

## Fragmentation functions for lepton pairs

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(Received 10 February 2001; published 9 January 2002)

We calculate the fragmentation function for a light quark to decay into a lepton pair to leading order in the QCD coupling constant. In the formal definition of the fragmentation function, a QED phase must be included in the eikonal factor to guarantee QED gauge invariance. We find that the longitudinal polarization fraction is a decreasing function of the factorization scale, in accord with the intuitive expectation that the virtual photon should behave more and more like a real photon as the transverse momentum of the fragmenting quark increases.

DOI: 10.1103/PhysRevD.65.034005

PACS number(s): 13.85.Qk, 12.38.Bx, 13.87.Fh, 13.88.+e

The QCD factorization theorems for inclusive single-particle production [1] guarantee that the differential cross section at sufficiently large transverse momentum  $P_T$  has the approximate scaling behavior  $d\sigma/dP_T^2 \sim 1/P_T^4$ . The scaling part of the differential cross section for producing the particle  $H$  with momentum  $P$  has the form

$$d\sigma[H(P)] = \sum_i \int_0^1 dz d\hat{\sigma}[i(P/z), \mu] D_{i \rightarrow H}(z, \mu), \quad (1)$$

where the sum is over partons  $i$  and  $d\hat{\sigma}$  is the differential cross section for producing an on-shell parton  $i$  with momentum  $P/z$ . The fragmentation function  $D_{i \rightarrow H}$  gives the probability for a virtual parton with invariant mass of order  $\mu$  to decay into a state that includes a particle  $H$  with the fraction  $z$  of the longitudinal momentum of the parton. The  $\mu$  dependence of the fragmentation functions is governed by Altarelli-Parisi evolution equations whose kernels can be calculated using perturbation theory. Large logarithms of  $z\mu/P_T$  in  $d\hat{\sigma}$  can be summed up by taking  $\mu$  of order  $P_T/z$ .

The factorization formula (1) can serve as an operational definition of the fragmentation functions. However, the fragmentation functions can also be given formal definitions as vacuum-to-vacuum matrix elements of operators that involve projections onto asymptotic states that include the particle  $H$ . Curci, Furmanski, and Petronzio defined them as matrix elements of bilocal operators in the light-cone gauge [2]. Collins and Soper introduced gauge-invariant definitions of the fragmentation functions [3]. They are defined as matrix elements of nonlocal gauge-invariant operators that involve a path-ordered exponential of the gluon field called an eikonal factor:

$$\mathcal{E} = \mathcal{P} \exp \left( ig \int dl n^\mu A_\mu^a T^a \right), \quad (2)$$

where  $T^a$  is a generator of the appropriate representation of the gauge group. In the light-cone gauge  $n^\mu A_\mu^a = 0$ , the eikonal factor collapses to 1 and the definition reduces to that of Curci, Furmanski, and Petronzio.

In the case of a light hadron  $H$ , the fragmentation function  $D_{i \rightarrow H}$  cannot be calculated using perturbative methods. However, once  $D_{i \rightarrow H}(z, \mu_0)$  has been measured as a function of  $z$  at some initial scale  $\mu_0$ , the Altarelli-Parisi equations can be used to evolve it to other scales  $\mu$ . An analysis of the fragmentation functions for light hadrons has recently been used to obtain a high-precision determination of the QCD coupling constant [4].

In the case of a heavy quarkonium state  $H$ , the fragmentation functions can be calculated using perturbation theory up to a few nonperturbative constants. The nonrelativistic QCD (NRQCD) factorization formalism [5] can be used to express the fragmentation function in the form

$$D_{i \rightarrow H}(z, \mu) = \sum_n d_{i \rightarrow (Q\bar{Q})_n}(z, \mu) \langle O_n^H \rangle, \quad (3)$$

where the sum is over color and angular momentum states of a  $Q\bar{Q}$  pair and the constants  $\langle O_n^H \rangle$  are called NRQCD matrix elements. The coefficients  $d_{i \rightarrow (Q\bar{Q})_n}$  of the NRQCD matrix elements can be calculated using perturbation theory. The first explicit calculations were the coefficients of  $\langle O_1(^1S_0) \rangle$  in the fragmentation functions for  $g \rightarrow \eta_c$  and  $c \rightarrow \eta_c$  and the coefficients of  $\langle O_1(^3S_1) \rangle$  in the fragmentation functions for  $g \rightarrow J/\psi$  and  $c \rightarrow J/\psi$  [6,7]. They were calculated by Braaten, Cheung, and Yuan using the operational definition of the fragmentation functions provided by the factorization theorem (1). The first explicit calculations using the formal definition of the fragmentation functions were carried out by Ma [8]. The formal definition is particularly useful for calculating the fragmentation functions beyond leading order in  $\alpha_s$  [9].

