PHYSICAL REVIEW D, VOLUME 65, 035001

Chargino and neutralino production in the 3-3-1 supersymmetric model in $e^{-}e^{-}$ scattering

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(Received 21 June 2001; published 21 December 2001)

The goal of this article is to derive the Feynman rules involving single charginos, neutralinos, double charged gauge bosons, and sleptons in a 3-3-1 supersymmetric model. Using these Feynman rules we calculate the production of double charged charginos with neutralinos and also the production of a pair of single charged charginos, both in an electron-electron linear collider.

DOI: 10.1103/PhysRevD.65.035001 PACS number(s): 12.60.Jv, 13.85.Lg

I. INTRODUCTION

The standard model is exceedingly successful in describing leptons, quarks, and their interactions. Nevertheless, the standard model is not considered as the ultimate theory since neither the fundamental parameters, masses and couplings, nor the symmetry pattern are predicted. Even though many aspects of the standard model are experimentally supported to a high degree of accuracy, the embedding of the model into a more general framework is to be expected. The possibility of a gauge symmetry based on the symmetry $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ (3-3-1) [1] is particularly interesting, because it explains some fundamental questions that have not been eluded in the context of the standard model [2].

Recently one of us proposed the supersymmetric extension of the 3-3-1 model [2]; in this kind of model if we want to remain in a perturbative regime at all energies, then we need, in a natural way, a broken supersymmetry at a TeV scale, and the lightest scalar boson has an upper limit, which is 124.5 GeV for a given set of parameters at the tree level. We must remember that in the minimal supersymmetric standar model (MSSM) the bound using radiative corrections to the mass of the lightest Higgs scalar is 130 GeV [3]. On the other hand, no direct observation of a Higgs boson has been made yet, and current direct searches made at the CERN $e^{+}e^{-}$ collider accelerator leads to the condition $M_h > 113$ GeV [4] in the MSSM.

Linear colliders would be most versatile tools in experimental high-energy physics. A large international effort is currently under way to study the technical feasibility and physics possibilities of linear $e^{+}e^{-}$ colliders in the TeV range. A number of designs have already been proposed [Next Linear Collider (NLC), Japan Linear Collider (JLC), DESY TeV Energy Superconducting Linear Accelerator TESLA, CERN Linear Collider CLIC, VLEPP, . . .] and several workshops have recently been devoted to this subject. They can provide not only $e^{+}e^{-}$ collisions and high luminosities, but also very energetic beams of real photons. One could thus exploit $\gamma\gamma$, $e^{-}\gamma$, and even $e^{-}e^{-}$ collisions for physics studies.

The last exciting prospects have prompted a growing number of theoretical studies devoted to the investigation of the physics potential of such $e^{-}e^{-}$ accelerator experiments. Of course, in the realm of the standard model this option is not particularly interesting because Møller scattering and bremsstrahlung events are mainly observed. However, it is just for that reason that $e^{-}e^{-}$ collisions can provide crucial information on exotic processes, in particular on processes involving lepton and/or fermion number violation. Therefore, new perspectives emerge in detecting new physics beyond the standard model in processes having nonzero initial electric charge (and nonzero lepton number) like in electron-electron $e^{-}e^{-}$ process.

In this paper we will explicitly work out two $e^{-}e^{-}$ processes and we will show the difference between the MSSM and the 3-3-1 supersymmetric model in the chargino production. This paper is organized as follows. The model is outlined in Sec. II, where leptons, sleptons, scalars, higgsinos, gauge bosons, and gauginos are defined. We derive the mass spectrum in Sec. III, while the interactions are in Sec. IV. In order to calculate cross sections, we derive the Feynman rules in the Sec. V, while the productions of charginos and neutralinos are worked out in Sec. VI. Our conclusions are given in Sec. VII.

The Lagrangian is given in Appendix A, while in Appendix B we show the mass matrices of the charginos and neutralinos. The procedure to write two-component spinors in terms of four-component spinors is given in Appendix C. The differential cross sections of the processes we have calculated are given in the Appendix D.

II. PARTICLE CONTENT

First of all, let us outline the model, following the notation given in [2]. We are writing here the particle content that we are using in this study, so it means that we are not going to discuss quarks and squarks.

The leptons and sleptons are given by

$$L_l=(\nu_l l l^c)_L^t, \quad \tilde{L}_l=(\tilde{\nu}_l \tilde{l} \tilde{l}^c)_L^t, \quad l=e,\mu,\tau. \tag{2.1}$$

The scalars of our theory are

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$$\eta=\begin{pmatrix}\eta^0\\ \eta_1^-\\ \eta_2^+\end{pmatrix},\quad \rho=\begin{pmatrix}\rho^+\\ \rho^0\\ \rho^{++}\end{pmatrix},\quad \chi=\begin{pmatrix}\chi^-\\ \chi^{--}\\ \chi^0\end{pmatrix},$$

$$S=\begin{pmatrix}\sigma_1^0 & \frac{h_2^+}{\sqrt{2}} & \frac{h_1^-}{\sqrt{2}}\\ \frac{h_2^+}{\sqrt{2}} & H_1^{++} & \frac{\sigma_2^0}{\sqrt{2}}\\ \frac{h_1^-}{\sqrt{2}} & \frac{\sigma_2^0}{\sqrt{2}} & H_2^{--}\end{pmatrix}, \tag{2.2}$$

the higgsinos, the superpartners of the scalars, are defined as

$$\tilde{\eta}=\begin{pmatrix}\tilde{\eta}^0\\ \tilde{\eta}_1^-\\ \tilde{\eta}_2^+\end{pmatrix},\quad \tilde{\rho}=\begin{pmatrix}\tilde{\rho}^+\\ \tilde{\rho}^0\\ \tilde{\rho}^{++}\end{pmatrix},\quad \tilde{\chi}=\begin{pmatrix}\tilde{\chi}^-\\ \tilde{\chi}^{--}\\ \tilde{\chi}^0\end{pmatrix},$$

$$\tilde{S}=\begin{pmatrix}\tilde{\sigma}_1^0 & \frac{\tilde{h}_2^+}{\sqrt{2}} & \frac{\tilde{h}_1^-}{\sqrt{2}}\\ \frac{\tilde{h}_2^+}{\sqrt{2}} & \tilde{H}_1^{++} & \frac{\tilde{\sigma}_2^0}{\sqrt{2}}\\ \frac{\tilde{h}_1^-}{\sqrt{2}} & \frac{\tilde{\sigma}_2^0}{\sqrt{2}} & \tilde{H}_2^{--}\end{pmatrix}. \tag{2.3}$$

To cancel chiral anomalies generated by these higgsinos we have to add the fields $\tilde{\eta}'$, $\tilde{\rho}'$, $\tilde{\chi}'$, and $\tilde{S}'$ and their respective scalars, see [2].

When we break the 3-3-1 symmetry to the $SU(3)_C \otimes U(1)_{EM}$, the scalars get the following vacuum expectation values (VEVs):

$$\langle\eta\rangle=\begin{pmatrix}v\\0\\0\end{pmatrix},\quad \langle\rho\rangle=\begin{pmatrix}0\\u\\0\end{pmatrix},\quad \langle\chi\rangle=\begin{pmatrix}0\\0\\w\end{pmatrix},$$

$$\langle S\rangle=\begin{pmatrix}0&0&0\\ 0&0&\frac{z}{\sqrt{2}}\\ 0&\frac{z}{\sqrt{2}}&0\end{pmatrix}, \tag{2.4}$$

and similar expressions to the primed fields, where $v=v_\eta/\sqrt{2}$, $u=v_\rho/\sqrt{2}$, $w=v_\chi/\sqrt{2}$, $z=v_{\sigma_2}/\sqrt{2}$, $v'=v_{\eta'}/\sqrt{2}$, $u'=v_{\rho'}/\sqrt{2}$, $w'=v_{\chi'}/\sqrt{2}$, and $z'=v_{\sigma_2'}/\sqrt{2}$.

In the 3-3-1 supersymmetric model the bosons of the symmetry $SU(3)_L$ are V^a and their superpartners, the gauginos, are λ_A^a, with $a=1,\ldots,8$. For the $U(1)_N$ symmetry, we have the boson V and its superpartner written as λ_B (which we call gaugino too).

In the usual 3-3-1 model [1] the gauge bosons are defined as

$$W_m^{\pm}(x)=-\frac{1}{\sqrt{2}}(V_m^1(x)\mp iV_m^2(x)),$$

$$V_m^{\pm}(x)=-\frac{1}{\sqrt{2}}(V_m^4(x)\pm iV_m^5(x)),$$

$$U_m^{\pm\pm}(x)=-\frac{1}{\sqrt{2}}(V_m^6(x)\pm iV_m^7(x)),$$

$$A_m(x)=\frac{1}{\sqrt{1+4t^2}}[(V_m^3(x)-\sqrt{3}V_m^8(x))t+V_m],$$

$$Z_m^0(x)=-\frac{1}{\sqrt{1+4t^2}}\left[\sqrt{1+3t^2}V_m^3(x)+\frac{\sqrt{3}t^2}{\sqrt{1+3t^2}}V_m^8(x)-\frac{t}{\sqrt{1+3t^2}}V_m(x)\right],$$

$$Z'^0_m(x)=\frac{1}{\sqrt{1+3t^2}}(V_m^8(x)+\sqrt{3}tV_m(x)), \tag{2.5}$$

where $t\equiv\tan\theta=g'/g$ where g' and g are the gauge coupling constants of $U(1)$ and $SU(3)$, respectively. We can define the charged gauginos, in analogy with the gauge bosons, in the following way

$$\lambda_W^{\pm}(x)=-\frac{1}{\sqrt{2}}(\lambda_A^1(x)\mp i\lambda_A^2(x)),$$

$$\lambda_V^{\pm}(x)=-\frac{1}{\sqrt{2}}(\lambda_A^4(x)\pm i\lambda_A^5(x)),$$

$$\lambda_U^{\pm\pm}(x)=-\frac{1}{\sqrt{2}}(\lambda_A^6(x)\pm i\lambda_A^7(x)). \tag{2.6}$$

III. MASS SPECTRUM

The higgsino mass term comes from $\mathcal{L}_{HMT}$, see Eq. (A7), the gauginos mass term is given by $\mathcal{L}_{GMT}$, see Eq. (A8) and the mixing term between gauginos and higgsinos is given by $\mathcal{L}^{\text{scalar}}_{H\tilde{H}\tilde{V}}$, Eq. (A6). After the mixture the physical states are double charged charginos, single charged charginos, and neutralinos. Now we will discuss these states.

A. Double charged chargino

Doing the calculations we obtain the mass matrix of the double charged charginos in the following[1] way:

$$\mathcal{L}^{\text{double}}_{\text{mass}}=-m_\lambda\lambda_U^{--}\lambda_U^{++}-\frac{\mu_\rho}{2}\tilde{\rho}'^{--}\tilde{\rho}^{++}-\frac{\mu_\chi}{2}\tilde{\chi}^{--}\tilde{\chi}'^{++}-\frac{\mu_S}{2}(\tilde{H}_1'^{--}\tilde{H}_1^{++}+\tilde{H}_2^{--}\tilde{H}'^{++}_2)$$
$$-ig\left(u\tilde{\rho}^{++}-w'\tilde{\chi}'^{++}-\frac{z}{\sqrt{2}}\tilde{H}_1^{++}+\frac{z'}{\sqrt{2}}\tilde{H}'^{++}_2\right)\lambda_U^{--}-ig\left(w\tilde{\chi}^{--}-u'\tilde{\rho}'^{--}-\frac{z}{\sqrt{2}}\tilde{H}_2^{--}+\frac{z'}{\sqrt{2}}\tilde{H}_1'^{--}\right)\lambda_U^{++}$$
$$+\frac{f_1 v}{3}\tilde{\chi}^{--}\tilde{\rho}^{++}-\frac{f_3}{3}u\tilde{\chi}^{--}\tilde{H}_1^{++}-\sqrt{2}\frac{f_3}{3}z\tilde{\chi}^{--}\tilde{\rho}^{++}-\frac{f_3}{3}w\tilde{H}_2^{--}\tilde{\rho}^{++}+\frac{f_1'v'}{3}\tilde{\chi}'^{++}\tilde{\rho}'^{--}-\frac{f_3'}{3}u'\tilde{\chi}'^{++}\tilde{H}'^{--}_1$$
$$-\sqrt{2}\,\frac{f_3'}{3}z'\tilde{\chi}'^{++}\tilde{\rho}'^{--}-\frac{f_3'}{3}w'\tilde{H}'^{++}_2\tilde{\rho}'^{--}+\text{H.c.},\tag{3.1}$$

which can be written in analogy with the MSSM,[2] see Appendix B 1, as follows

$$\mathcal{L}^{\text{double}}_{\text{mass}}=-\frac{1}{2}(\Psi^{\pm\pm})^t Y^{\pm\pm}\Psi^{\pm\pm}+\text{H.c.}\tag{3.2}$$

The double chargino mass matrix is diagonalized using two rotation matrices, A and B, defined by

$$\tilde{\chi}_i^{++}=A_{ij}\Psi_j^{++},\quad \tilde{\chi}_i^{--}=B_{ij}\Psi_j^{--},\quad i,j=1,\ldots,5,\tag{3.3}$$

where A and B are unitary matrices such that

$$M_{DCC}=B^*TA^{-1},\tag{3.4}$$

the matrix T is defined in Eq. (B3). To determine A and B, we note that

$$M_{DCC}^2=AT^t\cdot TA^{-1}=B^*T\cdot T^t(B^*)^{-1},\tag{3.5}$$

which means that A diagonalizes $T^t\cdot T$ while B diagonalizes $T\cdot T^t$.

