Scales of gravity

Gia Dvali,[1,*] Gregory Gabadadze,[2,†] Marko Kolanović,[1,‡] and Francesco Nitti[1,§]
[1]*Department of Physics, New York University, New York, New York 10003*
[2]*Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455*

We propose a framework in which the quantum gravity scale can be as low as 10^{-3} eV. The key assumption is that the standard model ultraviolet cutoff is much higher than the quantum gravity scale. This ensures that we observe conventional weak gravity. We construct an explicit brane-world model in which the brane-localized standard model is coupled to strong 5D gravity of infinite-volume flat extra space. Because of the high ultraviolet scale, the standard model fields generate a large graviton kinetic term on the brane. This kinetic term ''shields'' the standard model from the strong bulk gravity. As a result, an observer on the brane sees weak 4D gravity up to astronomically large distances beyond which gravity becomes five dimensional. Modeling quantum gravity above its scale by the closed string spectrum we show that the shielding phenomenon protects the standard model from an apparent phenomenological catastrophe due to the exponentially large number of light string states. The collider experiments, astrophysics, cosmology and gravity measurements *independently* point to the same lower bound on the quantum gravity scale, 10^{-3} eV. For this value the model has experimental signatures both for colliders and for submillimeter gravity measurements. Black holes reveal certain interesting properties in this framework.

I. INTRODUCTION

One of the great mysteries about gravity is its inexplicable weakness compared to all the other known forces of nature. For instance, the magnitude of the Newton gravitational force between two protons is 10^{36} times smaller than the magnitude of the Coulomb force between them. In the language of the low energy field theory the Newton force is mediated via the exchange of a virtual massless spin-2 particle, the graviton $h_{\mu\nu}$, which couples to the matter energy-momentum tensor $T_{\mu\nu}$ as follows:

$$\frac{1}{M_P} h_{\mu\nu} T^{\mu\nu}. \tag{1.1}$$

The corresponding coupling constant has the dimensionality and magnitude of the Planck mass, $M_P \sim 10^{18}$ GeV. Therefore, the dimensionless ratio that governs the strength of the Newton force between the two protons is

$$\alpha_{\text{gravity}} \sim \frac{m_{\text{proton}}^2}{M_P^2}. \tag{1.2}$$

This has to be confronted with the electromagnetic coupling constant $\alpha_{EM} = 1/137$ when one compares the Newton and Coulomb forces. In this language gravity is weak because of the huge value of M_P compared to the particle masses.

What is the meaning of the scale M_P? For some time the answer to this question has been known as what is called the

‾‾‾‾‾‾‾‾
*Email address: gd23@nyu.edu
†Email address: gabadadz@physics.umn.edu
‡Email address: mk679@nyu.edu
§Email address: fn230@nyu.edu

''standard paradigm.'' According to this paradigm M_P is the fundamental scale of quantum gravity, i.e., the scale at which the classical theory of gravitation, as we know it, should cease to be valid description of nature. This is based on the assumption that the effective low-energy description in terms of Newtonian or Einstein gravity breaks down at the Planck energies $\sim M_P$ or, equivalently, at the Planck distances $l_P = 1/M_P \sim 10^{-33}$ cm, and therefore, quantum gravity effects become important. On the other hand, the validity of the Newtonian interactions is experimentally measured only for distances bigger than ~ 0.2 millimeter [1]. The standard paradigm uses the following assumptions:

It assumes that nothing is happening with gravity all the way down to the distances of order 10^{-33} cm, or equivalently, all the way up until the energies of order M_P. Therefore, it extrapolates the known experimental result of the gravitational measurements by 31 orders of magnitude.

It also assumes that M_P is a natural field theory cutoff not only for gravity, but for the whole particle theory including the standard model (SM) and its extensions [e.g., grand unified theories (GUT), etc.].

The existence of an energy ''desert'' stretched over 17 orders of magnitude gave rise to the hierarchy problem. As seen from such an angle, the assumptions of the standard paradigm look somewhat unnatural. After all, if we think of known field theory examples such as, e.g., electrodynamics, quantum effects become important at scales at which the coupling is still very weak and perturbation theory is valid. There is no *a priori* reason for a theory to ''wait'' until the classical interactions blow up, in order for the quantum effects to become important. From this perspective, it is natural to question this paradigm and ask whether the gravitational cutoff might be much lower in reality.

This was done in Refs. [2,3]. The approach of [2] was mainly motivated by the hierarchy problem. It assumed that the quantum gravity scale, referred hereafter as M_*, and the

65 024031-1

field theory cutoff of SM and its possible extensions, referred below as M_{SM}, were around the electroweak scale, i.e., ~ 1 TeV.

Although the quantum gravity scale M_* is very low in the framework of [2], it nevertheless shares one common assumption with the standard paradigm: in both these approaches $M_* \sim M_{SM}$.

