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Are there ν_μ or ν_τ in the flux of solar neutrinos on Earth?

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Using the model independent method of Villante, Fiorentini, Lisi, Fogli, and Palazzo, and the rates measured in the SNO and Super-Kamiokande solar neutrino experiment, we calculate the amount of active ν_μ or ν_τ present in the flux of solar neutrinos on Earth. We show that the probability of $\nu_e \to \nu_{\mu,\tau}$ transitions is larger than zero at 99.89% C.L. We find that the averaged flux of $\nu_{\mu,\tau}$ on Earth is larger than 0.17 times the ^{8}B ν_e flux predicted by the BPB 2000 Standard Solar Model at 99% C.L. We discuss also the consequences of possible $\nu_e \to \bar{\nu}_{\mu,\tau}$ or $\nu_e \to \bar{\nu}_e$ transitions of solar neutrinos. We derive a model-independent lower limit of 0.52 at 99% C.L. for the ratio of the ^{8}B ν_e flux produced in the Sun and its value in the BPB 2000 Standard Solar Model.

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The first results of the SNO solar neutrino experiment [1] have beautifully confirmed the existence of the solar neutrino problem. A comparison of the neutrino flux measured through charged-current interactions in the SNO experiment with the flux measured through elastic scattering interactions in the Super-Kamiokande experiment [2] shows evidence of the presence of active ν_μ or[1] ν_τ in the solar neutrino flux measured by the Super-Kamiokande experiment [1,3]. Such a presence represents a very interesting indication in favor of neutrino physics beyond the standard model, most likely neutrino mixing that generates oscillations between different flavors (see [4]).

The purpose of this paper is to quantify the amount of this flux of active ν_μ or ν_τ in a model-independent way in the framework of frequentist statistics.[2]

The authors of Refs. [6,7] have noted that the response functions of the SNO and Super-Kamiokande (SK) experiments to solar neutrinos can be made approximately equal with a proper choice of the energy thresholds of the detected electrons. It turns out that given the threshold $T_e^{\mathrm{SNO}} = 6.75$ MeV, the two response functions are approximately equal for $T_e^{\mathrm{SK}} = 8.60$ MeV [3]. In this case the SNO and Super-Kamiokande event rates normalized to the 2000 Bahcall-Pinsonneault-Basu (BPB 2000) standard solar model (SSM) prediction [8] can be written in a model-independent way as [3]

$$R_{\mathrm{SNO}} = f_{\mathrm{B}} \langle P_{\nu_e\to\nu_e} \rangle , \tag{1}$$

$$R_{\mathrm{SK}} = f_{\mathrm{B}} \langle P_{\nu_e\to\nu_e} \rangle + f_{\mathrm{B}} \frac{\langle \sigma_{\nu_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} \langle P_{\nu_e\to\nu_{\mu,\tau}} \rangle , \tag{2}$$

where f_{B} is the ratio of the ^{8}B ν_e flux produced in the Sun and its value in the SSM [8], $\langle P_{\nu_e\to\nu_e} \rangle$ is the survival probability of solar ν_e's averaged over the common SNO and Super-Kamiokande response function,

$$\frac{\langle \sigma_{\nu_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} = 0.152 \tag{3}$$

is the ratio of the averaged $\nu_{\mu,\tau}$ and ν_e cross sections in the Super-Kamiokande experiment, and $\langle P_{\nu_e\to\nu_{\mu,\tau}} \rangle$ is the averaged probability of $\nu_e \to \nu_{\mu,\tau}$ transitions.

Calling

$$R_A \equiv R_{\mathrm{SK}} - R_{\mathrm{SNO}} , \tag{4}$$

from Eqs. (1) and (2) we have

$$R_A = f_{\mathrm{B}} \frac{\langle \sigma_{\nu_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} \langle P_{\nu_e\to\nu_{\mu,\tau}} \rangle . \tag{5}$$

Therefore, R_A is the rate of $\nu_{\mu,\tau}$-induced events in the Super-Kamiokande experiment, relative to the ν_e-induced rate predicted by the SSM.

Considering the data of the Super-Kamiokande experiment above the energy threshold $T_e^{\mathrm{SK}} = 8.60$ MeV and the BPB 2000 standard solar model [8], the measured values of R_{SNO} and R_{SK} are

$$R_{\mathrm{SNO}}^{\mathrm{exp}} = 0.347 \pm 0.029, \tag{6}$$

see Ref. [1], and

$$R_{\mathrm{SK}}^{\mathrm{exp}} = 0.451 \pm 0.017, \tag{7}$$

see Refs. [2,3].