We define the following Dirac spinors to represent the mass eigenstates:

$$\Psi(\tilde{\chi}_i^{++})=(\tilde{\chi}_i^{++}\overline{\tilde{\chi}_i^{--}})^t,\quad \Psi^c(\tilde{\chi}_i^{--})=(\tilde{\chi}_i^{--}\overline{\tilde{\chi}_i^{++}})^t,\tag{3.6}$$

where $\tilde{\chi}_i^{++}$ is the particle and $\tilde{\chi}_i^{--}$ is the antiparticle (we are using the same notation as in [5]).

B. Single Charged Chargino

We can write the mass matrix of the charged chargino in the following way

$$\mathcal{L}^{\text{single}}_{\text{mass}}=-m_\lambda(\lambda_V^-\lambda_V^++\lambda_W^-\lambda_W^+)-\frac{\mu_\eta}{2}(\tilde{\eta}_1^-\tilde{\eta}_1'^++\tilde{\eta}_2'^-\tilde{\eta}_2^+)-\frac{\mu_\rho}{2}\tilde{\rho}'^-\tilde{\rho}^+-\frac{\mu_\chi}{2}\tilde{\chi}^-\tilde{\chi}'^+-\frac{\mu_S}{2}(\tilde{h}_1^-\tilde{h}_1'^++\tilde{h}_2'^-\tilde{h}_2^+)$$
$$+ig\left(v\tilde{\eta}_2^+-w'\tilde{\chi}'^+-\frac{z}{2}\tilde{h}_2^+\right)\lambda_V^-+ig\left(w\tilde{\chi}^--v'\tilde{\eta}'^-_2+\frac{z'}{2}\tilde{h}_2'^-\right)\lambda_V^+-ig\left(u\tilde{\rho}^+-v'\tilde{\eta}'^+_1+\frac{z'}{2}\tilde{h}_1'^+\right)\lambda_W^-$$
$$-ig\left(v\tilde{\eta}_1^--u'\tilde{\rho}'^--\frac{z}{2}\tilde{h}_1^-\right)\lambda_W^+-\frac{f_1u}{3}\tilde{\chi}^-\tilde{\eta}_2^++\frac{f_1w}{3}\tilde{\eta}_1^-\tilde{\rho}^+-\frac{f_1'u'}{3}\tilde{\eta}_2'^-\tilde{\chi}'^++\frac{f_1'w'}{3}\tilde{\rho}'^-\tilde{\eta}_1'^+-\frac{f_3u}{3\sqrt{2}}\tilde{\chi}^-\tilde{h}_2^+-\frac{f_3w}{3\sqrt{2}}\tilde{h}_1^-\tilde{\rho}^+$$
$$-\frac{f_3'u'}{3\sqrt{2}}\tilde{\chi}'^+\tilde{h}_2'^--\frac{f_3'w'}{3\sqrt{2}}\tilde{h}_1'^+\tilde{\rho}'^-+\text{H.c.},\tag{3.7}$$

but Eq. (3.7), see Appendix B 2, takes the form

$$\mathcal{L}^{\text{single}}_{\text{mass}}=-\frac{1}{2}(\Psi^{\pm})^tY^{\pm}\Psi^{\pm}+\text{H.c.}\tag{3.8}$$

[1]Here we are considering the conservation of the quantum number $\mathcal{F}$, defined as $\mathcal{F}=L+B$, where L is the lepton number and B is the baryon number, then $f_2,f_2'=0$ [2].

[2]In the Appendix B we show the mass matrices of the charginos and neutralinos.

The chargino mass matrix is diagonalized using two rotation matrices, D and E, defined by

$$\tilde{\chi}_i^+ = D_{ij}\Psi_j^+, \quad \tilde{\chi}_i^- = E_{ij}\Psi_j^-, \quad i,j=1,\dots,8. \quad (3.9)$$

Then we can write the diagonal mass matrix (D and E are unitary) as

$$M_{SCC} = E^* X D^{-1}, \quad (3.10)$$

the matrix X is defined in Eq. (B8).

As in the previous subsection we will define the following Dirac spinors:

$$\Psi(\tilde{\chi}_i^+) = (\tilde{\chi}_i^+ \overline{\tilde{\chi}_i^-})^t, \quad \Psi^c(\tilde{\chi}_i^-) = (\tilde{\chi}_i^- \overline{\tilde{\chi}_i^+})^t, \quad (3.11)$$

where $\tilde{\chi}_i^+$ is the particle and $\tilde{\chi}_i^-$ is the antiparticle. This structure is the same as in the chargino sector of the MSSM [5].

C. Neutralino

For the neutralino case we have

$$\begin{aligned}\mathcal{L}^{\text{neutralino}}_{\text{mass}} =& -\frac{m_\lambda}{2}(\lambda_A^3\lambda_A^3+\lambda_A^8\lambda_A^8) - \frac{m'}{2}\lambda_B\lambda_B - \frac{\mu_\eta}{2}\tilde{\eta}^0\tilde{\eta}'^0 - \frac{\mu_\rho}{2}\tilde{\rho}^0\tilde{\rho}'^0 - \frac{\mu_\chi}{2}\tilde{\chi}^0\tilde{\chi}'^0 - \frac{\mu_S}{2}(\tilde{\sigma}_1'^0\tilde{\sigma}_1^0+\tilde{\sigma}_2'^0\tilde{\sigma}_2^0) - \frac{ig'}{\sqrt{2}}(w\tilde{\chi}^0 - u\tilde{\rho}^0 \\ & + u'\tilde{\rho}'^0 - w'\tilde{\chi}'^0)\lambda_B - \frac{ig}{\sqrt{2}}\left(u\tilde{\rho}^0 - v\tilde{\eta}^0 + v'\tilde{\eta}'^0 - u'\tilde{\rho}'^0 + \frac{z}{2}\tilde{\sigma}_2^0 - \frac{z'}{2}\tilde{\sigma}_2'^0\right)\lambda_A^3 - \frac{ig}{\sqrt{6}}\left(2w\tilde{\chi}^0 - u\tilde{\rho}^0 - v\tilde{\eta}^0 + u'\tilde{\rho}'^0\right. \\ & \left. + v'\tilde{\eta}'^0 - 2w'\tilde{\chi}'^0 + \frac{z}{2}\tilde{\sigma}_2^0 - \frac{z'}{2}\tilde{\sigma}_2'^0\right)\lambda_A^8 + \frac{f_1}{3}(u\tilde{\chi}^0\tilde{\eta}^0 - v\tilde{\chi}^0\tilde{\rho}^0 - w\tilde{\eta}^0\tilde{\rho}^0) + \frac{f_1'}{3}(u'\tilde{\chi}'^0\tilde{\eta}'^0 - v'\tilde{\chi}'^0\tilde{\rho}'^0 - w'\tilde{\eta}'^0\tilde{\rho}'^0) \\ & - \frac{f_3}{3\sqrt{2}}(u\tilde{\chi}^0\tilde{\sigma}_2^0 + w\tilde{\rho}^0\tilde{\sigma}_2^0 + 2z\tilde{\chi}^0\tilde{\rho}^0) - \frac{f_3'}{3\sqrt{2}}(u'\tilde{\chi}'^0\tilde{\sigma}_2'^0 + w'\tilde{\rho}'^0\tilde{\sigma}_2'^0 + 2z'\tilde{\chi}'^0\tilde{\rho}'^0) + \text{H.c.} \end{aligned} \quad (3.12)$$

Equation (3.12) can be put in the following form, see Appendix B3:

$$\mathcal{L}^{\text{neutralino}}_{\text{mass}} = -\frac{1}{2}(\Psi^0)^t Y^0 \Psi^0 + \text{H.c.} \quad (3.13)$$

The neutralino mass matrix is diagonalized by a 13×13 rotation matrix N, a unitary matrix satisfying

$$M_N = N^* Y^0 N^{-1}, \quad (3.14)$$

and the mass eigenstates are

$$\tilde{\chi}_i^0 = N_{ij}\Psi_j^0, \quad j=1,\dots,13. \quad (3.15)$$

We can define the following Majorana spinor to represent the mass eigenstates

$$\Psi(\tilde{\chi}_i^0) = (\tilde{\chi}_i^0 \overline{\tilde{\chi}_i^0})^t. \quad (3.16)$$

In supersymmetric left-right models, there are double charged higgsinos too [6]. There and in the present model the diagonalization of the chargino and neutralino sectors are numerically performed.

D. Sleptons

We can write the slepton mass term as

$$\begin{aligned}\mathcal{L}^{\text{slepton}}_{\text{mass}} &= \mathcal{L}_{\text{SMT}} + \mathcal{L}^{\text{slepton}}_F + \mathcal{L}^{\text{slepton}}_D \\ &= -\tilde{m}_{\tilde{\nu}}^2 \tilde{\nu}_l^* \tilde{\nu}_l \\ &\quad - \left(\tilde{m}_L^2 + \frac{4m_l^2}{9}\right)\tilde{l}^*\tilde{l} - \left(\tilde{m}_R^2 + \frac{4m_l^2}{9}\right)\tilde{l}^{c*}\tilde{l}^c \\ &\quad + \tilde{m}_{LR}^2(\tilde{l}\tilde{l}^c - \tilde{l}^{c*}\tilde{l}^*), \end{aligned} \quad (3.17)$$

where

$$\begin{aligned} \tilde{m}_{\tilde{\nu}}^2 &= m_{\tilde{\nu}}^2 + \frac{z^2+2v^2-u^2-w^2}{6}, \\ \tilde{m}_L^2 &= m_{\tilde{l}}^2 + \lambda_2^2 v^2 + \frac{z^2 - 2(w^2-v^2-u^2)}{12}, \\ \tilde{m}_R^2 &= m_{\tilde{l}^c}^2 + \lambda_2^2 v^2 + \frac{z^2-2(u^2-2w^2-v^2)}{12}, \\ \tilde{m}_{LR}^2 &= \frac{9m_l}{4}\left[\left(\frac{\mu_S}{\sqrt{2}} + \frac{f_3 u w}{9z}\right) - \frac{4}{9}\zeta_0 z\right]. \end{aligned} \quad (3.18)$$

Performing the diagonalization we find the following states:

$$\begin{pmatrix} \tilde{l}_1^- \\ \tilde{l}_2^- \end{pmatrix} = \begin{pmatrix} \cos\theta_f & \sin\theta_f \\ -\sin\theta_f & \cos\theta_f \end{pmatrix} \begin{pmatrix} \tilde{l} \\ \tilde{l}^{c*} \end{pmatrix},$$

$$(\tilde{l}_1^+ \quad \tilde{l}_2^+) = (\tilde{l}^* \quad \tilde{l}^c) \begin{pmatrix} \cos\theta_f & -\sin\theta_f \\ \sin\theta_f & \cos\theta_f \end{pmatrix}, \tag{3.19}$$

where the mixing angle is given by

$$\tan 2\theta_f = \frac{2\tilde{m}_{LR}^2}{\tilde{m}_L^2 - \tilde{m}_R^2}, \tag{3.20}$$

and the masses of these states are

$$m_{1,2}^2 = \frac{4m_l^2}{9} + \frac{1}{2}[\tilde{m}_L^2 + \tilde{m}_R^2 \pm \sqrt{(\tilde{m}_L^2 - \tilde{m}_R^2)^2 + 4\tilde{m}_{LR}^4}]. \tag{3.21}$$

Substituting the diagonal slepton states, given in Eq. (3.19), in Eq. (3.17), we obtain the following diagonal Lagrangian:

$$\mathcal{L}_{\text{mass}}^{\text{slepton}} = -\tilde{m}_{\tilde{\nu}} \tilde{\nu}_l^* \tilde{\nu}_l - m_1^2 \tilde{l}_1^+ \tilde{l}_1^- - m_2^2 \tilde{l}_2^+ \tilde{l}_2^- . \tag{3.22}$$

We wish to stress that the slepton sector of this model is the same as in the MSSM [5].