The factorization theorems for inclusive single-hadron production can be applied equally well to photon production and to lepton-pair production. They imply that at sufficiently large  $Q_T$ , where  $Q_T$  is the transverse momentum of the photon or lepton pair, the differential cross section has the approximate scaling behavior  $d\sigma/dQ_T^2 \sim 1/Q_T^4$ . The scaling part of the differential cross section has the form (1). At leading order in the QED interaction, only a photon term

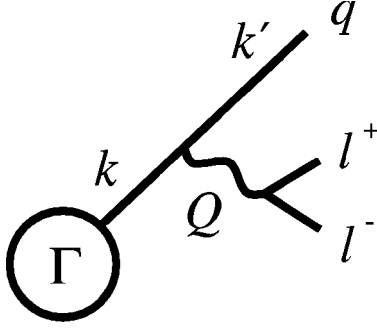


FIG. 1. The class of Feynman diagrams that in the light-cone gauge for QED gives the contribution to the production of lepton pairs associated with fragmentation of the light quark  $q$  at leading order in  $\alpha_s$ .

needs to be added to the sum over partons  $i$ . The fragmentation functions  $D_{q \rightarrow \gamma}$  and  $D_{g \rightarrow \gamma}$  for real photons are nonperturbative, and only their evolution with  $\mu$  is calculable in perturbation theory. The same is true of the fragmentation functions for a lepton pair with invariant mass  $Q$  comparable to the scale  $\Lambda$  associated with nonperturbative effects in QCD. On the other hand, if  $Q$  is much larger than  $\Lambda$ , the fragmentation functions  $D_{q \rightarrow l^+ l^-}$  and  $D_{g \rightarrow l^+ l^-}$  are completely calculable using perturbative QCD. Qiu and Zhang have recently introduced formal definitions for “virtual photon fragmentation functions”  $D_{q \rightarrow \gamma^*}$  and  $D_{g \rightarrow \gamma^*}$  that are equivalent to fragmentation functions for lepton pairs [10].

In this paper, we calculate the fragmentation function  $D_{q \rightarrow l^+ l^-}$  for a quark to decay into a lepton pair with large invariant mass at leading order in QCD perturbation theory. We calculate the fragmentation function using both dimensional regularization and an upper limit on the invariant mass of the fragmenting quark. We find that the fragmentation function defined by dimensional regularization has unphysical behavior except at asymptotic values of the factorization scale. The fragmentation function defined by the upper limit on the invariant mass gives a longitudinal polarization fraction that decreases as the factorization scale increases, in accord with the intuitive expectation that the virtual photon should behave more and more like a real photon as the transverse momentum of the fragmenting parton increases.

The fragmentation function for  $q \rightarrow l^+ l^-$  can be calculated using the operational definition provided by the factorization theorem, as in Refs. [6,7]. The Feynman diagrams for producing a lepton pair with large transverse momentum  $Q_T$  include some with the topology shown in Fig. 1, where the blob represents the parts of the diagram that involve the production of the virtual quark. The terms in the matrix element that corresponding to the fragmentation of a light quark with electric charge  $e_q$  have the form

$$\mathcal{M} = \frac{1}{k^2} \bar{u}(k') (i e_q e \gamma_\mu) \Gamma \frac{-i G^{\mu\nu}}{Q^2} \bar{u}(q_1) (-i e \gamma_\nu) v(q_2), \quad (4)$$

where  $k, k', q_1, q_2$ , and  $Q = q_1 + q_2$  are the momenta of the virtual quark, the final-state quark, the positive lepton, the

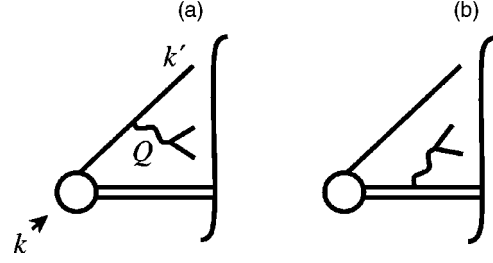


FIG. 2. Feynman diagrams for the fragmentation function  $D_{q \rightarrow l^+ l^-}$  at leading order in  $\alpha_s$ .

