 \mu_\rho v'_\eta$$
$$-\sqrt{2}f'_3 \mu_\rho v'_{\sigma_2}+2f_1 \mu_\chi v_\eta-\sqrt{2}\mu_\chi v_{\sigma_2}v'_\chi),$$

$$\mathbf{M}_{23}=-\left(\frac{g^2}{6}+g'^2\right)v_\rho v_\chi+\frac{1}{12\sqrt{2}}(12k_1 v_\eta+6\sqrt{2}k_3 v_{\sigma_2}$$
$$+2f'\mu_\eta v'_\eta+\sqrt{2}f_3 \mu_s v'_{\sigma_2})+\frac{v_\rho v_\chi}{9}(f_1^2+f_3^2),$$

$$\mathbf{M}_{24}=\frac{g^2}{12}v_\rho v_\chi+\frac{1}{18\sqrt{2}}\left(-2f_1 f_3 v_\eta v_\rho+\sqrt{2}f_3^2 v_{\sigma_2}v_\rho\right.$$
$$\left.+9\sqrt{2}k_3 v_\chi+\frac{3}{\sqrt{2}}f_3 \mu_\chi v'_\chi\right),$$

$$\mathbf{M}_{25}=\frac{g^2}{6}v'_\eta v_\rho+\frac{1}{6\sqrt{2}}(f_1 \mu_\eta v_\chi-f'_1 \mu_\rho v'_\chi),$$

$$M_{26} = -\left(\frac{g^2}{3} + g'^2\right)\frac{v_\rho v'_\rho}{3},$$

$$M_{27} = \left(\frac{g^2}{6} + g'^2\right)v_\rho v'_\chi - \frac{\mu_\rho}{12\sqrt{2}}(2f'_1 v'_\eta - f'_3 v'_{\sigma_2})$$
$$- \frac{\mu_\chi}{12\sqrt{2}}(2f_1 v_\eta - f_3 v_{\sigma_2}),$$

$$M_{28} = -\frac{g^2 v_\rho v'_{\sigma_2}}{12} + \frac{1}{12}(f_3\mu_s v_\chi + f'_3\mu_\rho v'_\chi),$$

$$M_{33} = \left(\frac{g^2}{3} + g'^2\right)v_\chi^2 - \frac{1}{12\sqrt{2}v_\chi}\left(12k_1 v_\eta v_\rho + \frac{12}{\sqrt{2}}k_3 v_\rho v_{\sigma_2}\right.$$
$$+ 2f_1\mu_\eta v'_\eta v_\rho - 2f_1\mu_\rho v_\eta v'_\rho + \sqrt{2}f_3\mu_\rho v'_\rho v_{\sigma_2}$$
$$\left. + \sqrt{2}f_3\mu_s v_\rho v'_{\sigma_2} - 2f'_1\mu_\chi v'_\eta v'_\rho + \sqrt{2}f_3\mu_\chi v'_\rho v'_{\sigma_2}\right),$$

$$M_{34} = \frac{g^2}{12}v_{\sigma_2}v_\chi + \frac{1}{18\sqrt{2}}\left(\frac{9}{\sqrt{2}}k_3 v_\rho + \frac{3}{\sqrt{2}}f_3\mu_\rho v'_\rho\right.$$
$$\left. - 2f_1 f_3 v_\eta v_\chi + \sqrt{2}f_3^2 v_{\sigma_2}v_\chi\right),$$

$$M_{35} = \frac{g^2}{6}v'_\eta v_\chi + \frac{1}{6\sqrt{2}}(f_1\mu_\rho v_\rho - f'_1\mu_\chi v'_\rho),$$

$$M_{36} = \left(\frac{g^2}{6} + g'^2\right)v'_\rho v_\chi + \frac{1}{12\sqrt{2}}(-2f_1\mu_\rho v_\eta + \sqrt{2}f_3\mu_\rho v_{\sigma_2}$$
$$- 2f'_1\mu_\chi v'_\eta + \sqrt{2}f'_3\mu_\chi v'_{\sigma_2}),$$

$$M_{37} = -\left(\frac{g^2}{3} + g'^2\right)v_\chi v'_\chi,$$

$$M_{38} = -\frac{g^2}{12}v'_{\sigma_2}v_\chi + \frac{1}{12}(f_3\mu_s v_\rho + f'_3\mu_\chi v'_\rho),$$

$$M_{44} = \frac{g^2}{12}v_{\sigma_2}^2 + \frac{1}{18\sqrt{2}v_{\sigma_2}}\left(f_1 f_3 v_\eta v_\rho^2 - \frac{18}{\sqrt{2}}k_3 v_\rho v_\chi\right.$$
$$- \frac{3}{\sqrt{2}}f_3\mu_\rho v'_\rho v_\chi + f_1 f_3 v_\eta v_\rho^2 + \frac{3}{\sqrt{2}}f'_3\mu_s v'_\rho v'_\chi$$
$$\left. - \frac{3}{\sqrt{2}}f_3\mu_\chi v_\rho v'_\chi\right),$$

