Scalar-tensor bimetric brane world cosmology

Donam Youm*
ICTP, Strada Costiera 11, 34014 Trieste, Italy
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We study a scalar-tensor bimetric cosmology in the Randall-Sundrum model with one positive tension brane, where the biscalar field is assumed to be confined on the brane. The effective Friedmann equations on the brane are obtained and analyzed. We comment on the resolution of cosmological problems in this bimetric model.

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Variable-speed-of-light (VSL) cosmologies were proposed [1,2] as possible solutions to the initial value problems in the standard big bang (SBB) model. VSL models assume that the speed of light initially took a larger value and then decreased to the present day value at an early time. The VSL models solve various cosmological problems of the SBB model associated with the initial value problems, including those solved by the inflationary scenario [3–5]. In the original models by Moffat [1] and by Albrecht and Magueijo [2], the speed of light c (and possibly Newton's constant G_4) in the action, which is a fundamental constant of the nature, is just assumed to vary with time during an early period of cosmic evolution and thereby the Lorentz symmetry becomes explicitly broken. So, it becomes necessary to assume that there exists a preferred frame in which the laws of physics take simple forms. Later, Clayton and Moffat [6–9] proposed an ingenious mechanism by which the speed of light can vary with time in a diffeomorphism invariant manner and without explicitly breaking the Lorentz symmetry. (See also Ref. [10] for an independent development.) In their models, two metrics are introduced into the spacetime manifold (thereby their models are called bimetric), one being associated with gravitons and the other providing the geometry on which matter fields, including photons, propagate. Since these two metrics are nonconformally related by a scalar field (called a biscalar) or a vector field (called a bivector), photons and gravitons propagate at different speeds.

It has been shown [11–18] that brane world models manifest the Lorentz violation, which is a necessary requirement for the VSL models. Therefore, it would be of interest to study the VSL cosmologies within the context of the brane world scenarios. In particular, the VSL models may provide a possible mechanism for bringing the quantum corrections to the fine-tuned brane tensions under control, since the VSL models generally solve the cosmological constant problem. In our previous work [19], we studied the VSL cosmologies in the Randall-Sundrum (RS) scenarios [20,21], following the approach of the earlier VSL models [1,2,22–26] with varying fundamental constants. In this paper, we follow the approach of Clayton and Moffat to study the bimetric cosmology in the RS2 model [21].

In the bimetric models it is usually assumed that gravitons and the biscalar[1] (or the bivector) propagate on the geometry described by the "gravity metric," whereas all the matter fields propagate on the geometry described by the "matter metric." So, a natural bimetric modification of a brane world model would be just to modify the brane matter field action to be constructed out of the matter metric. We consider the bimetric model with a biscalar, which is assumed to be confined on the brane world volume. The action for the bimetric RS2 model with the brane matter fields therefore takes the following form:

$$S=\int d^5x\sqrt{-G}\left[\frac{c^4}{16\pi G_5}\mathcal{R}-\Lambda\right]+\int d^4x\sqrt{-\hat{g}}\mathcal{L}_{\rm mat} -\int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi+V(\Phi)+\sigma\right], \tag{1}$$

where Φ is the biscalar field with the potential $V(\Phi)$ and σ is the tension of the brane assumed to be located at the origin $y=0$ of the extra spatial coordinate y. Here, the gravity metric $g_{\mu\nu}$ and the matter metric $\hat{g}_{\mu\nu}$ on the brane are given in terms of the bulk metric G_{MN} and Φ by

$$g_{\mu\nu}=G_{\mu\nu}(x^\rho,0),\quad \hat{g}_{\mu\nu}=g_{\mu\nu}-B\partial_\mu\Phi\partial_\nu\Phi, \tag{2}$$

where a dimensionless constant B is assumed to be positive. The Lagrangian $\mathcal{L}_{\rm mat}$ for the brane matter fields is constructed out of $\hat{g}_{\mu\nu}$.