E. Neutral gauge bosons

The gauge mass term is given by $\mathcal{L}_{HHVV}^{\text{scalar}}$, see Eq. (A6), which we can divided in $\mathcal{L}_{\text{mass}}^{\text{charged}}$ and $\mathcal{L}_{\text{mass}}^{\text{neutral}}$.

The neutral gauge boson mass is given by

$$\mathcal{L}_{\text{mass}}^{\text{neutral}} = \begin{pmatrix} V_{3m} & V_{8m} & V_m \end{pmatrix} M^2 \begin{pmatrix} V_3^m \\ V_8^m \\ V^m \end{pmatrix}, \tag{3.23}$$

where

$$M^2 = \frac{g^2}{2} \begin{pmatrix} (v^2+u^2+z^2) & \frac{1}{\sqrt{3}}(v^2-u^2+z^2) & -2tu^2 \\ \frac{1}{\sqrt{3}}(v^2-u^2+z^2) & \frac{1}{3}(v^2+u^2+4w^2+z^2) & \frac{2t}{\sqrt{3}}(u^2+2w^2) \\ -2tu^2 & \frac{2t}{\sqrt{3}}(u^2+2w^2) & 4t^2(u^2+w^2) \end{pmatrix}, \tag{3.24}$$

with $t = g'/g$.

In the approximation that $w^2 \gg v^2$, u^2, and z^2, the masses of the neutral gauge bosons are: 0, M_Z^2, and $M_{Z'}^2$, and the masses are given by

$$M_Z^2 \approx \frac{1}{2}\frac{g^2+4g'^2}{g^2+3g'^2}(v^2+u^2+z^2+v'^2+u'^2+z'^2),$$

$$M_{Z'}^2 \approx \frac{1}{3}(g^2+3g'^2)(2w^2+2w'^2). \tag{3.25}$$

F. Charged gauge bosons

The charged gauge boson mass term, $\mathcal{L}_{\text{mass}}^{\text{neutral}}$, is

$$\mathcal{L}_{\text{mass}}^{\text{charged}} = M_W^2 W_m^- W^{+m} + M_V^2 V_m^- V^{+m} + M_U^2 U_m^{--} U^{++m}, \tag{3.26}$$

where

$$M_U^2 = \frac{g^2}{4}(v_\rho^2 + v_\chi^2 + 4v_{\sigma_2}^2 + v_{\rho'}^2 + v_{\chi'}^2 + 4v_{\sigma_2'}^2),$$

$$M_W^2 = \frac{g^2}{4}(v_\eta^2 + v_\rho^2 + 2v_{\sigma_2}^2 + v_{\eta'}^2 + v_{\rho'}^2 + 2v_{\sigma_2'}^2),$$

$$M_V^2 = \frac{g^2}{4}(v_\eta^2 + v_\chi^2 + 2v_{\sigma_2}^2 + v_{\eta'}^2 + v_{\chi'}^2 + 2v_{\sigma_2'}^2). \tag{3.27}$$

Using M_W given in Eq. (3.27) and M_Z in Eq. (3.25) we get the following relation:

$$\frac{M_Z^2}{M_W^2} = \frac{1+4t^2}{1+3t^2} = \frac{1}{1-\sin^2\theta_W}, \tag{3.28}$$

therefore we obtain

$$t^2 = \frac{\sin^2\theta_W}{1-4\sin^2\theta_W}. \tag{3.29}$$

We want to mention that the gauge boson sector is exactly the same as in the nonsupersymmetric 3-3-1 model [1].

IV. INTERACTIONS

In the previous section we have analyzed the physical spectrum of the model. Now we are going to get interactions between these particles. The procedure to write two-component spinors in terms of four-component spinors is given in Appendix C.

A. Lepton gauge boson interactions

We will define the following Dirac spinors for the leptons:

$$\Psi(l)=(l\overline{l^c})^t, \quad \Psi^c(l)=(l^c\overline{l})^t, \quad \Psi(\nu_l)=(\nu_l 0)^t. \tag{4.1}$$

The leptons and the gauge bosons are defined in the Eqs. (2.1) and (2.5) and using Eqs. (4.1), we can rewrite Eq. (A3) as follows:

$$\begin{aligned}\mathcal{L}^{\rm lep}_{llV}=&-\frac{g}{\sqrt{2}}[\overline{\Psi}^c(l)\gamma^m L\Psi(l)U^{++}_m+\overline{\Psi}^c(l)\gamma^m L\Psi(\nu_l)V^+_m+\overline{\Psi}(\nu_l)\gamma^m L\Psi(l)W^+_m+\text{H.c.}]\\ &-\frac{gt}{\sqrt{1+4t^2}}\overline{\Psi}(l)\gamma^m\Psi(l)A_m-\frac{g}{2}\sqrt{\frac{1+4t^2}{1+3t^2}}\overline{\Psi}(\nu_l)\gamma^m L\Psi(\nu_l)\left[Z_m-\frac{1}{\sqrt{3}}\frac{1}{\sqrt{1+4t^2}}Z'_m\right]\\ &-\frac{g}{4}\sqrt{\frac{1+4t^2}{1+3t^2}}\left[\left(\frac{-1}{1+4t^2}\overline{\Psi}(l)\gamma^m\Psi(l)+\overline{\Psi}(l)\gamma^m\gamma^5\Psi(l)\right)Z_m\right.\\ &\left.+\left(\frac{-\sqrt{3}}{\sqrt{1+4t^2}}\overline{\Psi}(l)\gamma^m\Psi(l)-\frac{1}{\sqrt{3}}\frac{1}{\sqrt{1+4t^2}}\overline{\Psi}(l)\gamma^m\gamma^5\Psi(l)\right)Z'_m\right]\\ =&\mathcal{L}^{NC}_{llV}+\mathcal{L}^{CC}_{llV},\end{aligned} \tag{4.2}$$

where t is defined in Eq. (3.29), and L and R are the usual chiral projectors, see Eq. (C2).

Let us define also the following parameters:

$$\begin{aligned}&e=\frac{g\sin\theta}{\sqrt{1+3\sin^2\theta}}, \quad h(t)=1+4t^2, \quad v_l=-\frac{1}{h(t)},\\ &a_l=1, \quad v'_l=-\frac{\sqrt{3}}{\sqrt{h(t)}}, \quad a'_l=\frac{v'_l}{3}, \quad g^2=\frac{8G_F M_W^2}{\sqrt{2}}.\end{aligned} \tag{4.3}$$

Then we can now rewrite the neutral part of Eq. (4.2):

$$\begin{aligned}\mathcal{L}^{NC}_{llV}=&-e\overline{\Psi}(l)\gamma^m\Psi(l)A_m-\frac{g}{2}\frac{M_Z}{M_W}\overline{\Psi}(\nu_l)\gamma^m L\Psi(\nu_l)\\ &\times\left[Z_m-\frac{1}{\sqrt{3}}\frac{1}{\sqrt{h(t)}}Z'_m\right]-\frac{g}{4}\frac{M_Z}{M_W}[\overline{\Psi}(l)\gamma^m(v_l+a_l\gamma^5)\\ &\times\Psi(l)Z_m+\overline{\Psi}(l)\gamma^m(v'_l+a'_l\gamma^5)\Psi(l)Z'_m].\end{aligned} \tag{4.4}$$

The charged part of Eq. (4.2) is

$$\mathcal{L}^{CC}_{llV}=-\frac{g}{\sqrt{2}}[\overline{\Psi}^c(l)\gamma^m L\Psi(l)U^{++}_m+\overline{\Psi}^c(l)\gamma^m L\Psi(\nu_l)V^+_m+\overline{\Psi}(\nu_l)\gamma^m L\Psi(l)W^+_m+\text{H.c.}], \tag{4.5}$$

where g is defined in Eq. (4.3). The Lagrangian in the Eqs. (4.4) and (4.5) is the same that was shown in Ref. [1].

B. Chargino (neutralino) U^{--} interactions

Using Eq. (C2) in the Eqs. (A5) and (A6), we can write the interaction between the double charged vector field with the double charged charginos and the neutralinos, in the following way:

$$\mathcal{L}_{U\tilde{\chi}^{++}\tilde{\chi}^0}=-\frac{g}{2}\{[\bar{\tilde{W}}_3L\gamma^m R\tilde{U}-\bar{\tilde{U}}^cL\gamma^m R\tilde{W}_3+\sqrt{3}(\bar{\tilde{U}}^cL\gamma^m R\tilde{W}_8-\bar{\tilde{W}}_8L\gamma^m R\tilde{U})$$
$$+\sqrt{2}(\bar{\tilde{T}}^0_5L\gamma^m R\tilde{T}^{++}_2+\bar{\tilde{T}}^0_4L\gamma^m R\tilde{T}^{++}_1+\bar{\tilde{T}}^{c++}_2L\gamma^m R\tilde{T}^0_6+\bar{\tilde{T}}^{c++}_1L\gamma^m R\tilde{T}^0_3)+\bar{\tilde{S}}^{c++}_2L\gamma^m R\tilde{S}^0_4+\bar{\tilde{S}}^{c++}_1L\gamma^m R\tilde{S}^0_3$$
$$+\bar{\tilde{S}}^0_4L\gamma^m R\tilde{S}^{++}_1+\bar{\tilde{S}}^0_3L\gamma^m R\tilde{S}^{++}_2]U^{--}_m+\text{H.c.}\}$$
$$=-\frac{g}{2}\{\{[(N^*_{i1}-\sqrt{3}N^*_{i2})B_{j1}+\sqrt{2}(N^*_{i8}B_{j3}+N^*_{i7}B_{j2})+N^*_{i12}B_{j5}+N^*_{i13}B_{j4}]\overline{\Psi}(\tilde{\chi}^0_i)L\gamma^m R\Psi(\tilde{\chi}^{++}_j)$$
$$+[A^*_{i1}(\sqrt{3}N_{j2}-N_{j1})+\sqrt{2}(A^*_{i3}N_{j9}+A^*_{i2}N_{j6})+A^*_{i5}N_{j13}+A^*_{i4}N_{j12}]\overline{\Psi}^c(\tilde{\chi}^{--}_i)L\gamma^m R\Psi(\tilde{\chi}^0_j)\}U^{--}_m+\text{H.c.}\}. \tag{4.6}$$