In the present paper we relax this assumption and show that the quantum gravity scale M_* can be much lower than the field theory cutoff M_{SM} without conflicting with any of the existing laboratory, astrophysical or cosmological bounds.

We consider a theory in which gravity, becoming "soft" above the scale M_*, is coupled to the SM which remains consistent field theory up to the scale $M_{SM} \gg M_*$. We show that despite the small quantum gravity scale, the weakness of an observable gravity is guaranteed by the high cutoff of the SM. In this framework, the Planck mass M_P is not a fundamental scale but is rather a derived parameter. The large value of the observable M_P is determined by M_{SM} rather than M_*.

In the present work we will mostly be concentrated on a brane-world model of Ref. [4]. However, the similar consideration is applicable in a conventional four-dimensional theory which we will discuss in Sec. II. As an example of this phenomenon in four dimensions consider the following action (we will clarify the origin of this action in Sec. II):

$$S = \int d^4x \sqrt{|g|} \left(M_P^2 R + \sum_{n=1}^{\infty} c_n \frac{R^{n+1}}{M_*^{2(n-1)}} \right). \quad (1.3)$$

Here $M_* \ll M_P$, R denote the four-dimensional Ricci scalar and higher powers of R denote all possible higher-dimensional invariants (c_n's are some constants). Suppose that this gravitational theory is coupled to the SM fields in a conventional way. Let us consider the gravitational interactions of the SM particles at the distances much bigger than $1/M_*$. Since the coefficient in front of the Einstein-Hilbert term is M_P^2, the gravitational coupling of the SM fields is proportional to $1/M_P$ and is very weak. However, the higher derivative terms in Eq. (1.3) become important and the gravitational self-interactions become of order 1 at the distances below $1/M_*$. Thus, the effective low-energy approximation to gravity breaks down at an energy scale of order M_*. Above this scale quantum gravity corrections should be taken into account. If these corrections are "soft," i.e., if they do not lead to an effective increase of the coupling of the SM to gravity, then this breakdown of the effective gravitational theory would not be observable in any present day high-energy particle physics experiments which are insensitive to the effects of the gravitational strength (defined by $1/M_P$). Therefore, the gravitational interactions for the SM particles with energies above M_* will still remain very weak compared to the SM gauge interactions.

The idea that effective field theory description of gravitation can brake down at distances smaller than, but close to, a millimeter was put forward by Sundrum [5]. Motivated by toy QCD examples he discussed the "compositness scale"

(string scale) for gravity at an inverse millimeter with the purpose to solve the cosmological constant problem.

In theories with a super low quantum gravity scale (i.e., when $M_* \ll M_{SM}$) there are at least three important issues to be addressed. First, it is not clear *a priori* why the gravitational constant is determined by the scale $1/M_P^2$ if its field-theory description breaks down at the scale M_*; one would rather expect in this case that the gravitational coupling is determined by $1/M_*^2$. As a resolution of this puzzle, we will show that the loops of the SM with the UV scale M_{SM} renormalize the strong gravitational coupling $1/M_*^2$ to make it weak, that is $1/M_P^2$. The second issue deals with the huge hierarchy between the scales M_* and M_{SM}. Although this hierarchy is stable by itself (i.e., the hierarchy is technically natural), still it is desirable to have some dynamical realization for it. We will argue in Sec. III that brane-world scenarios can offer such a realization. In fact, we present a toy brane-world model which has a string theory realization and naturally gives rise to the hierarchy $M_* \ll M_{SM}$. The third issue concerns the assumed "softness" of gravity above the scale M_*. In order to understand this issue in more details one should have a model for quantum gravity. At present, a candidate for quantum gravity is string theory which is formulated in higher dimensions. Therefore, the latter two issues motivate us to go to higher dimensional theories. An attractive possibility for this, as we mentioned above, is the brane-world scenario.

A 5D brane-world framework which explicitly realizes this idea was introduced in Ref. [4]. The model has a 3-brane embedded in flat uncompactified 5-dimensional space where gravity propagates. The SM fields are assumed to be confined to the brane. The field theory cutoff of the bulk gravity is M_*. The effective world-volume theory on the brane is a field theory with a very high cutoff M_{SM}. In this framework $M_{SM} \gg M_*$ (see discussion of an example of this type in Sec. III). The world-volume theory is coupled to the bulk gravity.

