

## Membranes wrapped on holomorphic curves

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We construct supergravity solutions dual to the twisted field theories arising when  $M$ -theory membranes wrap holomorphic curves in Calabi-Yau  $n$ -folds. The solutions are constructed in an Abelian truncation of maximal  $D=4$  gauged supergravity and then uplifted to  $D=11$ . For fourfolds and fivefolds we find new smooth AdS/conformal field theory examples and for all cases we analyze the nature of the singularities that arise. Our results provide an interpretation of certain charged topological AdS black holes. We also present the generalized calibration two-forms for the solutions.

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### I. INTRODUCTION

An interesting generalization of the AdS/conformal field theory (CFT) correspondence [1] is the construction of supergravity solutions dual to the field theories that arise on branes wrapping supersymmetric cycles. To preserve supersymmetry it is necessary for the field theory to be twisted in the sense that there is an identification of the spin connection on the cycle with certain external  $R$ -symmetry gauge fields [2]. It was argued in [3] that this implies that dual supergravity solutions can be found in the appropriate gauged supergravity and then, ideally, uplifted to 10 or 11 dimensions. This two-step approach enables one to find highly nontrivial supergravity solutions and has been further developed in a number of papers [4–11]. Here we will extend these investigations by considering the theories that arise when  $M$ -theory membranes wrap two-cycles in Calabi-Yau two-, three-, four- and fivefolds.

For unwrapped membranes recall that there is a decoupling limit in which one obtains a  $D=3$   $N=8$  super conformal field theory (SCFT) that is dual to  $M$  theory on  $\text{AdS}_4 \times S^7$ , with the  $\text{SO}(8)$  isometries of the seven-sphere corresponding to the  $\text{SO}(8)_R$   $R$  symmetries of the SCFT [1]. When a membrane wraps a two-cycle  $\Sigma$  in a Calabi-Yau  $n$ -fold it will preserve some supersymmetry if the two-cycle is holomorphic. The relevant twisting mentioned above then depends on the structure of the normal bundle of the cycle in the  $n$ -fold, as we shall discuss. The twistings we shall consider involve only the gauge fields of the maximal Abelian subgroup  $\text{U}(1)^4$  of  $\text{SO}(8)_R$ . As a consequence we are able to construct the solutions in a  $\text{U}(1)^4$  truncated version of  $D=4$   $N=8$  gauged supergravity [12] and then use the results of [13] to uplift to find  $D=11$  solutions.

The gauged supergravity solutions have an  $\text{AdS}_4$ -type region, specified more precisely below, that describes the UV physics of the decoupled  $D=3$  twisted field theory arising on the wrapped membranes. The appropriate decoupling limit involves letting  $l_{pl} \rightarrow 0$  while keeping the volume of the

cycle fixed and implies that only the local geometry of the holomorphic curve in the Calabi-Yau manifold is relevant [3]. In the IR, at energies small compared to the energy scale set by the inverse size of the two-cycle, the theory reduces to a one-dimensional supersymmetric quantum mechanics. The solutions we obtain describe the flow from the dual  $\text{AdS}_4$ -type UV region to the gravity dual descriptions of the IR physics. For twistings corresponding to holomorphic curves in Calabi-Yau four- and fivefolds, we find an IR fixed point with geometry  $\text{AdS}_2 \times \Sigma$  when the curvature of the two-cycle  $\Sigma$  is negative. Lifted to  $D=11$ , these solutions are dual to a superconformal quantum mechanics and thus provide new AdS/CFT examples. Interestingly, for the fivefold case, the full lifted solution describing the flow from the UV is an embedding into 11 dimensions of the supersymmetric “topological” charged  $D=4$  AdS black hole of [14]. Our analysis thus indicates the proper interpretation of this solution. In addition we are able to lift the rotating generalization of the  $D=4$  black hole discussed in [14] to obtain a new supersymmetric  $D=11$  solution that describes supersymmetric waves on the wrapped membranes.

Since the  $D=11$  solutions describe the geometry arising when membranes wrap the two-cycle  $\Sigma$  and also preserve supersymmetry, on general grounds we would expect that a probe membrane wrapping the same cycle  $\Sigma$  will also be supersymmetric. Given that our backgrounds are static and have nonvanishing four-forms, this means that our solutions should admit generalized Kähler calibration two-forms [15], which we shall explicitly present.

The solutions presented here for wrapped membranes and in related work for other wrapped branes go well beyond the intersecting brane solutions found using the “harmonic function rule” [16–18] or the “generalized harmonic function rule” [19,20]. In [21,22] a set of more general Bogomol’nyi-Prasad-Sommerfield (BPS) equations were derived for certain smoothed intersections of five-branes corresponding to five-branes wrapped on Riemann surfaces and some solutions were found. Further solutions were presented in [23,24]. This approach was generalized to obtain BPS equations for five-branes wrapped on Kähler four-cycles in six dimensions and also to membranes wrapped on Riemann surfaces in [25] where the connection with generalized calibrations was exploited. In order to clarify the connection of this alternative approach with ours we will present a change of

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coordinates that recasts our solutions into the form considered in [25]. Similar coordinate transformations also exist for the wrapped five-brane solutions constructed in [7].

The plan of the rest of the paper is as follows. In the next section we recall the Abelian truncation of  $N=8$  gauged supergravity and how it relates to  $D=11$  supergravity. We then present and analyze the BPS equations for membranes wrapping two-cycles in Calabi-Yau two-, three-, four- and five-folds. Section IV presents the generalized Kähler calibration and the coordinate transformations mentioned in the last paragraph. Section V concludes.

## II. $S^7$ REDUCTION AND $U(1)^4$ GAUGED SUPERGRAVITY

The  $S^7$  reduction of 11-dimensional supergravity gives rise to  $d=4$ ,  $N=8$  gauged  $SO(8)$  supergravity. We will use the reduction ansatz presented in [13] which retains only  $U(1)^4$  gauge fields. The 11-dimensional metric is given by

$$ds^2 = \Delta^{2/3} ds_4^2 + \frac{2}{e^2} \Delta^{-1/3} \sum_{\alpha} X_{\alpha}^{-1} [d\mu_{\alpha}^2 + \mu_{\alpha}^2 (d\phi_{\alpha} + 2eA_{\alpha})^2], \quad (2.1)$$

where  $ds_4^2$  is the four-dimensional metric,  $X_{\alpha}$  and  $A_{\alpha}$ , with  $\alpha=1, \dots, 4$ , are scalars and one-forms, respectively, on the four-dimensional space, and  $\Delta = \sum_{\alpha} X_{\alpha} \mu_{\alpha}^2$ . The coordinates  $\mu_{\alpha}$  and  $\phi_{\alpha}$  parametrize a seven-sphere: the  $\mu_{\alpha}$  are constrained to satisfy  $\sum_{\alpha} \mu_{\alpha}^2 = 1$  and  $0 \leq \phi_{\alpha} < 2\pi$  are angles. The four scalar fields  $X_{\alpha}$ , satisfying  $X_1 X_2 X_3 X_4 = 1$ , deform the round sphere metric generically breaking the  $SO(8)$  symmetry to  $U(1)^4$  parametrized by rotations in the four angles  $\phi_{\alpha}$ . In addition these directions are twisted by the four  $U(1)$  gauge fields  $A_{\alpha}$ .

