How far can the SO(10) two Higgs model describe the observed neutrino masses and mixings?

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Can the SO(10) model with one **10** and one **126** Higgs scalar give the observed masses and mixings of quarks and leptons without any other additional Higgs scalars? Recently, at least for quarks and charged leptons, it has been demonstrated that this is possible. However, for neutrinos, it is usually said that the parameters that are determined from the quark and charged lepton masses cannot give the observed large neutrino mixings. This problem is systematically investigated, and it is concluded that the present data cannot exclude the SO(10) model with two Higgs scalars although this model cannot give the best fit values of the data.

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I. INTRODUCTION

The SO(10) ground unified theory (GUT) model seems to us the most attractive model when we take the unification of the quarks and leptons into consideration. However, in order to reproduce the observed quark and lepton masses and mixings, usually many Higgs scalars are brought into the model. So it is a very crucial problem to know the minimum number of Higgs scalars that can give the observed fermion mass spectra and mixings. A model with one Higgs scalar is obviously ruled out for the description of realistic quark and lepton mass spectra. Two Higgs models were initially discussed by Mohapatra and co-workers [1].

In a previous paper [2], we discussed two Higgs scalars, the {**10** and **126**} and {**10** and **120**} cases, and showed that they reproduce quark-lepton mass matrices, unlike the conventional results [3]. One of the new points of our approach is that we adopt general forms of Yukawa couplings allowable in the SO(10) framework. However, we did not consider the neutrino mass matrix there, since it may incorporate additional assumptions such as like the seesaw mechanism, etc.

One of the merits of the SO(10) model is that it includes right-handed Majorana neutrinos in the fundamental representation and naturally leads to the seesaw mechanism. Also, some papers claimed that the two Higgs model {**10** and $\mathbf{126}+\overline{\mathbf{126}}$} does not reproduce the large mixing angle of the atmospheric neutrino deficit [4]. So in this paper we apply the method developed in [2] to the neutrino mass matrix, fitting the other parameters of the quark-lepton mass matrices. Our model has the two Higgs scalars {**10** and **126**}, both of which are symmetric with respect to the family index. Therefore those mass matrices are symmetric whose entries are complex valued. We do not adopt the other choice {**10** and **120**}, because it does not involve the mass term of the right-handed Majorana neutrinos which are the ingredients of the seesaw mechanism.

We begin with a short review of our previous work [2]. In the case where two Higgs scalars ϕ_{10} and ϕ_{126} are incorporated in the SO(10) model, the mass matrices of quarks and charged leptons have the following forms:

$$M_u=c_0M_0+c_1M_1,\quad M_d=M_0+M_1,\quad M_e=M_0-3M_1. \tag{1.1}$$

Here M_0 and M_1 are the mass matrices generated by the Higgs scalars ϕ_{10} and ϕ_{126}, respectively. c_0 and c_1 are the ratios of the vacuum expectation values

$$c_0=v_0^u/v_0^d=\langle\phi_{10}^{u0}\rangle/\langle\phi_{10}^{d0}\rangle,$$
$$c_1=v_1^u/v_1^d=\langle\phi_{126}^{u0}\rangle/\langle\phi_{126}^{d0}\rangle, \tag{1.2}$$

and ϕ^u and ϕ^d denote the Higgs scalar components that couple with up and down quarks, respectively. Eliminating M_0 and M_1 from Eq. (1.1), we obtain

$$M_e=c_dM_d+c_uM_u, \tag{1.3}$$

where

$$c_d=-\frac{3c_0+c_1}{c_0-c_1},\quad c_u=\frac{4}{c_0-c_1}. \tag{1.4}$$

Since M_u, M_d, and M_e are complex symmetric matrices, they are diagonalized by the unitary matrices U_u, U_d, and U_e, respectively, as

$$U_u^T M_u U_u = D_u, \quad U_d^T M_d U_d = D_d, \quad U_e^T M_e U_e = D_e, \tag{1.5}$$

where D_u, D_d, and D_e are diagonal matrices given by

$$D_u \equiv \mathrm{diag}(m_u, m_c, m_t), \quad D_d \equiv \mathrm{diag}(m_d, m_s, m_b).$$

$$D_e \equiv \mathrm{diag}(m_e, m_\mu, m_\tau), \tag{1.6}$$

Since the Cabibbo-Kobayashi-Maskawa (CKM) matrix V_q is given by

$$V_q = U_u^T U_d^*, \tag{1.7}$$

the relation (1.3) is rewritten as follows:

$$(U_e^\dagger U_u)^T D_e (U_e^\dagger U_u) = c_d V_q D_d V_q^T + c_u D_u. \tag{1.8}$$

