Infrared and ultraviolet cutoffs of quantum field theory

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Quantum gravity arguments and the entropy bound for effective field theories proposed by Cohen, Kaplan, and Nelson [Phys. Rev. Lett. **82**, 4971 (1999)] lead us to consider two correlated scales which parametrize departures from relativistic quantum field theory at low and high energies. A simple estimate of their possible phenomenological implications leads us to identify a scale of around 100 TeV as an upper limit on the domain of validity of a quantum-field-theory description of nature. This fact agrees with recent theoretical developments in large extra dimensions. Phenomenological consequences in the beta-decay spectrum and cosmic-ray physics associated with possible Lorentz invariance violations induced by the infrared scale are discussed. It is also suggested that this scale might produce new unexpected effects at the quantum level.

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Local quantum-field-theory (QFT) is a good effective description of nature at low energies. However, it is usually assumed that QFT's have a high-energy limitation of applicability. This is obvious for nonasymptotically free theories, such as QED and scalar ϕ^4 theory, which seem to fail at very high energy [1], but it appears to be an inevitable general conclusion when trying to incorporate gravity due to the nonrenormalizability of a QFT of gravitation. Moreover, it has been proposed that gravitational stability of the vacuum sets a limit on the shortest scale of any QFT compatible with gravity, which is somewhat above the Planck length [2].

As a consequence, QFT's would be low-energy approximations to a more fundamental theory that may not be a field theory at all [3], and are valid up to energies below a certain scale Λ, which then represents an ultraviolet (UV) cutoff of the effective field theory. The failure in obtaining a QFT of gravity indicates that this scale is lower than the Planck mass M_P, the scale at which the gravitational strength is comparable to that of the rest of the fundamental interactions. Can we tell anything more about the order of magnitude of the scale Λ?

In a QFT, the maximum entropy $S_{\max}$ scales extensively, with the space volume [4], which is a reasonable guess for a local theory. For a QFT in a box of size L with UV cutoff Λ, $S_{\max}\sim L^3\Lambda^3$ [5]. However, Bekenstein arguments [6], based on black-hole gedanken experiments and the validity of the generalized second law of thermodynamics [7], lead to think that, in a quantum theory of gravity, the maximum entropy should be proportional to the area and not to the volume [4]:

$$S \lesssim L^2 M_P^2. \tag{1}$$

Equation (1) is usually called the holography entropy bound. This bound suggests that conventional field theories over count degrees of freedom [8,4], and implies the breakdown of any effective field theory with an UV cutoff to describe systems which exceed a certain critical volume L^3 which depends on the UV cutoff. From Eq. (1), it is straightforward that $L \lesssim M_P^2\Lambda^{-3}$. This observation lead to Cohen, Kaplan, and Nelson [5] to endow every effective field theory with an infrared (IR) cutoff correlated to its UV one. However, using very plausible arguments [5], they noticed that conventional QFT should fail at an entropy well below the holography bound. A QFT should not attempt to describe systems containing a black hole. Therefore it should not include states with an energy greater than LM_P^2. Since the maximum energy density in a QFT with UV cutoff Λ is Λ^4, one such a state can be formed when the size of the system is

$$L \sim \frac{M_P}{\Lambda^2}. \tag{2}$$

In Ref. [5], the scales L and Λ are not considered as absolute. They signal the range of validity of QFT calculations when applied to a phenomenon of a certain energy scale. However, as noted above, it is believed that QFT has an absolute UV limit of validity Λ which is probably related to gravitational effects. Therefore, this necessarily implies the existence of a related absolute IR cutoff λ which, according to Eq. (2), will be given by

$$\lambda \sim \frac{\Lambda^2}{M_P}. \tag{3}$$

If one now considers that $\Lambda \sim M_P$, Eq. (3) gives the absurd result $\lambda \sim M_P$. It is natural to think that λ should be low enough to make compatible the breakdown of QFT at low energies with its success in particle physics. This is so, however, unless the departures from QFT through effects produced by the scale λ were completely suppressed by a factor proportional to the gravitational coupling. If this were the case, one could not derive any relevant bounds for the IR and UV scales. This scenario is indeed a possibility. However in the following we will assume that this is not the case, and that the λ-induced effects do not have such a suppression factor and may have observable consequences.