$$M_{45} = \frac{g^2}{6}v'_\eta v_{\sigma_2},$$

$$M_{46} = -\frac{g^2}{12}v'_\rho v_{\sigma_2} + \frac{1}{12}(f_3\mu_\rho v_\chi + f'_3\mu_s v'_\chi),$$

$$M_{47} = -\frac{g^2}{12}v_{\sigma_2}v'_\chi + \frac{1}{12}(f'_3\mu_s v'_\rho + f_3\mu_\chi v_\rho),$$

$$M_{48} = -\frac{g^2}{12}v_{\sigma_2}v'_{\sigma_2},$$

$$M_{55} = \frac{g^2}{3}v'^2_\eta + \frac{1}{18\sqrt{2}v'_\eta}(f'_1 f'_3 v'^2_\rho v'_{\sigma_2} - 3f_1\mu_\rho v_\rho v_\eta$$
$$+ 3f'_1\mu_\chi v'_\rho v_\chi - 18k'_1 v'_\rho v'_\chi + 3f'_1\mu_\rho v_\rho v'_\chi$$
$$+ f'_1 f'_3 v'_{\sigma_2} v'^2_\chi),$$

$$M_{56} = -\frac{g^2}{6}v'_\eta v'_\rho + \frac{1}{9\sqrt{2}}\left(\sqrt{2}f'^2_1 v'_\eta v'_\rho - f'_1 f'_3 v'_\rho v'_{\sigma_2}\right.$$
$$\left. - \frac{3}{2}f'_1\mu_\chi v_\chi + 9k'_1 v'_\chi\right),$$

$$M_{57} = -\frac{g^2}{6}v'_\eta v'_\chi + \frac{1}{9\sqrt{2}}\left(9k'_1 v'_\rho - \frac{3}{2}f'_1\mu_\rho v_\rho + \sqrt{2}f'^2_1 v'_\eta v'_\chi\right.$$
$$\left. - f'_1 f'_3 v_{\sigma_2}v'_\chi\right),$$

$$M_{58} = -\frac{g^2}{6}v'_\eta v'_{\sigma_2} - \frac{f'_1 f'_3}{18\sqrt{2}}(v'^2_\rho + v'^2_\chi),$$

$$M_{66} = \left(\frac{g^2}{3} + g'^2\right)v'^2_\rho + \frac{1}{12\sqrt{2}v'_\rho}\left(2f_1\mu_\rho v_\eta v_\chi\right.$$
$$- \sqrt{2}f_3\mu_\rho v_{\sigma_2}v_\chi + 2f'_1\mu_\chi v'_\eta v_\chi - \sqrt{2}f'_3\mu_\chi v'_{\sigma_2}v_\chi$$
$$- 12k'_1 v'_\eta v'_\chi - \frac{12}{\sqrt{2}}k'_3 v'_{\sigma_2}v'_\chi - 2f'_1\mu_\eta v'_\eta v'_\chi$$
$$\left. - \sqrt{2}f'_3\mu_s v_{\sigma_2}v'_\chi\right),$$

$$M_{67} = -\left(\frac{g^2}{6} + g'^2\right)v'_\rho v'_\chi + \frac{1}{12\sqrt{2}}\left(12k'_1 v'_\eta + \frac{12}{\sqrt{2}}k'_3 v'_{\sigma_2}\right.$$
$$+ 2f'_1\mu_\eta v_\eta + \sqrt{2}f'_3\mu_s v_{\sigma_2} + \frac{3}{\sqrt{2}}f'^2_1 v'_\rho v'_\chi$$
$$\left. + \frac{3}{\sqrt{2}}f'^2_3 v'_\rho v'_\chi\right),$$

$$M_{68} = \frac{g^2}{12}v'_\rho v'_{\sigma_2} + \frac{f'_3 v'_\rho}{18\sqrt{2}}(\sqrt{2}f'_3 v'_{\sigma_2} - 2f'_1 v'_\eta),$$

$$\mathbf{M_{77}} = \left(\frac{g^2}{3} + g'^2\right)v'^2_\chi - \frac{1}{12\sqrt{2}v'_\chi}(12k'_1 v'_\eta v'_\rho + 12k'_3 v'_\rho v'_{\sigma_2}$$

$$+ 2f'_1\mu_\eta v_\eta v'_\rho - 2f'_1\mu_\rho v'_\eta v_\rho + \sqrt{2}f'_3\mu_\rho v_\rho v_{\sigma_2}$$