In a similar way we get for the charginos

$$\mathcal{L}_{U\tilde{\chi}^+\tilde{\chi}^+}=-\frac{g}{\sqrt{2}}\left[\left(\bar{\tilde{W}}^cL\gamma^m R\tilde{V}-\bar{\tilde{V}}^cL\gamma^m R\tilde{W}+\bar{\tilde{T}}^{c+}_2L\gamma^m R\tilde{T}^+_1+\bar{\tilde{T}}^{c+}_1L\gamma^m R\tilde{T}^+_2+\frac{1}{2}\bar{\tilde{S}}^{c+}_1L\gamma^m R\tilde{S}^+_2+\frac{1}{2}\bar{\tilde{S}}^{c+}_2L\gamma^m R\tilde{S}^+_1\right)U^{--}_m+\text{H.c.}\right]$$
$$=\frac{g}{\sqrt{2}}\left[\left(D^*_{i1}E_{j2}-D^*_{i2}E_{j1}+D^*_{i4}E_{j3}+D^*_{i3}E_{j4}+\frac{1}{2}D^*_{i7}E_{j8}+\frac{1}{2}D^*_{i8}E_{j7}\right)\overline{\Psi}^c(\tilde{\chi}^-_i)L\gamma^m R\Psi(\tilde{\chi}^+_j)U^{--}_m+\text{H.c.}\right]. \tag{4.7}$$

C. Lepton slepton chargino (neutralino) interactions

The interactions between lepton slepton chargino (neutralino) are given by

$$\mathcal{L}_{l\tilde{l}\tilde{\chi}}=\{-g\cos\theta_f A_i\overline{\Psi}(\tilde{\chi}^0_i)L\Psi(l)-g\sin\theta_f A_{i1}\overline{\Psi}(l)R\Psi^c(\tilde{\chi}^{--}_i)+2\lambda_3\sin\theta_f[A^*_{i5}\overline{\Psi}^c(\tilde{\chi}^{--}_i)L\Psi^c(l)+N^*_{i8}\overline{\Psi}(\tilde{\chi}^0_i)L\Psi(l)]\}\tilde{l}^+_1$$
$$+\{g\sin\theta_f A^*_i\overline{\Psi}(\tilde{\chi}^0_i)L\Psi(l)-g\cos\theta_f A_{i1}\overline{\Psi}(l)R\Psi^c(\tilde{\chi}^{--}_i)+2\lambda_3\cos\theta_f[A^*_{i5}\overline{\Psi}^c(\tilde{\chi}^{--}_i)L\Psi^c(l)+N^*_{i8}\overline{\Psi}(\tilde{\chi}^0_i)L\Psi(l)]\}\tilde{l}^+_2$$
$$+\left\{-g\frac{2\lambda_3}{\sqrt{2}}[E^*_{i8}\overline{\Psi}(\tilde{\chi}^+_i)L\Psi(l)+D^*_{i7}\overline{\Psi}^c(\tilde{\chi}^-_i)L\Psi^c(l)+\sqrt{2}N^*_{i7}\overline{\Psi}(\tilde{\chi}^0_i)L\Psi(\nu_l)]\right\}\tilde{\nu}_l+\text{H.c.}, \tag{4.8}$$

where $A_i=(N^*_{i1}/\sqrt{2}+N^*_{i2}/\sqrt{6})$.

V. FEYNMAN RULES

In the previous section we got the interaction Lagrangians, mainly the following vertices: lepton-lepton-gauge boson; chargino-chargino (neutralino)-gauge boson; slepton-lepton-chargino (neutralino).

In the Table I we give the Feynman rules for the interactions mentioned above, and we have defined the following operators:

$$O^1_{ij}=A^*_{i1}(\sqrt{3}N_{j2}-N_{j1})+\sqrt{2}(A^*_{i3}N_{j9}+A^*_{i2}N_{j6})+A^*_{i5}N_{j13}+A^*_{i4}N_{j12}, \tag{5.1}$$

$$O^2_{ij}=-\left(D^*_{i1}E_{j2}-D^*_{i2}E_{j1}+D^*_{i4}E_{j3}+D^*_{i3}E_{j4}+\frac{1}{2}D^*_{i7}E_{j8}+\frac{1}{2}D^*_{i8}E_{j7}\right). \tag{5.2}$$

To derive the differential and total cross sections, we will assume that the mass of the lightest particle of this model coincides with the mass parameter coming from the MSSM. In Table II we shown two different solutions got in mSUGRA [7], and they are the values that we used in our calculations.

VI. APPLICATIONS

In this section, we will perform the calculation of differential and total-cross-sections of the lightest single charged chargino, double charged chargino, and neutralino, in e^-e^- scattering. First we present the chargino production.

In the two subsections below, we neglect the electron mass and we assume that electrons have energy $E/2$. We will consider that P_1 and P_2 are the four momenta of the incoming electrons while K_1 and K_2 are the four momenta of the outgoing particles.

A. Charginos production $e^-e^-\to\tilde{\chi}^-\tilde{\chi}^-$

Considering the Table I, we have five diagrams at the tree level corresponding to this process, see Fig. 1. The first and the third diagrams that appear in Fig. 1 exist in the MSSM, but the other ones are new contributions coming from the 3-3-1 supersymmetric model.

TABLE I. Feynman rules derived from Eqs. (4.5), (4.6), (4.7), and (4.8).

Vertices	Feynman rules
$l^- l^- U^{--}$	$-\frac{ig}{\sqrt{2}} C\gamma^m L$
$\tilde{\chi}_j^{--}\tilde{\chi}_i^0 U^{--}$	$\frac{ig}{2} O^1_{ij} C\gamma^m R$
$\tilde{\chi}_i^- \tilde{\chi}_j^- U^{--}$	$\frac{ig}{2} O^2_{ij} C\gamma^m R$
$\tilde{l}_1^- l^- \tilde{\chi}_i^{--}$	$-2i\lambda_3 A_{i5} \sin\theta_f R$
$\tilde{l}_2^- l^- \tilde{\chi}_i^{--}$	$-2i\lambda_3 A_{i5} \cos\theta_f R$
$\tilde{l}_1^- l^- \tilde{\chi}_i^0$	$i\left[g\left(\frac{N_{i1}}{\sqrt{2}}+\frac{N_{i2}}{\sqrt{6}}\right)\cos\theta_f R-\lambda_3\frac{2}{\sqrt{2}}\sin\theta_f N_{i8} R\right]$
$\tilde{l}_2^- l^- \tilde{\chi}_i^0$	$i\left[g\left(\frac{N_{i1}}{\sqrt{2}}+\frac{N_{i2}}{\sqrt{6}}\right)\sin\theta_f R+\lambda_3\frac{2}{\sqrt{2}}\cos\theta_f N_{i8} R\right]$
$\tilde{l}_1^+ l^- \tilde{\chi}_i^0$	$i\left[g\left(\frac{N^*_{i1}}{\sqrt{2}}+\frac{N^*_{i2}}{\sqrt{6}}\right)\cos\theta_f L-\lambda_3\frac{2}{\sqrt{2}}\sin\theta_f N^*_{i8} L\right]$
$\tilde{l}_2^+ l^- \tilde{\chi}_i^0$	$i\left[g\left(\frac{N^*_{i1}}{\sqrt{2}}+\frac{N^*_{i2}}{\sqrt{6}}\right)\sin\theta_f L+\lambda_3\frac{2}{\sqrt{2}}\cos\theta_f N^*_{i8} L\right]$
$\tilde{l}_1^+ l^- \tilde{\chi}_i^{--}$	$-igA_{i1}\sin\theta_f RC$
$\tilde{l}_2^+ l^- \tilde{\chi}_i^{--}$	$-igA_{i1}\cos\theta_f RC$
$\tilde{\nu}_l l^- \tilde{\chi}_i^-$	$-i\lambda_3\frac{2}{\sqrt{2}} D^*_{i7} L$
$\tilde{\nu}_l^* l^- \tilde{\chi}_i^-$	$-i\lambda_3\frac{2}{\sqrt{2}} D_{i7} R$

We have calculated the differential cross section, see Eq. (D1), and the total cross section, and we have displayed several plots of the total cross section with $M_U \geqslant 0.5$ TeV and $\sqrt{s}=0.5, 1.0$, and 2.0 TeV.

The plots show that outside the U resonance, the total-cross-section is of the order of pb, like in the MSSM [8]. This result is displayed in Figs. 2, 3, and 4.

B. Double chargino and neutralino production $e^-e^- \to \tilde{\chi}^{--}\tilde{\chi}^0$

Considering the Table I, we have at the tree level nine diagrams for this process, see Fig. 5. In the supersymmetric

TABLE II. Mass values of the lightest chargino and neutralino, sleptons, and of the sneutrino at electroweak scale corresponding to the mSUGRA solution.

$\tilde{m}$ [GeV]	RR1: $\tan\beta=3$	RR2: $\tan\beta=30$
$\tilde{\chi}_1^\pm$	128	132
$\tilde{\chi}_1^0$	70	72
$\tilde{e}_L^-$	176	217
$\tilde{e}_R^-$	132	183
$\tilde{\nu}$	166	206

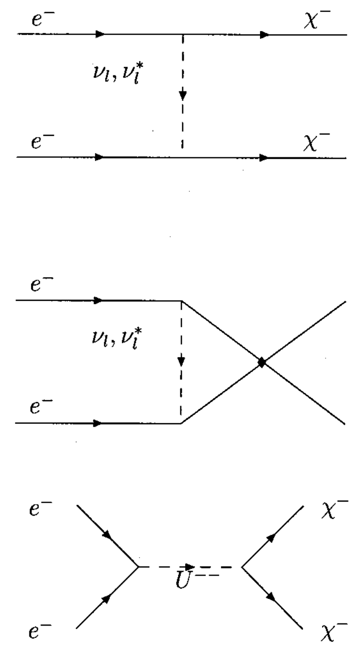

FIG. 1. Feynman diagrams for the process $e^-e^- \to \tilde{\chi}^-\tilde{\chi}^-$.

left-right model [6] the production of a double charged higgsino occurs via a selectron exchange in the t-channel, like in the third diagram of Fig. 5.

As in the previous subsection, we have calculated the differential cross section, see Eq. (D2), and the total cross section. We have done several plots using M_{χ^0}, $M_{\tilde{l}_1}$, and $M_{\tilde{l}_2}$, given in Table II, and $M_U=0.5$ TeV. Some of our results are shown in Figs. 6, 7, and 8.

VII. CONCLUSIONS

Because of the low level of standard model backgrounds, the total-cross-section $\sigma \approx 10^{-3}$ nb at $\sqrt{s}=500$ GeV [9], e^-e^- collisions are an appropriate reaction for discovering

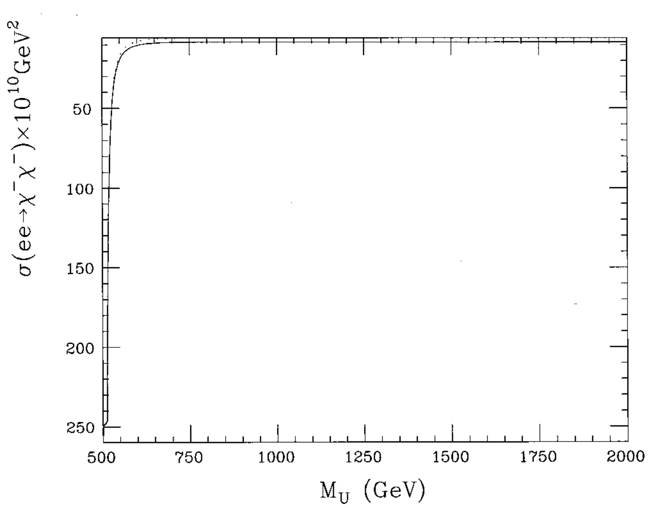

FIG. 2. Total cross section $e^-e^- \to \tilde{\chi}^-\tilde{\chi}^-$ at $\sqrt{s}=0.5$ TeV and $O_2=1$, $A_{i1}=10^{-1}$.

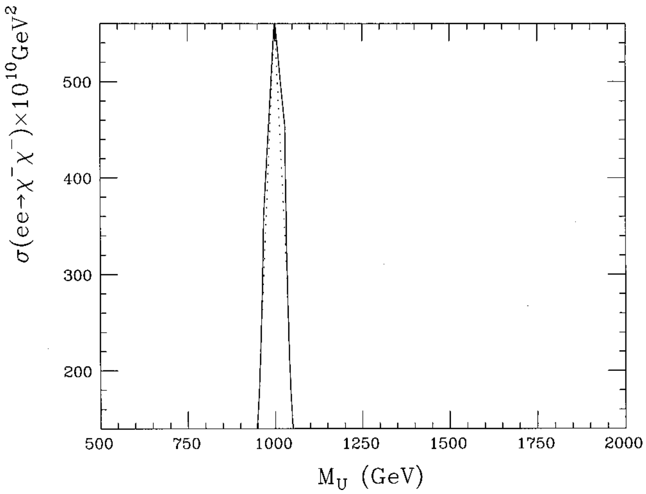

FIG. 3. Total cross section $e^-e^-\rightarrow\tilde{\chi}^-\tilde{\chi}^-$ at $\sqrt{s}=1.0$ TeV and $D_{i7}=10^{-1}$, $O_2=10^{-1}$.