Coming back to relation (3), note that a low value of λ implies a limit of validity of QFT at high energies much

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lower than the Planck mass M_P. Precision tests of the electroweak standard model put the lower bound for Λ in the TeV range. On the other hand, as we will see below, a value of λ higher than the scale of eV (and even smaller values depending on the assumptions used to estimate the effects of the IR scale) could have observable effects in certain experiments, like the tritium beta decay. These two observations restrict very much the possible ranges of the two cutoffs: $\Lambda \sim (1-100)$ TeV and $\lambda \sim (10^{-4}-1)$ eV. Therefore QFT would be an effective theory valid only up to an energy scale of the order of 100 TeV. If one takes this cutoff as the fundamental short distance scale then one has the possibility to explore a new framework for solving the hierarchy problem which does not rely on either supersymmetry or technicolor. A disadvantage of this scenario is that one would loose the successful prediction of grand unification theories for $\sin \theta_W$, but anyway the correct prediction might come out in the end in a more complicated way.

If, as it is commonly accepted, the origin of the limitation at high energies of QFT is the gravitational interaction, it is natural to identify Λ with the fundamental scale of gravity. In fact, the existence of a fundamental scale of the gravitational interaction well below the Planck mass scale and just a few orders of magnitude above the electroweak scale, a very attractive idea to solve the hierarchy problem, has been proposed in some recent innovative works [9,10] on extra dimensions. In the approach of large extra dimensions [9], the observed hierarchy between the electroweak and Planck scales is explained by postulating a fundamental scale $M \sim 10-100$ TeV of gravity along with Kaluza-Klein compactification with large radius R. The Planck scale is then an effective four-dimensional scale. The case of two extra dimensions is particularly interesting. In this case, the radius of the compact extra dimensions is given by

$$R \simeq \frac{M_P}{M^2}, \tag{4}$$

which is close to the present limit of validity of Newton's gravitational law. It is surprising to note the similarities between Eqs. (3) and (4) if one identifies the UV limit with the fundamental scale of gravity in $4+2$ dimensions ($\Lambda \sim M$) and the IR limit with the inverse of the radius of compactification of the large extra dimensions ($\lambda \sim 1/R$), a relation to which we find no simple interpretation. Alternatives to the hierarchy problem in terms of finite, but noncompact extra dimensions also require a fundamental scale of the order of 100 TeV [10].

In the following we will explore both the phenomenological and theoretical consequences of these stringent limits of a QFT description of nature, and the compatibility of these limits [and therefore, of the arguments which lead to Eq. (2)] with present experimental status.

Let us examine first the deviations from QFT at low energies. We will consider two different scenarios for the effects of the IR scale. In the first scenario we will assume that the neutrino is particularly sensitive to the presence of the IR scale. This is a natural assumption in the framework of large extra dimensions where the neutrino is the only particle (besides the graviton) which propagates in the extra dimensions. Then any dependence on the IR scale (compactification radius) for the remaining particles requires to consider the gravitational coupling and will then be suppressed. For an IR cutoff of the order of the eV, effects should be seen in experiments sensitive to neutrinos with energies in the $10-100$ eV scale. Indeed, it is in this range where recent experiments of the tritium beta decay have observed an anomaly which consists in an excess of electron events at the end of the spectrum, at about 20 eV below the end point [11]. A value of λ higher than few eV would produce a signal in the tritium beta-decay spectrum in a larger range than that of the observed anomaly, so this gives the bound $\lambda < 1$ eV in this scenario, which produces the severe limit $\Lambda < 100$ TeV on the UV cutoff of QFT. But, in addition to putting an upper limit on λ, the tritium beta-decay spectrum could also allow to identify a correction induced by the IR cutoff. Indeed a modification of the dispersion relation for the neutrino, of the form

$$E^2 = \boldsymbol{p}^2 + m^2 + \mu |\boldsymbol{p}|, \tag{5}$$

has been used to explain the tritium beta-decay anomaly [12]. Matching with experimental results requires a value of μ in Eq. (5) of the order of the eV, and it can be seen that this does not contradict any other experimental result [12].