$$+ \sqrt{2}f'_3\mu_s v'_\rho v_{\sigma_2} - 2f_1\mu_\chi v_\eta v_\rho + \sqrt{2}f_3\mu_\chi v_\rho v_{\sigma_2}),$$

$$\mathbf{M_{78}} = \frac{g^2}{12}v'_{\sigma_2}v'_\chi + \frac{1}{12}(6k'_3 v'_\rho + f'_3\mu_\rho v_\rho)$$

$$+ \frac{f'_3 v'_\chi}{18\sqrt{2}}(f'_3 v'_{\sigma_2} - f'_1 v'_\eta),$$

$$\mathbf{M_{88}} = \frac{g^2}{12}v'^2_{\sigma_2} - \frac{1}{18\sqrt{2}v'_{\sigma_2}}\left(f'_1 f'_3 v'_\eta v'^2_\rho + \frac{3}{\sqrt{2}}f_3\mu_s v_\rho v_\chi\right.$$

$$+ \frac{3}{\sqrt{2}}f'_3\mu_\chi v'_\rho v_\chi + \frac{18}{\sqrt{2}}k'_3 v_\rho v'_\chi + \frac{3}{\sqrt{2}}f'_3\mu_\chi v'_\rho v_\chi$$

$$+ \left.\frac{18}{\sqrt{2}}k'_3 v'_\rho v'_\chi + \frac{3}{\sqrt{2}}f'_3\mu_\rho v_\rho v'_\chi - f'_1 f'_3 v'_\eta v'^2_\chi\right).$$

This mass matrix has no Goldstone bosons and eight mass eigenstates. Some typical values of the masses of these scalars, for a set of values of the parameters, are given in the text. In Fig. 1 we show the behavior of the mass of the lightest scalar H_1 with the v_χ, the largest VEV in the model.

APPENDIX B: MASS MATRIX OF THE PSEUDOSCALAR NEUTRAL FIELDS

The complete symmetric mass matrix in the CP-odd scalar sector, with the constraint equations of Appendix C taken into account, is given by

$$\mathcal{M}_{11} = \frac{1}{18\sqrt{2}v_\eta}(f_1 f_3 v^2_\rho v_{\sigma_2} - 18k_1 v_\rho v_\chi + 3f_1\mu_\rho v'_\rho v'_\chi),$$

$$\mathcal{M}_{12} = \frac{1}{6\sqrt{2}}(f_1\mu_\chi - 6f_1 v_\chi),$$

$$\mathcal{M}_{13} = \frac{1}{6\sqrt{2}}(f_1\mu_\rho v'_\rho - 6k_1 v_\rho),$$

$$\mathcal{M}_{14} = \frac{f_1 f_3}{18\sqrt{2}}(v^2_\rho + v^2_\chi),$$

$$\mathcal{M}_{15} = 0,$$

$$\mathcal{M}_{16} = \frac{1}{6\sqrt{2}}(f_1\mu_\eta v'_\chi - f'_1\mu_\rho v_\chi),$$

$$\mathcal{M}_{17} = \frac{1}{6\sqrt{2}}(f'_1\mu_\eta v'_\rho - f_1\mu_\chi v_\rho), \quad \mathcal{M}_{18} = 0,$$

$$\mathcal{M}_{22} = \frac{1}{6\sqrt{2}v_\rho}\left(-6k_1 v_\eta v_\chi - \frac{6}{\sqrt{2}}k_3 v_{\sigma_2}v_\chi - f_1\mu_\eta v'_\eta v_\chi\right.$$

$$- \frac{1}{\sqrt{2}}f_3\mu_s v'_{\sigma_2}v_\chi + f'_1\mu_\rho v'_\eta v'_\chi - \frac{1}{\sqrt{2}}f'_3\mu_\rho v'_{\sigma_2}v'_\chi$$

$$+ \left.f_1\mu_\chi v_\eta v'_\chi - \frac{1}{\sqrt{2}}f_3\mu_\chi v_{\sigma_2}v'_\chi\right),$$

$$\mathcal{M}_{23} = \frac{1}{6\sqrt{2}}\left(-6k_1 v_\eta - \frac{6}{\sqrt{2}}k_3 v_{\sigma_2}\right.$$

$$- \left.f_1\mu_\eta v'_\eta - \frac{1}{\sqrt{2}}\mu_s v'_{\sigma_2}\right),$$

$$\mathcal{M}_{24} = \frac{1}{12}(6k_3 v_\chi + f_3\mu_\chi v'_\chi),$$

$$\mathcal{M}_{25} = \frac{1}{6\sqrt{2}}(f_1\mu_\eta v_\chi - f'_1\mu_\rho v'_\chi), \quad \mathcal{M}_{26} = 0,$$