Therefore, we obtain the three independent equations:

$$\mathrm{Tr}\, D_e D_e^\dagger = |c_d|^2 \mathrm{Tr}[(V_q D_d V_q^T + \kappa D_u)(V_q D_d V_q^T + \kappa D_u)^\dagger], \tag{1.9}$$

$$\mathrm{Tr}(D_e D_e^\dagger)^2 = |c_d|^4 \mathrm{Tr}\{[(V_q D_d V_q^T + \kappa D_u) \times (V_q D_d V_q^T + \kappa D_u)^\dagger]^2\}, \tag{1.10}$$

$$\det D_e D_e^\dagger = |c_d|^6 \det[(V_q D_d V_q^T + \kappa D_u)(V_q D_d V_q^T + \kappa D_u)^\dagger], \tag{1.11}$$

where $\kappa = c_u / c_d$. By eliminating the parameter c_d, we have two equations for the parameter κ:

$$\frac{(m_e^2 + m_\mu^2 + m_\tau^2)^3}{m_e^2 m_\mu^2 m_\tau^2} = \frac{(1.9)^3}{(1.11)}, \tag{1.12}$$

$$\frac{(m_e^2 + m_\mu^2 + m_\tau^2)^2}{2(m_e^2 m_\mu^2 + m_\mu^2 m_\tau^2 + m_\tau^2 m_e^2)} = \frac{(1.9)^2}{(1.9)^2 - (1.10)}, \tag{1.13}$$

where $(1.9)^3$, for instance, means the right-hand side of Eq. (1.9) to the third power. Let us denote the parameter values of κ evaluated from Eqs. (1.12) and (1.13) as κ_A and κ_B, respectively. If κ_A and κ_B coincide with each other, then we have the possibility that the SO(10) GUT model can reproduce the observed quark and charged lepton mass spectra. If κ_A and κ_B do not coincide, the SO(10) model with one **10** and one **126** Higgs scalars is ruled out, and we must bring more Higgs scalar into the model.

Note that Eqs. (1.9)–(1.11) can constrain only the absolute value of $c_d \equiv |c_d| e^{i\sigma}$. The argument of the parameter c_d can be determined by taking the neutrino sector into consideration. In our previous paper [2], we found that only for the signs of the masses

$$(m_t, m_c, m_u; m_b, m_s, m_d; m_\tau, m_\mu, m_e) = (+,-,+;+,-,-;+,\pm,\pm) \quad \text{(a)} \tag{1.14}$$

and

$$= (+,-,-;+,-,-;+,\pm,\pm) \quad \text{(b)} \tag{1.15}$$

are there solutions that give $\kappa_A = \kappa_B$, and the corresponding parameter values $(|c_d|, \kappa)$ are, for (a),

$$(|c_d|, \kappa) = (3.15698, -0.019296 e^{2.64172° i}), \tag{1.16}$$

or

$$(3.03577, -0.019398 e^{2.99570° i}) \tag{1.17}$$

and, for (b),

$$(|c_d|, \kappa) = (3.13307, -0.019314 e^{2.71464° i}) \tag{1.18}$$

or

$$(3.00558, -0.019420 e^{3.10014° i}). \tag{1.19}$$

Here $m_s = 76.3$ MeV for input $\theta_{23} = 0.0420$ rad and $\delta = 60°$ at $\mu = m_Z$ (m_Z is the neutral weak boson mass). For the relation between the values at $\mu = m_Z$ and those at $\mu = \Lambda_X$ (Λ_X is a unification scale), see Ref. [2]. The purpose of the present paper is to investigate whether or not these solutions can give reasonable values for observed neutrino masses and mixings.

II. THE NUMBER OF PARAMETERS IN THE SO(10) MODEL WITH TWO HIGGS SCALARS

As we discussed in the previous section, among the four degrees of freedom of the complex $\{c_0, c_1\}$ or $\{c_d, \kappa\}$, we have been able to fix three of them, κ and $|c_d|$. This is not accidental. Let us discuss the situation in detail in the SO(10) two Higgs model.

In the previous paper [2], by using the relation (1.8), we investigated whether or not there is a set of parameters that can give the 13 observable quantities D_e, D_u, D_d, and V_q. We can rewrite Eq. (1.8) as

$$A_e^T D_e A_e = c_d (V_q D_d V_q^T + \kappa D_u), \tag{2.1}$$

where

$$A_e = U_e^\dagger U_u, \tag{2.2}$$

$$c_d = |c_d| e^{i\sigma}. \tag{2.3}$$

The quantities D_e, D_u, D_d, and V_q are inputs, and the quantities $|c_d|$, κ, and A_e are the parameters that should be fixed from these observed quantities. In general, an $n \times n$ unitary matrix for n generations has n^2 parameters. Therefore, the number of parameters is

$$N(\mathrm{pmt}) = N(A_e) + N(c_d) + N(\kappa) = n^2 + 2 + 2. \tag{2.4}$$

On the other hand, the number of equations is

$$N(\mathrm{eqs}) = n(n+1), \tag{2.5}$$

because Eq. (2.1) is symmetric. Therefore, the number of unfixed parameters is given by

$$N_{\mathrm{free}} = N(\mathrm{pmt}) - N(\mathrm{eqs}) = 4 - n = 1, \tag{2.6}$$

for $n=3$, i.e., the 13 observed quantities fix the parameters $|c_d|$, κ, and A_e, but one parameter σ remains unknown.