negative lepton, and the lepton pair, respectively. The factor  $\Gamma$  is the  $4 \times 1$  Dirac spinor associated with the blob in Fig. 1, from which the virtual quark emerges. If we choose a covariant gauge for QED, the numerator factor in the photon propagator reduces to  $G_{\mu\nu} = g_{\mu\nu}$ . However, if we square the matrix element (4) and integrate over phase space, isolating those terms with the proper scaling behavior at large  $Q_T$ , we will not get the correct fragmentation function. The reason is that in a covariant gauge, there are diagrams that do not have the topology of Fig. 1, but still contribute to the scaling part of the cross section. In order to extract the complete scaling contribution, it is necessary to use a light-cone gauge in which the numerator of the photon propagator has the form

$$G_{\mu\nu} = g_{\mu\nu} - \frac{Q_\mu n_\nu + n_\mu Q_\nu}{n \cdot Q}, \quad (5)$$

where  $n$  is the light-like vector that defines the longitudinal momentum fraction of the  $l^+ l^-$  pair:  $z = n \cdot Q / n \cdot k$ . Note that the  $n_\mu Q_\nu$  term in Eq. (5) does not contribute to the matrix element (4), because the leptons are on-shell. However, the  $Q_\mu n_\nu$  term does contribute, because the decaying quark is virtual. The calculations of the fragmentation functions for  $c \rightarrow \eta_c, J/\psi$  in Ref. [7] involved diagrams with the topology of Fig. 1, except they had a virtual gluon instead of a virtual photon. The authors calculated the diagram using the light-cone gauge propagator for the gluon. In a calculation using the formal definition, the contributions corresponding to the additional terms in the light-cone gauge propagator come from diagrams in which the virtual gluon is emitted by a gluon field from the eikonal factor (2).

If the fragmentation function for a lepton pair is calculated using the formal definition, it is necessary to generalize the QCD eikonal factor in Eq. (2) to include a phase from the photon field. In the fragmentation function for a quark with electric charge  $e_q$ , the eikonal factor becomes

$$\mathcal{E} = \mathcal{P} \exp \left( i g \int d \ln^\mu A_\mu^a T^a + i e_q e \int d \ln^\mu A_\mu \right). \quad (6)$$

At leading order in  $\alpha_s$ , the  $q \rightarrow l^+ l^-$  fragmentation function is given by the square of the sum of the Feynman diagrams in Fig. 2. The circles represent quark operators, and the double lines represent the eikonal factor. A convenient set of Feynman rules for calculating fragmentation functions is given in Ref. [3]. The contributions that correspond to the additional term in the light-cone gauge propagator (5) for the photon are provided by the diagram in Fig. 2b in which the virtual photon is emitted by a photon field from the eikonal factor (6).

In the calculation of the fragmentation function for  $q \rightarrow l^+ l^-$ , the integral over the relative momenta of the lepton pair for fixed total momentum  $Q$  gives the tensor  $-g^{\mu\nu} + Q^\mu Q^\nu / Q^2$ . In Ref. [10], Qiu and Zhang decomposed this tensor into two terms that correspond to the polarization tensors for virtual photons that are transversely and longitudinally polarized with respect to the direction of the momentum of the lepton pair in a particular frame:

$$-g^{\mu\nu} + \frac{Q^\mu Q^\nu}{Q^2} = \left( -g^{\mu\nu} + \frac{n^\mu \bar{n}^\nu + \bar{n}^\mu n^\nu}{2} \right) + \left( \frac{Q^\mu Q^\nu}{Q^2} - \frac{n^\mu \bar{n}^\nu + \bar{n}^\mu n^\nu}{2} \right), \quad (7)$$

where  $\bar{n}$  is a conjugate light-like vector satisfying  $n \cdot \bar{n} = 2$ . The frame chosen in Ref. [10] is the one in which the perpendicular momenta of the decaying quark and the lepton pair are  $k'_\perp = -Q_\perp / z$  and  $Q'_\perp = 0$ . In this frame, the second tensor on the right side of Eq. (7) can be expressed as  $\epsilon_L^\mu \epsilon_L^\nu$ , where the polarization vector  $\epsilon_L^\mu$  is proportional to  $(n \cdot Q)^2 \bar{n}^\mu - Q^2 n^\mu$ . This choice for the decomposition into transverse and longitudinal contributions is rather artificial, but we will adopt it in order to compare our results with those of Ref. [10].