Despite the existence of an infinite-volume 5D space with a strong gravitational constant proportional to $1/M_*^3$, an observer on the brane measures the 4D weak gravity with the conventional Newton coupling $G_N = 1/16\pi M_P^2$ within the following intermediate distances:

$$M_*^{-1} \ll r \ll r_c \equiv \frac{M_P^2}{M_*^3}. \quad (1.4)$$

However, for the distances $r \gg r_c$ gravity becomes five dimensional. The reason for such an unusual behavior is as follows. Consider the renormalization of the graviton kinetic term due to matter loops on the brane (Fig. 1). The diagram with massive states in the loop gives rise to the renormalization of the 4D graviton kinetic term which is dominated by the states with the masses close to the SM cutoff M_{SM}. The resulting term in the action has the form

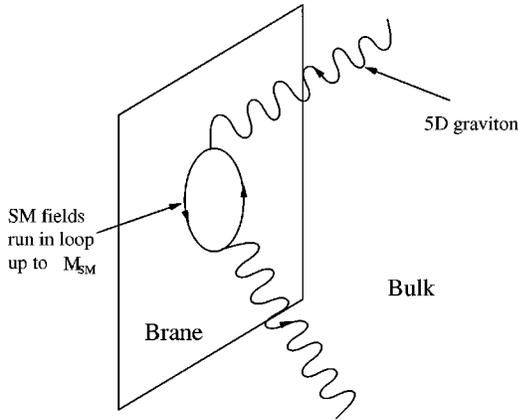$$S_{ind} = \gamma M_{SM}^2 \int d^4x \sqrt{|g|} R(x), \quad (1.5)$$

FIG. 1. SM fields on the brane induce localized kinetic term for the bulk fields.

where g is the induced metric on the brane and $R(x)$ is the corresponding four dimensional Ricci scalar. The coefficient of this term, γ, depends on the number of states and particle content of the SM running in the loop. The multiplier in Eq. (1.5), that is γM_{SM}^2, should be set equal to the 4D Planck mass M_{P} due to phenomenological requirements. Indeed, we will see that the effective Newton constant measured by the brane observer is equal to the inverse value of this constant, $G_N = 1/(16\pi\gamma M_{\mathrm{SM}}^2)$. Therefore, the Planck scale in this framework is not a fundamental quantity but is rather a derived scale which is related to the SM cutoff M_{SM} (or to the GUT cutoff) and its particle content.

The crucial point is that the 4D Newton constant is set by the cutoff of the SM which is much bigger than M_*. The induced term plays a crucial role in what follows. It ensures that a brane observer measures the weak 4D gravity at distances $r \ll r_c$. For the values of M_* that are of our interests, r_c is astronomically large. Thus the crucial question follows: what is the lower bound on M_*?

It was already noticed in Ref. [6] that in the effective field theory picture there is no phenomenological constraint that would forbid M_* to take any small value all the way down to 10^{-3} eV. The reason is as follows [6]: Because of the induced term (1.5) gravity on a brane becomes more and more four-dimensional as we increase energy and all the high energy reactions with graviton emission from the SM states proceed as in the conventional weak 4D gravity. Thus the rigid SM "shields" itself against the strong bulk gravity. Nevertheless, as it was suggested in Ref. [7], the low value of M_* may still manifest itself in gravitational measurements at scales $r \ll M_*^{-1}$. This constrains the value of M_*^{-1} to be smaller than the distance at which the gravitational interactions between static sources are presently measured, i.e., $M_*^{-1} \lesssim 0.2$ mm [1]. Therefore, $M_* > 10^{-3}$ eV.

In the discussions above the graviton momenta were effectively cut off at M_* in all the high energy SM processes due to the lack of the knowledge of the precise theory of quantum gravity above the scale M_*. However, the behavior of gravity above this scale may dramatically change the conclusions. At present, the construction of a realistic brane world model from string theory which would possess all the desired phenomenological properties is difficult.

For this reason, we attack this problem from a somewhat pragmatic point of view. We assume that above the scale M_* gravity is described by a model which mimics in many respects the crucial properties of string theory. In particular, we assume that the bulk theory has the mass spectrum and multiplicity which is similar to that of a closed bosonic string theory in critical dimension (neglecting tachyon). Doing so, we will be able to construct a toy model with some of the crucial features of string theory (most importantly, the huge multiplicity of states) which would naively invalidate any proposal with a low quantum gravity scale.

In the present model the massive stringy states become important at the scale M_*. Furthermore, we assume that the stringy tower of bulk states couples to the fields of the standard model which are localized on a brane. Such a setup, although far from being a self-consistent string theory, nevertheless serves our purpose of putting phenomenological constraints on M_* and studying possible signatures.

We discover that the lower bound on M_* is still 10^{-3} eV. This, surprisingly enough, comes from several rather independent considerations:

The model, as we discussed, predicts the modification of Newtonian gravity at distances $r < M_*^{-1}$. This constrains $1/M_*$ to be smaller than ~ 0.2 mm, i.e., M_* to be bigger than 10^{-3} eV.