In the present paper, we will try to predict the neutrino masses

$$D_\nu = U_\nu^T M_\nu U_\nu \tag{2.7}$$

and mixing matrix

$$V_l = U_e^T U_\nu^* \tag{2.8}$$

by using the observed quantities D_e, D_u, D_d, and V_q and the parameter values $|c_d|$, κ, and A_e fixed by Eq. (2.1).

The SO(10) GUT asserts that the Dirac neutrino mass matrix M_D is given by the form

$$M_D = c_0 M_0 - 3c_1 M_1, \tag{2.9}$$

and the Majorana mass matrices of the left-handed and right-handed neutrinos M_L and M_R are proportional to the matrix M_1:

$$M_L = c_L M_1, \quad M_R = c_R M_1, \tag{2.10}$$

where M_0 and M_1 are related to the quark and charged lepton mass matrices M_u, M_d, and M_e as follows:

$$M_0 = \frac{3M_d + M_e}{4}, \tag{2.11}$$

$$M_1 = \frac{M_d - M_e}{4}. \tag{2.12}$$

Then the neutrino mass matrix derived from the seesaw mechanism becomes

$$\begin{aligned} M_\nu &= M_L - M_D M_R^{-1} M_D^T \\ &= c_L M_1 - c_R^{-1}(c_0 M_0 - 3c_1 M_1) \\ &\quad \times M_1^{-1}(c_0 M_0 - 3c_1 M_1)^T. \end{aligned} \tag{2.13}$$

In the present paper we adopt $c_L=0$. Also we may ignore the phase of c_R, which does not affect the observed values. Therefore, we can rewrite Eq. (2.13) as

$$|c_R| A_\nu^T D_\nu A_\nu = \tilde{M}_D \tilde{M}_1^{-1} \tilde{M}_D^T, \tag{2.14}$$

similarly to Eq. (2.1), where

$$\tilde{M}_D = c_0 \tilde{M}_0 - 3c_1 \tilde{M}_1, \tag{2.15}$$

$$\tilde{M}_0 = \frac{1}{4}(3\tilde{M}_d + \tilde{M}_e), \tag{2.16}$$

$$\tilde{M}_1 = \frac{1}{4}(\tilde{M}_d - \tilde{M}_e), \tag{2.17}$$

with

$$\tilde{M}_d = U_u^T M_d U_u = V_q D_d V_q^T, \tag{2.18}$$

$$\begin{aligned} \tilde{M}_e &= U_u^T M_e U_u = A_e^T D_e A_e \\ &= c_d (V_q D_d V_q^T + \kappa D_u). \end{aligned} \tag{2.19}$$

In contrast to the previous work, the quantities D_ν and V_l are unknown parameters at the present stage. Since

$$V_l = A_e^* A_\nu^T, \tag{2.20}$$

and A_e is fixed from Eq. (2.1), the number of unknown parameters in Eq. (2.20) is

$$N(A_\nu) = N(V_l) = n^2. \tag{2.21}$$

Of course, the unknown parameters in A_ν contain n unphysical parameters which cannot be determined because of the rephasing in the fields e_L. Therefore, the number of unknown parameters is

$$\begin{aligned} N(\mathrm{pmt}) &= N(D_\nu) + N(A_\nu) + N(|c_R|) + N(\sigma) \\ &= n + n^2 + 1 + 1 = n^2 + n + 2 \end{aligned} \tag{2.22}$$

and from the number of equations $N(\mathrm{eqs}) = n(n+1)$ in Eq. (2.14) we obtain the number of unfixed parameters as

$$\begin{aligned} N_{\mathrm{free}} &= N(\mathrm{pmt}) - N(\mathrm{eqs}) \\ &= (n^2 + n + 2) - n(n+1) = 2. \end{aligned} \tag{2.23}$$

This means that we can predict the neutrino masses and mixing completely if we have the two values $|c_R|$ and σ. The numerical predictions will be investigated in the next section.