The four-form field strength is given in terms of the same fields via

$$G = \sqrt{2}e \sum_{\alpha} (X_{\alpha}^2 \mu_{\alpha}^2 - \Delta X_{\alpha}) \epsilon_4 - \frac{1}{\sqrt{2}e} \sum_{\alpha} X_{\alpha}^{-1} * dX_{\alpha} \wedge d\mu_{\alpha}^2 - \frac{2\sqrt{2}}{e^2} \sum_{\alpha} X_{\alpha}^{-2} d\mu_{\alpha}^2 \wedge (d\phi_{\alpha} + 2eA_{\alpha}) \wedge *F_{\alpha}, \quad (2.2)$$

where  $*$  is the Hodge dual operator on the four-dimensional space and  $\epsilon_4$  is the corresponding volume form.

Reducing with this ansatz leads to a four-dimensional theory with bosonic action

$$\mathcal{L} = \frac{1}{2\kappa^2} \sqrt{-g} \left[ R - \frac{1}{2} (\partial \vec{\phi})^2 - 2 \sum_{\alpha} e^{\vec{a}_{\alpha} \cdot \vec{\phi}} F_{\alpha}^2 - V \right], \quad (2.3)$$

where

$$V = -4e^2 (\cosh \phi_{12} + \cosh \phi_{13} + \cosh \phi_{14}). \quad (2.4)$$

Here we have introduced a new basis for the constrained scalar fields  $X_{\alpha}$  in terms of a vector of scalar fields  $\phi_{\alpha\beta}$

$$\vec{\phi} = (\phi_{12}, \phi_{13}, \phi_{14}), \quad (2.5)$$

where under the  $\alpha$  and  $\beta$  indices  $\phi_{\alpha\beta}$  is symmetric and  $\phi_{34} = \phi_{12}$ ,  $\phi_{24} = \phi_{13}$  and  $\phi_{23} = \phi_{14}$ , while  $\phi_{11} = \phi_{22} = \phi_{33} = \phi_{44} = 0$ . The  $X_{\alpha}$  are then given by

$$X_{\alpha} = \exp(-\vec{a}_{\alpha} \cdot \vec{\phi}/2), \quad (2.6)$$

where

$$\begin{aligned} \vec{a}_1 &= (1, 1, 1), & \vec{a}_2 &= (1, -1, -1), \\ \vec{a}_3 &= (-1, 1, -1), & \vec{a}_4 &= (-1, -1, 1). \end{aligned} \quad (2.7)$$

As discussed in [13], this is the bosonic action of  $d=4$   $N=2$  gauged  $U(1)^4$  supergravity with the three axions set to zero. Thus the reduction ansatz can be used to embed  $d=4$  solutions with vanishing axions into solutions of 11-dimensional supergravity. The same bosonic action can also be obtained by truncating  $N=8$  gauged  $SO(8)$  supergravity as in [12]. [This is where the peculiar index structure of  $\phi_{\alpha\beta}$  comes from: these fields really parametrize a self-dual four-form under  $SO(8)$ .] The corresponding  $N=8$  fermionic supersymmetry transformations can be written as follows. First, note that the  $N=8$  fermions consist of the gravitini  $\psi_{\mu}^I$  and the spin-half fields  $\chi^{IJK}$  where  $I, J$  and  $K$  are  $SU(8)$  indices. Given the ansatz for the scalar and vector fields, the index  $I$  is equivalent to the pair  $(\alpha, i)$  where  $\alpha=1, \dots, 4$  as above and  $i=1, 2$ . With this notation the variations of the gravitini are given by [12]

$$\begin{aligned} \delta \psi_{\mu}^{\alpha i} &= \nabla_{\mu} \epsilon^{\alpha i} - 2e \sum_{\beta} \Omega_{\alpha\beta} A_{\beta, \mu} \epsilon^{ij} \epsilon^{\beta j} \\ &+ \frac{e}{4\sqrt{2}} \sum_{\beta} e^{-\vec{a}_{\beta} \cdot \vec{\phi}/2} \gamma_{\mu} \epsilon^{\alpha i} \\ &+ \frac{1}{2\sqrt{2}} \sum_{\beta} \Omega_{\alpha\beta} e^{\vec{a}_{\beta} \cdot \vec{\phi}/2} F_{\beta, \nu\lambda} \gamma^{\nu\lambda} \gamma_{\mu} \epsilon^{ij} \epsilon^{\alpha j}, \end{aligned} \quad (2.8)$$

where  $\Omega_{\alpha\beta}$  is the matrix

$$\Omega = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}. \quad (2.9)$$

For the spin-1/2 fermions, one finds  $\delta \chi^{\alpha i \beta j \gamma k} = \delta \chi^{\alpha \gamma k} \delta^{\alpha\beta} \epsilon^{ij} + \delta \chi^{\beta \alpha i} \delta^{\beta\gamma} \epsilon^{jk} + \delta \chi^{\gamma \beta j} \delta^{\gamma\alpha} \epsilon^{ki}$  with [12]

$$\begin{aligned}
\delta\chi^{\alpha\beta i} = & -\frac{1}{\sqrt{2}}\gamma^\mu\partial_\mu\phi_{\alpha\beta}\epsilon^{ij}\epsilon^{\beta j} \\
& -e\sum_{\gamma\delta}\Sigma_{\alpha\beta\gamma}\Omega_{\gamma\delta}e^{-\vec{a}_\delta\cdot\vec{\phi}/2}\epsilon^{ij}\epsilon^{\beta j} \\
& +\sum_{\delta}\Omega_{\alpha\delta}e^{\vec{a}_\delta\cdot\vec{\phi}/2}F_{\delta\mu\nu}\gamma^{\mu\nu}\epsilon^{\beta i}. \quad (2.10)
\end{aligned}$$

The tensor  $\Sigma_{\alpha\beta\gamma}$  selects a particular  $\gamma$  depending on  $\alpha$  and  $\beta$ , and is defined by

$$\Sigma_{\alpha\beta\gamma} = \begin{cases} |\epsilon_{\alpha\beta\gamma}| & \text{for } \alpha, \beta, \gamma \neq 1, \\ \delta_{\beta\gamma} & \text{for } \alpha = 1, \\ \delta_{\alpha\gamma} & \text{for } \beta = 1, \\ 0 & \text{otherwise.} \end{cases} \quad (2.11)$$

### III. BPS EQUATIONS

Our strategy for constructing supersymmetric  $D=11$  solutions describing wrapped membranes is to first construct four-dimensional BPS solutions of Eq. (2.3) such that the supersymmetry variations (2.8) and (2.10) vanish. These are then uplifted to  $D=11$  using ansatz (2.1) and (2.2). For orientation note that the vacuum  $\text{AdS}_4$  solution

$$ds^2 = \frac{1}{2e^2r^2}(-dt^2 + dx^2 + dy^2 + dr^2), \quad (3.1)$$

with the gauge fields and the scalars set to zero, uplifts to  $\text{AdS}_4 \times S^7$ , the supergravity dual of the SCFT for flat unwrapped membranes.