$$\mathcal{M}_{27} = \frac{1}{12\sqrt{2}}(-2f'_1\mu_\rho v'_\eta + \sqrt{2}f'_3\mu_\rho v'_{\sigma_2}$$

$$- 2f_1\mu_\chi v_\eta + \sqrt{2}f_3\mu_\chi v_{\sigma_2}),$$

$$\mathcal{M}_{28} = \frac{1}{12}(f_3\mu_s v_\chi + f'_3\mu_\rho v'_\chi),$$

$$\mathcal{M}_{33} = \frac{1}{6\sqrt{2}v_\chi}\left(-6f_1 k_1 v_\eta v_\rho - \frac{6}{\sqrt{2}}k_3 v_\rho v_{\sigma_2} - f_1\mu_\eta v'_\eta v_\rho\right.$$

$$+ f_1\mu_\rho v_\eta v'_\rho - \frac{1}{\sqrt{2}}f_3\mu_\rho v'_\rho v_{\sigma_2} - \frac{1}{\sqrt{2}}f_3\mu_s v_\rho v'_{\sigma_2}$$

$$+ \left.f'_1\mu_\chi v'_\eta v'_\rho - \frac{1}{\sqrt{2}}f'_3\mu_\chi v'_\rho v'_{\sigma_2}\right),$$

$$\mathcal{M}_{34} = \frac{1}{12}(6k_3 v_\rho + f_3\mu_\rho v'_\rho),$$

$$\mathcal{M}_{35} = \frac{1}{\sqrt{2}}(f_1\mu_\eta v_\rho - f'_1\mu_\chi v'_\chi),$$

$$\mathcal{M}_{36} = \frac{1}{12\sqrt{2}}(-2f_1\mu_\rho v_\eta + \sqrt{2}f_3\mu_\rho v_{\sigma_2}$$

$$- 2f'_1\mu_\chi v'_\eta + \sqrt{2}f'_3\mu_\chi v'_{\sigma_2}),$$

$$\mathcal{M}_{37} = 0, \quad \mathcal{M}_{38} = \frac{1}{12}(f_3\mu_s v_\rho + f'_3\mu_\chi v'_\rho),$$

$$\mathcal{M}_{44} = \frac{1}{18\sqrt{2}v_{\sigma_2}}\left(f_1 f_3 v_\eta v_\rho^2 - \frac{18}{\sqrt{2}}k_3 v_\rho v_\chi - \frac{3}{\sqrt{2}}\mu_\rho v_\rho' v_\chi\right.$$

$$+ \frac{18}{\sqrt{2}}k_3 v_\rho v_\chi - \frac{3}{\sqrt{2}}f_3 \mu_\rho v_\rho' v_\chi + f_1 f_3 v_\eta v_\chi^2$$

$$\left. - \frac{3}{\sqrt{2}}f_3' \mu_s v_\rho' v_\chi' - \frac{3}{\sqrt{2}}f_3 \mu_\chi v_\rho v_\chi'\right),$$

$$\mathcal{M}_{45} = 0, \quad \mathcal{M}_{46} = -\frac{1}{12}(f_3 \mu_\rho v_\chi + f_3' \mu_s v_\chi'),$$

$$\mathcal{M}_{47} = -\frac{1}{12}(f_3' \mu_\rho v_\rho' + f_3 \mu_\chi v_\rho'), \quad \mathcal{M}_{48} = 0,$$

$$\mathcal{M}_{55} = \frac{1}{18\sqrt{2}v_\eta'}(f_1' f_3' v_\rho'^2 v_{\sigma_2}' - 3f_1 \mu_\eta v_\rho v_\chi + 3f_1' \mu_\chi v_\rho' v_\chi$$

$$- 18k_1' v_\rho' v_\chi' + 3f_1' \mu_\rho v_\rho v_\chi' + f_1' f_3' v_{\sigma_2}' v_\chi'^2$$

$$+ 3f_1' \mu_\rho v_\rho v_\chi' + f_1 f_3' v_{\sigma_2}' v_\chi'^2),$$

$$\mathcal{M}_{56} = \frac{f_1 f_3}{18\sqrt{2}}(\mu_\chi v_\chi - k_1' v_\chi'),$$

$$\mathcal{M}_{57} = \frac{1}{6\sqrt{2}}(-6k_1' v_\rho' + f_1' \mu_\rho v_\rho),$$