III. NUMERICAL RESULTS

Here we discuss the numerical results for the neutrino mass spectrum and neutrino mass matrix. For our example, we use the set in Eq. (1.18). Even if other sets are used, our results are scarcely changed. The allowed values of the neutrino mass square differences and lepton flavor mixing angles depict complicated tracks with moving $\sigma \equiv \arg c_d$ (Fig. 1). This figure shows a general tendency for the lepton flavor mixing angles θ_{12} and θ_{23} to get larger as σ approaches $3\pi/2$. For illustration we take $\sigma = 149\pi/100$; then these values become

$$\begin{gathered} \frac{\Delta m_{12}^2}{\Delta m_{13}^2} = 0.15, \quad \frac{\Delta m_{23}^2}{\Delta m_{13}^2} = 0.85, \\ \sin^2(2\theta_{12}) = 0.76, \quad \sin^2(2\theta_{23}) = 0.75, \\ \sin^2(2\theta_{13}) = 0.16. \end{gathered} \tag{3.1}$$

There still remain some discrepancies between our results and experiments. However our results are much improved in comparison with those of Babu and Mohapatra [1], who obtained $\sin\theta_{12} = 0$–0.3, $\sin\theta_{13} = 0.05$, and $\sin\theta_{23} = 0.12$–0.16.

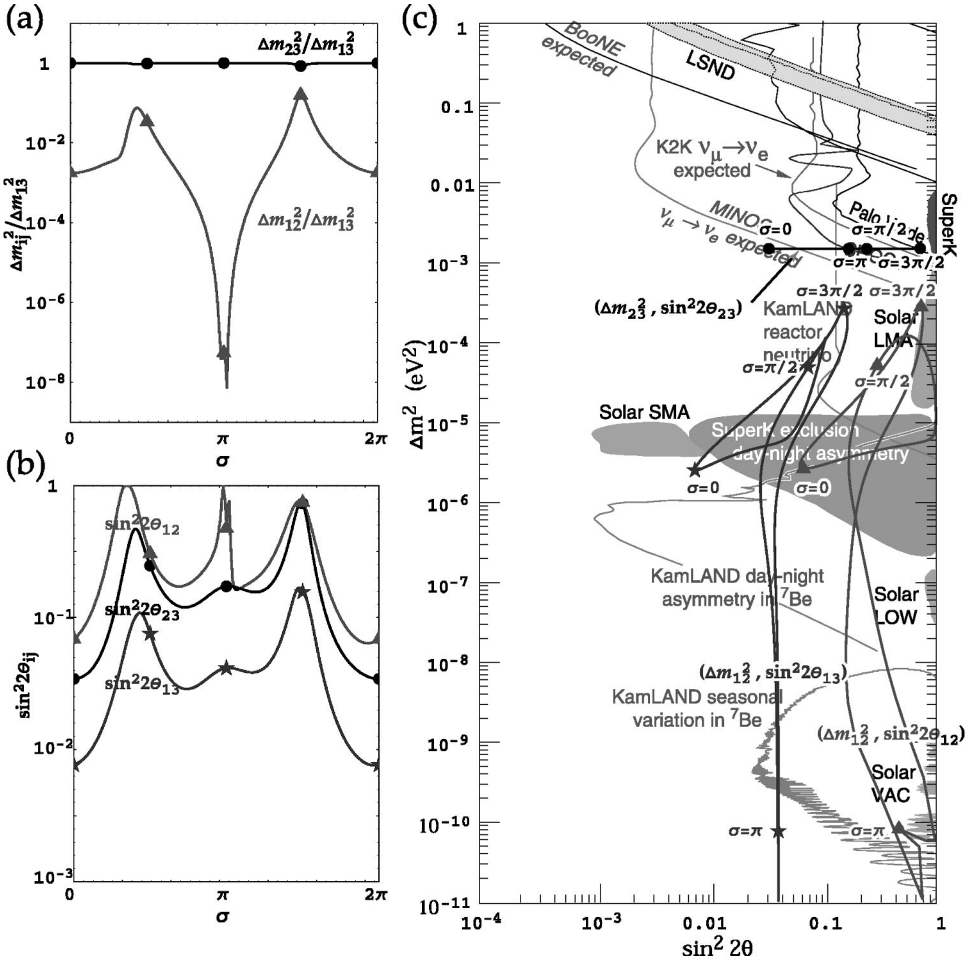

FIG. 1. The relation between our results and the two-flavor oscillation analysis [14] when σ is moved. (a) The circles and triangles indicate the values of $\Delta m_{23}^2/\Delta m_{13}^2$ and $\Delta m_{12}^2/\Delta m_{13}^2$ at every $\pi/2$ of σ. (b) The circles, triangles, and stars indicate the values of $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{12}$, and $\sin^2 2\theta_{13}$ at every $\pi/2$ of σ. (c) The circles, triangles, and stars indicate the values of $(\Delta m_{23}^2, \sin^2 2\theta_{23})$, $(\Delta m_{12}^2, \sin^2 2\theta_{12})$, and $(\Delta m_{12}^2, \sin^2 2\theta_{13})$ at every $\pi/2$ of σ. Here we have set $\Delta m_{23}^2 = 1.5\times 10^{-3}$ eV^2 in every case.

The purpose of the present paper is to study the general tendency of the fitting and not to pursue a precise data fitting, for the data themselves are not definitive, and there are theoretical ambiguities not incorporated in the present data fitting, such as the renormalization group effect.