To describe wrapped membranes, we take the ansatz for the four-dimensional metric to be

$$ds^2 = e^{2f}(-dt^2 + dr^2) + e^{2g}ds^2(\Sigma), \quad (3.2)$$

where  $ds^2(\Sigma)$  is the metric on the two-cycle  $\Sigma$ . The ansatz for the gauge fields is determined as follows. First recall that the normal bundle to a holomorphic two-cycle in a Calabi-Yau  $(n+1)$ -fold is  $U(n) \subset SO(2n)$ . Moreover, for the Calabi-Yau manifold to have only  $SU(n+1)$  holonomy, the  $U(1)$  spin connection of  $\Sigma$  must be identified with the diagonal  $U(1)$  factor in  $U(n)$ . When a membrane probe wraps the two-cycle, the  $SO(8)$   $R$  symmetry of the normal directions of an unwrapped membrane in flat space is naturally split into  $SO(2n) \times SO(8-2n)$ , reflecting the split of the directions normal to the two-cycle within the Calabi-Yau manifold and the rest. The structure of the normal bundle of the Calabi-Yau manifold then automatically requires an identification of the  $U(1)$  spin connection on the cycle with the corresponding diagonal  $U(1) \subset U(n) \subset SO(2n)$  part of the  $R$  symmetry: this is what is meant by ‘‘twisting.’’ The upshot of these observations is that in the supergravity ansatz the only nonvanishing  $SO(8)$  gauge fields should lie in  $U(n) \subset SO(2n)$  and we must identify the  $U(1)$  spin connection of  $\Sigma$  in Eq. (3.2) with the diagonal  $U(1) \subset U(n)$ . Now

the above supergravity truncation keeps only Abelian gauge fields  $U(1)^4 \subset SO(8)$ . We thus want to identify each of the  $n$   $U(1)$  gauge fields in  $SO(2n)$  with the  $U(1)$  spin connection, and set the remaining  $U(1)$  gauge fields to zero. Note that the restriction to Abelian gauged supergravity means, for  $n \geq 2$ , that we are considering twistings corresponding to non-generic Calabi-Yau  $(n+1)$ -folds for which the normal bundle to the two-cycle is restricted to be only  $U(1)$  rather than the full  $U(n)$ . A familiar Calabi-Yau threefold example is provided by the resolved conifold.

The supergravity ansatz is completed by specifying scalar fields consistent with the  $SO(2n) \times SO(8-2n)$  split. In each case this is achieved by writing  $\vec{\phi}$  in terms of a single scalar field  $\phi$ .

Requiring the fermionic variations (2.8) and (2.10) to vanish with this ansatz leads to a set of BPS first-order differential equations for the metric functions  $f$  and  $g$  and the scalar field  $\phi$ . These will be presented in the subsequent subsections. In all cases, we find that the Killing spinors  $\epsilon^{\alpha i}$  satisfy

$$\gamma^3\epsilon^{\alpha i} = \epsilon^{\alpha i}, \quad \epsilon^{\alpha i} = e^{f/2}\epsilon_0^{\alpha i}, \quad (3.3)$$

where  $\epsilon_0^{\alpha i}$  is a constant spinor and  $\gamma^3$  points in the radial direction and is defined using the orthonormal frame

$$e^m = (e^f dt, e^g \bar{e}^1, e^g \bar{e}^2, e^f dr), \quad (3.4)$$

with  $(\bar{e}^1, \bar{e}^2)$  giving an orthonormal frame for  $\Sigma$ . The first condition breaks half of the supersymmetry. In each case, there are then different additional restrictions, breaking more supersymmetry. The second condition comes from the  $r$  component of the gravitino variation (2.8). Otherwise the Killing spinors are independent of coordinates. As in previous studies [3–7], it is necessary that the metric on the two-cycle is Einstein, which means that it has constant curvature. The cycle is either a sphere  $S^2$ , hyperbolic space  $H^2$  or flat. The Ricci tensor is given by

$$R(\Sigma)_{ab} = l g_{ab}, \quad (3.5)$$

where  $a$  and  $b$  are indices on  $\Sigma$  and the volume of the cycle is normalized so that  $l=1$  for  $\Sigma=S^2$ ,  $l=0$  for  $\Sigma=\mathbb{R}^2$  and  $l=-1$  for  $\Sigma=H^2$ . Since the  $D=4$  Killing spinors are independent of the coordinates of the cycle, we can also take quotients of these spaces while preserving supersymmetry. In particular  $\Sigma$  can be a compact Riemann surface of any genus.

Before discussing the different cases let us comment on the flat case,  $l=0$ . The fact that we are considering gauge fields corresponding to Calabi-Yau manifolds with nongeneric normal bundles means that if  $\Sigma$  is flat then the whole normal bundle is in fact trivial. Thus the Calabi-Yau manifold is locally just flat  $\mathbb{C}^{d+1}$ , and in all cases we are simply considering the embedding of a flat M2-brane in flat space. The corresponding supergravity solution is then very well known and can be written in familiar form in terms of a harmonic function. For this case solutions to our BPS equations simply correspond to a choice of harmonic function preserving  $SO(2n) \times SO(8-2n)$  symmetry in the eight transverse dimensions. In fact it is straightforward to solve

the  $l=0$  BPS equations exactly by introducing isotropic coordinates as in the Calabi-Yau fivefold case discussed below. As a consequence we shall not dwell on the  $l=0$  case in the following. Note that these solutions are the direct analogues of the D3-brane solutions given in [26].

### A. Calabi-Yau twofold

For this case there is a natural split of  $SO(8)$  into  $SO(2) \times SO(6)$ . The nonvanishing gauge fields are in the  $SO(2)$  gauge group and so we have

$$F_1 = -\frac{l}{2e} \text{Vol}(\Sigma), \quad F_2 = F_3 = F_4 = 0, \quad (3.6)$$

where  $\text{Vol}(\Sigma)$  is the volume two-form on the two-cycle. The scalar fields are given by  $\vec{\phi} = (\phi, \phi, \phi)$  so that  $X_1 = e^{-3\phi/2}$ ,  $X_2 = X_3 = X_4 = e^{\phi/2}$ .