$$\mathcal{M}_{58} = \frac{f_1' f_3'}{18\sqrt{2}}(v_\rho'^2 - v_\chi'^2),$$

$$\mathcal{M}_{66} = \frac{1}{12\sqrt{2}v_\rho'}\left(2f_1 \mu_\rho v_\eta v_\chi - \sqrt{2}f_3 \mu_\rho v_{\sigma_2} v_\chi\right.$$

$$+ 2f_1' \mu_\chi v_\eta' v_\chi - \sqrt{2}f_3' \mu_\chi v_{\sigma_2} v_\chi - 12k_1' v_\eta' v_\chi'$$

$$\left. - \frac{12}{\sqrt{2}}k_3' v_{\sigma_2}' v_\chi' - 2f_1' \mu_\eta v_\eta v_\chi' - \sqrt{2}\mu_s v_{\sigma_2} v_\chi'\right),$$

$$\mathcal{M}_{67} = \frac{1}{12\sqrt{2}}\left(-12k_1' v_\eta' - \frac{12}{\sqrt{2}}k_3' v_{\sigma_2}' - 2f_1' \mu_\eta v_\eta\right.$$

$$\left. - \sqrt{2}f_3' \mu_s v_{\sigma_2}\right),$$

$$\mathcal{M}_{68} = \frac{1}{12}(-f_3' \mu_\chi v_\chi + 6k_3' v_\chi'),$$

$$\mathcal{M}_{77} = \frac{1}{12\sqrt{2}v_\chi'}\left(-12k_3' v_\eta' v_\rho' - \frac{12}{\sqrt{2}}k_3' v_\rho' v_{\sigma_2}\right.$$

$$\left. - 2f_1' \mu_\eta v_\eta v_\rho' + 2f_1' \mu_\rho v_\eta' v_\rho - \sqrt{2}f_3' \mu_\rho v_\rho v_{\sigma_2}'\right),$$

$$\mathcal{M}_{78} = \frac{1}{12}(6k_3' v_\rho' - f_3' \mu_\rho v_\rho),$$

$$\mathcal{M}_{88} = \frac{1}{18\sqrt{2}v_{\sigma_2}'}\left(f_1' f_3' v_\eta' v_\rho'^2 - \frac{3}{\sqrt{2}}\mu_s v_\rho v_\chi - \frac{3}{\sqrt{2}}\mu_\chi v_\rho' v_\chi\right.$$

$$\left. - \frac{18}{\sqrt{2}}k_3' v_\rho' v_\chi' - \frac{3}{\sqrt{2}}f_3' \mu_\rho v_\rho v_\chi' + f_1' f_3' v_\eta' v_\chi'^2\right).$$

This mass matrix has two Goldstone bosons and six mass eigenstates. In Fig. 1 we show the behavior of the mass of the lightest pseudoscalar scalar A_1 as a function of v_χ. Typical values for the masses in this sector for a set of values of the parameters are given in the text.

APPENDIX C: CONSTRAINT EQUATIONS

The constraint equations are

$$\frac{t_\eta}{v_\eta} = \frac{g^2}{12}(2v_\eta^2 - 2v_\eta'^2 - v_\rho^2 + v_\rho'^2 - v_{\sigma_2}^2 + v_{\sigma_2}'^2 - v_\chi^2 + v_\chi'^2) + m_\eta^2$$

$$+ \frac{1}{4}\mu_\eta^2 + \frac{f_1^2}{18}(v_\rho^2 + v_\chi^2) + \frac{f_1 f_3}{18\sqrt{2}}\frac{v_{\sigma_2}}{v_\eta}(v_\rho^2 + v_\chi^2)$$

$$+ \frac{k_1}{\sqrt{2}}\frac{v_\rho v_\chi}{v_\eta} + \frac{1}{6\sqrt{2}v_\eta}(f_1 \mu_\rho v_\rho' v_\chi + f_1 \mu_\chi v_\rho v_\chi'$$

$$+ f_1' \mu_\eta v_\rho' v_\chi'),$$

$$\frac{t_\rho}{v_\rho} = \frac{g^2}{12}\left(-v_\eta^2 + v_\eta'^2 + 2v_\rho^2 - 2v_\rho'^2 + \frac{1}{2}v_{\sigma_2}^2 - \frac{1}{2}v_{\sigma_2}'^2 - v_\chi^2\right.$$

$$\left. + v_\chi'^2\right) + \frac{g'^2}{2}(v_\rho^2 - v_\rho'^2 - v_\chi^2 + v_\chi'^2) + \frac{f_1^2}{18}(v_\eta^2 + v_\chi^2)$$

$$+ \frac{f_3^2}{36}(v_{\sigma_2}^2 + 2v_\chi^2) - \frac{f_1 f_3}{9\sqrt{2}}v_\eta v_{\sigma_2} + m_\rho^2 + \frac{1}{4}\mu_\rho^2$$