In fact a deviation from QFT at low energies due to the IR scale λ may well be expected to violate relativistic invariance. The reason is that it has been shown [3] that *any* theory incorporating quantum mechanics (QM) and special relativity, with an additional "cluster" condition [3], must reduce to a QFT at low energies. A modified dispersion relation of the form of Eq. (5) is a simple way of incorporating effects beyond QFT that violate relativistic invariance at low energies. Note that, together with the dispersion relation (5), one should indicate the "preferred" frame in which this relation is valid. There is however another possibility, which is to extend our concept of relativistic invariance to a more general framework, in which Eq. (5) would be an observer-invariant relation. The possibility to have a modification of Lorentz transformations compatible with the presence of an observer-independent scale of length has recently only been explored [13].

In order for the dispersion relation Eq. (5) to be compatible with the very stringent limits on CPT violation [14] it is necessary to have the same scale μ in the particle-antiparticle dispersion relation. Even with this limitation a modified dispersion relation for any particle with $\mu \sim \lambda$ is not compatible with experimental limits. For example, Eq. (5) for the electron would slightly modify the energy levels of the hydrogen atom. Given the extraordinary agreement between theory and the experimental measurement of the Lamb shift (one part in 10^5 [15,3]), one has $\mu < 10^{-6}-10^{-7}$ eV. Therefore, an identification of the scale μ which parametrizes the Lorentz invariance violation (LIV) at low energies in the dispersion relation with the IR scale λ is incompatible with the arguments which lead to Eq. (2). A way to reconcile these arguments with a LIV at low energies is in the framework of the scenario described above, in which one expects a suppression of the dependence of LIV effects on the IR scale for all particles except for the neutrino. This would explain why no signal of these LIV's has been observed. In the case of the neutrino no such suppression is present and then $\mu \sim \lambda$; besides that the neutrino mass is not larger than the IR scale and this makes it possible to observe the consequences of a Lorentz noninvariant term in the neutrino dispersion relation. Then the anomaly in tritium beta decay, if confirmed as a real physical effect, could be the first manifestation of the IR cutoff of QFT. If the anomaly results to be a consequence of some systematic effect not taken into account [11] then one can obtain a stronger upper limit on the IR cutoff.

We also note that the new dispersion relation Eq. (5) might have important effects in cosmic rays through threshold effects which become relevant when $\mu|\boldsymbol{p}_{\rm th}|\sim m^2$. Here m^2 is an ''effective'' mass squared which controls the kinematic condition of allowance or prohibition of an specific process. Indeed a consequence of these threshold effects could be that neutrons and pions become stable particles at energies close to the knee of the cosmic-ray spectrum [12], which would drastically alter the composition of cosmic rays. It is quite remarkable that cosmic-ray phenomenology could be sensitive to the presence of an IR scale.

Since the IR scale was introduced by general arguments reflecting the apparent incompatibility of QM with a complete theory which contains gravity, it is natural to consider a second scenario in which the effects of the IR scale are due to the quantum fluctuations of the vacuum and then affect all the particles. In this scenario the most stringent limits on the IR scale come from the high precision tests of QED, in particular from the anomalous magnetic moment of the electron. In this case, the characteristic physical scale is the electron mass, but the precision achieved makes the experiment sensitive to much lower scales. In fact, it gives the most precise test of QED. We should therefore ask whether it is compatible with the presence of an IR cutoff. A simple estimate of the correction to the usual calculation imposed by the IR scale is

$$\delta a_e \sim \frac{\alpha}{\pi}\left(\frac{\lambda}{m_e}\right) \sim 4\times10^{-9}\,\frac{\lambda}{(1\ \ \mathrm{eV})}. \qquad (6)$$

If we ask this correction to be smaller than the uncertainty of the theoretical prediction for a_e in QED caused by the uncertainty in the determination of α [15], we get a bound for the IR scale $\lambda \lesssim 10^{-2}$ eV. Other tests of QED, like shifts of energy levels in hydrogen atom, positronium, etc., lead in this case to less stringent bounds on the scale λ. The previous bound on the IR scale corresponds to an UV scale $\Lambda \lesssim 10$ TeV which is very close to the present and near future energies available in accelerator physics.