Adding in quadrature the uncertainties of R_{SNO} and R_{SK}, for R_A we obtain

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[1] In this paper the conjunction ''or'' is used as a logical inclusive disjunction (the sentence is true when either or both of its constituent propositions are true).

[2] Since the results that we obtain are not too close to physical boundaries for the quantities under discussion and we assume a normal distribution for the errors, the numerical values in the framework of Bayesian probability theory with a flat prior are close to those obtained here, but their meaning is different (see, for example, Ref. [5]).

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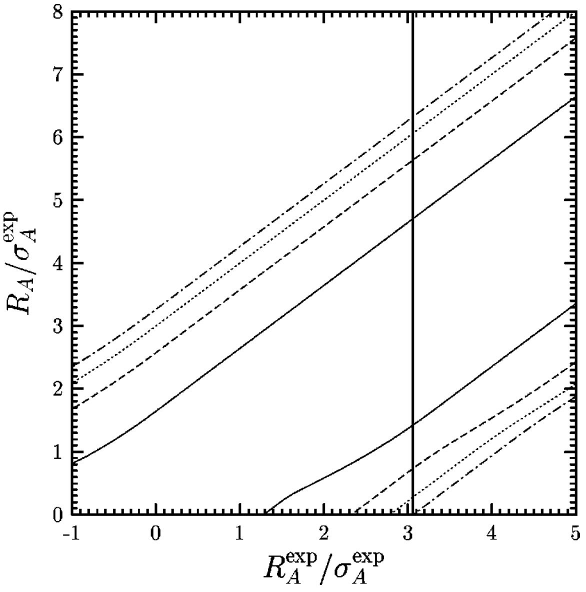

FIG. 1. Confidence belts in the unified approach [9] for a normal distribution with unit standard deviation. The regions between the solid, long-dashed, dotted and dash-dotted lines correspond, respectively, to 90% (1.64σ), 99% (2.58σ), 99.73% (3σ) and 99.89% (3.06σ) C.L. The thick solid vertical line represents the measured value of $R_A^{\mathrm{exp}}/\sigma_A^{\mathrm{exp}}$ [Eq. (10)].

$$R_A^{\mathrm{exp}} = 0.104 \pm 0.034. \tag{8}$$

The standard deviation of R_A^{exp} is

$$\sigma_A^{\mathrm{exp}} = 0.034, \tag{9}$$

and we have

$$\frac{R_A^{\mathrm{exp}}}{\sigma_A^{\mathrm{exp}}} = 3.06 \pm 1. \tag{10}$$

Hence, the central value of R_A is 3.06σ away from zero, implying an evidence of solar $\nu_e \to \nu_{\mu,\tau}$ transitions [1,3]. Our purpose is to quantify the probability of these transitions and possibly derive a lower limit.

The authors of Ref. [1] calculate the probability of a fluctuation larger than the observed one assuming $R_A = 0$: for normally distributed errors the probability of a fluctuation larger than 3.06σ from the mean is 0.11%.

Recently some frequentist methods have been proposed that allow us to obtain always meaningful confidence intervals with correct coverage for quantities like R_A that are bound to be positive by definition [9–12]. In particular, the unified approach proposed in Ref. [9] has been widely publicized by the Particle Data Group [13] and used by several experimental collaborations.

Using the unified approach we can derive confidence intervals for R_A. Figure 1 shows the confidence belts in the unified approach for a normal distribution with unit standard deviation for 90% (1.64σ), 99% (2.58σ), 99.73% (3σ), and 99.89% (3.06σ) C.L. One can see that the measured value (10) of $R_A^{\mathrm{exp}}/\sigma_A^{\mathrm{exp}}$ implies that

$$0 < \frac{R_A}{\sigma_A^{\mathrm{exp}}} < 6.32 \quad \text{at} \quad 99.89\% \ \text{C.L.}, \tag{11}$$

i.e. active ν_μ or ν_τ are present in the solar neutrino flux on Earth at 99.89% C.L. Equation (11) implies that there is a 0.11% probability that the true value of $R_A/\sigma_A^{\mathrm{exp}}$ is zero or larger than 6.32. This probability is the same as the probability of a fluctuation larger than 3.06σ calculated in Ref. [1] assuming $R_A = 0$. However, our result has been derived without making any assumption on the true unknown value of R_A and has a well defined meaning in the framework of frequentist statistics: whatever the true value of R_A, the interval (11) belongs to a set of intervals that could be obtained in the same way from repeated measurements and have the property that 99.89% of these intervals cover the true value of $R_A/\sigma_A^{\mathrm{exp}}$.