$$+ \frac{k_1}{\sqrt{2}}\frac{v_\eta v_\chi}{v_\rho} + \frac{k_3}{2}\frac{v_{\sigma_2} v_\chi}{v_\rho} + \frac{f_1}{6\sqrt{2}v_\rho}(\mu_\eta v_\eta' v_\chi + \mu_\chi v_\eta v_\chi')$$

$$+ \frac{f_3}{12v_\rho}(\mu_s v_{\sigma_2}' v_\chi + \mu_\chi v_{\sigma_2} v_\chi') + \frac{f_3'}{12v_\rho}\mu_\rho v_{\sigma_2}' v_\chi'$$

$$-\frac{f_1'}{6\sqrt{2}v_\rho}\mu_\rho v_\eta' v_\chi'$$

$$\frac{\mathbf{t}_\chi}{\mathbf{v}_\chi}=\frac{g^2}{12}(-v_\eta^2+v_\eta'^2-v_\rho^2+v_\rho'^2+v_{\sigma_2}^2-v_{\sigma_2}'^2+2v_\chi^2-2v_\chi'^2)$$

$$+\frac{g'^2}{2}(v_\chi^2-v_\chi'^2-v_\rho^2+v_\rho'^2)+\frac{f_1^2}{18}(v_\chi^2+v_\eta^2)$$

$$+\frac{f_3^2}{36}(2v_\rho^2+v_{\sigma_2}^2)-\frac{f_1 f_3}{9\sqrt{2}}v_\eta v_{\sigma_2}+m_\chi^2+\frac{1}{4}\mu_\chi^2$$

$$-\frac{1}{6\sqrt{2}v_\chi}(f_1\mu_\rho v_\eta v_\rho'-f_1\mu_\eta v_\eta' v_\rho+f_1'\mu_\chi v_\eta' v_\rho')$$

$$+\frac{1}{12v_\chi}(f_3\mu_\rho v_\rho' v_{\sigma_2}+f_3\mu_s v_\rho v_{\sigma_2}+f_3'\mu_\chi v_\rho' v_{\sigma_2}')$$

$$+\frac{k_1}{\sqrt{2}}\frac{v_\eta v_\rho}{v_\chi}+\frac{k_3}{2}\frac{v_\rho v_{\sigma_2}}{v_\chi},$$

$$\frac{\mathbf{t}_{\sigma_2}}{\mathbf{v}_{\sigma_2}}=\frac{g^2}{24}(-2v_\eta^2+2v_\eta'^2+v_\rho^2-v_\rho'^2-v_{\sigma_2}'^2+v_{\sigma_2}^2+v_\chi^2-v_\chi'^2)$$

$$+\frac{f_3^2}{36}(v_\rho^2+v_\chi^2)+m_s^2+\frac{1}{4}\mu_s^2-\frac{f_1 f_3 v_\eta}{18\sqrt{2}v_{\sigma_2}}(v_\rho^2+v_\chi^2)$$

$$+\frac{1}{12v_{\sigma_2}}(f_3'\mu_s v_\rho' v_\chi'+f_3\mu_\chi v_\rho v_\chi'+f_3\mu_\rho v_\rho' v_\chi)$$

$$+\frac{k_3}{2}\frac{v_\rho v_\chi}{v_{\sigma_2}},$$

$$\frac{\mathbf{t}_{\eta'}}{v_\eta'}=\frac{g^2}{12}(-2v_\eta^2+v_\eta'^2+v_\rho^2-v_\rho'^2+v_{\sigma_2}^2-v_{\sigma_2}'^2+v_\chi^2-v_\chi'^2)$$

$$+m_{\eta'}^2+\frac{1}{4}\mu_\eta^2+\frac{f_1^2}{18}(v_\rho'^2+v_\chi'^2)-\frac{f_1 f_3}{18\sqrt{2}}\frac{v_{\sigma_2}'}{v_\eta'}(v_\rho'^2+v_\chi'^2)$$

$$+\frac{k_1'}{\sqrt{2}}\frac{v_\rho' v_\chi'}{v_\eta'}+\frac{1}{6\sqrt{2}v_\eta'}(f_1\mu_\eta v_\rho v_\chi-f_1'\mu_\chi v_\rho' v_\chi$$

$$-f_2'\mu_\rho v_\rho v_\chi'),$$

$$\frac{\mathbf{t}_{\rho'}}{\mathbf{v}_\rho'}=\frac{g^2}{12}\left(v_\eta^2-v_\eta'^2-2v_\rho^2+2v_\rho'^2-\frac{1}{2}v_{\sigma_2}^2+\frac{1}{2}v_{\sigma_2}'^2+v_\chi^2-v_\chi'^2\right)$$