We proceed to calculate the fragmentation functions from the square of the sum of the two Feynman diagrams in Fig. 2. We use dimensional regularization in  $4 - 2\epsilon$  space-time dimensions to regularize ultraviolet divergences. The fragmentation function can be expressed as an integral over the invariant mass  $Q^2$  of the lepton pair:

$$D_{q \rightarrow l^+ l^-}^{(0)}(z) = \frac{C e_q^2 \alpha^2}{6 \pi^2} \int \frac{dQ^2}{Q^2} \left( \frac{4\pi\mu^2}{Q^2} \right)^{2\epsilon} \times \left\{ \frac{2(1-z) + (1-\epsilon)z^2}{z^2} [z f_1(z) - f_2(z)] + \frac{2(1-z)}{z^2} f_2(z) \right\}, \quad (8)$$

where  $C$  is a constant that reduces to 1 as  $\epsilon \rightarrow 0$ :

$$C = \frac{(1-\epsilon) 2^{2\epsilon} \Gamma\left(\frac{1}{2}\right)}{(1-2\epsilon) \left(1 - \frac{2}{3}\epsilon\right) \Gamma\left(\frac{1}{2} - \epsilon\right)}. \quad (9)$$

In Eq. (8), there is an implicit lower limit on  $Q^2$  that is large enough that a perturbative calculation of the QCD corrections would be reliable. The two terms in the integrand correspond to transversely and longitudinally polarized photons, respectively. The functions  $f_n(z)$  can be expressed as integrals over the perpendicular components  $\mathbf{Q}_\perp$  of the momentum of the lepton pair. Equivalently, they can be expressed as integrals over the invariant mass  $s$  of the decaying quark, which is related to  $\mathbf{Q}_\perp$  by light-cone energy conservation:

$$s = \frac{Q^2 + Q_\perp^2}{z} + \frac{Q_\perp^2}{1-z}. \quad (10)$$

This implies the lower limit  $s > Q^2/z$ . The functions  $f_n(z)$  in Eq. (8) are given by

$$f_n(z) = \frac{1}{\Gamma(1-\epsilon)} [z(1-z)]^{-\epsilon} \int_{1/z}^{\infty} dy \frac{(y-1/z)^{-\epsilon}}{y^n}, \quad (11)$$

where  $y = s/Q^2$ . They can be evaluated analytically:

$$f_n(z) = \frac{\Gamma(n-1+\epsilon)}{\Gamma(n)} z^{n-1} (1-z)^{-\epsilon}. \quad (12)$$

The function  $f_1(z)$  in Eq. (8) has a pole in  $\epsilon$ . This ultraviolet divergence is cancelled by the renormalization of the composite operator in the definition of the fragmentation function. The renormalized fragmentation function in the modified minimal subtraction ( $\overline{\text{MS}}$ ) scheme is

$$D_{q \rightarrow l^+ l^-}^{\overline{\text{MS}}}(z, \mu) = D_{q \rightarrow l^+ l^-}^{(0)}(z) - \frac{(4\pi e^{-\gamma})^\epsilon e_q^2 \alpha}{\epsilon 2\pi} \times \int_z^1 \frac{dy}{y} P_{q \rightarrow \gamma}(z/y) D_{\gamma \rightarrow l^+ l^-}(y), \quad (13)$$

where  $P_{q \rightarrow \gamma}$  is the splitting function

$$P_{q \rightarrow \gamma} = \frac{1 + (1-z)^2}{z}, \quad (14)$$

and  $D_{\gamma \rightarrow l^+ l^-}$  is the fragmentation function for a photon to decay into a lepton pair:

$$D_{\gamma \rightarrow l^+ l^-}(z) = \frac{C \alpha}{3\pi} \int \frac{dQ^2}{Q^2} \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon \delta(1-z). \quad (15)$$

The coefficient  $C$  is given in Eq. (9). Our final result for the  $\overline{\text{MS}}$  fragmentation function is obtained by taking the limit  $\epsilon \rightarrow 0$  in Eq. (13):

$$D_{q \rightarrow l^+ l^-}^{\overline{\text{MS}}}(z, \mu) = \frac{e_q^2 \alpha^2}{6 \pi^2} \int \frac{dQ^2}{Q^2} \left\{ \left[ \frac{1 + (1-z)^2}{z} \times \left( \ln \frac{\mu^2}{(1-z)Q^2} - 1 \right) - z \right] + \frac{2(1-z)}{z} \right\}. \quad (16)$$

The two terms in the integrand correspond to transversely and longitudinally polarized photons, respectively.

In Ref. [10], Qiu and Zhang defined their fragmentation functions by imposing the constraint  $s < \mu_F^2$  on the invariant mass of the decaying quark, which sets the upper limit  $y < \mu_F^2/Q^2$  on the integral in Eq. (11). This eliminates ultraviolet divergences, so we can set  $\epsilon = 0$ . The results for the integrals are then

$$f_0(z) = \frac{1}{z} \left( \frac{z\mu_F^2}{Q^2} - 1 \right) \theta(\mu_F^2 - Q^2/z), \quad (17)$$

$$f_1(z) = \ln \frac{z\mu_F^2}{Q^2} \theta(\mu_F^2 - Q^2/z), \quad (18)$$

$$f_2(z) = z \left( 1 - \frac{Q^2}{z\mu_F^2} \right) \theta(\mu_F^2 - Q^2/z). \quad (19)$$