For a two-cycle in a Calabi-Yau twofold, we expect to preserve eight supercharges. Demanding that the supersymmetry variations are zero, we find that, in addition to Eq. (3.3), the Killing spinor  $\epsilon^{\alpha i}$  must satisfy

$$\gamma^{12} \epsilon^{\alpha i} = \epsilon^{ij} \epsilon^{\alpha j} \quad \text{for } \alpha = 1, \dots, 4, \quad (3.7)$$

where 1 and 2 are tangent space indices in the  $\Sigma$  directions, as defined by the orthonormal frame (3.4). Together these conditions do indeed give eight independent Killing spinors. The variations then vanish provided we satisfy the BPS equations

$$\begin{aligned} e^{-f} f' &= -\frac{e}{2\sqrt{2}} (e^{-3\phi/2} + 3e^{\phi/2}) + \frac{l}{2\sqrt{2}e} e^{3\phi/2-2g}, \\ e^{-f} g' &= -\frac{e}{2\sqrt{2}} (e^{-3\phi/2} + 3e^{\phi/2}) - \frac{l}{2\sqrt{2}e} e^{3\phi/2-2g}, \\ e^{-f} \phi' &= -\frac{e}{\sqrt{2}} (e^{-3\phi/2} - e^{\phi/2}) + \frac{l}{\sqrt{2}e} e^{3\phi/2-2g}. \end{aligned} \quad (3.8)$$

These equations can be partially integrated to give

$$e^{2g+\phi} = C \left( 2e^{2g-\phi} + \frac{l}{e^2} \right)^{1/2} + e^{2g-\phi} + \frac{l}{e^2}, \quad (3.9)$$

where  $C$  is a constant. It is also straightforward to determine the full asymptotic behavior of the solutions. In different limits, different terms in the BPS equations (3.8) dominate. For example, for large  $e^{2g}$ , the final terms proportional to  $l$  are small and the leading behavior, valid at small  $r$ , is given by

$$ds^2 \approx \frac{1}{2e^2 r^2} [-dt^2 + dr^2 + ds^2(\Sigma)], \quad (3.10)$$

with  $\phi \approx cr + (l - \frac{1}{2}c^2)r^2$ , for an arbitrary constant  $c$ . This is almost  $\text{AdS}_4$  except that the metric on  $\mathbb{R}^{1,2}$  has been replaced

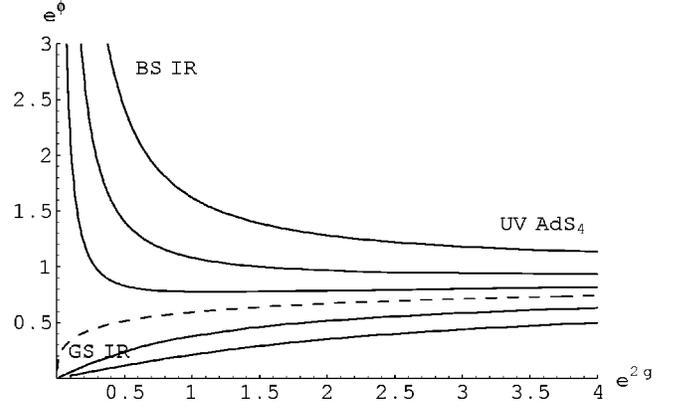


FIG. 1. Behavior of the flows for an  $S^2$  cycle in a Calabi-Yau twofold.

by one on  $\mathbb{R} \times \Sigma$ . This limit specifies the UV behavior of the membrane wrapped on  $\Sigma$ , and is universal in the sense that, as we will see, it is present independent of the dimension of the Calabi-Yau manifold and of the curvature of  $\Sigma$ . The leading and subleading terms in  $\phi$  are interpreted as either the insertion of a boundary operator in the UV theory, due to the curvature of the two-cycle, or the expectation value of this operator [27].

The behavior of the solutions are illustrated in Figs. 1 and 2 for the case of  $\Sigma = S^2$  and  $\Sigma = H^2$ , respectively, for different values of  $C$  in Eq. (3.9). These are interpreted as describing the flows from the UV region to gravity duals of the IR physics. In each case, we have noted whether the resulting singularities encountered in the IR are of good or bad type according to the criteria of [3]. Recall that for “good” singularities the time component of the uplifted 11-dimensional metric  $g_{00}^{(11)}$  goes to zero for the strong form of “good” and to a constant for the weak form. For a “bad” singularity  $g_{00}^{(11)}$  is unbounded. The physical idea behind the criteria of good singularities is that one expects that, as one goes to the IR, fixed proper energies should correspond to smaller or nonincreasing energies in the dual field theory. The good singularities should correspond to different physical branches of the dual IR quantum mechanics theory and the bad singularities

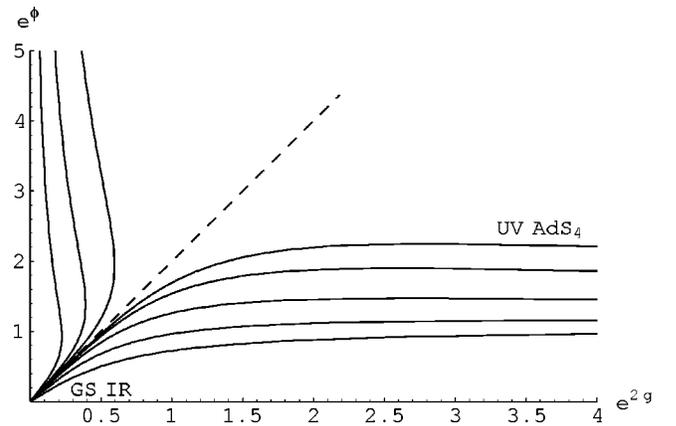


FIG. 2. Behavior of the flows for an  $H^2$  cycle in a Calabi-Yau twofold.

to nonphysical solutions. We shall return briefly to this in the final section.

Explicitly, for both  $\Sigma = S^2$  and  $\Sigma = H^2$  there is a bad singularity when  $e^{2g} \rightarrow 0$  and  $e^\phi \rightarrow \infty$ , where the BPS equations are dominated by the final terms proportional to  $l$ . (This limit is also universal, present whatever the dimension of the Calabi-Yau manifold and the value of  $l$ . However, in all cases, only for  $\Sigma = S^2$  is there a flow from the UV AdS<sub>4</sub>-type region to this singularity.) For either value of  $l$  there is also a good singularity when both  $e^{2g}$  and  $e^\phi$  tend to zero, where one can neglect the  $e^{\phi/2}$  terms in the BPS equations. For  $l = 1$ , the asymptotic solution has the form

$$ds^2 = - \left( \frac{z^2 - z_0^2}{4e^{-1}z} \right)^{1/2} dt^2 + \left( \frac{4e^{-1}z}{z^2 - z_0^2} \right)^{1/2} \times [dz^2 + (z^2 - z_0^2)ds^2(\Sigma)],$$

$$e^\phi = \left( \frac{z^2 - z_0^2}{e^{-1}z} \right)^{1/2}, \quad (3.11)$$

where  $z_0$  is a constant, such that  $z_0 \geq 0$  and the solution is valid only in leading order with  $z \rightarrow z_0$  from above. For  $l = -1$  the solution has the same form but with  $z^2 - z_0^2$  replaced by  $z_0^2 - z^2$ , with  $z_0 > 0$  and  $z \rightarrow z_0$  from below. In terms of the parameter  $C$  in Eq. (3.9), for  $\Sigma = S^2$ , solutions with  $C \leq -\sqrt{|l|e^2}$  flow to the good singularity, while those with  $C > -\sqrt{|l|e^2}$  flow to the bad singularity. For  $\Sigma = H^2$ , whatever the value of  $C$  all solutions flow to the good singularity.