Collider phenomenology puts the same constraint on M_* since the rate of the production of the bulk stringy Regge recurrences would become significant at the energies of order $\sqrt{M_* M_{\mathrm{P}}}$ which would be less than a TeV if M_* was smaller than 10^{-3} eV.

Astrophysical bounds arising from constraints on the rate of star cooling due to the emission of stringy Regge states put the same bound on M_*.

The Hagedorn type phenomenon can strongly affect the early universe and in particular the big bang nucleosynthesis. These cosmological considerations also constrain M_* to be bigger than 10^{-3} eV or so.

For the value of M_* which saturates the bound 10^{-3} eV, the model has a number of very distinctive experimental signatures including the deviation from the Newtonian gravity at sub-millimeter scales, as well as the collider signatures due to the production of stringy Regge recurrences with the mass gap of the size of an inverse millimeter. Surprisingly enough, these prediction are somewhat similar to those obtained in the models of Ref. [2] with two sub-millimeter extra dimensions. However, both the modification of Newtonian potential as well as the spectrum of missing energy in collider experiments are different.

In the present scenario the behavior of black holes is rather peculiar. Elementary particles heavier than M_* can turn into long-lived black holes if emitted from the brane into the bulk.

One interesting range for the parameter M_* is $M_* \sim 10$ MeV. In this case, our model predicts in addition the modification of gravitational laws at scales comparable with the present cosmological horizon. This gives rise to a possibility to accommodate an accelerated 4D Universe [8] which is in agreement with the recent supernovae and cosmic mi-

crowave background observations [9]. The remarkable feature of this scenario is that is does not require a small non-zero cosmological constant, instead, the acceleration takes place due to the presence of an infinite volume fifth dimension.

One more attractive feature of the present scenario is that the SM fields are confined to the brane, and, therefore, if supplemented by low-energy supersymmetry, the conventional logarithmic gauge coupling unification of a 4D theory [10] holds unchanged.

Furthermore, since quantum gravity in the present model becomes important at a millimeter, it is natural to explore the idea of Ref. [5] on the cosmological constant in this context [7]. This will be discussed in Ref. [7].

The paper is organized as follows: In Sec. II we discuss a 4D theory with no branes or extra dimensions. We show how the scale of quantum gravity can be much smaller than the SM ultraviolet cutoff. Moreover, we show how the SM fields "shield" themselves from strong gravity. In Sec. III we describe the basic ingredients of the 5D brane-world model. In addition, we propose a mechanism for localization of massive fields in the model; we also study the tensorial structure of the graviton propagator. In Sec. IV we describe qualitatively why the presence of light Regge recurrences in a theory with low M_* cannot affect strongly the 4D physics on the brane. In Sec. V we develop a model which mimics basic properties of the string spectrum. We show how the SM can "shield" itself from the huge multiplicity of the Regge states. In Sec. VI we study high-energy, astrophysical and cosmological constraints on M_* which arise due to the high multiplicity stringy Regge states. Section VII discusses some curious aspects of black hole physics in the present context. In Sec. VIII we study the processes of baryon number violation due to quantum gravity effects. Conclusions are given in Sec. IX. Some useful derivations and formulas are collected in the Appendix.

II. A FOUR-DIMENSIONAL EXAMPLE

Before we discuss the five-dimensional brane model, we are going to show in this section how the gravitational coupling constant can be determined by "non-gravitational" physics already in a simple four-dimensional example.

Let us consider a model with the following two scales:

$$M_* \ll M_{SM}. \qquad (2.1)$$

Here we assume that the SM cutoff M_{SM} can be as large as the conventional GUT scale. In the latter case one needs to stabilize the Higgs boson mass against radiative corrections. Therefore, above 1 TeV the standard model should be embedded in some bigger theory (supersymmetry, extended technicolor or something else). In the paper we use for convenience the name standard model for this theory.

Note that the hierarchy between the scales M_* and M_{SM} (2.1) is *stable*; this is similar to the stability of the QCD scale with respect to the electroweak scale.

Consider the following gravitational action

$$S_G = \int d^4x \sqrt{|g|} \left(M_*^2 R + \sum_{n=1}^{\infty} c_n \frac{R^{n+1}}{M_*^{2(n-1)}} \right), \qquad (2.2)$$

where R^n, $n=2,3,\ldots$, stand for all possible higher derivative curvature invariants. So far the only scale in this model is M_*. Since this scale is small, i.e., $M_* \ll 10^{19}$ GeV, the self-interactions of gravitons are strong. The corresponding Newton constant is defined as follows:

$$G_* = \frac{1}{16 \pi M_*^2}. \qquad (2.3)$$

Furthermore, the effective field-theory description of gravity in Eq. (2.2) ceases to be valid for energies above M_*.