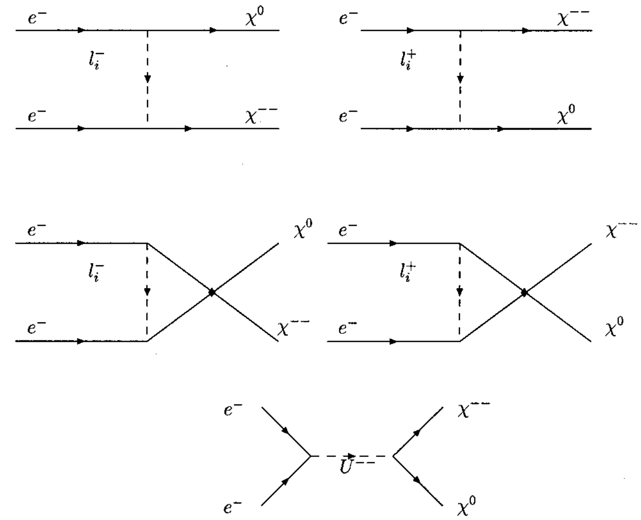

FIG. 5. Feynman diagrams for the process $e^-e^-\rightarrow\tilde{\chi}^{--}\tilde{\chi}^0$ and $i=1,2$.

and investigating new physics at linear colliders.

We have shown in this work that the production of single charged charginos have more contributions in this model than in the MSSM. Although in the MSSM the chargino pairs can be only produced through e^-e^- collisions by sneutrino exchanges in the u and t channels, in the 3-3-1 supersymmetric model we also have a s channel contribution due to the exchange of U^{--}. This new contribution induces a peak at $\sqrt{s}\simeq M_U$, where M_U is the mass of this boson, and gives a clear signal. Near the resonance the dominant term in the total-cross-section is given by $|O_2|^2(m^4_{\tilde{\chi}^+}-8sm^2_{\tilde{\chi}^+}+4s^2)$ coming from the U contribution. The total-cross-section outside the U resonance has the same order of magnitude than the cross section in the MSSM, as we should expected, because in this case we do not have an enhancement due to the s- channel contribution.

It was also shown that in this model we have double charged charginos, which are not present in the MSSM framework. Therefore, it is a very useful way to distinguish the 3-3-1 supersymmetric model from the MSSM, and also from the usual 3-3-1 model because in this case we do not have double charged leptons. We have considered the double chargino mass in the range $700\leqslant M_{\tilde{\chi}^{++}}\leqslant 800$ GeV, and we could get cross section of the order of pb outside the U resonance, while in the resonance we have an enhancement in the cross section. We believe that these new states can be discovered, if they really exist, in linear colliders like NLC, JLC, TESLA, CLIC or VLEPP.

ACKNOWLEDGMENTS

This research was financed by Fundação de Amparo à Pesquisa do Estado de São Paulo (M.C.R.). One of us (M.C.R.) would like to thank the Laboratoire de Physique Mathématique et Théorique (Montpellier) for its kind hospitality and also G. Moultaka for useful discussions.

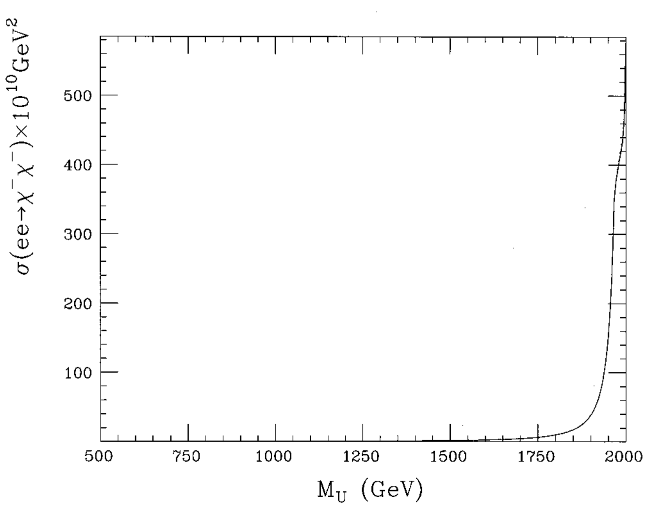

FIG. 4. Total cross section $e^-e^-\rightarrow\tilde{\chi}^-\tilde{\chi}^-$ at $\sqrt{s}=2.0$ TeV and $D_{i7}=1$, $O_2=10^{-1}$.

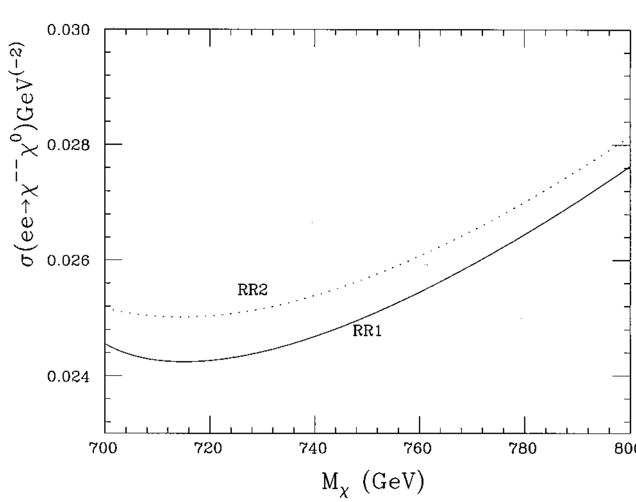

FIG. 6. Total cross section $e^-e^-\rightarrow\tilde{\chi}^{--}\tilde{\chi}^0$ at $\sqrt{s}=0.5$ TeV and $X_1\cos\theta_f=X_2\cos\theta_f=X_3\sin\theta_f=X_4\sin\theta_f=10^{-1}$, $O^1=10^{-2}$.

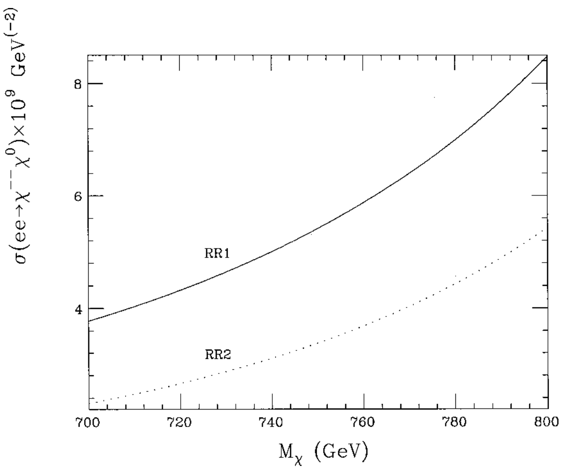

FIG. 7. Total cross section $e^-e^-\rightarrow\tilde{\chi}^{--}\tilde{\chi}^0$ at $\sqrt{s}=1.0$ TeV and $X_1\cos\theta_f=X_2\cos\theta_f=X_3\sin\theta_f=X_4\sin\theta_f=10^{-1}$, $O^1=10^{-1}$.

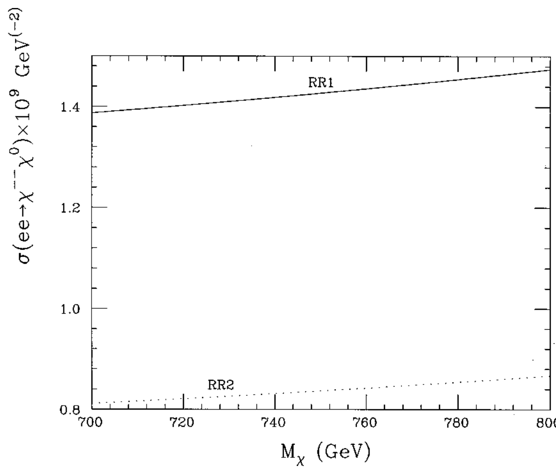

FIG. 8. Total cross section $e^-e^-\rightarrow\tilde{\chi}^{--}\tilde{\chi}^0$ at $\sqrt{s}=2.0$ TeV and $X_1\cos\theta_f=X_2\cos\theta_f=X_3\sin\theta_f=X_4\sin\theta_f=10^{-2}$, $O^1=10^{-2}$.

APPENDIX A: LAGRANGIAN

With the fields introduced in Sec. II, we can built the following Lagrangian [2]

$$\mathcal{L}_{331S}=\mathcal{L}_{\rm SUSY}+\mathcal{L}_{\rm soft} \tag{A1}$$

where

$$\mathcal{L}_{\rm SUSY}=\mathcal{L}_{\rm lepton}+\mathcal{L}_{\rm gauge}+\mathcal{L}_{\rm scalar}, \tag{A2}$$

is the supersymmetric part while $\mathcal{L}_{\rm soft}$ breaks supersymmetry. Now we are going to present all terms in the Lagrangian of the model, which we have used in this work.

1. Lepton Lagrangian

In the $\mathcal{L}_{\rm Lepton}$ term, the interactions between leptons and gauge bosons are given in components by

$$\mathcal{L}^{\rm lep}_{llV}=\frac{g}{2}\overline{L\sigma^m}\lambda^a L V^a_m, \tag{A3}$$

where λ^a are the usual Gell-Mann matrices.The next parts we are interested in are lepton-slepton-gaugino interactions:

$$\mathcal{L}^{\rm lep}_{l\tilde{l}\tilde{V}}=-\frac{ig}{\sqrt{2}}(\overline{L}\lambda^a\tilde{L}\overline{\lambda}^a_A-\bar{\tilde{L}}\lambda^a L\lambda^a_A). \tag{A4}$$

2. Gauge Lagrangian

The part we are interested in $\mathcal{L}_{\rm gauge}$ is the interaction between higgsino and gauge boson; that is

$$\mathcal{L}^{\rm gauge}_{\lambda\lambda V}=-igf^{abc}\overline{\lambda}^a_A\lambda^b_A\sigma^m V^c_m, \tag{A5}$$

where f^{abc} are the structure constants of the gauge group $SU(3)$.