We study the brane world cosmology associated with the above action. The general metric ansatz for the expanding brane universe where the principle of homogeneity and isotropy in the three-dimensional space on the three-brane is satisfied is given by

$$G_{MN}dx^Mdx^N=-n^2(t,y)c^2dt^2+a^2(t,y)\gamma_{ij}dx^idx^j +b^2(t,y)dy^2, \tag{3}$$

where γ_{ij} is the metric for the maximally symmetric three-dimensional space given in the Cartesian and the spherical coordinates by

[1]As shown in Ref. [7], one can also rewrite the biscalar equation of motion in such a way that the biscalar field appears to propagate on a different geometry described by the metric expressed in terms of the gravity metric and the matter field energy-momentum tensor.

*Email address: youmd@ictp.trieste.it

$$\gamma_{ij}dx^i dx^j = \left(1+\frac{k}{4}\delta_{mn}x^m x^n\right)^{-2}\delta_{ij}dx^i dx^j$$
$$=\frac{dr^2}{1-kr^2}+r^2(d\theta^2+\sin^2\theta d\phi^2), \tag{4}$$

with $k=-1,0,1$ respectively for the three-dimensional spaces with the negative, zero and positive spatial curvatures. Making use of the fact that it is always possible to choose a gauge so that $n(t,0)$ is constant without introducing the cross term G_{ty}, we scale the time coordinate such that $n(t,0)=1$. With the assumption of homogeneity and isotropy on the three-brane, the biscalar field Φ does not depend on the spatial coordinates x^i $(i=1,2,3)$ of the three-brane.

In obtaining the Einstein's equations by varying the action with respect to the metric, we have to keep in mind that $\hat{g}_{\mu\nu}$ is the physical metric for matter fields on the brane. So, the energy-momentum tensor for the matter fields are defined in terms of $\hat{g}_{\mu\nu}$:

$$\hat{T}^{\mu\nu}\equiv\frac{2}{\sqrt{-\hat{g}}}\frac{\delta(\sqrt{-\hat{g}}\mathcal{L}_{\rm mat})}{\delta\hat{g}_{\mu\nu}}. \tag{5}$$

Modeling the brane matter fields as perfect fluid, we can put this energy-momentum tensor into the following standard form:

$$\hat{T}^{\mu\nu}=\left(\varrho+\frac{\wp}{c^2}\right)U^\mu U^\nu+\wp\hat{g}^{\mu\nu}, \tag{6}$$

where ϱ, $\wp$ and U^μ are respectively the mass density, the pressure and the four-velocity of the fluid, and the inverse $\hat{g}^{\mu\nu}$ of $\hat{g}_{\mu\nu}$ is given by

$$\hat{g}^{\mu\nu}=g^{\mu\nu}+\frac{B}{1-Bg^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi}g^{\mu\mu'}g^{\nu\nu'}\partial_{\mu'}\Phi\partial_{\nu'}\Phi. \tag{7}$$

The four-velocity is normalized as $\hat{g}_{\mu\nu}U^\mu U^\nu=-c^2$. So, in the comoving coordinates, the nonzero component of U^μ is given by

$$U^t=\frac{1}{\sqrt{1+B\dot{\Phi}^2/c^2}}, \tag{8}$$

where the overdot denotes derivative with respect to t. The nonzero components of the energy-momentum tensor for the brane matter fields are therefore

$$\hat{T}^{tt}=\frac{\varrho}{1+B\dot{\Phi}^2/c^2}, \quad \hat{T}^{ij}=\frac{\wp}{a_0^2}\gamma^{ij}. \tag{9}$$

Since we assume that the matter field action is constructed out of $\hat{g}_{\mu\nu}$, the equations of motion of the brane matter fields imply the conservation law for the brane matter fields energy-momentum tensor:

$$\frac{1}{\sqrt{-\hat{g}}}\partial_\mu(\sqrt{-\hat{g}}\hat{T}^{\mu\nu})=0, \tag{10}$$

which takes the following standard form after the above expressions for the energy-momentum tensor and the matter metric are substituted:

$$\dot{\varrho}+3\left(\varrho+\frac{\wp}{c^2}\right)\frac{\dot{a}_0}{a_0}=0, \tag{11}$$

where the subscript 0 denotes quantities evaluated at $y=0$, i.e., $a_0(t)\equiv a(t,0)$. This conservation equation can be derived independently from the effective four-dimensional Friedmann equations on the brane, implying that Eq. (10) is consistent with the Einstein's equations (12) in the below.