As a next step, let us couple the standard model fields to the gravity described by the action (2.2). For this we introduce the action of the SM fields:

$$S_{SM} = \int d^4x \sqrt{|g|} \mathcal{L}_{SM}(\Psi, M_{SM}), \qquad (2.4)$$

where M_{SM} is the ultraviolet cutoff of particle physics described by Eq. (2.4) and Ψ collectively denotes all the SM fields. The total action we deal with is the sum

$$S = S_G + S_{SM}. \qquad (2.5)$$

Gravity in Eqs. (2.2), (2.5) is considered as an effective low-energy field theory up to energies of order M_*. The SM, on the other hand, is supposed to be treated as a quantum field theory up to the scale M_{SM}. This is the classical picture.

The crucial point is that at the quantum level the standard model loops renormalize the gravitational action (2.2). This renormalization is due to perturbative [11] as well as nonperturbative [12] SM contributions. For the illustrative purposes consider a one-loop polarization diagram with two external graviton legs and only SM heavy particles in the loop.[1] Let us set the momenta in the graviton external legs to be smaller than M_* so that Eq. (2.2) provides valid classical description of external graviton lines. On the other hand, the momentum in the loop in which the SM fields are running can take any value from zero all the way up to M_{SM}. As a result, this diagram gives rise to the renormalization of the graviton kinetic term [11] which generically is determined by the mass square of the heaviest SM particle in the loop (the latter we set to be of the order of M_{SM}^2). On the other hand, the similar diagrams with the gravitational lines in the loop cannot be calculated within the effective field theory approximation given by Eq. (2.2). However, given the assumption of the "softness" of gravity above M_* these corrections become sub-dominant (see detailed discussions below and in Sec. IV). In general, while dealing with the renormalization of the graviton kinetic term, one should take into account nonper-

---

[1]For simplicity of arguments we do not discuss here the other diagrams in the same order in which two graviton lines join at the same point.

turbative SM contributions as well. All these contributions can be summarized by adding to the total action (2.5) the following induced terms:

$$\Delta S_{\text{ind}} = M_{\text{ind}}^2 \int d^4x \sqrt{|g|} \left(R + \mathcal{O}\left(\frac{R^2}{M_{\text{SM}}^2}\right) + \cdots \right), \quad (2.6)$$

where the induced scale is defined as follows [12,13]:

$$M_{\text{ind}}^2 = \frac{i}{96} \int d^4x \, x^2 [\langle 0| T \, T_\mu^\mu(x) T_\nu^\nu(0) |0\rangle$$
$$- (\langle 0| T_\mu^\mu(0) \ |0\rangle)^2], \quad (2.7)$$

and T_μ^μ denotes the trace of the energy-momentum tensor of the fields of the particle theory (standard model).[2] Generically M_{ind}^2 is expected to be of the order of the cutoff of SM, that is $M_{\text{ind}}^2 \sim M_{\text{SM}}^2$.

The higher derivative terms which are also induced via the loop diagrams are suppressed by powers of M_{SM}. Since the latter scale is much bigger than M_*, we can neglect these higher derivative terms in comparison with the ones which are suppressed by the smaller scale M_* and are already present in Eq. (2.2).

Therefore, the total action takes the form

$$S_{\text{total}} = S_G + S_{\text{SM}} + \Delta S_{\text{ind}}. \quad (2.8)$$

The net result is that due to the induced term the coupling of gravity to the SM fields is renormalized. In fact, this coupling becomes weaker. The physical interpretation of this phenomenon will be given at the end of this section. Here we write the resulting Newton constant:

$$16 \pi G_N = \frac{1}{M_*^2 + M_{\text{ind}}^2} \simeq \frac{1}{M_{\text{ind}}^2} \propto \frac{1}{M_{\text{SM}}^2}. \quad (2.9)$$

For phenomenological reasons we have to put $M_{\text{ind}}^2 \simeq M_{\text{P}}^2$. Therefore, the Planck scale is a derived parameter and is completely defined by the content and dynamics of the corresponding particle physics theory.

Let us now turn to the higher derivative terms. As before, they are suppressed by the scale M_*:

$$\sqrt{|g|} \frac{R^{n+1}}{M_*^{2(n-1)}}. \quad (2.10)$$

As a result, the effective field theory approximation to gravity breaks down at the scale M_*.

Based on these considerations we can draw the following conclusions:

_____

[2] In the one-loop approximation scalars and spin-1/2 particles give rise to a positive contributions to the induced kinetic term while gauge bosons lead to negative terms. We will assume that in the present model the overall sign of M_{ind}^2 is positive. Moreover, through the paper we neglect (i.e., we fine-tune to zero) the 4D cosmological constant which is also induced by loops.

The matter fields are coupled to gravitons very weakly, via the ordinary 4D Newton constant $G_N = 1/16\pi M_{\text{P}}^2$.