In order to derive a lower limit for the averaged flux of $\nu_{\mu,\tau}$ on Earth, we consider in the following 99% confidence intervals. From Fig. 1 we obtain

$$0.74 < \frac{R_A}{\sigma_A^{\mathrm{exp}}} < 5.63 \quad (99\% \ \text{C.L.}), \tag{12}$$

whose meaning is that there is a 99% probability that the interval (12) covers the true unknown value of $R_A/\sigma_A^{\mathrm{exp}}$.

For $f_{\mathrm{B}} \langle P_{\nu_e \to \nu_{\mu,\tau}} \rangle$, which gives the flux of active $\nu_{\mu,\tau}$ averaged over the common Super-Kamiokande and SNO response function, relative to the SSM ^{8}B ν_e flux, we find

$$0.17 < f_{\mathrm{B}} \langle P_{\nu_e \to \nu_{\mu,\tau}} \rangle < 1.26 \quad (99\% \ \text{C.L.}). \tag{13}$$

Hence, we can say that the averaged flux of $\nu_{\mu,\tau}$ on Earth is larger than 0.17 times the ^{8}B ν_e flux predicted by the standard solar model at 99% C.L. This is an evidence in favor of relatively large $\nu_e \to \nu_{\mu,\tau}$ transitions if f_B is not too large.

One could argue that it is possible to derive a more stringent lower limit for $f_{\mathrm{B}} \langle P_{\nu_e \to \nu_{\mu,\tau}} \rangle$ by calculating a confidence belt without a left edge, instead of the one in Fig. 1 calculated in the unified approach. Such a procedure is not acceptable because it would lead to undercoverage if not chosen *a priori* independently from the data, as shown in Ref. [9] for the case of upper limits. The correct procedure is to choose *a priori* a method like the unified approach that always gives sensible results and apply it to the data, as we have done here. *A priori* one could have chosen another method, as those presented in Refs. [10–12], that may have even better properties than the unified approach [14,15], but we have verified that the intervals (11)–(13) do not change significantly.

Unfortunately, we cannot derive a model independent lower limit for the averaged $\nu_e \to \nu_{\mu,\tau}$ probability $\langle P_{\nu_e \to \nu_{\mu,\tau}} \rangle$, because f_{B} could be large. However, from Fig. 1 we can say that $R_A/\sigma_A^{\mathrm{exp}} > 0$ at 99.89% C.L. [see Eq. (11)], and hence

$$P_{\nu_e \to \nu_{\mu,\tau}} > 0 \quad \text{at} \quad 99.89\% \ \text{C.L.} \tag{14}$$

in the range of neutrino energies covered by the common SNO and Super-Kamiokande response function presented in Ref. [3].

On the other hand, it is interesting to note that the relations (1) and (2) allow us to derive a model-independent lower limit for f_{B}, taking into account that

$$\langle P_{\nu_e \to \nu_{\mu,\tau}} \rangle \leqslant 1 - \langle P_{\nu_e \to \nu_e} \rangle . \tag{15}$$

Using this inequality, from Eqs. (1) and (2) we obtain

$$f_{\mathrm{B}} \geqslant \frac{\langle \sigma_{\nu_e} \rangle}{\langle \sigma_{\nu_{\mu,\tau}} \rangle} R_{\mathrm{SK}} - \left(\frac{\langle \sigma_{\nu_e} \rangle}{\langle \sigma_{\nu_{\mu,\tau}} \rangle} - 1 \right) R_{\mathrm{SNO}} \equiv f_{\mathrm{B,min}} . \tag{16}$$

From Eqs. (3), (6) and (7), the experimental value of $f_{\mathrm{B,min}}$ is

$$f_{\mathrm{B,min}}^{\mathrm{exp}} = 1.031 \pm 0.197 . \tag{17}$$

Since the central value of $f_{\mathrm{B,min}}^{\mathrm{exp}}$ is 5.2σ away from zero, we can calculate the resulting 99% C.L. interval for $f_{\mathrm{B,min}}$ using the central intervals method (see [13]), which gives the same result as the unified approach far from the physical boundary $f_{\mathrm{B,min}} > 0$. Since in the central intervals method 99% C.L. corresponds to 2.58σ, we obtain the confidence interval

$$0.52 < f_{\mathrm{B,min}} < 1.54 \qquad (99\% \ \ \mathrm{C.L.}). \tag{18}$$

Therefore, we can conclude that the SNO and Super-Kamiokande data imply the model-independent lower limit

$$f_{\mathrm{B}} > 0.52 \qquad (99\% \quad \mathrm{C.L.}). \tag{19}$$

This is very interesting information for the physics of the Sun.