Taking the variation of the action S with respect to the metric, we obtain the following Einstein's equations:

$$\mathcal{G}^{MN}=\frac{8\pi G_5}{c^4}\mathcal{T}^{MN}, \tag{12}$$

with the total energy-momentum tensor given by

$$\mathcal{T}^{MN}=-G^{MN}\Lambda+\delta^M_\mu\delta^N_\nu\left[\hat{T}^{\mu\nu}\frac{\sqrt{-\hat{g}}}{\sqrt{-G}}+\left\{g^{\mu\mu'}g^{\nu\nu'}\partial_{\mu'}\Phi\partial_{\nu'}\Phi\right.\right.$$
$$\left.\left.-\frac{1}{2}g^{\mu\nu}\partial_\alpha\Phi\partial^\alpha\Phi-g^{\mu\nu}V(\Phi)-g^{\mu\nu}\sigma\right\}\frac{\sqrt{-g}}{\sqrt{-G}}\right]\delta(y). \tag{13}$$

The equation of motion for the biscalar is

$$\nabla^2\Phi-V'(\Phi)+B\frac{\sqrt{-\hat{g}}}{\sqrt{-g}}\hat{T}^{\mu\nu}\hat{\nabla}_\mu\hat{\nabla}_\nu\Phi=0, \tag{14}$$

where $\nabla^2\Phi=g^{\mu\nu}\nabla_\mu\nabla_\nu\Phi$ and the covariant derivative ∇_μ $[\hat{\nabla}_\mu]$ is defined in terms of the metric $g_{\mu\nu}$ $[\hat{g}_{\mu\nu}]$. We made use of the conservation law (10) to achieve the simplified form of the last term on the LHS and the prime on the biscalar potential V denotes derivative with respect to Φ. The above biscalar equation has dependence on $\hat{T}^{\mu\nu}$ due to the fact that the matter metric $\hat{g}_{\mu\nu}$, of which the brane matter fields action is made, depends on $\partial_\mu\Phi$.

After the above ansatz for the metric and the biscalar field are substituted, the Einstein's equations (12) take the following forms:

$$\frac{3}{n^2c^2}\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}\right)-\frac{3}{b^2}\left[\frac{a'}{a}\left(\frac{a'}{a}-\frac{b'}{b}\right)+\frac{a''}{a}\right]+\frac{3k}{a^2}$$
$$=\frac{8\pi G_5}{c^4}\left[\Lambda+\left(\varrho_\Phi c^2+\frac{\varrho c^2}{\sqrt{I}}+\sigma\right)\frac{\delta(y)}{b}\right], \tag{15}$$

$$\frac{1}{b^2}\left[\frac{a'}{a}\left(2\frac{n'}{n}+\frac{a'}{a}\right)-\frac{b'}{b}\left(\frac{n'}{n}+2\frac{a'}{a}\right)+2\frac{a''}{a}+\frac{n''}{n}\right] + \frac{1}{n^2c^2}\left[\frac{\dot{a}}{a}\left(2\frac{\dot{n}}{n}-\frac{\dot{a}}{a}\right)+\frac{\dot{b}}{b}\left(\frac{\dot{n}}{n}-2\frac{\dot{a}}{a}\right)-2\frac{\ddot{a}}{a}-\frac{\ddot{b}}{b}\right]-\frac{k}{a^2} = \frac{8\pi G_5}{c^4}\left[-\Lambda+(\wp_\Phi+\sqrt{I}\wp-\sigma)\frac{\delta(y)}{b}\right], \tag{16}$$

$$\frac{n'}{n}\frac{\dot{a}}{a}+\frac{a'}{a}\frac{\dot{b}}{b}-\frac{\dot{a}'}{a}=0, \tag{17}$$