Using the values of Eq. (3.1), we have

$$|c_d| = 3.16, \tag{3.2}$$

$$c_0 = \frac{1-c_d}{c_u} = 54.84 e^{-20.24° i}, \tag{3.3}$$

$$c_1 = -\frac{3+c_d}{c_u} = 70.54 e^{+41.90° i}. \tag{3.4}$$

In this case, Eqs. (2.11)–(2.13) are rewritten on the basis of $M_u = D_u$ [see Eq. (1.8)] as

$$M_0 = \frac{3V_q D_d V_q^T + c_d(\kappa D_u + V_q D_d V_q^T)}{4}$$
$$= 2.1646\times 10^3 e^{+10.48° i}\begin{pmatrix} -0.00405e^{-57.29° i} & -0.00753e^{-56.24° i} & -0.00533e^{+65.46° i} \\ -0.00753e^{-56.24° i} & -0.02986e^{-51.59° i} & +0.06358e^{-57.64° i} \\ -0.00533e^{+65.46° i} & +0.06358e^{-57.64° i} & +1.00000 \end{pmatrix} \text{ MeV}, \tag{3.5}$$

$$M_1 = \frac{V_q D_d V_q^T - c_d(\kappa D_u + V_q D_d V_q^T)}{4}$$
$$= 9.5127\times 10^2 e^{-24.44° i}\begin{pmatrix} -0.00715e^{+95.23° i} & -0.01333e^{+96.54° i} & +0.00944e^{+38.23° i} \\ -0.01333e^{+96.54° i} & -0.04878e^{+90.73° i} & +0.11247e^{+95.13° i} \\ +0.00944e^{+38.23° i} & +0.11247e^{+95.13° i} & +1.00000 \end{pmatrix} \text{ MeV}, \tag{3.6}$$

$$|c_R|M_\nu=(c_0M_0-3c_1M_1)M_1^{-1}(c_0M_0-3c_1M_1)^T$$
$$=-4.6628\times10^6e^{-52.17°i}\begin{pmatrix} +0.1163e^{+26.89°i} & +0.2165e^{+28.06°i} & -0.1536e^{-30.53°i} \\ +0.2165e^{+28.06°i} & +0.8193e^{+28.00°i} & -1.9276e^{+29.52°i} \\ -0.1536e^{-30.53°i} & -1.9276e^{+29.52°i} & +1.0000 \end{pmatrix}\ \text{MeV}. \quad (3.7)$$

Let us choose the free parameter $|c_R|$ so as to result in small neutrino masses, for example when $|c_R|=3.2\times10^{14}$, we have $\Delta m_{23}^2=1.5\times10^{-3}\ \text{eV}^2$.

Here there arises the question of what makes the two flavor mixing angles large. We need to investigate the mixing matrices U_e and U_ν that diagonalize M_e and M_ν, respectively. These are obtained as

$$U_e=\begin{pmatrix} +0.863 & +0.504e^{+9.46°i} & -0.022e^{+56.66°i} \\ -0.493e^{-9.82°i} & +0.834 & -0.248e^{+16.63°i} \\ -0.110e^{-21.40°i} & +0.223e^{-18.10°i} & +0.969 \end{pmatrix}, \quad (3.8)$$

$$U_\nu=\begin{pmatrix} +0.992 & -0.092e^{-15.94°i} & -0.088e^{+12.86°i} \\ +0.049e^{+76.86°i} & +0.724 & -0.688e^{-16.08°i} \\ +0.117e^{+9.80°i} & +0.683e^{+16.74°i} & +0.721 \end{pmatrix}. \quad (3.9)$$

Here, $|U_{e11}|,|U_{e12}|,|U_{e21}|,|U_{e22}|\gtrsim0.5$ for the charged lepton mass matrix and $|U_{\nu22}|,|U_{\nu23}|,|U_{\nu32}|,|U_{\nu33}|\gtrsim0.7$ for the neutrino mass matrix. Therefore the components of the lepton flavor mixing matrix become $|V_{l11}|$, $|V_{l12}|$, $|V_{l21}|$, $|V_{l22}|$, $|V_{l23}|$, $|V_{l32}|$, $|V_{l33}|\gtrsim0.5$:

$$V_l=\begin{pmatrix} +0.844e^{+2.10°i} & -0.494e^{-9.95°i} & +0.206e^{+23.61°i} \\ +0.527e^{+3.26°i} & +0.696e^{-8.84°i} & -0.488e^{+24.97°i} \\ +0.098e^{-15.78°i} & +0.521e^{-27.43°i} & +0.848e^{+6.32°i} \end{pmatrix}. \quad (3.10)$$

The mixing angle θ_{23} becomes larger and the mixing angle θ_{12} smaller if we take a smaller value of $|m_t|$ or a $|m_d|$, or a larger $|m_c|$, $|m_b|$, or $|m_s|$ than their center values.