At low momenta, i.e., for $p \ll M_*$, gravity couples to itself via the ordinary Newton constant G_N. However, at high momenta, $p \gtrsim M_*$, the higher derivative terms in Eq. (2.2) become important and there are additional contributions of the type p^n/M_*^n in the graviton self-interaction vertices.

For low momenta, $p \ll M_*$, the graviton propagator is that of a normal 4D massless particle. However, if $p \gtrsim M_*$ the propagator is modified by higher derivative terms.

As we discussed before, we assume that gravity becomes "soft" above the scale M_* so that the coupling of matter fields to gravity remains weak at any reasonable energies below M_{SM}. From the practical point of view this means that the "softening" of gravity due to quantum effects could be modeled by some kind of formfactors in the gravitational vertices and propagators (see Secs. IV, V).

What we have seen in this section is that the SM particles renormalize their own gravitational couplings and make it weaker. The mass squared parameter in front of the Ricci scalar in the Einstein-Hilbert action is similar in this respect to the Higgs boson mass in the standard model: no matter how small it is in the classical theory, the quantum loops drive its value all the way up to the corresponding ultraviolet cutoff.

Let us try to understand this phenomenon in terms of a simple physical picture. Suppose there is a single heavy scalar field which has a mass of the order of M_{SM} and which is the only state that runs in the SM loop. The corresponding (additive) renormalization of the gravitational constant is

$$M_{\text{SM}}^2 \log\left(\frac{M_{\text{SM}}^2}{\mu^2}\right). \quad (2.11)$$

Here μ is the energy scale (normalization point) in the SM process which is less than M_{SM} and bigger than the infrared cutoff of the SM. Thus, the renormalization of the gravitational coupling is proportional to the mass square of the particle. This can be understood as follows. Consider a heavy static source the gravitational pull of which we are measuring at some distance bigger than $1/M_*$. If the SM massive particles are present, they create virtual particle-antiparticle pairs and "polarize" the vacuum around the source. The pair consists of a virtual positive energy state which is gravitationally attracted to the source and a virtual negative energy state which is repelled from the source. Therefore, the vacuum is polarized with virtual "gravitational dipoles." As a result, these dipoles screen the original gravitational interactions. Thus, standard model particles "shield" sources (and themselves as well) from strong gravity. The heavier the particle, the more effective is the shielding.

In the following sections we consider the higher dimensional framework in which we will study the effects of quantum gravity modes on the observable 4D physics. The "shielding" phenomenon described above plays the crucial role in our considerations. The main motivation for going to higher dimensions, as was already mentioned in the Intro-

duction, is that some brane-world scenarios can provide an opportunity to produce the hierarchy between the scales M_* and M_{SM} dynamically.

III. THE FIVE DIMENSIONAL FRAMEWORK

In this section we shall set the 5D framework which allows to lower the fundamental Planck scale M_* much below the field theory cutoff of the SM. In fact, this is the model of Ref. [4] which we shall discuss briefly.

Consider five-dimensional Minkowski space with a standard bulk gravitational action

$$S_{bulk} = \int d^{(4+1)} X \sqrt{|G|} \mathcal{L}(G_{AB}, \mathcal{R}_{ABCD}, \Phi), \quad (3.1)$$

where the capital Latin indexes run over $D = (4+1)$-dimensional space-time. G_{AB} denotes the metric of 5-dimensional space, $\mathcal{R}_{ABCD}$ is the 5-dimensional Riemann tensor and Φ collectively denotes other possible fields. We shall assume that there is a 3-brane in this space. Although the 3-brane can be realized as a soliton of the corresponding field equations, at this point we shall keep our discussion as general as possible, and will simply treat the 3-brane as a hyper-surface that breaks five-dimensional translational invariance. We split the coordinates in 5-dimensions as follows: $X^A = (x^\mu, y)$, where Greek indexes run over the four-dimensional world-volume, $\mu = 0,1,2,3$, and y is the coordinate transverse to the brane. In order to reduce our discussion to its main point the brane will be taken to have zero width.[3] In this approximation the brane classical world volume action takes the form:

$$S_{3\text{-brane}} = -T \int d^4 x \sqrt{|g|}, \quad (3.2)$$

where T stands for the brane tension and $g_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B G_{AB}$ denotes the induced metric on the brane. For simplicity of arguments we neglect brane fluctuations.[4] and go to the coordinate system where the induced metric takes the following form:

$$g_{\mu\nu}(x) = G_{\mu\nu}(x, y=0). \quad (3.3)$$

We assume that the world volume theory on the brane is some gauge theory that includes standard model and its possible high-energy extensions (such as models of grand unification), with a field theory cutoff M_{SM}. The effective low-energy theory in the bulk is just 5D gravity with a fundamental Planck scale M_*, above which quantum gravity effects become important. The crucial assumption is that $M_* \ll M_{SM}$, and in fact we will be mostly interested in the case when $M_* \ll 1$ TeV.