$$\frac{3}{b^2}\frac{a'}{a}\left(\frac{a'}{a}+\frac{n'}{n}\right)-\frac{3}{n^2c^2}\left[\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}-\frac{\dot{n}}{n}\right)+\frac{\ddot{a}}{a}\right]-\frac{3k}{a^2} = -\frac{8\pi G_5}{c^4}\Lambda, \tag{18}$$

where the primes on the metric components denote derivative with respect to y. Here, the biscalar field mass density ϱ_Φ and pressure $\wp_\Phi$ and I are defined as

$$\varrho_\Phi=\left(\frac{1}{2}\frac{\dot{\Phi}^2}{c^2}+V\right)\frac{1}{c^2}, \quad \wp_\Phi=\frac{1}{2}\frac{\dot{\Phi}^2}{c^2}-V, \quad I\equiv 1+B\frac{\dot{\Phi}^2}{c^2}. \tag{19}$$

The biscalar field equation (14) takes the following form:

$$\frac{1}{c^2}\left(1-\frac{c^2B}{I^{3/2}}\varrho\right)\ddot{\Phi}+\frac{3}{c^2}\frac{\dot{a}_0}{a_0}\dot{\Phi}\left(1+\frac{B}{\sqrt{I}}\wp\right)+V'(\Phi)=0. \tag{20}$$

In the above equations of motion, we made use of the assumption $n(t,0)=1$ to simplify the expressions.

The derivatives of the metric components with respect to y are discontinuous at $y=0$ due to the δ-function like brane source there. From Eqs. (15),(16), we obtain the following boundary conditions on the first derivatives of a and n at $y=0$:

$$\frac{[a']_0}{a_0b_0}=-\frac{8\pi G_5}{3c^4}(\sigma+\varrho_{\rm tot}c^2), \tag{21}$$

$$\frac{[n']_0}{n_0b_0}=-\frac{8\pi G_5}{3c^4}(\sigma-3\wp_{\rm tot}-2\varrho_{\rm tot}c^2), \tag{22}$$

where

$$\varrho_{\rm tot}\equiv\varrho_\Phi+\varrho/\sqrt{I}, \quad \wp_{\rm tot}\equiv\wp_\Phi+\sqrt{I}\wp. \tag{23}$$

Here, $[F]_0\equiv F(0^+)-F(0^-)$ denotes the jump of a function $F(y)$ across $y=0$.

The effective four-dimensional Friedmann equations on the three-brane world volume can be obtained [27] by taking the jumps and the mean values of the above five-dimensional Einstein's equations across $y=0$ and then applying the boundary conditions (21),(22) on the first derivatives of the metric components. Here, the mean value of a function F across $y=0$ is defined as $\sharp F\sharp\equiv[F(0^+)+F(0^-)]/2$. In this paper, we assume that the brane universe is invariant under the $\mathbf{Z}_2$ symmetry, $y\to -y$. Then, the mean value of the first derivative of a function across $y=0$ vanishes. We also define the y-coordinate to be proportional to the proper distance along the y-direction with b being the constant of proportionality, so $b'=0$. We further assume that the radius of the extra space is stabilized, i.e., $\dot{b}=0$, due to some mechanism involving for example a bulk scalar field with a stabilizing potential (cf. Refs. [28,29]). Making use of these assumptions, we define the y-coordinate such that $b=1$. The resulting effective four-dimensional Friedmann equations take the following forms:

$$\left(\frac{\dot{a}_0}{a_0}\right)^2=\frac{16\pi^2G_5^2}{9c^6}(\varrho_{\rm tot}^2c^4+2\sigma\varrho_{\rm tot}c^2) +c^2\frac{a''_{R\,0}}{a_0}+\frac{8\pi G_5}{3c^2}\left(\Lambda+\frac{2\pi G_5}{3c^4}\sigma^2\right)-\frac{kc^2}{a_0^2}, \tag{24}$$

$$\frac{\ddot{a}_0}{a_0}=-\frac{16\pi^2G_5^2}{9c^6}(2\varrho_{\rm tot}^2c^4+\sigma\varrho_{\rm tot}c^2+3\sigma\wp_{\rm tot}+3\wp_{\rm tot}\varrho_{\rm tot}c^2) -c^2\frac{a''_{R\,0}}{a_0}+\frac{16\pi^2G_5^2}{9c^6}\sigma^2, \tag{25}$$

where the subscript R denotes the regular part of a function (note, a'' has a δ-function like singularity at $y=0$).