The resulting expression for the fragmentation function is

$$D_{q \rightarrow l^+ l^-}(z, \mu_F) = \frac{e_q^2 \alpha^2}{6\pi^2} \int \frac{dQ^2}{Q^2} \theta(\mu_F^2 - Q^2/z) \times \left\{ \frac{1 + (1-z)^2}{z} \left( \ln \frac{z\mu_F^2}{Q^2} - 1 + \frac{Q^2}{z\mu_F^2} \right) + \frac{2(1-z)}{z} \left( 1 - \frac{Q^2}{z\mu_F^2} \right) \right\}. \quad (20)$$

This result has been confirmed by Qiu and Zhang [10]. If we take the formal limit  $\mu_F^2 \gg Q^2/z$  in Eq. (20), the  $\theta$  function becomes 1 and the resulting expression differs from Eq. (16) only in the argument of the logarithm and in the coefficient of the  $z$  term. The arguments of the logarithm are the same if we make the identification  $\mu_F^2 = \mu^2/(z(1-z))$ . This can be understood by examining the expression (10) for the invariant mass of the decaying virtual quark. It suggests that the scale  $\mu$  of dimensional regularization should be identified not with a cutoff on the invariant mass of the virtual quark, but with a cutoff on the perpendicular momentum  $Q_\perp$  of the lepton pair. The difference between the coefficients of the  $z$  term comes from the  $\epsilon z f_1(z)$  term in Eq. (8), which reduces to  $z$  in the limit  $\epsilon \rightarrow 0$ . On the other hand, if we impose an invariant-mass cutoff  $\mu_F$  and set  $\epsilon = 0$ , this term vanishes. Taking the limit  $\mu_F \gg Q$  with  $z$  fixed in Eq. (20) corresponds simply to an alternative ultraviolet cutoff. The fragmentation function should then differ from Eq. (16) by a finite renormalization of the composite operator in the definition of the fragmentation function. This finite renormalization corresponds to adding the term  $-\epsilon z$  to the splitting function  $P_{q \rightarrow \gamma}$  in Eq. (13).

One advantage of the fragmentation function (20) defined by an upper limit on the invariant mass is that it builds in threshold effects and the constraints of energy conservation associated with the decay of a virtual quark with invariant mass  $\mu_F$ . This might be useful for quantitative applications of the fragmentation function. In Fig. 3, we compare the differential fragmentation functions  $Q^2 dD(z)/dQ^2$  corresponding to Eq. (20) with invariant mass cutoff  $\mu_F$  and Eq. (16) with renormalization scale  $\mu^2 = z(1-z)\mu_F^2$ . We set  $e_q = +\frac{2}{3}$  and  $\alpha = \frac{1}{137}$ . We choose  $Q = 5$  GeV and consider 2 values of  $\mu_F$ :  $\mu_F = 10$  GeV in Fig. 3a and  $\mu_F = 50$  GeV in Fig. 3b. The fragmentation function defined by  $s < \mu_F^2$  is 0 below the threshold at  $z = Q^2/\mu_F^2$  and is positive for larger values of  $z$ . The fragmentation function defined by dimensional regularization has unphysical negative values over