### B. Calabi-Yau threefold

For this case, there is a natural split of SO(8) into SO(4) × SO(4). The diagonal U(1) gauge fields of U(2) ⊂ SO(4) are nonvanishing and so we take

$$F_1 = F_2 = -\frac{l}{4e} \text{Vol}(\Sigma), \quad F_3 = F_4 = 0, \quad (3.12)$$

where, as before, Vol(Σ) is the volume two-form on the two-cycle. The scalar fields are given by  $\vec{\phi} = (\phi, 0, 0)$ , so that  $X_1 = X_2 = e^{-\phi/2}$  and  $X_3 = X_4 = e^{\phi/2}$ . We expect a theory with four supercharges, and indeed find that setting the fermionic variations to zero requires, in addition to Eq. (3.3), the projections

$$\gamma^{12} \epsilon^{ai} = \epsilon^{ij} \epsilon^{ai} \quad \text{for } \alpha = 1, 2,$$

$$\epsilon^{ai} = 0 \quad \text{for } \alpha = 3, 4. \quad (3.13)$$

The variations then vanish provided we satisfy the BPS equations

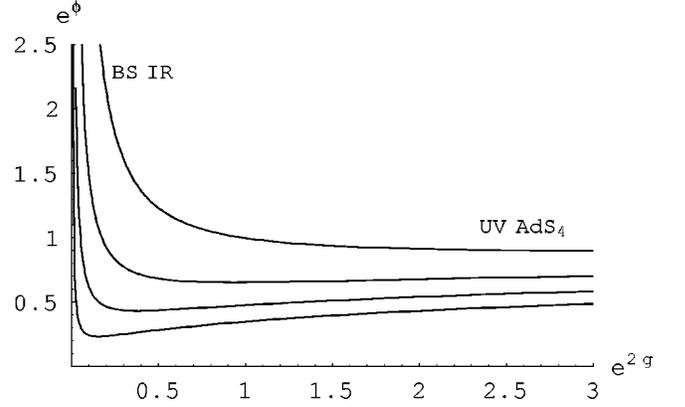


FIG. 3. Behavior of the flows for an  $S^2$  cycle in a Calabi-Yau threefold.

$$e^{-f} f' = -\frac{e}{\sqrt{2}} (e^{-\phi/2} + e^{\phi/2}) + \frac{l}{2\sqrt{2}e} e^{\phi/2-2g},$$

$$e^{-f} g' = -\frac{e}{\sqrt{2}} (e^{-\phi/2} + e^{\phi/2}) - \frac{l}{2\sqrt{2}e} e^{\phi/2-2g},$$

$$e^{-f} \phi' = -\sqrt{2}e (e^{-\phi/2} - e^{\phi/2}) + \frac{l}{\sqrt{2}e} e^{\phi/2-2g}. \quad (3.14)$$

To analyze these equations, one can introduce the variables  $x = e^{2g-\phi}$ ,  $F = e^{g+\phi/2}$  which then satisfy

$$\frac{dF}{dx} = \frac{F}{2F\sqrt{x} + l/e^2}. \quad (3.15)$$

It is possible to solve this equation exactly, but the resulting form is not very illuminating. It is anyway again straightforward to determine the asymptotic behavior of the solutions.

For large  $e^{2g}$  we have the AdS<sub>4</sub>-type region (3.10) describing the UV physics, now with  $\phi \approx cr + lr^2$ , where  $c$  is an arbitrary constant. We have illustrated the flows to the IR and the resulting types of singularities in Figs. 3 and 4. As

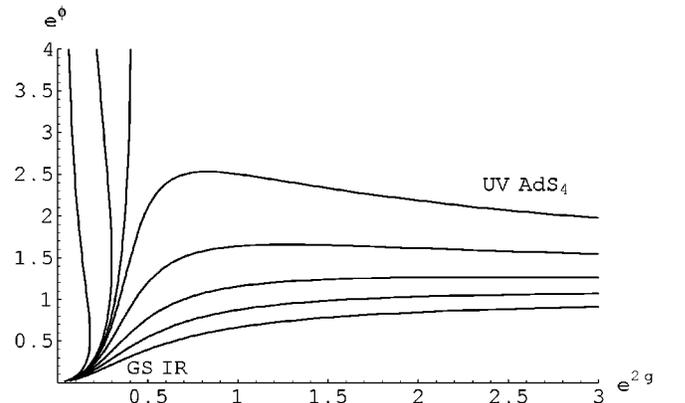


FIG. 4. Behavior of the flows for an  $H^2$  cycle in a Calabi-Yau threefold.

before, there are bad singularities in the regions of large  $e^\phi$  and small  $e^{2g}$  where the BPS equations (3.14) are dominated by the terms proportional to  $l$ . There is a good singularity only for  $\Sigma=H^2$ , in the region of small  $e^\phi$  and  $e^{2g}$  where one can neglect the  $e^{\phi/2}$  terms in Eq. (3.14). The asymptotic solution is given by

$$ds^2 = \left(\frac{r_0}{r}\right)^{1/2} e^{-2er} [-dt^2 + dr^2 + e^{-1} r ds^2(\Sigma)],$$

$$e^\phi = \left(\frac{r_0}{4r}\right)^{1/2} e^{-2er}, \quad (3.16)$$

where  $r_0$  is a constant.