---

[3]In fact, we assume that the brane width is of the order of $1/M_{SM} \ll 1/M_*$, see discussions below.

[4]One could do this by, e.g., putting the brane onto an orbifold fixed point in extra dimension. In this case the brane is just an "end" of the infinite extra dimension.

Before we proceed further let us make a digression and address the issue whether it is possible to have the scale of the worldvolume theory M_{SM} much bigger than the bulk fundamental scale M_* within any dynamically realizable framework. As an existence proof of such a scenario we give an example of 4D $\mathcal{N} = 1$ supersymmetric $SU(N)$ Yang-Mills theory with large number of colors, $N \gg 1$. In this model there are BPS domain walls [14] which in many respect resemble D2-branes of string theory [15]. In particular, the tension of this wall scales as $N\Lambda_{SYM}^3$ [14] and the width of the wall scales as $1/(N\Lambda_{SYM})$ (Λ_{SYM} being the strong interaction scale) [16] as it would for a D2-brane. The world volume theory of this toy $(2+1)$-dimensional "braneworld" provides a precise realization of the scenario which we are alluding to in the present work. Indeed, the bulk fundamental scale in this model is Λ_{SYM} (the counterpart of our M_*). However, there is much higher scale present in the theory, that is $N\Lambda_{SYM} \gg \Lambda_{SYM}$ (the counterpart of our M_{SM}). In other words, there is the distance scale in the model, $1/N\Lambda_{SYM}$, which is much smaller than the fundamental length scale $1/\Lambda_{SYM}$. The brane width in this theory is determined by the shorter scale $1/N\Lambda_{SYM}$ and not by the fundamental scale $1/\Lambda_{SYM}$ [16]. Furthermore, as was argued in Ref. [17] there should exist in the theory nonperturbative states the mass of which scale as $N\Lambda_{SYM}$. These states can be present as in the bulk as well as in the world volume theory. Moreover, the world volume states consist of the localized Goldstone particles and the heavy states the masses of which scale as $N\Lambda_{SYM}$. Thus, the true ultraviolet cutoff of the worldvolume theory should be $N\Lambda_{SYM}$, which is much bigger than the bulk fundamental scale Λ_{SYM} at which the bulk theory changes dramatically its regime due to the confinement.

We might hope that a similar scenario can be realized in string theory. Here the origin of the huge scale separations between M_* and M_{SM} could be provided, for instance, by a very small string coupling constant g_s. The small string coupling gives rise to a new nonperturbative scale in string theory which is related to the fundamentals string length as $g_s l_s$ [18]. The g_s should play the role of the small number similar to that played by the parameter $1/N$ in the aforementioned example of supersymmetric gluodynamics. These issues will not be discussed in this paper, but will rather be postponed until further investigations.

After this digression let us turn back to our main discussions. Thus, the unified tree-level bulk-brane action can be written as follows:

$$S = M_*^3 \int d^4 x \, dy \sqrt{|G|} \left\{ \mathcal{R} + \mathcal{O}\left(\frac{\mathcal{R}^2}{M_*^2}\right) \right\}$$

$$+ \int d^4 x \sqrt{|g|} \mathcal{L}_{SM}(\Psi, M_{SM}). \quad (3.4)$$

Here the first term is the standard 5D Einstein-Hilbert action, whereas the last term describes coupling of bulk gravity to the brane world-volume SM field theory which has the cutoff $M_{SM} \gg M_*$. We assume that the SM fields are confined to the brane. Moreover, in what follows we will imply, without

manifestly writing it that the Gibbons-Hawking surface term is included in the brane action as it gives rise to the correct bulk Einstein equations.

What would be the observational consequences of the action (3.4)? Naively, the theory based on such an action is ruled out by everyday gravitational observations; the extra dimension is not compactified, it has an infinite volume and a brane-localized observer would measure the strong five-dimensional gravity with the small Planck scale M_* rather than the weak 4D gravity with $M_P \sim 10^{18}$ GeV. However, the above naive argument is false; the reason being that the important terms which are compatible with all the symmetries of the action have been left out in Eq. (3.4). Such terms, even if not included in the classical action, will be generated by quantum loops on the brane. To see this let us concentrate on the one-loop diagram of Fig. 1.

This diagram describes the renormalization of the graviton kinetic term, due to the SM matter loops localized on the brane. Just as in the 4D case, the corresponding operator which is induced by this correction has the form [4,19]

$$S_{\mathrm{ind}} = M_{\mathrm{ind}}^2 \int d^4 x \sqrt{|g|} R(x), \tag{3.5}$$

where g is the higher dimensional metric evaluated at the position of the brane defined in Eq. (3.3), and $R(x)$ is the corresponding four dimensional Ricci scalar. As in the 4D case of the previous section the induced gravitational constant has to equal to the 4D Planck mass, $M_{\mathrm{ind}}^2 = M_P^2$. This term should be added to the action (3.4).