So far we have not considered the possible existence of exotic mechanisms that produce $\nu_e \to \bar{\nu}_{\mu,\tau}$ or $\nu_e \to \bar{\nu}_e$ transitions (in addition to or in alternative to $\nu_e \to \nu_{\mu,\tau}$ transitions), such as resonant spin-flavor precession of Majorana neutrinos[3] [16,17]. In this case, Eq. (2) must be replaced with

$$\begin{aligned} R_{\mathrm{SK}} = {} & f_{\mathrm{B}} \langle P_{\nu_e \to \nu_e} \rangle + f_{\mathrm{B}} \left[\frac{\langle \sigma_{\nu_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} \langle P_{\nu_e \to \nu_{\mu,\tau}} \rangle \right. \\ & \left. + \frac{\langle \sigma_{\bar{\nu}_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} \langle P_{\nu_e \to \bar{\nu}_{\mu,\tau}} \rangle + \frac{\langle \sigma_{\bar{\nu}_e} \rangle}{\langle \sigma_{\nu_e} \rangle} \langle P_{\nu_e \to \bar{\nu}_e} \rangle \right], \end{aligned} \tag{20}$$

and Eq. (5) with

$$\begin{aligned} R_A = f_{\mathrm{B}} & \left[\frac{\langle \sigma_{\nu_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} \langle P_{\nu_e \to \nu_{\mu,\tau}} \rangle + \frac{\langle \sigma_{\bar{\nu}_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} \langle P_{\nu_e \to \bar{\nu}_{\mu,\tau}} \rangle \right. \\ & \left. + \frac{\langle \sigma_{\bar{\nu}_e} \rangle}{\langle \sigma_{\nu_e} \rangle} \langle P_{\nu_e \to \bar{\nu}_e} \rangle \right]. \end{aligned} \tag{21}$$

Using the ^{8}B neutrino spectrum given in Ref. [18], the neutrino-electron elastic scattering cross section calculated in Ref. [19], taking into account radiative corrections, and the Super-Kamiokande energy resolution given in Ref. [20], we obtain the following values for the ratios of the averaged cross sections in the Super-Kamiokande experiment for the threshold energy $T_e^{\mathrm{SK}} = 8.60$ MeV:

$$\frac{\langle \sigma_{\bar{\nu}_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} = 0.114, \qquad \frac{\langle \sigma_{\bar{\nu}_e} \rangle}{\langle \sigma_{\nu_e} \rangle} = 0.120. \tag{22}$$

Hence, we have the useful inequalities

$$\frac{\langle \sigma_{\bar{\nu}_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} < \frac{\langle \sigma_{\bar{\nu}_e} \rangle}{\langle \sigma_{\nu_e} \rangle} < \frac{\langle \sigma_{\nu_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} . \tag{23}$$

The lower bound in Eq. (11) implies the existence of solar $\nu_e \to \nu_{\mu,\tau}$ or $\nu_e \to \bar{\nu}_{\mu,\tau}$ or $\nu_e \to \bar{\nu}_e$ transitions at 99.89% C.L. The inequalities in Eq. (12) imply that the quantity on the right-hand side of Eq. (21) is limited in the interval (0.025,0.19) at 99% C.L. Using the inequalities (23), we obtain

$$\begin{aligned} 0.17 < {} & f_{\mathrm{B}} [\langle P_{\nu_e \to \nu_{\mu,\tau}} \rangle + \langle P_{\nu_e \to \bar{\nu}_{\mu,\tau}} \rangle + \langle P_{\nu_e \to \bar{\nu}_e} \rangle] \\ & < 1.67 \qquad (99\% \ \ \mathrm{C.L.}). \end{aligned} \tag{24}$$

Therefore, the averaged flux of ν_μ, ν_τ, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$ and $\bar{\nu}_e$ on Earth is larger than 0.17 times the ^{8}B ν_e flux predicted by the BPB 2000 standard solar model at 99% C.L.