$$+\frac{g'^2}{2}(-v_\rho^2+v_\rho'^2+v_\chi^2-v_\chi'^2)+\frac{f_1'^2}{18}(v_\eta'^2+v_\chi'^2)$$

$$+\frac{f_3'^2}{36}(v_{\sigma_2}'^2+2v_\chi'^2)-\frac{f_1' f_3'}{9\sqrt{2}}v_\eta' v_{\sigma_2}'+m_{\rho'}^2+\frac{1}{4}\mu_\rho^2$$

$$+\frac{1}{6\sqrt{2}v_\rho}(f_1\mu_\rho v_\eta v_\chi-f_1'\mu_\chi v_\eta' v_\chi+f_1'\mu_\eta v_\eta v_\chi')$$

$$+\frac{1}{12v_\rho'}(f_3\mu_\rho v_{\sigma_2} v_\chi+f_3'\mu_\chi v_{\sigma_2}' v_\chi+f_3'\mu_s v_{\sigma_2} v_\chi')$$

$$+\frac{k_1'}{\sqrt{2}}\frac{v_\eta' v_\chi'}{v_\rho'}+\frac{k_3'}{3}\frac{v_{\sigma_2}' v_\chi'}{v_\rho'},$$

$$\frac{\mathbf{t}_{\chi'}}{\mathbf{v}_{\chi'}}=\frac{g^2}{12}\left(v_\eta^2-v_\eta'^2+v_\rho^2-v_\rho'^2+\frac{1}{2}v_{\sigma_2}'^2-\frac{1}{2}v_{\sigma_2}^2-v_\chi^2+v_\chi'^2\right)$$

$$+\frac{g'^2}{2}(v_\rho^2-v_\rho'^2-v_\chi^2+v_\chi'^2)+\frac{f_1'^2}{18}(v_\eta'^2+v_\rho^2)$$

$$+\frac{f_3'^2}{36}(2v_\rho'^2+v_{\sigma_2}'^2)+\frac{f_1' f_3'}{9\sqrt{2}}v_\eta' v_{\sigma_2}'+m_{\chi'}^2+\frac{1}{4}\mu_\chi^2$$

$$+\frac{1}{6\sqrt{2}v_\chi'}(f_1'\mu_\eta v_\eta v_\rho'-f_1'\mu_\rho v_\eta' v_\rho+f_1\mu_\chi v_\eta v_\rho)$$

$$+\frac{1}{12v_\chi'}(f_3'\mu_\rho v_\rho v_{\sigma_2}'+f_3'\mu_s v_\rho' v_{\sigma_2}+f_3\mu_\chi v_\rho v_{\sigma_2})$$

$$+\frac{k_1'}{\sqrt{2}}\frac{v_\eta' v_\rho'}{v_\chi'}+\frac{k_3'}{2}\frac{v_\rho' v_{\sigma_2}'}{v_\chi'},$$

$$\frac{\mathbf{t}_{\sigma_2'}}{\mathbf{v}_{\sigma_2'}}=\frac{g^2}{24}(2v_\eta^2-2v_\eta'^2-v_\rho^2+v_\rho'^2-v_{\sigma_2}^2+v_{\sigma_2}'^2-v_\chi^2+v_\chi'^2)$$

$$+\frac{f_3'^2}{36}(v_\rho'^2+v_\chi'^2)+m_{\sigma_2'}^2+\frac{1}{4}\mu_s^2-\frac{f_1' f_3'}{18\sqrt{2}}\frac{v_\eta'}{v_{\sigma_2}'}(v_\rho'+v_\chi'^2)$$

$$+\frac{1}{12v_{\sigma_2}'}(f_3\mu_s v_\rho v_\chi+f_3'\mu_\chi v_\rho' v_\chi+f_3'\mu_\rho v_\rho v_\chi')$$

$$+\frac{k_3'}{2}\frac{v_\rho' v_\chi'}{v_{\sigma_2}'}.$$

[1] B. Kayser, in Proceedings of the XXXth International Conference on High Energy Physics, Osaka, Japan, 2000; hep-ph/00110206.

[2] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992); R. Foot, O.F. Hernandez, F. Pisano, and V. Pleitez, *ibid.* **47**, 4158 (1993).

[3] P.H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992); D. Ng, Phys. Rev. D **49**, 4805 (1994).

[4] P. Jain and S.D. Joglekar, Phys. Lett. B **407**, 151 (1997).

[5] P. Frampton, Int. J. Mod. Phys. A **13**, 2345 (1998).

[6] J.C. Montero, F. Pisano, and V. Pleitez, Phys. Rev. D **47**, 2918 (1993).

[7] K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. **63**, 938 (1989).