As a simple example, a shift of $|m_d|$ and $|m_s|$ causes a change of mixing angles and neutrino mass square differences as depicted in Fig. 2. Figure 2 shows that θ_{23} and θ_{13} can approach the 99% C.L. of superkamiokande (SK) [5] and CHOOZ [6] but data θ_{12} and Δm_{12}^2 are out of the range of 99%–99.9% C.L. of SOLAR [7] and CHOOZ data.

IV. DISCUSSION

Since there are only two basic matrices M_0 and M_1 in this model, the number of parameters in Eqs. (2.1) and (2.14) is

$D_u,\ D_d,\ D_e,\ D_\nu$	$3\times4=12$
$c_d,\ \lvert c_R\rvert,\ \kappa$	$2+1+2=\ 5$
$V_q,\ A_e,\ A_\nu$	$4+9+9=22$
sum	39

(4.1)

and the number of equations is $N(\text{eqs})=12\times2=24$. Therefore the number of free parameters is $N(\text{pmt})-N(\text{eqs})=39-24=15$. On the other hand, the number of physical parameters that can be determined by experiments is

m_u, m_c, m_t	3
m_d, m_s, m_b	3
CKM: $\theta_{12}\ \theta_{23}, \theta_{13}, \delta,$	4
m_e, m_μ, m_τ	3
$m_{\nu_e}, m_{\nu_\mu}\ m_{\nu_\tau}$	3
MNS: $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \beta, \rho$	6
sum	22

(4.2)

where β and ρ are Majorana phases in the Maki-Nakagawa-Sakata (MNS) matrix because there is no rephasing in the neutrino fields ν_L. To sum up the matter, we discuss the consistency test for 22 physical parameters by using only 15 free parameters. The consistency test in the quark sector is good, as shown in our previous paper. In the lepton sector, the test is not so bad when we adopt the MSW large mixing angle solution of the solar neutrino deficit, and this model favors the normal hierarchy of the neutrino mass spectrum.

We can also predict as yet unobserved values such as the average neutrino masses $\langle m\rangle_{\alpha\beta}$ and the Jarlskog parameter

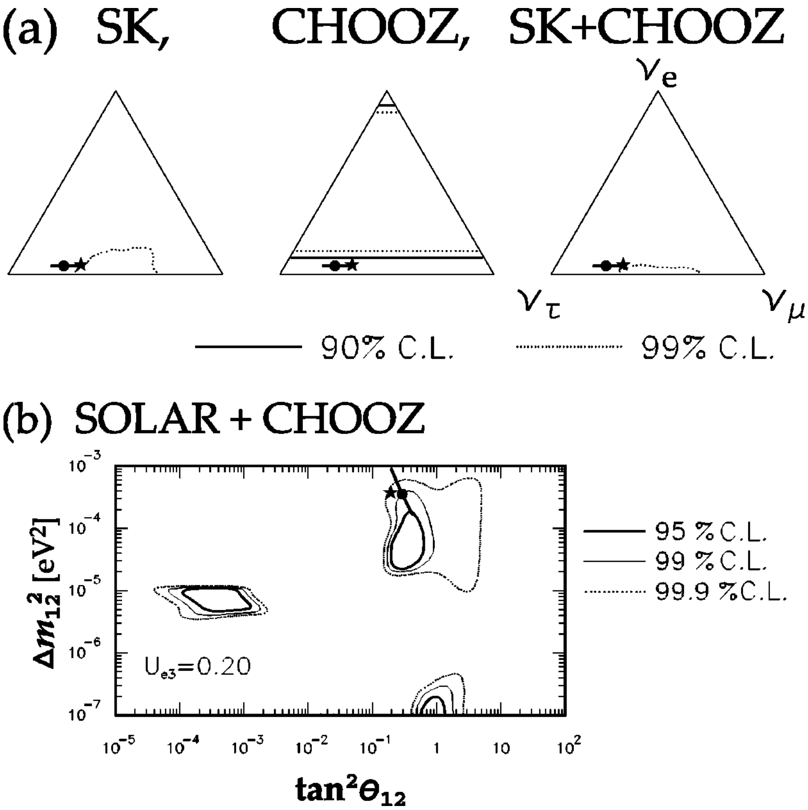

FIG. 2. The relation between 3ν oscillation analyses by Fogli *et al.* [15] and by us for $\Delta m^2_{23} = 1.5\times 10^{-3}$ eV^2 (QVO). (a) For SK+CHOOZ. (b) For SOLAR+CHOOZ. The circles indicate our solutions for Eq. (3.1). The solid line through them is the track as m_d is varied. From the experimental limits, $|m_d|$ moves over the range 4.03–5.29 MeV [16]. $|m_s|$ simultaneously changes over the range 76.3–76.2 MeV so as to satisfy the relations (1.12) and (1.13). If we take a smaller $|m_d|$ with a fixed σ, the solution in (a) moves rightward and the solution in (b) moves left and upward [Table I(i)]. Since the minimum $|m_d|$ for (b) gives a poor fit, we have changed σ from $149\pi/100$ to $146\pi/100$, which is denoted by the star [Table I(ii)]. Thus our result approaches the 99% C.L. of SK+CHOOZ and the 99.9% C.L. of SOLAR+CHOOZ.