The $a''_{R\,0}$ term (called ''dark radiation'' term) in the above Friedmann equations originates from the Weyl tensor of the bulk and thus describes the back reaction of the bulk gravitational degrees of freedom on the brane [27,30–33]. This term can be evaluated by solving a''_R as a function of y from the following equation obtained from the Einstein's equations (15),(16),(18):

$$3\frac{a''_R}{a}+\frac{n''_R}{n}=-\frac{16\pi G_5}{3c^4}\Lambda, \tag{26}$$

along with the following relation obtained from the (t,y)-component Einstein's equation (17) with the assumed $\dot{b}=0$ condition:

$$n(t,y)=\lambda(t)\dot{a}(t,y), \tag{27}$$

where $\lambda(t)$ is an arbitrary function of t. The resulting expression is

$$a''_R=\frac{\mathcal{C}}{a^3}-\frac{4\pi G_5}{3c^4}\Lambda a, \tag{28}$$

where $\mathcal{C}$ is an integration constant.

To make contact with conventional cosmology having the Hubble parameter proportional to $\sqrt{\varrho}$, we assume that σ

$\gg \varrho_{\rm tot}c^2, \wp_{\rm tot}$ [34,35]. To the leading order, the effective Friedmann equations (24),(25) along with Eq. (28) then take the forms

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{32\pi^2 G_5^2\sigma}{9c^4\sqrt{I}}\varrho + \frac{32\pi^2 G_5^2\sigma}{9c^6}\left(\frac{1}{2}\frac{\dot{\Phi}^2}{c^2}+V\right) + \frac{4\pi G_5}{3c^2}\left(\Lambda + \frac{4\pi G_5}{3c^4}\sigma^2\right) + \frac{\mathcal{C}c^2}{a_0^4} - \frac{kc^2}{a_0^2}, \tag{29}$$

$$\frac{\ddot{a}_0}{a_0} = -\frac{16\pi^2 G_5^2\sigma}{9c^4\sqrt{I}}\left(\varrho + 3I\frac{\wp}{c^2}\right) - \frac{32\pi^2 G_5^2\sigma}{9c^6}\left(\frac{\dot{\Phi}^2}{c^2}-V\right) + \frac{4\pi G_5}{3c^2}\left(\Lambda + \frac{4\pi G_5}{3c^4}\sigma^2\right) - \frac{\mathcal{C}c^2}{a_0^4}. \tag{30}$$

These effective Friedmann equations for the bimetric brane world cosmology have the same forms as the Friedmann equations for the scalar-tensor bimetric model of Clayton and Moffat except for the dark radiation term $\mathcal{C}c^2/a_0^4$.

Note, the overdots in the above effective equations denote derivatives with respect to the time coordinate t, with which the matter metric takes the form

$$\hat{g}_{\mu\nu}dx^\mu dx^\nu = -[c^2 + B\dot{\Phi}^2]dt^2 + a_0^2(t)\gamma_{ij}dx^i dx^j, \tag{31}$$

and the gravity metric on the brane is given by

$$g_{\mu\nu}dx^\mu dx^\nu = -c^2 dt^2 + a_0^2(t)\gamma_{ij}dx^i dx^j. \tag{32}$$

Namely, the above effective equations are written in a comoving frame for the gravity metric. As can be seen from these metric expressions, with a choice of time coordinate t, a graviton travels with a constant speed $c_{\rm grav}=c$ and a photon, which is coupled to $\hat{g}_{\mu\nu}$, travels with variable speed $c_{\rm ph}=\sqrt{c^2+B\dot{\Phi}^2}=c\sqrt{I}$. So, a photon is observed to travel faster than the present day speed in this frame, while the biscalar field varies with t.