### C. Calabi-Yau four-fold

In this case the  $SO(8)$  splits into  $SO(6) \times SO(2)$ . The only nonzero gauge fields are in the diagonal  $U(1)$  in  $U(3) \subset SO(6)$  and so we have

$$F_1 = F_2 = F_3 = -\frac{l}{6e} \text{Vol}(\Sigma), \quad F_4 = 0, \quad (3.17)$$

with  $\text{Vol}(\Sigma)$  the volume two-form on  $\Sigma$ . The scalar fields are given by  $\vec{\phi} = (\phi, \phi, -\phi)$ , so that  $X_1 = X_2 = X_3 = e^{-\phi/2}$  and  $X_4 = e^{3\phi/2}$ . Requiring the fermionic variations to vanish implies in addition to Eq. (3.3) the projections

$$\gamma^{12} \epsilon^{1i} = \epsilon^{ij} \epsilon^{1i},$$

$$\epsilon^{\alpha i} = 0 \quad \text{for } \alpha = 2, 3, 4, \quad (3.18)$$

which gives a theory with two supercharges, as expected. In addition, we find the BPS equations

$$e^{-f} f' = -\frac{e}{2\sqrt{2}} (3e^{-\phi/2} + e^{3\phi/2}) + \frac{l}{2\sqrt{2}e} e^{\phi/2-2g},$$

$$e^{-f} g' = -\frac{e}{2\sqrt{2}} (3e^{-\phi/2} + e^{3\phi/2}) - \frac{l}{2\sqrt{2}e} e^{\phi/2-2g},$$

$$e^{-f} \phi' = -\frac{e}{\sqrt{2}} (e^{-\phi/2} - e^{3\phi/2}) + \frac{l}{3\sqrt{2}e} e^{\phi/2-2g}. \quad (3.19)$$

As before it is straightforward to analyze the asymptotic behavior. For large  $e^{2g}$  there is the UV  $\text{AdS}_4$ -type region (3.10) now with  $\phi \approx cr + (\frac{1}{3}l + \frac{1}{2}c^2)r^2$  where  $c$  is an arbitrary constant. For  $\Sigma=S^2$  there is only one other asymptotic region at small  $e^{2g}$  and large  $e^\phi$ , where there is a bad singularity and the BPS equations are dominated by the terms proportional to  $l$ . The general flows are shown in Fig. 5.

For  $\Sigma=H^2$  the situation is considerably more complicated, as shown in Fig. 6. Aside from the UV  $\text{AdS}_4$ -type

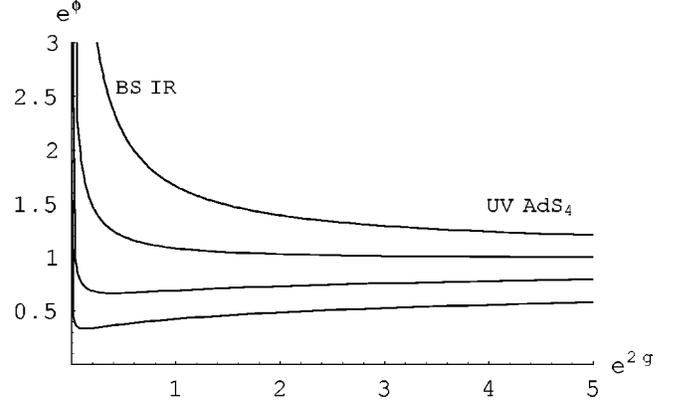


FIG. 5. Behavior of the flows for an  $S^2$  cycle in a Calabi-Yau fourfold.

region, there is also, for the first time, an  $\text{AdS}_2 \times H^2$  fixed point. Explicitly, there is an exact solution of Eq. (3.19) given by

$$ds^2 = \frac{1}{6\sqrt{3}e^2 r^2} (-dt^2 + dr^2) + \frac{1}{2\sqrt{3}e^2} ds^2(H^2),$$

$$e^\phi = \sqrt{3}. \quad (3.20)$$

This provides an example of “flow across dimensions” from a three-dimensional UV theory to a superconformal quantum mechanics IR fixed point.

In addition, there are two regions of bad singularities at both small  $e^{2g}$  and large  $e^\phi$ . One is the universal bad singularity where the terms proportional to  $l$  in Eq. (3.19) dominate. The other corresponds to a region where the  $l$  and  $e^{-\phi/2}$  terms are proportional and dominate. There is also a region at small  $e^{2g}$  and  $e^\phi$  where one can neglect the  $e^{3\phi/2}$  terms and which gives a good singularity. The asymptotic solution is

$$ds^2 = \frac{r_0^3}{r^3} \left[ -dt^2 + dr^2 + \frac{3}{4} r^2 ds^2(\Sigma) \right],$$

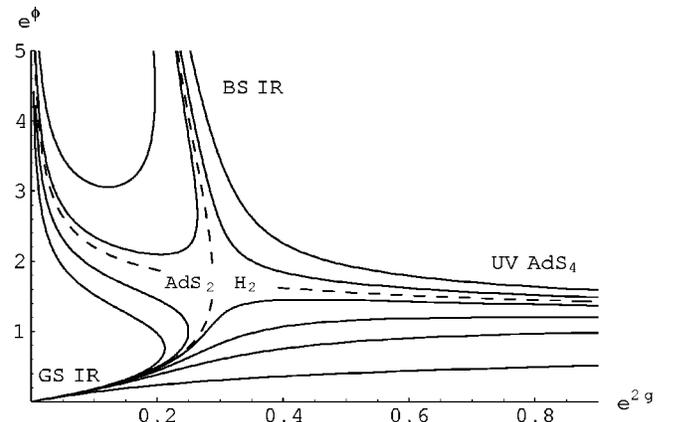


FIG. 6. Behavior of the flows for an  $H^2$  cycle in a Calabi-Yau fourfold.

$$e^\phi = \frac{9e^2 r_0^3}{8r}, \quad (3.21)$$

where  $r_0$  is a constant.

#### D. Calabi-Yau fivefold

For this case the diagonal  $U(1)$  of  $U(4) \subset SO(8)$  is the only nonvanishing gauge field giving rise to

$$F_1 = F_2 = F_3 = F_4 = -\frac{l}{8e} \text{Vol}(\Sigma), \quad (3.22)$$

where  $\text{Vol}(\Sigma)$  is the volume two-form on  $\Sigma$ . The scalar fields are all zero so that  $X_i = 1$ . The projections on the Killing spinors are exactly the same as in the Calabi-Yau fourfold case (3.3), (3.18), leading to preservation of two supercharges. The BPS equations then read

$$\begin{aligned} e^{-f} f' &= -\sqrt{2}e + \frac{l}{2\sqrt{2}e} e^{-2g}, \\ e^{-f} g' &= -\sqrt{2}e - \frac{l}{2\sqrt{2}e} e^{-2g}, \end{aligned} \quad (3.23)$$

We can find the general solution by first introducing a new radial variable defined by

$$\frac{d\rho}{dr} = e^{2f}. \quad (3.24)$$

We then obtain, after absorbing an integration constant into the definition of the coordinate  $t$ ,

$$\begin{aligned} ds^2 &= -\left(\sqrt{2}e\rho + \frac{l}{2\sqrt{2}e\rho}\right)^2 dt^2 + \left(\sqrt{2}e\rho + \frac{l}{2\sqrt{2}e\rho}\right)^{-2} d\rho^2 \\ &\quad + \rho^2 ds^2(\Sigma). \end{aligned} \quad (3.25)$$

When  $l=1$  we obtain a bad IR singularity in the IR. When  $l=-1$  the solution interpolates from the  $\text{AdS}_4$  type region to the superconformal  $\text{AdS}_2 \times H^2$  fixed point in the IR specified by

$$ds^2 = \frac{1}{8e^2 r^2} (-dt^2 + dr^2) + \frac{1}{4e^2} ds^2(H^2). \quad (3.26)$$

This IR fixed point is the gravity dual of a superconformal quantum mechanics.