[8] R. Foot, G.C. Joshi, H. Lee, and R.R. Volkas, Mod. Phys. Lett. A **5**, 2721 (1990); R. Foot, *ibid.* **6**, 527 (1991); R. Foot, H. Lee, and R.R. Volkas, J. Phys. G **19**, 269 (1993).

[9] J. Sladkowski and M. Zralek, Phys. Rev. D **45**, 1701 (1992).

[10] M. Nowakowski and A. Pilaftsis, Phys. Rev. D **48**, 259 (1993).

[11] C.A. de S. Pires and O. Ravinez, Phys. Rev. D **58**, 035008 (1998).

[12] K.S. Babu and R.N. Mohapatra, Phys. Rev. D **41**, 271 (1990).

[13] C.A. de S. Pires, Phys. Rev. D **60**, 075013 (1999).

[14] V. Pleitez and M.D. Tonasse, Phys. Rev. D **48**, 5274 (1993).

[15] F. Pisano and V. Pleitez, Phys. Rev. D **51**, 3865 (1995).

[16] V. Pleitez, Phys. Rev. D **53**, 514 (1996).

[17] V. Pleitez and M.D. Tonasse, Phys. Rev. D **48**, 2353 (1993).

[18] J.W. Valle and M. Singer, Phys. Rev. D **28**, 540 (1983).

[19] R. Foot, H.N. Long, and T.A. Tran, Phys. Rev. D **50**, R34 (1994).

[20] P.B. Pal, Phys. Rev. D **52**, 1659 (1995).

[21] R.D. Peccei and H.R. Quinn, Phys. Rev. D **16**, 1791 (1977).

[22] J.C. Montero, C.A. de S. Pires, and V. Pleitez, Phys. Rev. D **64**, 096001 (2001).

[23] L. Epele, H. Fanchiotti, C. García Canal, and D. Gómez Dumm, Phys. Lett. B **343**, 291 (1995).

[24] D.G. Dumm, Int. J. Mod. Phys. A **11**, 887 (1996).

[25] J.T. Liu and D. Ng, Phys. Rev. D **50**, 548 (1994).

[26] J.C. Montero, V. Pleitez, and O. Ravinez, Phys. Rev. D **60**, 076003 (1999).

[27] D.G. Dumm, Phys. Lett. B **411**, 313 (1997).

[28] J.F. Gunion, H.E. Haber, G.L. Kane, and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, New York, 1989), and references therein.

[29] Y. Chikashige, R.N. Mohapatra, and R.D. Peccei, Phys. Lett. **98B**, 265 (1981).

[30] A. Zee, Phys. Lett. **93B**, 389 (1980).

[31] J.C. Montero, C.A. de S. Pires, and V. Pleitez, Phys. Rev. D **60**, 115003 (1999).

[32] F. Pisano, V. Pleitez, and M.D. Tonasse, hep-ph/9310230.

[33] For several ways to have massive neutrinos at the tree level or via radiative corrections in 3-3-1 models see P.H. Frampton, P.I. Krastev, and J.T. Liu, Mod. Phys. Lett. A **9**, 761 (1994); Y. Okamoto and M. Yasue, Phys. Lett. B **466**, 267 (1999); T. Kitabayashi and M. Yasue, hep-ph/0006040; hep-ph/0011153; J.C. Montero, C.A. de S. Pires, and V. Pleitez, Phys. Lett. B **502**, 167 (2001); hep-ph/0103096.

[34] T.V. Duong and E. Ma, Phys. Lett. B **316**, 307 (1993).

[35] H.N. Long and P.B. Pal, Mod. Phys. Lett. A **13**, 2355 (1998).

[36] H.E. Haber and G.L. Kane, Phys. Rep. **117**, 75 (1985).

[37] A. Salam and J. Strathdee, Nucl. Phys. **B83**, 85 (1975); P. Fayet, *ibid.* **B90**, 104 (1975); G.R. Farrar and S. Weinberg, Phys. Rev. D **27**, 2732 (1983).

[38] M. Capdequi-Peyranère and M.C. Rodriguez, Phys. Rev. D (to be published), hep-ph/0103013.

[39] H.E. Haber, Eur. Phys. J. C **15**, 817 (2000).

[40] E. Ma, Phys. Rev. D **39**, 1922 (1989).

[41] K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, Prog. Theor. Phys. **67**, 1889 (1982); **68**, 927 (1982).

[42] G.L. Kane, C. Kolda, and J.D. Wells, Phys. Rev. Lett. **70**, 2686 (1993).

[43] R. Flores and M. Sher, Ann. Phys. (N.Y.) **148**, 95 (1983).

[44] J.R. Espinosa and M. Quirós, Phys. Lett. B **279**, 92 (1992).