Since all the matter fields on the brane are coupled to the matter metric $\hat{g}_{\mu\nu}$, it would be more natural to consider the comoving frame for the matter metric in order to make a connection with standard cosmology. By defining the cosmic time τ of the brane universe in the following way

$$d\tau^2 \equiv (1 + B\dot{\Phi}^2/c^2)dt^2, \tag{33}$$

we can bring the matter metric into the following standard comoving frame form for the Robertson-Walker metric:

$$\hat{g}_{\mu\nu}dx^\mu dx^\nu = -c^2 d\tau^2 + a_0^2(\tau)\gamma_{ij}dx^i dx^j. \tag{34}$$

In this new frame, the gravity metric (32) takes the form

$$g_{\mu\nu}dx^\mu dx^\nu = -[c^2 - B\dot{\Phi}^2]d\tau^2 + a_0^2(\tau)\gamma_{ij}dx^i dx^j, \tag{35}$$

where the overdot from now on stands for derivative with respect to τ. So, in the matter metric comoving frame with the time coordinate τ, a photon travels with a constant speed $c_{\rm ph}=c$ and a graviton travels with a time-variable speed $c_{\rm grav}=\sqrt{c^2-B\dot{\Phi}^2}=c/\sqrt{I}$. Note, $I=1/(1-B\dot{\Phi}^2/c^2)$ when the overdot stands for derivative w.r.t. τ. So, a graviton is observed to travel slower than the present day speed in this new frame, while Φ varies with τ. In this new frame, the effective Friedmann equations (29),(30) take the following forms:

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{32\pi^2 G_5^2\sigma}{9c^4 I^{3/2}}\varrho + \frac{32\pi^2 G_5^2\sigma}{9c^6 I}\left(\frac{I}{2}\frac{\dot{\Phi}^2}{c^2}+V\right) + \frac{4\pi G_5}{3c^2 I}\left(\Lambda + \frac{4\pi G_5}{3c^4}\sigma^2\right) + \frac{\mathcal{C}c^2}{a_0^4 I} - \frac{kc^2}{a_0^2 I}, \tag{36}$$

$$\frac{\ddot{a}_0}{a_0} + \frac{1}{2}\frac{\dot{I}}{I}\frac{\dot{a}_0}{a_0} = -\frac{16\pi^2 G_5^2\sigma}{9c^4 I^{3/2}}\left(\varrho + 3I\frac{\wp}{c^2}\right) - \frac{32\pi^2 G_5^2\sigma}{9c^6 I}\left(I\frac{\dot{\Phi}^2}{c^2}-V\right) + \frac{4\pi G_5}{3c^2 I}\left(\Lambda + \frac{4\pi G_5}{3c^4}\sigma^2\right) - \frac{\mathcal{C}c^2}{a_0^4 I}, \tag{37}$$

and the biscalar equation (20) takes the form

$$\frac{I^2}{c^2}\left(1 - \frac{c^2 B}{I^{3/2}}\varrho\right)\ddot{\Phi} + \frac{3I}{c^2}\frac{\dot{a}_0}{a_0}\dot{\Phi}\left(1 + \frac{B}{\sqrt{I}}\wp\right) + V'(\Phi) = 0. \tag{38}$$

The first Friedmann equation (36) can be put into the following "sum-rule" form:

$$1 + I^{-1}\Omega_k = I^{-3/2}\Omega_\varrho + I^{-1}\Omega_\Phi + I^{-3/2}\Omega_{\Lambda_4} + I^{-1}\Omega_{\mathcal{C}}, \tag{39}$$

where the density parameters are defined as

$$\Omega_k \equiv \frac{kc^2}{a_0^2 H^2}, \quad \Omega_\varrho \equiv \frac{32\pi^2 G_5^2\sigma\varrho}{9c^4 H^2}, \quad \Omega_\Phi \equiv \frac{32\pi^2 G_5^2\sigma\varrho_\Phi}{9c^4 H^2},$$

$$\Omega_{\Lambda_4} \equiv \frac{32\pi^2 G_5^2\sigma\varrho_{\Lambda_4}}{9c^4 H^2}, \quad \Omega_{\mathcal{C}} = \frac{\mathcal{C}c^2}{a_0^4 H^2}. \tag{40}$$