in the lepton part. The average neutrino masses appear in reactions where Majorana neutrinos propagate in the intermediate states. They are

$$\langle m_\nu \rangle_{\alpha\beta} \equiv \left| \sum_{j=1}^{3} U_{\alpha j} U_{\beta j} m_j \right|, \tag{4.3}$$

where α and β are (e,μ,τ). They correspond to neutrinoless double beta decay [8] for $\alpha=\beta=e$, μ-e conversion $[\mu^- + (A,Z) \rightarrow e^+ + (A,Z-2)]$ for $\alpha=\mu$, $\beta=e$, and K decay $(K^- \rightarrow \pi^+ \mu^- \mu^-)$ for $\alpha=\beta=\mu$ [9], etc. In Fig. 3 we have depicted the σ dependence of $\langle m_\nu \rangle_{\alpha\beta}/\sqrt{\Delta m^2_{23}}$. In the case of Eq. (3.1), these values become as follows:

TABLE I. Our solution (the second and third lines) from the input parameters (the first line). The result (i) is obtained when we move $|m_d|$ from 4.69 to 4.03 eV. (ii) is the result when we move $|m_d|$ as in (i) and, furthermore, change σ from $149\pi/100$ to $146\pi/100$. These data fittings correspond to Fig. 2.

(i)
$\lvert m_d\rvert = 4.03$ (MeV), $\lvert m_s\rvert = 76.3$ (MeV), $\sigma = 149\pi/100$,
$(\Delta m^2_{12})/(\Delta m^2_{13}) = 0.43$, $(\Delta m^2_{23})/(\Delta m^2_{13}) = 0.57$,
$\sin^2(2\theta_{12}) = 0.52$, $\sin^2(2\theta_{23}) = 0.91$, $\sin^2(2\theta_{13}) = 0.17$
(ii)
$\lvert m_d\rvert = 4.03$ (MeV), $\lvert m_s\rvert = 76.3$ (MeV), $\sigma = 146\pi/100$,
$(\Delta m^2_{12})/(\Delta m^2_{13}) = 0.20$, $(\Delta m^2_{23})/(\Delta m^2_{13}) = 0.80$,
$\sin^2(2\theta_{12}) = 0.54$, $\sin^2(2\theta_{23}) = 0.88$, $\sin^2(2\theta_{13}) = 0.20$

$$\frac{\langle m \rangle_{\alpha\beta}}{\sqrt{\Delta m^2_{23}}} \simeq \begin{pmatrix} 0.87 & 0.35 & 0.048 \\ & 0.50 & 0.20 \\ & & 0.14 \end{pmatrix}. \tag{4.4}$$

For instance, if we input $\Delta m^2_{23} = 1.5\times 10^{-3}$ eV^2, $\langle m \rangle_{ee}$ becomes 0.034 eV. This value is accessible to the next genera-

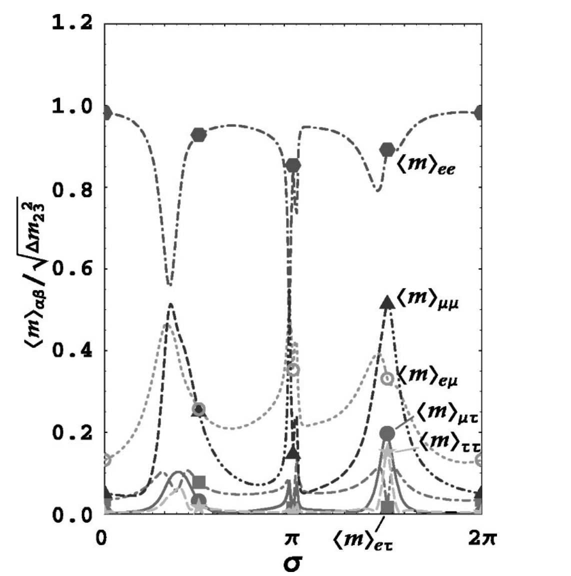

FIG. 3. The relations between the averaged neutrino masses of lepton number violating processes and σ. The hexagons, white circles, boxes, triangles, black circles, and stars indicate the values of $\langle m \rangle_{ee}/\sqrt{\Delta m^2_{23}}$, $\langle m \rangle_{e\mu}/\sqrt{\Delta m^2_{23}}$, $\langle m \rangle_{e\tau}/\sqrt{\Delta m^2_{23}}$, $\langle m \rangle_{\mu\mu}/\sqrt{\Delta m^2_{23}}$, $\langle m \rangle_{\mu\tau}/\sqrt{\Delta m^2_{23}}$, and $\langle m \rangle_{\tau\tau}/\sqrt{\Delta m^2_{23}}$ at every $\pi/2$ of σ.