We note that when  $l=-1$  the full solution (3.25) is the supersymmetric magnetically charged ‘‘topological’’ AdS black hole discussed in [14]. The term topological refers to the unusual feature that black holes in AdS space can admit spatial sections that are flat or have constant negative curvature. Here we see that this solution can be interpreted, after being uplifted to  $D=11$ , as the gravity dual corresponding to wrapped membranes in a Calabi-Yau fivefold. It was also observed in [14] that the four-dimensional solution with  $l$

$= -1$  can be generalized to include rotation while maintaining supersymmetry. In our conventions it is given by

$$\begin{aligned} ds^2 &= -\frac{\Delta_r}{\Xi^2 \rho^2} [dt + a \sinh^2 \theta d\phi]^2 + \frac{\rho^2}{\Delta_r} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 \\ &\quad + \frac{\Delta_\theta \sinh^2 \theta}{\Xi^2 \rho^2} [adt - (r^2 + a^2) d\phi]^2, \end{aligned} \quad (3.27)$$

with

$$\begin{aligned} \rho^2 &= r^2 + a^2 \cosh^2 \theta, \quad \Xi = 1 + 2e^2 a^2, \\ \Delta_r &= r^2 \left[ \sqrt{2}er - \frac{1}{2\sqrt{2}er} (1 - 2e^2 a^2) \right]^2, \\ \Delta_\theta &= 1 + 2e^2 a^2 \cosh^2 \theta, \end{aligned} \quad (3.28)$$

and

$$A_1 = -\frac{(1 + 2e^2 a^2) \cosh \theta}{8e \Xi \rho^2} [adt - (r^2 + a^2) d\phi]. \quad (3.29)$$

An interesting feature of this solution is that it is regular provided that the rotation parameter  $a$  satisfies  $a < \sqrt{2}e$ . We can use this four-dimensional solution to obtain a new  $D=11$  supersymmetric solution by uplifting using the ansatz (2.1) and (2.2). This  $D=11$  solution can be interpreted as the gravity dual corresponding to supersymmetric waves on wrapped membranes. The bound on the angular momentum parameter is reminiscent of the ‘‘stringy exclusion principle’’ observed in [28] and it would be interesting to explore this in more detail.

#### IV. GENERALIZED CALIBRATIONS

Our  $D=11$  solutions correspond to the near-horizon geometry of membranes wrapping holomorphic two-cycles in a Calabi-Yau  $(n+1)$ -fold. Being supersymmetric we expect that a probe membrane will be static and supersymmetric if it wraps the same two-cycle, or more precisely a holomorphic cycle in the same homology class. In the language of [15] this means that we expect that the  $D=11$  solutions admit generalized Kähler two-form calibrations  $\Omega$ . A static, supersymmetric probe brane then minimizes the pullback of  $\Omega$  integrated over the two-cycle. The calibration  $\Omega$  can be constructed from  $D=11$  Killing spinors  $\epsilon$  via  $\Omega_{MN} = \bar{\epsilon} \Gamma_{MN} \epsilon$ .

Let us give an explicit expression for  $\Omega$  for our solutions and show that it is indeed a calibration. First it is useful to introduce a slightly nonobvious  $D=11$  orthonormal frame

$$\begin{aligned} e^0 &= \Delta^{1/3} e^f dt, \\ e^1 &= \Delta^{1/3} e^g e^{-1}, \\ e^2 &= \Delta^{1/3} e^g e^{-2}, \\ e^{\rho\alpha} &= \Delta^{-1/6} [e^f X_\alpha^{1/2} \mu_\alpha dr - \sqrt{2} e^{-1} X_\alpha^{-1/2} d\mu_\alpha], \\ e^{\phi\alpha} &= \sqrt{2} e^{-1} \Delta^{-1/6} X_\alpha^{-1/2} \mu_\alpha (d\phi_\alpha + 2eA_\alpha), \end{aligned} \quad (4.1)$$

where, as before,  $\bar{e}^1$  and  $\bar{e}^2$  define an orthonormal frame for the two-cycle. We then have, in all cases,

$$\Omega = \Delta^{1/3} e^f \left[ e^1 \wedge e^2 + \sum_{\alpha} e^{\rho_{\alpha}} \wedge e^{\phi_{\alpha}} \right]. \quad (4.2)$$

For the directions  $\alpha$  with vanishing gauge fields we find  $d[\Delta^{1/3} e^f e^{\rho_{\alpha}} \wedge e^{\phi_{\alpha}}] = 0$ . Using this and the relevant BPS equations, we can show that

$$d\Omega = i_k G, \quad (4.3)$$

where  $k = \partial/\partial t$ . This is one of the conditions satisfied by the generalized calibration (and in fact follows from the spinorial construction). In addition, we note that  $\Omega$  naturally defines an almost complex structure  $J$  on the ten-dimensional spatial part of our solution. In the orthonormal frame,  $J$  simply pairs  $e^1$  with  $e^2$  and  $e^{\rho_{\alpha}}$  with  $e^{\phi_{\alpha}}$ . Formally it can be defined by raising one index of  $\Omega$  using the rescaled  $D = 10$  spatial metric:

$$d\tilde{s}^2 = \Delta^{1/3} e^f \left[ e^1 e^1 + e^2 e^2 + \sum_{\alpha} e^{\rho_{\alpha}} e^{\rho_{\alpha}} + \sum_{\alpha} e^{\phi_{\alpha}} e^{\phi_{\alpha}} \right]. \quad (4.4)$$

We have checked that it is in fact integrable (without using the BPS equations) and so the ten-dimensional spatial part of our solutions in fact describes a complex manifold. This and Eq. (4.3) establish that  $\Omega$  is indeed a generalized calibration [15]. Note that, in addition, as is easy to see in the orthonormal frame, the spatial part of our metric is Hermitian with respect to this complex structure.

An alternative approach to finding solutions corresponding to membranes wrapping holomorphic curves in Calabi-Yau  $(n+1)$ -folds was discussed in [25]. Building on the work of [21] and exploiting the existence of a generalized calibration an ansatz for the solutions was presented. The BPS equations were derived but no solutions were given. The ansatz has the form

$$ds^2 = -H^{-2n/3} dt^2 + H^{(n-3)/3} g_{AB} dy^A dy^B + H^{n/3} \delta_{IJ} dx^I dx^J, \quad (4.5)$$

$$A_{(3)} = \pm H^{-1} dt \wedge \omega,$$

where  $y^A$  with  $A = 1, \dots, 2n+2$  are real coordinates on a complex  $(n+1)$ -fold with a Hermitian metric  $g_{AB}$  (the analogue of the original Calabi-Yau manifold), and  $x^I$  with  $I = 1, \dots, 8-2n$  denote the remaining transverse directions. The two-form  $\omega$  is related to the Hermitian metric  $g_{AB}$  by  $\omega_{AB} = J^C_{A} g_{CB}$  where  $J$  is the complex structure on the  $(n+1)$ -fold. Both  $g_{AB}$  and  $\omega_{AB}$  are functions of  $y^A$  and  $x^I$  as is  $H$ . Supersymmetry then puts various constraints on  $g_{AB}$  and  $H$ , for instance implying that for fixed  $x^I$  the metric  $g_{AB}$  is Kähler.