Here, $H=\dot{a}_0/a_0$ is the Hubble parameter and $\varrho_{\Lambda_4} = \Lambda_4 c_{\rm grav}^2/(8\pi G_4) = (3c^2\sqrt{I}/8\pi G_5\sigma)[\Lambda + (4\pi G_5/3c^4)\sigma^2]$ is the vacuum energy density, where $\Lambda_4 = (4\pi G_5/c^4)[\Lambda + (4\pi G_5/3c^4)\sigma^2]$ is the effective four-dimensional cosmological constant. Unlike the case of conventional cosmology, the sum rule involves the additional factors of I. From the second Friedmann equation (37), we obtain the following expression for the deceleration parameter:

$$q=-\frac{a_0\ddot{a}_0}{\dot{a}_0^2}=\frac{\dot{I}}{2HI}+\frac{1}{2}(I^{-3/2}\Omega_\varrho+I^{-1}\Omega_\Phi)+\frac{16\pi^2G_5^2\sigma}{3c^6H^2}(I^{-1/2}\wp+I^{-1}\wp_\Phi+2I^{-1/2}\wp_{\Lambda_4})+I^{-1}\Omega_{\mathcal{C}}, \tag{41}$$

where $\wp_{\Lambda_4}=-c_{\rm grav}^2\varrho_{\Lambda_4}=-(3c^4/8\pi G_5\sigma\sqrt{I})[\Lambda+(4\pi G_5/3c^4)\sigma^2]$. We consider the special case describing the present day universe having $k=0$, $\Lambda_4=0$ and $\wp=0$. For such case, the sum-rule formula (39) takes the form

$$1=I^{-3/2}\Omega_\varrho+I^{-1}\Omega_\Phi+I^{-1}\Omega_{\mathcal{C}}. \tag{42}$$

So, the deceleration parameter (41) reduces to

$$q=\frac{\dot{I}}{2HI}+\frac{1}{2}+\frac{16\pi^2G_5^2\sigma}{3c^6H^2I}\wp_\Phi+\frac{1}{2I}\Omega_{\mathcal{C}}. \tag{43}$$

To be consistent with the observational data, the deceleration parameter has to be negative. Unlike the case of conventional cosmology, we have additional contribution from the dark radiation term. A negative value of $\mathcal{C}$ helps with achieving negative q. With positive $\mathcal{C}$, more rapid variation of the biscalar field with time is required in order to be consistent with the observational data.

From the effective Friedmann equations (36),(37) in the comoving frame for the matter metric, we can read off that the speed of a graviton and the effective four-dimensional Newton's constant on the brane are respectively given by

$$c_{\rm grav}=\frac{c}{\sqrt{I}},\quad G_4=\frac{4\pi G_5^2\sigma}{3c^4I^{3/2}}. \tag{44}$$

This expression for $c_{\rm grav}$ agrees with the value read off from the gravity metric (35). G_4 also varies with time and takes smaller value than the present day value while $\dot{\Phi}\neq 0$. In terms of these effective four-dimensional parameters, the effective Friedmann equations (36),(37) in the comoving frame for the matter metric take the forms

$$\left(\frac{\dot{a}_0}{a_0}\right)^2=\frac{8\pi G_4}{3}\varrho+\frac{4\pi G_4}{3c_{\rm grav}^4\sqrt{I}}\dot{\Phi}^2+\frac{c_{\rm grav}^2}{3}\Lambda_{\rm eff}+\frac{\mathcal{C}c_{\rm grav}^2}{a_0^4}-\frac{kc_{\rm grav}^2}{a_0^2}, \tag{45}$$

$$\frac{\ddot{a}_0}{a_0}+\frac{1}{2}\frac{\dot{I}}{I}\frac{\dot{a}_0}{a_0}=-\frac{4\pi G_4}{3}\left(\varrho+3\frac{\wp}{c_{\rm grav}^2}\right)-\frac{8\pi G_4}{3c_{\rm grav}^4\sqrt{I}}\dot{\Phi}^2+\frac{c_{\rm grav}^2}{3}\Lambda_{\rm eff}-\frac{\mathcal{C}c_{\rm grav}^2}{a_0^4}, \tag{46}$$