This second scenario suggests that QM, as we know it today, might fail not only above the 10 TeV scale, but also that one could find unexpected effects at the quantum level for phenomena with a characteristic scale of $10^{-2} - 10^{-4}$ eV, for example, diffraction experiments with wavelengths of the order of a millimeter. Quantum systems which are sensitive to wavelengths of this order of magnitude are candidates to show departures of QFT parametrized by the IR scale. The understanding of the transition between the quantum and classical regimes, going from a QFT description with departures parametrized by an IR scale to the classical theory, would require the use of the more fundamental theory which could provide a solution to the quantum mesurement paradox [16]. Finally, we mention that some aspects of the large scale structure of the Universe, which are related to quantum fluctuations in an early epoch of its evolution, could also include signals of the IR cutoff.

We turn now our attention to possible departures from QFT results for high-energy phenomenology above the $1-100$ TeV limit. A natural candidate to reveal new physics beyond this energy scale is cosmic-ray physics, where several anomalies are observed [17,18]. However, a detailed discussion of the expected effects requires specific formulations of the kind of limitations presented by QFT, like the ones explored in the context of extra dimensions [19].

Generally speaking, the presence of a cutoff Λ also induces nonrenormalizable corrections to the effective field theory. For example, conventional QED would include modifications produced by a Lorentz, gauge, and CP invariant Pauli term of dimension 5, of order $1/\Lambda$ or, considering extra symmetries that restrict the form of the nonrenormalizable interactions, of order m/Λ^2 [3]. In the first case, the theoretical and experimental agreement on the value of the electron magnetic moment gives $\Lambda \gtrsim 4\times10^7$ GeV, which would mean the invalidity of the arguments that lead to Eq. (2). In the second case, it is the value of the muon magnetic moment [20] rather than that of the electron which provides the most useful limit on Λ, $\Lambda \gtrsim 3\times10^3$ GeV [3].[1] This is why it is usually considered that conventional QFT gives a correct description of nature at least up to the scale of the TeV.[2] This lower bound for Λ is the origin of the bound $\lambda \gtrsim 10^{-4}$ eV on the IR cutoff.

Let us also consider the possibility of LIV's at high energies. Several attempts have been made to question Lorentz invariance [21]. The existence of LIV's at high energies is natural in the context of quantum gravity [22,23]. Quantum gravity fluctuations produce, in general, modifications in the dispersion relations which characterize the laws of particle propagations [23–26]. These modifications have been used [25,26] to explain the observed violations of the Greisen-Zatsepin-Kuzmin (GZK) cutoff limit [27]. In fact, it has been recently shown [28] that these violations of Lorentz invariance can at the same time offer a solution to the anomaly

[1] The Muon $(g-2)$ Collaboration has recently reported [H. Brown *et al.*, Phys. Rev. Lett. **86**, 2227 (2001)] a possible incompatibility between the experimental value of the muon magnetic moment and its theoretical value from the standard model. This difference could be explained by nonrenormalizable corrections induced by the presence of an ultraviolet cutoff Λ around $4-5$ TeV.

[2] The less stringent limit on Λ can also be understood without the need of additional symmetries in the framework of the two considered scenarios, in which the Pauli term will be suppressed, either by the gravitational scale in the first scenario, or by quantum fluctuations in the second.

observed [29] in the gamma-ray spectrum of Markarian 501, which extends well beyond 10 TeV.

As an example, let us consider the Lorentz-violating class of dispersion relations [25]