To connect this work to our solutions, we would like to show that they can be written in the form (4.5). Starting with our metric ansatz (2.1), it is useful to introduce new coordinates,

$$\rho_a = -\frac{\sqrt{2}}{e} \mu_a X_a^{-1/2} e^{f/2} e^{(g-f)/n},$$

$$\rho_i = -\frac{\sqrt{2}}{e} \mu_i X_i^{-1/2} e^{f/2}, \quad (4.6)$$

where  $a = 1, \dots, n$  labels the gauged directions and  $i = 1, \dots, 4-n$  the ungauged ones. One can then check using the BPS equations that the frame (4.1) can be written as

$$e^{\rho_a} = \Delta^{-1/6} e^{-f/2} e^{(f-g)/n} d\rho_a,$$

$$e^{\phi_a} = -\Delta^{-1/6} e^{-f/2} e^{(f-g)/n} \rho_a (d\phi_a + 2eA_a),$$

$$e^{\rho_i} = \Delta^{-1/6} e^{-f/2} d\rho_i,$$

$$e^{\phi_i} = -\Delta^{-1/6} e^{-f/2} \rho_i d\phi_i. \quad (4.7)$$

The  $D = 11$  metric (2.1) is then given by

$$ds_{11}^2 = -\Delta^{2/3} e^{2f} dt^2 + \Delta^{-1/3} e^{-f+2(f-g)/n} \times \sum_{a=1}^n [d\rho_a^2 + \rho_a^2 (d\phi_a + 2eA_a)^2] + \Delta^{2/3} e^{2g} ds^2(\Sigma) + \Delta^{-1/3} e^{-f} \sum_{i=1}^{4-n} (d\rho_i^2 + \rho_i^2 d\phi_i^2), \quad (4.8)$$

with the three-form potential

$$A_{(3)} = -dt \wedge \left[ -e^{2(f-g)/n} \sum_{a=1}^n \rho_a d\rho_a \wedge (d\phi_a + 2eA_a) + \Delta e^{f+2g} \text{Vol}(\Sigma) \right]. \quad (4.9)$$

Comparing with Eq. (4.5) we can then identify

$$H = \Delta^{-1/n} e^{-3f/n} \quad (4.10)$$

and

$$g_{AB} dy^A dy^B = \Delta^{1/n} e^{3f/n} \left[ e^{2(f-g)/n} \sum_{a=1}^n [d\rho_a^2 + \rho_a^2 (d\phi_a + 2eA_a)^2] + \Delta e^{f+2g} ds^2(\Sigma) \right]. \quad (4.11)$$

Given the arguments above that the spatial part of our solutions describes a complex manifold with a Hermitian metric, we see that  $g_{AB}$  is indeed Hermitian. Since our solution is

supersymmetric, preserving  $2^{-n}$  of the supersymmetry for  $n=1, 2, 3, 4$  and  $2^{-4}$  for  $n=5$ , we expect that  $H$  and  $g_{AB}$  satisfy the conditions given in [25]. One should note that in general our ansatz (2.1) and (2.2) is not equivalent to the ansatz (4.5). It was only for the particular BPS solutions that we were able to rewrite one as the other.

## V. DISCUSSION

We have presented BPS equations and constructed solutions of  $D=11$  supergravity that are dual to the twisted theories arising on membranes wrapping holomorphic curves in Calabi-Yau  $n$ -folds. For the fourfolds and fivefolds we found exact conformal fixed points when the membrane wraps a Riemann surface of genus  $g>1$ . It would be interesting to determine the physical reason for such fixed points being present only for these two cases. We also analyzed the BPS equations numerically and analyzed the types of singularity encountered in the IR. For the fivefold case we managed to find the most general solutions and it would be nice if the same could be achieved for the other cases.

In Sec. IV we elucidated some of the structure of the solutions by presenting the generalized calibrations. In addition a new set of coordinates was introduced that connects the solutions with other work in the literature. It is likely that the analysis of this section can be applied to other supergravity solutions describing wrapped branes that are obtained using the technique of [3].

For the case of Calabi-Yau fivefolds when the scalar fields are vanishing, we noted that the four-dimensional solution interpolating from the UV  $\text{AdS}_4$  region to the IR  $\text{AdS}_2 \times H_2$  fixed point is in fact the ‘‘topological AdS black hole’’ of [14]. For the Calabi-Yau fourfolds we numerically demonstrated a flow from the UV  $\text{AdS}_4$  region to the IR  $\text{AdS}_2 \times H_2$  fixed point. This can be similarly considered to be a ‘‘topological AdS black hole’’ with scalar hair. By analogy with what was found for the fivefold case, it seems likely that a rotating version also exists. More generally, it seems likely that the flows to IR AdS fixed points considered in [3,6,5,7,8] will also have rotating generalizations.

The focus of the paper has been on finding new solutions

and exploring some of their geometry. We now conclude by briefly discussing the interpretation of the flows from the UV to the IR from the dual field theory (quantum mechanics) perspective. The motion of the wrapped membranes transverse to the two-cycle and tangent to the Calabi-Yau correspond to possible ‘‘Higgs branches’’ while motion that is also transverse to the Calabi-Yau corresponds to ‘‘Coulomb branches.’’ Classically we do not expect Higgs branches for the case of membranes wrapping the two-sphere, as the corresponding scalar fields of the membrane theory, after twisting, will not have zero modes. On the other hand we do expect them for the case of membranes wrapped on Riemann surfaces with genus  $g$  greater than 1. Naively then one would expect good singularities in the IR of the supergravity solutions corresponding to each physical branch. For the case of membranes wrapping a two-sphere we thus interpret the good singularities that arise for the Calabi-Yau twofold case as corresponding to the Coulomb branch. For the remaining cases with  $l=1$  we see only bad singularities in the IR, which suggests that the Coulomb branches are not accessible in the limits we are considering. For the  $l=-1$  cases we always see a branch of good singularities which could correspond to either Coulomb or Higgs branches or both. We expect that any conformal fixed point should appear at the junction between the two branches. However, for the fourfold case we do have a fixed point but with good singularities only on one side. This suggests that in fact again only one branch is accessible in these solutions. It would naturally be interesting to investigate the gravity/field theory correspondence for the flows we have presented beyond these simple observations. Perhaps the cleanest direction is to focus on the superconformal quantum mechanics at the IR fixed points that we found for membranes wrapping Riemann surfaces with  $g>1$  in Calabi-Yau fourfolds and fivefolds.

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