3. Scalar Lagrangian

In the scalar sector, we are interested in the following three terms:

$$\begin{aligned}\mathcal{L}^{\rm scalar}_{H\tilde{H}\tilde{V}}=&-\frac{ig}{\sqrt{2}}[\bar{\tilde{\eta}}\lambda^a\eta\overline{\lambda}^a_A-\overline{\eta}\lambda^a\tilde{\eta}\lambda^a_A+\bar{\tilde{\rho}}\lambda^a\rho\overline{\lambda}^a_A-\overline{\rho}\lambda^a\tilde{\rho}\lambda^a_A+\bar{\tilde{\chi}}\lambda^a\chi\overline{\lambda}^a_A-\overline{\chi}\lambda^a\tilde{\chi}\lambda^a_A+\bar{\tilde{S}}\lambda^a S\overline{\lambda}^a_A-\overline{S}\lambda^a\tilde{S}\lambda^a_A-\bar{\tilde{\eta}}'\lambda^{*a}\eta'\overline{\lambda}^a_A\\&+\overline{\eta'}\lambda^{*a}\tilde{\eta}'\lambda^a_A-\bar{\tilde{\rho}}'\lambda^{*a}\rho'\overline{\lambda}^a_A+\overline{\rho'}\lambda^{*a}\tilde{\rho}'\lambda^a_A-\bar{\tilde{\chi}}'\lambda^{*a}\chi'\overline{\lambda}^a_A+\overline{\chi'}\lambda^{*a}\tilde{\chi}'\lambda^a_A-\bar{\tilde{S}}'\lambda^{*a}S'\overline{\lambda}^a_A\\&+\overline{S'}\lambda^{*a}\tilde{S}'\lambda^a_A]-\frac{ig'}{\sqrt{2}}[\bar{\tilde{\rho}}\rho\overline{\lambda}_B-\overline{\rho}\tilde{\rho}\lambda_B-\bar{\tilde{\chi}}\chi\overline{\lambda}_B+\overline{\chi}\tilde{\chi}\lambda_B-\bar{\tilde{\rho}}'\rho'\overline{\lambda}_B+\overline{\rho'}\tilde{\rho}'\lambda_B+\bar{\tilde{\chi}}'\chi'\overline{\lambda}_B-\overline{\chi'}\tilde{\chi}'\lambda_B],\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{\rm scalar}_{\tilde{H}\tilde{H}V}=&\frac{g}{2}[\bar{\tilde{\eta}}\overline{\sigma}^m\lambda^a\tilde{\eta}+\bar{\tilde{\rho}}\overline{\sigma}^m\lambda^a\tilde{\rho}+\bar{\tilde{\chi}}\overline{\sigma}^m\lambda^a\tilde{\chi}+\bar{\tilde{S}}\overline{\sigma}^m\lambda^a\tilde{S}-\bar{\tilde{\eta}}'\overline{\sigma}^m\lambda^{*a}\tilde{\eta}'-\bar{\tilde{\rho}}'\overline{\sigma}^m\lambda^{*a}\tilde{\rho}'-\bar{\tilde{\chi}}'\overline{\sigma}^m\lambda^{*a}\tilde{\chi}'-\bar{\tilde{S}}'\overline{\sigma}^m\lambda^{*a}\tilde{S}']V^a_m\\&+\frac{g'}{2}[\bar{\tilde{\rho}}\overline{\sigma}^m\tilde{\rho}-\bar{\tilde{\chi}}\overline{\sigma}^m\tilde{\chi}-\bar{\tilde{\rho}}'\overline{\sigma}^m\tilde{\rho}'+\bar{\tilde{\chi}}'\overline{\sigma}^m\tilde{\chi}']V_m,\end{aligned}$$

$$\mathcal{L}^{\rm Scalar}_{HHVV}=\frac{1}{4}[g^2V^a_mV^{bm}\overline{\eta}\lambda^a\lambda^b\eta+g^2V^a_mV^{bm}\overline{\rho}\lambda^a\lambda^b\rho+g^2V^a_mV^{bm}\overline{\chi}\lambda^a\lambda^b\chi+g^2V^a_mV^{bm}\overline{\eta'}\lambda^{*a}\lambda^{*b}\eta'$$
$$+g^2V^a_mV^{bm}\overline{\rho'}\lambda^{*a}\lambda^{*b}\rho'+g^2V^a_mV^{bm}\overline{\chi'}\lambda^{*a}\lambda^{*b}\chi'+g^2V^a_mV^{bm}(\lambda^a_{ik}\overline{S_{kj}}+\lambda^a_{jk}\overline{S_{ki}})(\lambda^a_{ik}S_{kj}+\lambda^a_{jk}S_{ki})$$
$$+g^2V^a_mV^{bm}(\lambda^{*a}_{ik}\overline{S'_{kj}}+\lambda^{*a}_{jk}\overline{S'_{ki}})(\lambda^{*a}_{ik}S'_{kj}+\lambda^{*a}_{jk}S'_{ki})+g'^2V^mV_m\overline{\rho}\rho+g'^2V^mV_m\overline{\chi}\chi+2gg'V^a_mV^m(\overline{\rho}\lambda^a\rho)$$
$$-2gg'V^a_mV^m(\overline{\chi}\lambda^a\chi)+g'^2V^mV_m\overline{\rho'}\rho'+g'^2V^mV_m\overline{\chi'}\chi'+2gg'V^a_mV^m(\overline{\rho'}\lambda^{*a}\rho')-2gg'V^a_mV^m(\overline{\chi'}\lambda^{*a}\chi')]. \quad \text{(A6)}$$

4. Superpotential

The terms of interest in this case are

$$\mathcal{L}_{HMT}=-\frac{\mu_\eta}{2}\tilde{\eta}_i\tilde{\eta}'_i-\frac{\mu_\rho}{2}\tilde{\rho}_i\tilde{\rho}'_i-\frac{\mu_\chi}{2}\tilde{\chi}_i\tilde{\chi}'_i-\frac{\mu_S}{2}\tilde{S}_{ij}\tilde{S}'_{ji}+\text{H.c.},$$

$$\mathcal{L}_F=\frac{1}{3}[3\lambda_1\epsilon F_L\tilde{L}\tilde{L}+\lambda_2\epsilon(2F_L\eta+F_\eta\tilde{L})\tilde{L}$$
$$+\lambda_3(2F_LS+F_S\tilde{L})\tilde{L}+f_1\epsilon(F_\rho\chi\eta+\rho F_\chi\eta+\rho\chi F_\eta)$$
$$+f_2(2F_\eta\eta S+\eta\eta F_S)+f_3(F_\rho\chi S+\rho F_\chi S+\rho\chi F_S)$$
$$+f'_1\epsilon(F_{\rho'}\chi'\eta'+\rho'F_{\chi'}\eta'+\rho'\chi'F_{\eta'})$$
$$+f'_2(2F_{\eta'}\eta'S'+\eta'\eta'F_{S'})$$
$$+f'_3(F_{\rho'}\chi'S'+\rho'F_{\chi'}S'+\rho'\chi'F_{S'})]+\text{H.c.},$$

$$\mathcal{L}_{H\tilde{H}\tilde{H}}=-\frac{1}{3}[f_1\epsilon(\tilde{\rho}\tilde{\chi}\eta+\rho\tilde{\chi}\tilde{\eta}+\tilde{\rho}\chi\tilde{\eta})$$
$$+f_2(\tilde{\eta}\tilde{\eta}S+\eta\tilde{\eta}\tilde{S}+\tilde{\eta}\eta\tilde{S})+f_3(\tilde{\rho}\tilde{\chi}S+\rho\tilde{\chi}\tilde{S}+\tilde{\rho}\chi\tilde{S})$$
$$+f'_1\epsilon(\tilde{\rho}'\tilde{\chi}'\eta'+\rho'\tilde{\chi}'\tilde{\eta}'+\tilde{\rho}'\chi'\tilde{\eta}')$$
$$+f'_2(\tilde{\eta}'\tilde{\eta}'S'+\eta'\tilde{\eta}'\tilde{S}'+\tilde{\eta}'\eta'\tilde{S}')$$
$$+f'_3(\tilde{\rho}'\tilde{\chi}'S'+\rho'\tilde{\chi}'\tilde{S}'+\tilde{\rho}'\chi'\tilde{S}')]+\text{H.c.} \quad \text{(A7)}$$

5. Soft term

The soft term of this model is

$$\mathcal{L}_{GMT}=-\frac{1}{2}\left[m_\lambda\sum_{a=1}^{8}(\lambda^a_A\lambda^a_A)+m'\lambda_B\lambda_B+\text{H.c.}\right],$$

$$\mathcal{L}_{\rm scalar}=-m^2_\eta\eta^\dagger\eta-m^2_\rho\rho^\dagger\rho-m^2_\chi\chi^\dagger\chi-m^2_STr(S^\dagger S)$$
$$-m^2_{\eta'}\eta'^\dagger\eta'-m^2_{\rho'}\rho'^\dagger\rho'-m^2_{\chi'}\chi'^\dagger\chi'$$
$$-m^2_{S'}Tr(S'^\dagger S')+(k_1\epsilon_{ijk}\rho_i\chi_j\eta_k+k_2\eta_i\eta_jS^\dagger_{ij}$$
$$+k_3\chi_i\rho_jS^\dagger_{ij}+k'_1\epsilon_{ijk}\rho'_i\chi'_j\eta'_k+k'_2\eta'_i\eta'_jS'^\dagger_{ij}$$
$$+k'_3\chi'_i\rho'_jS'^\dagger_{ij}+\text{H.c.}),$$

$$\mathcal{L}_{SMT}=-m^2_L\tilde{L}^\dagger\tilde{L}+\zeta_0\sum_{i=1}^{3}\sum_{j=1}^{3}(\tilde{L}_i\tilde{L}_jS_{ij}+\bar{\tilde{L}}_i\bar{\tilde{L}}_jS^*_{ij}). \quad \text{(A8)}$$

APPENDIX B: THE MASS MATRICES

In this appendix we display the mass matrices of the charginos and neutralinos.

1. Double charged chargino

Introducing the notation

$$\psi^{++}=(-i\lambda^{++}_U\tilde{\rho}^{++}\tilde{\chi}'^{++}\tilde{H}^{++}_1\tilde{H}'^{++}_2)^t,$$

$$\psi^{--}=(-i\lambda^{--}_U\tilde{\rho}'^{--}\tilde{\chi}^{--}\tilde{H}'^{--}_1\tilde{H}^{--}_2)^t,$$

and

$$\Psi^{\pm\pm}=(\psi^{++}\psi^{--})^t, \quad \text{(B1)}$$

we can write Eq. (3.1) as follows:

$$\mathcal{L}^{\rm double}_{\rm mass}=-\frac{1}{2}(\Psi^{\pm\pm})^tY^{\pm\pm}\Psi^{\pm\pm}+\text{H.c.}, \quad \text{(B2)}$$

where

$$Y^{\pm\pm}=\begin{pmatrix}0 & T^t\\ T & 0\end{pmatrix}, \quad \text{(B3)}$$

with

$$T=\begin{pmatrix} -m_\lambda & -gu & gw' & \frac{gz}{\sqrt{2}} & -\frac{gz'}{\sqrt{2}} \\ gu' & \frac{\mu_\rho}{2} & -\left(\frac{f_1' v'}{3}-\sqrt{2}\frac{f_3'}{3}z'\right) & 0 & \frac{f_3'}{3}w' \\ -gw & -\left(\frac{f_1 v}{3}-\sqrt{2}\,\frac{f_3}{3}z\right) & \frac{\mu_\chi}{2} & \frac{f_3}{3}u & 0 \\ -\frac{gz'}{\sqrt{2}} & 0 & \frac{f_3'}{3}u' & \frac{\mu_S}{2} & 0 \\ \frac{gz}{\sqrt{2}} & \frac{f_3}{3}w & 0 & 0 & \frac{\mu_S}{2} \end{pmatrix}. \tag{B4}$$

The matrix $Y^{\pm\pm}$ in Eq. (3.2) satisfies the following relation:

$$\det(Y^{\pm\pm}-\lambda I)=\det\left[\begin{pmatrix} -\lambda & T^t \\ T & -\lambda \end{pmatrix}\right]=\det(\lambda^2-T^t\cdot T), \tag{B5}$$

so we only have to calculate $T^t\cdot T$ to obtain the eigenvalues. Since $T^t\cdot T$ is a symmetric matrix, λ^2 must be real, and positive because $Y^{\pm\pm}$ is also symmetric.