$$E^2 = \boldsymbol{p}^2 + m^2 + \frac{|\boldsymbol{p}|^{2+n}}{M^n}, \tag{7}$$

where M is the characteristic scale of these violations (the fundamental scale of gravitation in the quantum gravity framework). It is then easy to see that M causes the appearance of threshold effects at momenta $|\boldsymbol{p}| \gtrsim |\boldsymbol{p}_{\rm th}|$, where $|\boldsymbol{p}_{\rm th}|^{2+n} \sim m^2 M^n$. It is these effects which allows to explore quantum gravity at energies much lower than M [30]. For $M \sim M_P$ and a typical hadronic process, one gets $|\boldsymbol{p}_{\rm th}| \sim 10^{15}$ eV in the case $n=1$ and $|\boldsymbol{p}_{\rm th}| \sim 10^{18}$ eV in the case $n=2$. In both cases one has modifications to relativistic kinematics at energies below the GZK cutoff, so that the observed violations of this cutoff in the cosmic-ray spectrum [18] could be a footprint of a LIV at high energies. The fact that these violations can also offer a solution to the Markarian 501 anomaly is easily seen if one considers the threshold of the reaction $\gamma + \gamma \to e^+ e^-$, which restrains the propagation of gamma rays in the IR background. The effective mass that controls this process is $m^2 \sim 1$ MeV^2, so that $n=1$ gives $|\boldsymbol{p}_{\rm th}| \sim 10$ TeV, and the conventional threshold is modified.

The characteristic scale M for LIV's at high energies does not have to coincide, in principle, with the UV scale Λ, which is defined as the maximum energy of the quantum fluctuations whose effects can be described using QFT, though M will surely depend on Λ. In fact, a scale M much larger than Λ can be justified for all particles except for the neutrino in the first scenario if one assumes that the suppression of the dependence on the IR scale applies also to the dependence on the UV scale. On the other hand in the case of the neutrino one expects that $M \sim \Lambda$ and then more clear signals of LIV's at high energies in reactions involving neutrinos. Alternatively, in the second scenario all the effects of the IR and UV scales are due to the quantum fluctuations of the vacuum and then one expects $\mu \ll \lambda$ and $M \gg \Lambda$ for the scales that parametrize LIV's at low and high energies for any particle. In both scenarios the scale M can be made sufficiently large to be consistent with the tight constraints from experiments on Lorentz and *CPT* violations [31,26].

In conclusion, quantum gravity, Bekenstein's and Cohen, Kaplan and Nelson's arguments, together with their phenomenological consequences, indicate that a QFT description of nature might not be valid above the scale $\Lambda \sim (1-100)$ TeV and that departures could be seen at low energies characterized by an IR scale $\lambda \sim (10^{-4}-1)$ eV. Interpreting Λ as the fundamental scale of gravity, the surprise is that the UV limit of QFT would not be the Planck scale, but 14 orders of magnitude lower. This would made quantum gravity phenomenology much more accessible, and agrees with recent theoretical development on extra dimensions. We have identified certain experiments that could reflect the limitations of QFT and explored in which scenarios these ideas are compatible with the present experimental status quo.

As a final comment, we would like to speculate with the hypothesis that the apparition of two correlated scales in QFT might be a general property of every extension of QFT which tried to incorporate the gravitational interaction. An example is the IR/UV connection in noncommutative gauge theories [32] which arise as effective field theories of the tachyon and gauge field degrees of freedom of the open string in the presence of D-branes [33]. Another example are large extra dimension models, where, besides the fundamental scale of gravity, one has to introduce another scale corresponding to the compactification radius of the extra dimensions. The presence of two correlated scales is particularly suggestive to give an answer to the cosmological constant problem, which seems to require a correlation between the Fermi energy scale, 300 GeV, and the cosmological constant scale, 10^{-2} eV, in order to explain the enormous precision in the cancellation of the vacuum energy density contribution of the standard model. It is noticeable that these two scales coincide very approximately with the UV and IR scales of QFT that we have identified through phenomenological arguments.