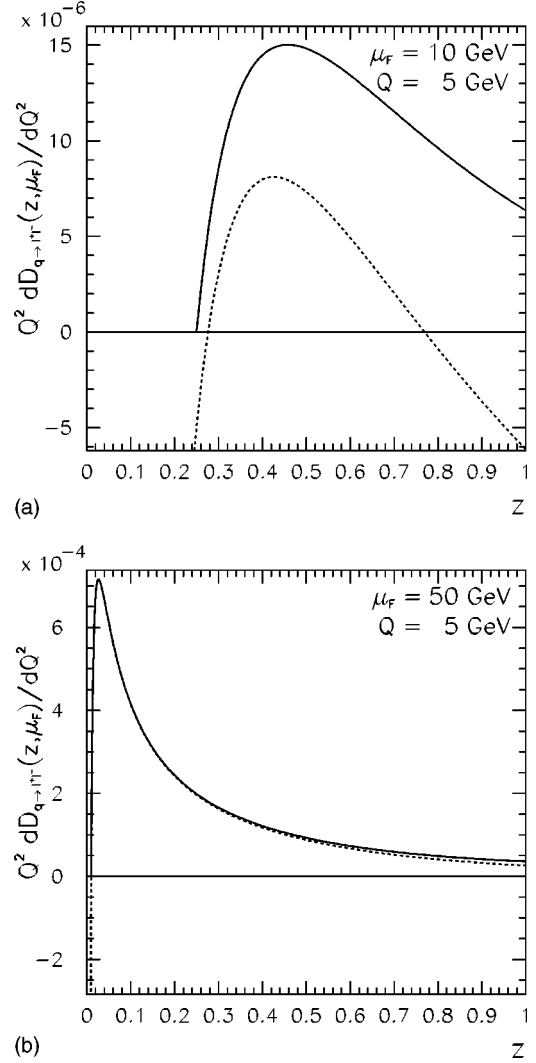


FIG. 3. The differential fragmentation function  $Q^2 dD(z)/dQ^2$  for  $q \rightarrow l^+ l^-$  for a lepton pair with invariant mass  $Q = 5$  GeV as a function of  $z$  for (a)  $\mu_F = 10$  GeV and (b)  $\mu_F = 50$  GeV. The solid curves are the fragmentation functions (20) defined by the invariant-mass cutoff  $\mu_F$ . The dashed curves are the fragmentation functions (16) defined by dimensional regularization with renormalization scale  $\mu^2 = z(1-z)\mu_F^2$ .

part of its range as can be seen in Fig. 3. As  $z \rightarrow 1$ , it remains positive only if  $\mu_F > \exp(1)Q$ . As  $z \rightarrow 0$ , it is negative for all  $\mu_F$ , diverging like  $\ln(z\mu_F^2/Q^2)/z$ . For large enough values of  $\mu_F/Q$  as in Fig. 3b, the two fragmentation functions look similar except that one vanishes for  $z < Q^2/\mu_F^2$  and the other becomes negative in that region. If we keep  $\mu$  fixed in Eq. (16) instead of  $\mu_F$ , the fragmentation function still exhibits unphysical behavior. It is positive-definite if  $\mu > Q$ , but it diverges like  $\ln(\mu^2/Q^2)/z$  as  $z \rightarrow 0$  and like  $\ln(\mu^2/[(1-z)Q^2])$  as  $z \rightarrow 1$ . We conclude that the fragmentation function (16) defined by dimensional regularization is of little practical use. It is essential to take into account threshold effects in some way, such as by imposing an upper limit on the invariant mass as in Eq. (20).

If the QED phase in the eikonal factor (6) is omitted in the formal definition of the fragmentation function,  $D_{q \rightarrow l^+ l^-}^{(0)}$  is given by the square of the Feynman diagram in Fig. 2a. The resulting expression is not gauge invariant, but it is independent of the gauge parameter for covariant gauges. Using dimensional regularization, we obtain

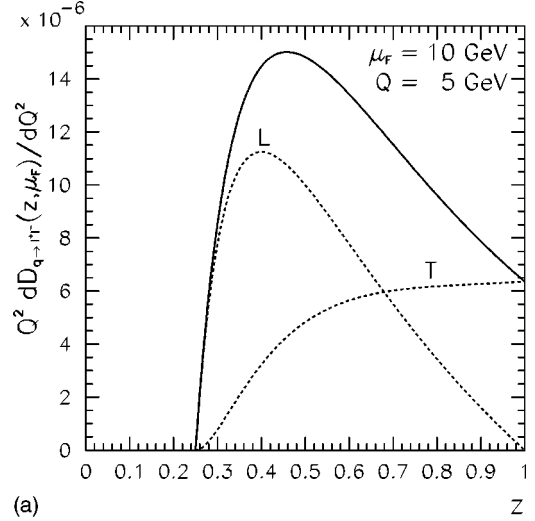
$$D_{q \rightarrow l^+ l^-}^{(0)}(z) = \frac{C e_q^2 \alpha^2}{6 \pi^2} \int \frac{dQ^2}{Q^2} \left( \frac{4 \pi \mu^2}{Q^2} \right)^{2\epsilon} \times \left\{ \frac{2(1-z) + (1-\epsilon)z^2}{z^2} [z f_1(z) - f_2(z)] + \frac{1-z}{2z^2} [z^2 f_0(z) - 4z f_1(z) + 4f_2(z)] \right\}. \quad (21)$$

The two terms in the integrand correspond to transversely and longitudinally polarized photons, respectively. The transverse term is identical to that in Eq. (16). This follows from the fact that the Feynman rule for the emission of a virtual photon from the eikonal line in Fig. 2b is proportional to  $n^\mu$ , which is orthogonal to the transverse tensor in Eq. (7). Thus the omission of the diagram in Fig. 2b can only affect the longitudinal term. With dimensional regularization, the function  $f_0(z)$  vanishes, as is evident from Eq. (12). The function  $f_1(z)$  has a pole in  $\epsilon$ . In the transverse term, the pole can be removed by the renormalization (13). However, there is also a pole in the longitudinal term that is not removed by renormalization of the composite operator. This failure of renormalization is the signal that the definition of the fragmentation function that omits the QED phase in the eikonal factor is inconsistent.