2. Single charged chargino

Introducing the notation

$$\psi^+=(-i\lambda_W^+ - i\lambda_V^+\,\tilde{\eta}_1'^+\,\tilde{\eta}_2^+\,\tilde{\rho}^+\,\tilde{\chi}'^+\,\tilde{h}_1'^+\,\tilde{h}_2^+)^t,$$

$$\psi^-=(-i\lambda_W^- - i\lambda_V^-\,\tilde{\eta}_1^-\,\tilde{\eta}_2'^-\,\tilde{\rho}'^-\,\tilde{\chi}^-\,\tilde{h}_1^-\,\tilde{h}_2'^-)^t,$$

and

$$\Psi^\pm=(\psi^+\,\psi^-)^t, \tag{B6}$$

Equation (3.7) takes the form

$$\mathcal{L}^{\text{unique}}_{\text{mass}}=-\frac{1}{2}(\Psi^\pm)^t Y^\pm \Psi^\pm+\text{H.c.}, \tag{B7}$$

where

$$Y^\pm=\begin{pmatrix} 0 & X^t \\ X & 0 \end{pmatrix}, \tag{B8}$$

with

$$X=\begin{pmatrix} -m_\lambda & 0 & gv' & 0 & -gu & 0 & -\frac{gz'}{2} & 0 \\ 0 & -m_\lambda & 0 & -gv & 0 & gw' & 0 & -\frac{gz}{2} \\ -gv & 0 & \frac{\mu_\eta}{2} & 0 & -\frac{f_1 w}{3} & 0 & 0 & 0 \\ 0 & gv' & 0 & \frac{\mu_\eta}{2} & 0 & \frac{f_1' u'}{3} & 0 & 0 \\ gu' & 0 & -\frac{f_1' w'}{3} & 0 & \frac{\mu_\rho}{2} & 0 & 0 & 0 \\ 0 & -gw & 0 & \frac{f_1 u}{3} & 0 & \frac{\mu_\chi}{2} & 0 & 0 \\ -\frac{gz}{2} & 0 & 0 & 0 & \frac{f_3 w}{3\sqrt{2}} & 0 & \frac{\mu_S}{2} & 0 \\ 0 & \frac{gz'}{2} & 0 & 0 & 0 & \frac{f_3' u'}{3\sqrt{2}} & 0 & \frac{\mu_S}{2} \end{pmatrix}. \tag{B9}$$

The matrix X satisfies the same properties as the matrix T [Eq. (B5)].

3. Neutralinos

Introducing the notation

$$\Psi^0=(-i\lambda_A^3 - i\lambda_A^8 - i\lambda_B\,\tilde{\eta}^0\,\tilde{\eta}'^0\,\tilde{\rho}^0\,\tilde{\rho}'^0\,\tilde{\chi}^0\,\tilde{\chi}'^0\,\tilde{\sigma}_1^0\,\tilde{\sigma}_1'^0\,\tilde{\sigma}_2^0\,\tilde{\sigma}_2'^0)^t,$$

Equation (3.12) takes the following form:

$$\mathcal{L}^{\text{neutralino}}_{\text{mass}}=-\frac{1}{2}(\Psi^0)^t Y^0 \Psi^0+\text{H.c.}, \tag{B10}$$

where

$$Y^0=\begin{pmatrix}
-m_\lambda & 0 & 0 & -\frac{gv}{\sqrt{2}} & \frac{gv'}{\sqrt{2}} & \frac{gu}{\sqrt{2}} & -\frac{gu'}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{gz}{2\sqrt{2}} & -\frac{gz'}{2\sqrt{2}} \\
0 & -m_\lambda & 0 & -\frac{gv}{\sqrt{6}} & \frac{gv'}{\sqrt{6}} & -\frac{gu}{\sqrt{6}} & \frac{gu'}{\sqrt{6}} & \frac{2}{\sqrt{6}}gw & -\frac{2}{\sqrt{6}}gw' & 0 & 0 & \frac{gz}{2\sqrt{6}} & -\frac{gz'}{2\sqrt{6}} \\
0 & 0 & -m' & 0 & 0 & -\frac{g'u}{\sqrt{2}} & \frac{g'u'}{\sqrt{2}} & \frac{g'w}{\sqrt{2}} & -\frac{g'w'}{\sqrt{2}} & 0 & 0 & 0 & 0 \\
-\frac{gv}{\sqrt{2}} & -\frac{gv}{\sqrt{6}} & 0 & 0 & \frac{\mu_\eta}{2} & \frac{f_1 w}{3} & 0 & -\frac{f_1 u}{3} & 0 & 0 & 0 & 0 & 0 \\
\frac{gv'}{\sqrt{2}} & \frac{gv'}{\sqrt{6}} & 0 & \frac{\mu_\eta}{2} & 0 & 0 & \frac{f_1' w'}{3} & 0 & -\frac{f_1' u'}{3} & 0 & 0 & 0 & 0 \\
\frac{gu}{\sqrt{2}} & -\frac{gu}{\sqrt{6}} & -\frac{g'u}{\sqrt{2}} & \frac{f_1 w}{3} & 0 & 0 & \frac{\mu_\rho}{2} & A & 0 & 0 & 0 & \frac{f_3 w}{3\sqrt{2}} & 0 \\
-\frac{g'u'}{\sqrt{2}} & \frac{g'u'}{\sqrt{6}} & \frac{g'u'}{\sqrt{2}} & 0 & \frac{f_1' w'}{3} & \frac{\mu_\rho}{2} & 0 & 0 & B & 0 & 0 & 0 & \frac{f_3' w'}{3\sqrt{2}} \\
0 & \frac{2}{\sqrt{6}}gw & \frac{g'w}{\sqrt{2}} & -\frac{f_1 u}{3} & 0 & A & 0 & 0 & \frac{\mu_\chi}{2} & 0 & 0 & \frac{f_3 u}{3\sqrt{2}} & 0 \\
0 & -\frac{2}{\sqrt{6}}gw' & -\frac{g'w'}{\sqrt{2}} & 0 & -\frac{f_1' u'}{3} & 0 & B & \frac{\mu_\chi}{2} & 0 & 0 & 0 & 0 & \frac{f_3' u'}{3\sqrt{2}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu_S}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu_S}{2} & 0 & 0 & 0 \\
\frac{gz'}{2\sqrt{2}} & \frac{gz}{2\sqrt{6}} & 0 & 0 & 0 & \frac{f_3 w}{3\sqrt{2}} & 0 & \frac{f_3 u}{3\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{\mu_S}{2} \\
-\frac{gz'}{2\sqrt{2}} & -\frac{gz'}{2\sqrt{6}} & 0 & 0 & 0 & 0 & \frac{f_3' w'}{3\sqrt{2}} & 0 & \frac{f_3' u'}{3\sqrt{2}} & 0 & 0 & \frac{\mu_S}{2} & 0
\end{pmatrix}, \tag{B11}$$

with

$$A=\frac{f_1 v}{3}+\sqrt{2}\,\frac{f_3}{3}z,\qquad B=\frac{f_1' v'}{3}+\sqrt{2}\,\frac{f_3'}{3}z'. \tag{B12}$$

APPENDIX C: CONNECTION BETWEEN THE TWO- AND FOUR- COMPONENT SPINORS

In this appendix we show the procedure to write two-component spinors in terms of four-component spinors.

1. Weak eigenstates

The weak interaction eigenstates are:

$$\tilde{U}=\begin{pmatrix} -i\lambda_U^{++} \\ i\overline{\lambda_U^{--}} \end{pmatrix},\quad \tilde{U}^c=\begin{pmatrix} -i\lambda_U^{--} \\ i\overline{\lambda_U^{++}} \end{pmatrix},$$

$$\tilde{T}_1^{++}=\begin{pmatrix} \tilde{\rho}^{++} \\ \bar{\tilde{\rho}}'^{--} \end{pmatrix},\quad \tilde{T}_1^{c++}=\begin{pmatrix} \tilde{\rho}'^{--} \\ \bar{\tilde{\rho}}^{++} \end{pmatrix},$$

$$\tilde{T}_2^{++}=\begin{pmatrix} \tilde{\chi}'^{++} \\ \bar{\tilde{\chi}}^{--} \end{pmatrix},\quad \tilde{T}_2^{c++}=\begin{pmatrix} \tilde{\chi}^{--} \\ \bar{\tilde{\chi}}'^{++} \end{pmatrix},$$

$$\tilde{S}_1^{++}=\begin{pmatrix} \tilde{H}_1^{++} \\ \bar{\tilde{H}}'^{--}_1 \end{pmatrix},\quad \tilde{S}_1^{c++}=\begin{pmatrix} \tilde{H}'^{--}_1 \\ \bar{\tilde{H}}_1^{++} \end{pmatrix},$$

$$\tilde{S}_2^{++}=\begin{pmatrix} \tilde{H}'^{++}_2 \\ \bar{\tilde{H}}_2^{--} \end{pmatrix},\quad \tilde{S}_2^{c++}=\begin{pmatrix} \tilde{H}_2^{--} \\ \bar{\tilde{H}}'^{++}_2 \end{pmatrix},$$

$$\tilde{W}=\begin{pmatrix} -i\lambda_W^{+} \\ i\overline{\lambda_W^{-}} \end{pmatrix},\quad \tilde{W}^c=\begin{pmatrix} -i\lambda_W^{-} \\ i\overline{\lambda_W^{+}} \end{pmatrix},$$

$$\tilde{V}=\begin{pmatrix} -i\lambda_V^{+} \\ i\overline{\lambda_V^{-}} \end{pmatrix},\quad \tilde{V}^c=\begin{pmatrix} -i\lambda_V^{-} \\ i\overline{\lambda_V^{+}} \end{pmatrix},$$

$$\tilde{T}_1^{+}=\begin{pmatrix} \tilde{\eta}'^{+}_1 \\ \bar{\tilde{\eta}}_1^{-} \end{pmatrix},\quad \tilde{T}_1^{c+}=\begin{pmatrix} \tilde{\eta}_1^{-} \\ \bar{\tilde{\eta}}'^{+}_1 \end{pmatrix},$$

$$\tilde{T}_2^{+}=\begin{pmatrix} \tilde{\eta}_2^{+} \\ \bar{\tilde{\eta}}'^{-}_2 \end{pmatrix},\quad \tilde{T}_2^{c+}=\begin{pmatrix} \tilde{\eta}'^{-}_2 \\ \bar{\tilde{\eta}}_2^{+} \end{pmatrix},$$

$$\tilde{T}_3^{+}=\begin{pmatrix} \tilde{\rho}^{+} \\ \bar{\tilde{\rho}}'^{-} \end{pmatrix},\quad \tilde{T}_3^{c+}=\begin{pmatrix} \tilde{\rho}'^{-} \\ \bar{\tilde{\rho}}^{+} \end{pmatrix},$$

$$\tilde{T}_4^{+}=\begin{pmatrix} \tilde{\chi}'^{+} \\ \bar{\tilde{\chi}}^{-} \end{pmatrix},\quad \tilde{T}_4^{c+}=\begin{pmatrix} \tilde{\chi}^{-} \\ \bar{\tilde{\chi}}'^{+} \end{pmatrix},$$

$$\tilde{S}_1^{+}=\begin{pmatrix} \tilde{h}_1^{-} \\ \bar{\tilde{h}}'^{+}_1 \end{pmatrix},\quad \tilde{S}_1^{c+}=\begin{pmatrix} \tilde{h}'^{+}_1 \\ \bar{\tilde{h}}_1^{-} \end{pmatrix},$$

$$\tilde{S}_2^{+}=\begin{pmatrix} \tilde{h}_2^{+} \\ \bar{\tilde{h}}'^{-}_2 \end{pmatrix},\quad \tilde{S}_2^{c+}=\begin{pmatrix} \tilde{h}'^{-}_2 \\ \bar{\tilde{h}}_2^{+} \end{pmatrix},$$

$$\tilde{W}_3=\begin{pmatrix} -i\lambda_A^{3} \\ i\overline{\lambda_A^{3}} \end{pmatrix},\quad \tilde{W}_8=\begin{pmatrix} -i\lambda_A^{8} \\ i\overline{\lambda_A^{8}} \end{pmatrix},$$

$$\tilde{B}=\begin{pmatrix} -i\lambda_B \\ i\overline{\lambda_B} \end{pmatrix},$$

$$\tilde{T}_1^{0}=\begin{pmatrix} \tilde{\eta}^{0} \\ \bar{\tilde{\eta}}^{0} \end{pmatrix},\quad \tilde{T}_2^{0}=\begin{pmatrix} \tilde{\eta}'^{0} \\ \bar{\tilde{\eta}}'^{0} \end{pmatrix},\quad \tilde{T}_3^{0}=\begin{pmatrix} \tilde{\rho}^{0} \\ \bar{\tilde{\rho}}^{0} \end{pmatrix},$$

$$\tilde{T}_4^{0}=\begin{pmatrix} \tilde{\rho}'^{0} \\ \bar{\tilde{\rho}}'^{0} \end{pmatrix},\quad \tilde{T}_5^{0}=\begin{pmatrix} \tilde{\chi}^{0} \\ \bar{\tilde{\chi}}^{0} \end{pmatrix},$$

$$\tilde{T}_6^{0}=\begin{pmatrix} \tilde{\chi}'^{0} \\ \bar{\tilde{\chi}}'^{0} \end{pmatrix},\quad \tilde{S}_1^{0}=\begin{pmatrix} \tilde{\sigma}_1^{0} \\ \bar{\tilde{\sigma}}_1^{0} \end{pmatrix},\quad \tilde{S}_2^{0}=\begin{pmatrix} \tilde{\sigma}'^{0}_1 \\ \bar{\tilde{\sigma}}'^{0}_1 \end{pmatrix},$$

$$\tilde{S}_3^{0}=\begin{pmatrix} \tilde{\sigma}_2^{0} \\ \bar{\tilde{\sigma}}_2^{0} \end{pmatrix},\quad \tilde{S}_4^{0}=\begin{pmatrix} \tilde{\sigma}'^{0}_2 \\ \bar{\tilde{\sigma}}'^{0}_2 \end{pmatrix}. \tag{C1}$$