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[1] See, e.g., D.J.E. Callaway, Phys. Rep. **167**, 5 (1988).
[2] R. Brustein, D. Eichler, and S. Foffa, hep-th/0009063.
[3] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, England, 1995).
[4] L. Susskind, J. Math. Phys. **36**, 6377 (1995).
[5] A.G. Cohen, D.B. Kaplan, and A.E. Nelson, Phys. Rev. Lett. **82**, 4971 (1999).
[6] J.D. Bekenstein, Phys. Rev. D **7**, 2333 (1973); **9**, 3292 (1974); **23**, 287 (1981); **49**, 1912 (1994).
[7] J.D. Bekenstein, gr-qc/0009019.
[8] G. 't Hooft, in *Salamfestschrift: A Collection of Talks*, edited by A. Ali, J. Ellis, and S. Randkbar-Daemi (World Scientific, Singapore, 1993).
[9] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *ibid.* **436**, 257 (1998); R. Sundrum, Phys. Rev. D **59**, 085010 (1999); N. Arkani-Hamed, S. Dimopoulos, and J. March-Russell, *ibid.* **63**, 064020 (2001).
[10] A.G. Cohen and D.B. Kaplan, Phys. Lett. B **470**, 52 (1999).
[11] V.M. Lobashev *et al.*, Phys. Lett. B **460**, 227 (1999); Ch. Weinheimer *et al.*, *ibid.* **460**, 219 (1999).
[12] J.M. Carmona and J.L. Cortés, Phys. Lett. B **494**, 75 (2000).
[13] G. Amelino-Camelia, gr-qc/0012051; hep-th/0012238.

[14] H. Dehmelt *et al.*, Phys. Rev. Lett. **83**, 4694 (1999); D.F. Phillips *et al.*, Phys. Rev. D **63**, 111101(R) (2001).
[15] T. Kinoshita, Rep. Prog. Phys. **59**, 1459 (1996).
[16] J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, England, 1987).
[17] A.V. Olinto, Phys. Rep. **333-334**, 329 (2000).
[18] A.A. Watson, Phys. Rep. **333-334**, 310 (2000).
[19] G. Giudice, R. Rattazzi, and J. Wells, Nucl. Phys. **B544**, 3 (1999); T. Han, J.D. Lykken, and R.-J. Zhang, Phys. Rev. D **59**, 105006 (1999); E.A. Mirabelli, M. Perelstein, and M.E. Peskin, Phys. Rev. Lett. **82**, 2236 (1999); J.L. Hewett, *ibid.* **82**, 4765 (1999); S. Nussinov and R. Shrock, Phys. Rev. D **59**, 105002 (1999); K. Cheung and G. Landsberg, *ibid.* **62**, 076003 (2000).
[20] H.N. Brown *et al.*, Phys. Rev. D **62**, 091101(R) (2000).
[21] D. Colladay and V.A. Kostelecký, Phys. Rev. D **55**, 6760 (1997); **58**, 116002 (1998) and earlier references therein.
[22] T. Jacobson and D. Mattingly, gr-qc/0007031; G. 't Hooft, Class. Quantum Grav. **13**, 1023 (1996).
[23] G. Amelino-Camelia, Int. J. Mod. Phys. D **10**, 1 (2001).
[24] R. Gambini and J. Pullin, Phys. Rev. D **59**, 124021 (1999); G. Amelino-Camelia, Mod. Phys. Lett. A **13**, 1319 (1998); J. Alfaro, H.A. Morales-Técotl, and L.F. Urrutia, Phys. Rev. Lett. **84**, 2318 (2000).
[25] G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, and S. Sarkar, Nature (London) **393**, 763 (1998).
[26] S. Coleman and S. Glashow, Phys. Rev. D **59**, 116008 (1999).
[27] K. Greisen, Phys. Rev. Lett. **16**, 748 (1966); G.T. Zatsepin and V.A. Kuzmin, Zh. Éksp. Teor. Fiz. Pis'ma Red. **4**, 414 (1966) [JETP Lett. **4**, 78 (1966)].
[28] R.J. Protheroe and H. Meyer, Phys. Lett. B **493**, 1 (2000); G. Amelino-Camelia and T. Piran, Phys. Rev. D **64**, 036005 (2001).
[29] F.A. Aharonian *et al.*, Astron. Astrophys. **349**, 11 (1999).
[30] R. Aloisio, P. Blasi, P.L. Ghia, and A. Grillo, Phys. Rev. D **62**, 053010 (2000).
[31] *CPT and Lorentz Symmetry*, edited by V. A. Kostelecký (World Scientific, Singapore, 1999).
[32] A. Matusis, L. Susskind, and N. Toumbas, J. High Energy Phys. **12**, 002 (2000).
[33] N. Seiberg and E. Witten, J. High Energy Phys. **09**, 032 (1999).