where $\Lambda_{\rm eff}$ is the effective total four-dimensional cosmological constant given by

$$\Lambda_{\rm eff}=\frac{4\pi G_5}{c^4}\left(\Lambda+\frac{4\pi G_5}{3c^4}\sigma^2\right)+\frac{32\pi^2G_5^2\sigma}{3c^8}V(\Phi). \tag{47}$$

This effective four-dimensional cosmological constant has contribution only from $V(\Phi)$, if the brane tension takes the fine-tuned value $\sigma=\sqrt{-3c^4\Lambda/4\pi G_5}$ of the RS2 model [21].

We discuss resolution of various cosmological problems within our bimetric model. First, we consider the horizon problem. The four-velocity vector V^μ of a photon, which is null with respect to the matter metric, i.e., $\hat{g}_{\mu\nu}V^\mu V^\nu=0$, is spacelike with respect to the gravity metric, i.e., $g_{\mu\nu}V^\mu V^\nu=B(V^\mu\partial_\mu\Phi)^2>0$ when $\partial_\mu\Phi\neq 0$ and $B>0$. So, photons and other matter fields propagate outside the lightcone of the gravity metric. The horizon problem is therefore resolved in our bimetric model, provided Φ varies rapid enough with time during an early period of cosmic evolution. Furthermore, the problem of unwanted relics such as magnetic monopoles, which requires a larger value of the light speed during an early period for its resolution in the VSL models, can also be resolved by our bimetric model. However, the flatness problem and the cosmological constant problem, which require the rapid enough decrease in the speed of a graviton to the present day value (in the Friedmann equations) for their resolution in the VSL models, cannot be resolved by our bimetric model, since the speed of a graviton takes a constant value c in the comoving frame for the gravity metric and takes a smaller value than c in the comoving frame for the matter metric, while $\dot{\Phi}\neq 0$. The flatness problem may be resolved by our bimetric model, provided the biscalar potential $V(\Phi)$ has a region satisfying the slow-roll approximation and thereby the biscalar can act as an inflaton. Detailed discussion on resolution of these cosmological problems within the VSL brane world cosmologies is given in Refs. [19,36].

We comment on the Planck problem of the VSL cosmologies pointed out in Ref. [37]. When the speed of a graviton and Newton's constant vary with time, so do the Planck mass $m_{pl}=\sqrt{\hbar c_{\rm grav}/G_4}$, the Planck length $l_{pl}=\sqrt{\hbar G_4/c_{\rm grav}^3}$ and the Planck time $t_{pl}=\sqrt{\hbar G_4/c_{grav}^5}$. By substituting $c_{\rm grav}$ and G_4 in Eq. (44), we see that the Planck mass takes a *larger* value than the present day value, the Planck length remains constant and the Planck time takes a *larger* value, while the biscalar varies with time. Since the Planck mass takes a larger value, our bimetric model makes the hierarchy problem worse. Furthermore, too much large value of I, which leads to the value of the Planck time ($\sim c_{grav}^{-5/2}\sim I^{5/4}$) larger than $\sim 10^{-20}$ sec would totally mess up the usual standard particle physics arguments, e.g., matter dominance over antimatter. Therefore, a judicial choice of the biscalar potential $V(\Phi)$ which leads to the value of I not exceeding $\sim 10^{20}$ and therefore the speed of light ($\sim I^{1/2}$) not exceeding $\sim 10^{10}$ times the present day value is necessary. This limit on the speed of light during the early stage of cosmological evolution may be insufficient for solving the cosmological problems. So, our bimetric model risks the above mentioned problem associated with large t_{pl}, if it is to solve the cosmological problems. However, since the Planck density ($\sim m_{pl}/l_{pl}^3 \sim I^{1/2}$) increases for our bimetric model, the Planck density problem may be resolved.

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