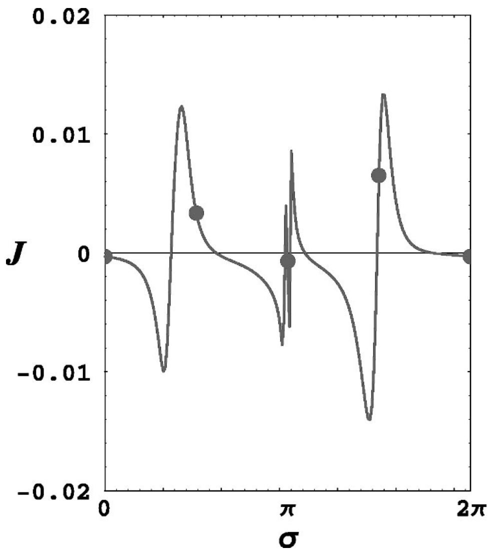

FIG. 4. The relation between the Jarlskog parameter J and σ. The circles indicate the values of J at every $\pi/2$ of σ.

tion of experiments such as GENIUS [10], CUORE [11], and MOON [12]. The Jarlskog parameter [13] appears in three generations:

$$P(\nu_e \to \nu_\mu) - P(\nu_\mu \to \nu_e) = J \frac{\Delta E_{21} \Delta E_{32} \Delta E_{31}}{\Delta E_{21}^M \Delta E_{32}^M \Delta E_{31}^M} \times \sin\left(\frac{\Delta E_{21}^M L}{2}\right) \sin\left(\frac{\Delta E_{32}^M L}{2}\right) \sin\left(\frac{\Delta E_{31}^M L}{2}\right) \tag{4.5}$$

with

$$J \equiv \mathrm{Im}(V_{l12} V_{l22}^* V_{l13}^* V_{l23}). \tag{4.6}$$

Here we have adopted the notation

$$\Delta E_{jk} \equiv E_j - E_k = \frac{\Delta m_{jk}^2}{2E},$$

TABLE II. The values of averaged neutrino masses and the Jarlskog parameter for the cases (i) and (ii) in Table I.

(i)
$\langle m \rangle_{ee} / \sqrt{\Delta m_{23}^2} = 1.16$, $\langle m \rangle_{e\mu} / \sqrt{\Delta m_{23}^2} = 0.32$,
$\langle m \rangle_{e\tau} / \sqrt{\Delta m_{23}^2} = 0.09$, $\langle m \rangle_{\mu\mu} / \sqrt{\Delta m_{23}^2} = 0.65$,
$\langle m \rangle_{\mu\tau} / \sqrt{\Delta m_{23}^2} = 0.40$, $\langle m \rangle_{\tau\tau} / \sqrt{\Delta m_{23}^2} = 0.36$,
$J = 0.0091$
(ii)
$\langle m \rangle_{ee} / \sqrt{\Delta m_{23}^2} = 0.94$, $\langle m \rangle_{e\mu} / \sqrt{\Delta m_{23}^2} = 0.36$,
$\langle m \rangle_{e\tau} / \sqrt{\Delta m_{23}^2} = 0.16$, $\langle m \rangle_{\mu\mu} / \sqrt{\Delta m_{23}^2} = 0.44$,
$\langle m \rangle_{\mu\tau} / \sqrt{\Delta m_{23}^2} = 0.23$, $\langle m \rangle_{\tau\tau} / \sqrt{\Delta m_{23}^2} = 0.15$,
$J = -0.014$

$$\Delta E_{jk}^M \equiv E_j^M - E_k^M \tag{4.7}$$

with

$$U \,\mathrm{diag}(E_1, E_2, E_3) U^{-1} + \mathrm{diag}(a,0,0) \equiv U^M \,\mathrm{diag}(E_1^M, E_2^M, E_3^M) (U^M)^{-1}. \tag{4.8}$$

The σ dependence of J is depicted in Fig. 4. For Eq. (3.1), we have

$$J \simeq 0.00015. \tag{4.9}$$

However, it needs careful consideration that J drastically changes at $\sigma \simeq 3\pi/2$. $\langle m \rangle_{\alpha\beta}$ and J in the cases of Tables I(i) and I(ii) discussed in Fig. 2 are also listed in Tables II(i) and II(ii). In this paper we have discussed to what extent the SO(10) two Higgs scalar model describes the quark-lepton masses and mixing parameters. We conclude that this model cannot be rejected within the existing data. It should be remarked that all the parameters can be determined in principle from the existing data.

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