Let us derive now the most general model-independent lower limit for f_{B} (assuming only that the Super-Kamiokande and SNO events are produced by neutrinos or antineutrinos generated as ν_e from ^{8}B decay in the Sun). Using the inequality

$$\langle P_{\nu_e \to \nu_{\mu,\tau}} \rangle + \langle P_{\nu_e \to \bar{\nu}_{\mu,\tau}} \rangle + \langle P_{\nu_e \to \bar{\nu}_e} \rangle \leqslant 1 - \langle P_{\nu_e \to \nu_e} \rangle \tag{25}$$

and those in Eq. (23), from Eqs. (1) and (20) we obtain again the limit in Eq. (16). Therefore, Eq. (19) gives the most general model-independent lower limit for f_{B} following from the SNO and Super-Kamiokande data.

In conclusion, we have considered the model-independent relations (1),(2) [3,6,7] [and (1),(20)] and the rates measured in the SNO [1] and Super-Kamiokande [2] solar neutrino experiments in the framework of frequentist statistics. We have shown that the probability of $\nu_e \to \nu_{\mu,\tau}$ (and $\nu_e \to \bar{\nu}_{\mu,\tau}, \nu_e \to \bar{\nu}_e$) transitions is larger than zero at 99.89% C.L. in the range of neutrino energies covered by the common SNO and Super-Kamiokande response function. We have found that the flux of $\nu_{\mu,\tau}$ (and $\bar{\nu}_{\mu,\tau}, \bar{\nu}_e$) on Earth averaged over the common SNO and Super-Kamiokande response function is larger than 0.17 times the ^{8}B ν_e flux predicted by the BPB 2000 standard solar model at 99% C.L. We have derived a model-independent lower limit of 0.52 at 99% C.L. for the ratio f_{B} of the ^{8}B ν_e flux produced in the Sun and its value in the BPB2000 standard solar model [8].

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[3] In the case of Majorana neutrinos the right-handed states are conventionally called antineutrinos.

[1] SNO Collaboration, Q.R. Ahmad *et al.*, Phys. Rev. Lett. **87**, 071301 (2001).
[2] SuperKamiokande Collaboration, S. Fukuda *et al.*, Phys. Rev. Lett. **86**, 5651 (2001).
[3] G.L. Fogli, E. Lisi, D. Montanino, and A. Palazzo, Phys. Rev. D **64**, 093007 (2001).
[4] S.M. Bilenky, C. Giunti, and W. Grimus, Prog. Part. Nucl. Phys. **43**, 1 (1999).
[5] G. D'Agostini, CERN Yellow Report No. 99-03, 1999.
[6] F.L. Villante, G. Fiorentini, and E. Lisi, Phys. Rev. D **59**, 013006 (1999).
[7] G.L. Fogli, E. Lisi, A. Palazzo, and F.L. Villante, Phys. Rev. D **63**, 113016 (2001).
[8] J.N. Bahcall, M.H. Pinsonneault, and S. Basu, Astrophys. J. **555**, 990 (2001).
[9] G.J. Feldman and R.D. Cousins, Phys. Rev. D **57**, 3873 (1998).
[10] S. Ciampolillo, Nuovo Cimento A **111**, 1415 (1998).
[11] C. Giunti, Phys. Rev. D **59**, 053001 (1999).
[12] M. Mandelkern and J. Schultz, J. Math. Phys. **41**, 5701 (2000).
[13] Particle Data Group, D.E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000); URL: http://pdg.lbl.gov.
[14] C. Giunti and M. Laveder, Int. J. Mod. Phys. C **12**, 1155 (2001).
[15] C. Giunti and M. Laveder, hep-ex/0011069.
[16] C.-S. Lim and W.J. Marciano, Phys. Rev. D **37**, 1368 (1988).
[17] E.K. Akhmedov, Phys. Lett. B **213**, 64 (1988).
[18] J.N. Bahcall *et al.*, Phys. Rev. C **54**, 411 (1996).
[19] J.N. Bahcall, M. Kamionkowski, and A. Sirlin, Phys. Rev. D **51**, 6146 (1995).
[20] Super-Kamiokande Collaboration, M. Nakahata *et al.*, Nucl. Instrum. Methods Phys. Res. A **421**, 113 (1999).