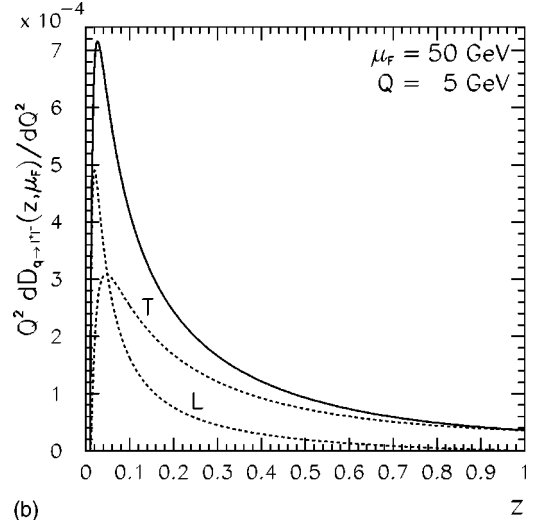
In Fig. 4, we compare the transverse and longitudinal contributions to the fragmentation function (20) calculated using an invariant-mass cutoff. We choose  $Q=5$  GeV and consider 2 values of  $\mu_F$ :  $\mu_F=10$  GeV in Fig. 4a and  $\mu_F=50$  GeV in Fig. 4b. The dashed curves labeled  $T$  and  $L$  are the transverse and longitudinal contributions given by the two terms in Eq. (20). Their sum is the solid curve. The longitudinal polarization dominates just above the threshold at  $z=Q^2/\mu_F^2$ , because the longitudinal term increases linearly in  $z-Q^2/\mu_F^2$  while the transverse term increases quadratically. The transverse polarization dominates at large  $z$ , because the longitudinal term vanishes at  $z=1$ .

We can define a longitudinal polarization fraction  $\xi$  by dividing the longitudinal fragmentation probability by the total fragmentation probability for lepton pairs of invariant mass  $Q^2$ . The fragmentation probability is proportional to the integral over  $z$  of  $dD(z)/dQ^2$ . The longitudinal polarization fraction for  $Q=5$  GeV decreases from  $\xi=0.56$  at  $\mu_F=10$  GeV to 0.34 at  $\mu_F=50$  GeV and to 0.23 at  $\mu_F=250$  GeV. This is in accord with the intuition that the virtual photon should behave more and more like a real photon as  $\mu_F$  increases.

The longitudinal polarization fraction defined by the ratio of fragmentation probabilities decreases rather slowly with  $\mu_F$ . However a more relevant measure of the polarization is



(a)



(b)

FIG. 4. The differential fragmentation function  $Q^2 dD(z)/dQ^2$  for  $q \rightarrow l^+ l^-$  for a lepton pair with invariant mass  $Q=5$  GeV as a function of  $z$  for the invariant-mass cutoffs (a)  $\mu_F=10$  GeV and (b)  $\mu_F=50$  GeV. The solid curve is the total fragmentation function. The dashed curves labeled  $T$  and  $L$  are the contributions from transverse and longitudinal virtual photons.

the ratio of the second moments of the fragmentation functions. The reason is that the hard-scattering cross section  $d\hat{\sigma}$  scales like  $1/(Q_T/z)^2$ , and this weights the fragmentation function by  $z^2$ . The longitudinal polarization fraction for  $Q=5$  GeV defined by the ratio of the second moments decreases from  $\xi=0.43$  at  $\mu_F=10$  GeV to 0.15 at  $\mu_F=50$  GeV and to 0.08 at  $\mu_F=250$  GeV.

An important observable in lepton pair production is the angular distribution of the momentum of one of the leptons in the rest frame of the lepton pair. The angular distribution is proportional to  $1 + \alpha \cos^2 \theta$ , where  $\theta$  is the angle between the momentum  $\mathbf{q}_1$  of the negative lepton and some quantization axis. The polarization variable  $\alpha$  is related to the fraction  $\xi$  of lepton pairs that come from longitudinally polarized virtual photons by  $\alpha = (1 - 3\xi)/(1 + \xi)$ . The fraction  $\xi$  depends on the choice of quantization axis. The choice adopted

in Ref. [10] is rather artificial, because it requires specifying the transverse momentum of the fragmenting quark which is not easily observed. In the case of a hadron collider, a more physical choice for the quantization axis is the direction of the momentum  $\mathbf{Q}$  of the lepton pair in the rest frame of the colliding hadrons. In this case, the longitudinal polarization vector  $\epsilon_L^\mu$  is a linear combination of  $Q$  and the total momentum  $K$  of the colliding hadrons:

$$\epsilon_L^\mu = \frac{Q^2 K^\mu - (K \cdot Q) Q^\mu}{[(K \cdot Q)^2 - K^2 Q^2]^{1/2} [Q^2]^{1/2}}. \quad (22)$$