With the states defined in Eqs. (C1) we get the following identities that allow us to write the two-component spinors in terms of the four-component weak eigenstates

$$\lambda_U^{--}\sigma^m\overline{\lambda_A^3}=-\bar{\tilde{U}}L\gamma^m R\tilde{W}_3,$$

$$\lambda_U^{++}\sigma^m\overline{\lambda_A^3}=-\bar{\tilde{U}}^c L\gamma^m R\tilde{W}_3,$$

$$\lambda_A^{8}\sigma^m\overline{\lambda_U^{++}}=-\bar{\tilde{W}}_8 L\gamma^m R\tilde{U}^c,$$

$$\lambda_U^{++}\sigma^m\overline{\lambda_A^8}=-\bar{\tilde{U}}^c L\gamma^m R\tilde{W}_8,$$

$$\tilde{\chi}'^{++}\sigma^m\bar{\tilde{\chi}}'^{0}=-\bar{\tilde{T}}_3^{c++}L\gamma^m R\tilde{T}_6^0,$$

$$\tilde{\rho}^{++}\sigma^m\bar{\tilde{\rho}}^{0}=-\bar{\tilde{T}}_1^{c++}L\gamma^m R\tilde{T}_3^0,$$

$$\tilde{\chi}^{0}\sigma^m\bar{\tilde{\chi}}^{--}=-\bar{\tilde{T}}_5^{0}L\gamma^m R\tilde{T}_2^{++},$$

$$\tilde{\rho}'^{0}\sigma^m\bar{\tilde{\rho}}'^{--}=-\bar{\tilde{T}}_4^{0}L\gamma^m R\tilde{T}_1^{++},$$

$$\tilde{\sigma}'^{0}_2\sigma^m\bar{\tilde{H}}'^{--}_1=-\bar{\tilde{S}}_4^{0}L\gamma^m R\tilde{S}_1^{++},$$

$$\tilde{H}_1^{++}\sigma^m\bar{\tilde{\sigma}}_2^{0}=-\bar{\tilde{S}}_1^{c++}L\gamma^m R\tilde{S}_3^0,$$

$$\tilde{H}'^{++}_2\sigma^m\bar{\tilde{\sigma}}'^{0}_2=-\bar{\tilde{S}}_2^{c++}L\gamma^m R\tilde{S}_4^0,$$

$$\tilde{\sigma}_2^{0}\sigma^m\bar{\tilde{H}}_2^{--}=-\bar{\tilde{S}}_3^{0}L\gamma^m R\tilde{S}_2^{++},$$

$$\lambda_V^{-}\sigma^m\overline{\lambda_W^{+}}=-\bar{\tilde{V}}L\gamma^m R\tilde{W}^c,\qquad \lambda_W^{+}\sigma^m\overline{\lambda_V^{-}}=-\bar{\tilde{W}}^c L\gamma^m R\tilde{V},$$

$$\tilde{\eta}_2^{+}\sigma^m\bar{\tilde{\eta}}_1^{-}=-\bar{\tilde{T}}_2^{c+}L\gamma^m R\tilde{T}_1^{+},$$

$$\tilde{h}'^{+}_{1}\sigma^{m}\bar{\tilde{h}}'^{-}_{2}=-\bar{\tilde{S}}^{c+}_{1}L\gamma^{m}R\tilde{S}^{+}_{2},$$

$$\tilde{\eta}'^{+}_{1}\sigma^{m}\bar{\tilde{\eta}}'^{-}_{2}=-\bar{\tilde{T}}^{c+}_{1}L\gamma^{m}R\tilde{T}^{+}_{2},$$

$$\tilde{h}^{+}_{2}\sigma^{m}\bar{\tilde{h}}^{-}_{1}=-\bar{\tilde{S}}^{c+}_{2}L\gamma^{m}R\tilde{S}^{+}_{1}. \quad \text{(C2)}$$

2. Mass eigenstates

From Eqs. (3.3), (3.9), and (3.15), we can write the inverse transformations as

$$\Psi^{++}_{k}=A^{*}_{ik}\tilde{\chi}^{++}_{i},\quad \Psi^{--}_{k}=B^{*}_{ik}\tilde{\chi}^{--}_{i},\quad \Psi^{+}_{k}=D^{*}_{ik}\tilde{\chi}^{+}_{i},$$

$$\Psi^{-}_{k}=E^{*}_{ik}\tilde{\chi}^{-}_{i},\quad \Psi^{0}_{k}=N^{*}_{ik}\tilde{\chi}^{0}_{i}. \quad \text{(C3)}$$

Equation(C2) above does not involve physical particles. From Eqs. (3.6) and (3.16) we can show the following relations:

$$\tilde{\chi}^{++}_{i}=L\Psi(\tilde{\chi}^{++}_{i}),\quad \bar{\tilde{\chi}}^{++}_{i}=R\Psi^{c}(\tilde{\chi}^{--}_{i}),$$

$$\bar{\tilde{\chi}}^{--}_{i}=R\Psi(\tilde{\chi}^{++}_{i}),\quad \tilde{\chi}^{--}_{i}=L\Psi^{c}(\tilde{\chi}^{--}_{i}),$$

$$\tilde{\chi}^{0}_{i}=L\Psi(\tilde{\chi}^{0}_{i}),\quad \bar{\tilde{\chi}}^{0}_{i}=R\Psi(\tilde{\chi}^{0}_{i}),$$

$$\tilde{\chi}^{+}_{i}=L\Psi(\tilde{\chi}^{+}_{i}),\quad \bar{\tilde{\chi}}^{+}_{i}=R\Psi^{c}(\tilde{\chi}^{-}_{i}),$$

$$\bar{\tilde{\chi}}^{-}_{i}=R\Psi(\tilde{\chi}^{+}_{i}),\quad \tilde{\chi}^{-}_{i}=L\Psi^{c}(\tilde{\chi}^{-}_{i}). \quad \text{(C4)}$$

APPENDIX D: DIFFERENTIAL CROSS SECTIONS

In this appendix we calculate the differential cross sections to the processes we have studied in Sec. VI.

1. $e^-e^-\to\tilde{\chi}^-\tilde{\chi}^-$ (single charged charginos production)

$$\frac{d\sigma}{d\Omega}(e^-e^-\to\tilde{\chi}^-\tilde{\chi}^-)=\frac{1}{128\pi^2 s}\sqrt{\frac{s}{\frac{s}{4}-m^2_{\tilde{\chi}^+}}}|\mathcal{M}_T|^2, \quad \text{(D1)}$$

where

$$|\mathcal{M}_T|^2=|D_{i7}|^4\left(\frac{1}{(t-m^2_{\tilde{\nu}})^2}+\frac{1}{(u-m^2_{\tilde{\nu}})^2}\right)$$
$$\times\left\{2m^4_{\tilde{\chi}^+}+2(E-2m^2_{\tilde{\chi}^+})m^2_{\tilde{\chi}^+}+2\left[\left(m^2_{\tilde{\chi}^+}-\frac{s}{2}\right)^2\right.\right.$$
$$\left.\left.+s\left(\frac{s}{4}-m^2_{\tilde{\chi}^+}\right)\cos\theta\right]\right\}-\frac{2g^2(s-M^2_U)O_2|D_{i7}|^2}{(s-M^2_U)^2+(\Gamma_U M_U)^2}$$
$$\times\left[m^4_{\tilde{\chi}^+}\left(\frac{1}{(t-m^2_{\tilde{\nu}})}+\frac{1}{(u-m^2_{\tilde{\nu}})}\right)+(2m^2_{\tilde{\chi}^+}+s)\right.$$
$$\left.\times\left(\frac{u}{(t-m^2_{\tilde{\nu}})}+\frac{t}{(u-m^2_{\tilde{\nu}})}\right)\right],$$
$$+\frac{g^4|O_2|^2}{(s-M^2_U)^2+(\Gamma_U M_U)^2}(m^4_{\tilde{\chi}^+}-8sm^2_{\tilde{\chi}^+}+4s^2)$$
$$+\frac{4s|D_{i7}|^2}{(t-m^2_{\tilde{\nu}})(u-m^2_{\tilde{\nu}})}.$$

2. $e^-e^-\to\tilde{\chi}^{--}\tilde{\chi}^0$ (double charged chargino neutralino production)

$$\frac{d\sigma}{d\Omega}(e^-e^-\to\tilde{\chi}^{--}\tilde{\chi}^0)=\frac{1}{64\pi^2 s}\sqrt{\frac{s}{E^2_{\chi^{++}}-m^2_{\chi^{++}}}}|\mathcal{M}_T|^2, \quad \text{(D2)}$$

where

$$|\mathcal{M}_T|^2=\left(\frac{X_1^2\cos^2\theta_f}{(u-m^2_{\tilde{l}_2})^2}+\frac{X_2^2\cos^2\theta_f}{(t-m^2_{\tilde{l}_2})^2}+\frac{X_3^2\sin^2\theta_f}{(t-m^2_{\tilde{l}_1})^2}+\frac{X_4^2\sin^2\theta_f}{(u-m^2_{\tilde{l}_1})^2}+\frac{X_1X_4\sin\theta_f\cos\theta_f}{(u-m^2_{\tilde{l}_2})(u-m^2_{\tilde{l}_1})}\right.$$
$$\left.+\frac{X_2X_3\sin\theta_f\cos\theta_f}{(t-m^2_{\tilde{l}_2})(t-m^2_{\tilde{l}_1})}\right)\cdot[2(m^4_{\chi^{++}}+m^4_{\chi^0})+s(m^2_{\chi^{++}}+m^2_{\chi^0}-s)+2t(t-2m^2_{\chi^{++}})+2u(u-2m^2_{\chi^0})]$$
$$+2m^4_{\chi^{++}}m^4_{\chi^0}O^1\cos\theta_f\left(\frac{X_1}{(u-m^2_{\tilde{l}_2})^2[(s-M^2_U)^2+(\Gamma_U M_U)^2]}+\frac{X_2}{(t-m^2_{\tilde{l}_2})^2[(s-M^2_U)^2+(\Gamma_U M_U)^2]}\right)$$
$$+2m^4_{\chi^{++}}m^4_{\chi^0}O^1\sin\theta_f\left(\frac{X_3}{(t-m^2_{\tilde{l}_1})^2[(s-M^2_U)^2+(\Gamma_U M_U)^2]}+\frac{X_4}{(u-m^2_{\tilde{l}_1})^2[(s-M^2_U)^2+(\Gamma_U M_U)^2]}\right)$$
$$+\frac{O^1}{(s-M^2_U)^2+(\Gamma_U M_U)^2}[2(m^4_{\chi^{++}}+m^2_{\chi^0})+4(2(m^2_{\chi^{++}}+m^2_{\chi^0}))s+s^2],$$

where

$$X_1=\frac{2}{\sqrt{2}}A_{i5}\lambda_3 N_{i8}, \quad X_2=\frac{2}{\sqrt{2}}A_{i1}gN_{i8},$$

$$X_3=A_{i1}g\left(\frac{N_{i1}}{\sqrt{2}}+\frac{N_{i2}}{\sqrt{6}}\right), \quad X_4=A_{i5}\lambda_3\left(\frac{N_{i1}}{\sqrt{2}}+\frac{N_{i2}}{\sqrt{6}}\right). \tag{D3}$$

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