The contribution to the fragmentation function from longitudinally polarized virtual photons can be obtained by replacing the lepton tensor  $-g^{\mu\nu} + Q^\mu Q^\nu / Q^2$  by  $\epsilon_L^\mu \epsilon_L^\nu$ . The resulting expression for the fragmentation function depends explicitly on  $k_+ = k \cdot n$ :

$$D_{q \rightarrow (l^+ l^-)_L}(z, \mu_F) = \frac{e_q^2 \alpha^2}{6\pi^2} \int \frac{dQ^2}{Q^2} \frac{2(1-z)}{z^2} \times f_2(z, \mu_F/Q, Q/k_+), \quad (23)$$

where  $f_2$  is a function of  $z$ ,  $\mu_F/Q$ , and  $Q/k_+$ :

$$f_2(z, \mu_F/Q, Q/k_+) = \int_{1/z}^{\mu_F^2/Q^2} \frac{dy}{y^2} \frac{[z - y(Q/k_+)^2]^2}{[z + (1+y-yz)(Q/k_+)^2]^2 - 4(Q/k_+)^2}. \quad (24)$$

In the limit  $k_+ \gg Q$ , this reduces to Eq. (19). The complete fragmentation function summed over polarizations is independent of  $Q/k_+$  and is given by Eq. (20):

$$D_{q \rightarrow l^+ l^-}(z, \mu_F) = \frac{e_q^2 \alpha^2}{6\pi^2} \int \frac{dQ^2}{Q^2} \theta(\mu_F^2 - Q^2/z) \times \left\{ \frac{1 + (1-z)^2}{z} \ln \frac{z\mu_F^2}{Q^2} - z \left( 1 - \frac{Q^2}{z\mu_F^2} \right) \right\}. \quad (25)$$

In the frame defined by  $\mathbf{K}=0$ ,  $k_+$  is the sum of the energy and momentum of an on-shell quark produced by some hard scattering. A reasonable choice for  $\mu_F$  is the transverse momentum of that quark, give or take a factor of 2.

The NRQCD factorization approach predicts that the  $1^{--}$  quarkonium states should become increasingly transversely polarized as their transverse momentum  $P_T$  increases [11,12]. Quantitative predictions of the polarization of the  $\psi(2S)$  [13,14],  $J/\psi$  [15], and  $\Upsilon(2S)$  [16] indicate that the increase in the polarization should set in at values of  $P_T$  that are accessible at the Tevatron. The present data on the polarization of  $J/\psi$  and  $\psi(2S)$  from the CDF Collaboration [17] seem to indicate a decrease in the transverse polarization at large  $P_T$ , although in both cases the discrepancy with the prediction is significant only in the largest  $P_T$  bin. The argument that the transverse polarization of  $J/\psi$  or  $\psi(2S)$  should increase with  $P_T$  is completely analogous to the corresponding argument for lepton pairs, except that it involves a virtual gluon instead of a virtual photon. There are many effects that could dilute the transverse polarization or delay the onset of the predicted increase. However no plausible mechanisms have been identified that could make it decrease with  $P_T$ . We expect that more accurate measurements from Run II of the Tevatron will reveal the increase in transverse polarization predicted by NRQCD.

In conclusion, we have calculated the fragmentation function for a light quark to decay into a lepton pair to leading order in  $\alpha_s$ . For renormalizability and for QED gauge invariance, it is essential to include a QED phase in the eikonal factor in the formal definition of the fragmentation function. Berger, Gordon, and Klasen [18] have shown that the distribution of the transverse momentum  $Q_T$  of lepton pairs in hadron collisions is dominated by parton processes initiated by gluons if  $Q_T > Q/2$ . The  $Q_T$  distribution can therefore provide useful constraints on the parton distribution for gluons. Our fragmentation function for  $q \rightarrow l^+ l^-$  may be useful for calculating the  $Q_T$  distribution in the limit  $Q_T \gg Q$ .

J.L. thanks the IPPP at University of Durham and the High Energy Theory Group at Ohio State University for their hospitality while this work was being carried out. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-91-ER40690 and by the KOSEF and the DFG through the German-Korean scientific exchange program DFG-446-KOR-113/137/0-1. We thank J.-W. Qiu for pointing out an error in the normalization of our fragmentation functions in a previous version of this paper.

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