Thus, the bulk graviton acquires a four-dimensional kinetic term which is localized on the brane. To realize the importance of this correction, note that this term is weighted by the factor M_P^2 which is much bigger than the bulk scale M_* that multiplies the bulk Einstein-Hilbert term. As we shall see the scale M_P will play the role of a 4D Planck scale for an observer on the brane. In this framework, similar to the 4D case, the Planck scale is determined by the cutoff of the standard model. The high SM cutoff M_{SM} makes its own gravitational coupling to be naturally small. Thus, the SM "shields" itself from strong 5D bulk gravity by means of the vacuum polarization effects described in the previous section.

Let us remark here that in addition the following 4D induced terms should be included in the world volume action:

$$S_{\mathrm{add}} = M_P^2 \int d^4 x \sqrt{|g|} [\Lambda + \mathcal{O}(R^2)], \tag{3.6}$$

where Λ in Eq. (3.6) denotes an induced four-dimensional cosmological constant. The role of this term is to renormalize the brane tension. In five-dimensional Minkowski space a brane with nonzero tension inflates [20–22]. Therefore, to avoid worldvolume inflation we fine tune the brane tension T and the brane world volume cosmological constant Λ so that the net tension is vanishing $T' \equiv T - \Lambda M_P^2 = 0$. This is a usual fine-tuning of the four-dimensional cosmological constant.

A. Four-dimensional gravity on a brane

Here we recall how the 4D gravity is obtained on the brane with uncompactified infinite volume extra dimension (see Ref. [4]). At the end of the subsection we obtain certain new results on the localization of massive fields on a brane.

We start by including in the action (3.4) the induced 4D Einstein-Hilbert term. Let us neglect for a moment the higher derivative terms (they will be discussed in the next subsection). The action takes the form

$$S = M_*^3 \int d^4 x \, dy \sqrt{|G|} \mathcal{R} + \int d^4 x \sqrt{|g|} \{ M_P^2 R(x)$$
$$+ \mathcal{L}_{\mathrm{SM}}(\Psi, M_{\mathrm{SM}}) \}. \tag{3.7}$$

As before we imply the presence of the Gibbons-Hawking surface term on the world volume. The graviton propagator resulting from such an action is quite peculiar. Ignoring the tensor structure for a moment we obtain the following expression for the two-point Green function [4]:

$$\tilde{G}(p, y) = \frac{1}{2 M_*^3 p + M_P^2 p^2} \exp\{-p|y|\}. \tag{3.8}$$

Here p^2 is the four-dimensional Euclidean momentum and $p \equiv \sqrt{p^2}$. For sources which are localized on the brane, i.e., for $y = 0$, this propagator reduces to a massless four-dimensional Green's function

$$\tilde{G}(p, y = 0) = \frac{1}{M_P^2 p^2} + \cdots, \tag{3.9}$$

provided that $p \gg 1/r_c \equiv M_*^3/M_P^2$. Thus, at distances $r \ll r_c$ we observe the correct Newtonian behavior of the potential created by a static source of mass M:

$$U(r \ll r_c) = \frac{M}{16 \pi M_P^2 r} + \cdots. \tag{3.10}$$

At large distances, $r \gg r_c$, however, the behavior of the Green function changes

$$\tilde{G}(p, y = 0) = \frac{1}{2 M_*^3 p} + \cdots. \tag{3.11}$$

This gives rise to the Newtonian potential which scales in accordance with the laws of a five-dimensional theory

$$U(r \gg r_c) = \frac{M}{16 \pi^2 M_*^3 \; r^2} + \cdots. \tag{3.12}$$

The explicit corrections to these expressions can be found in Ref. [4].[5] This somewhat unusual behavior can be understood in two equivalent ways which we briefly discuss.

First let us adopt the five-dimensional point of view. In this language, although there is no localized massless particle, there exists a localized unstable state in the spectrum (we call it a resonance state for convenience). The lifetime of this resonance is $\sim r_c$. The resonance decays into the continuum of modes. This can be manifestly seen using the Källen-Lehmann representation for the Green's function

$$\widetilde{G}(p,y=0)=\frac{1}{M_*^3}\left(\frac{1}{2\,p+r_c p^2}\right)=\int_0^\infty\frac{\rho(s)ds}{s+p^2}, \quad (3.13)$$

where the spectral density as a function of the Mandelstam variable s takes the form

$$\rho(s)\propto\frac{1}{\sqrt{s}}\frac{r_c}{4+s\,r_c^2}. \quad (3.14)$$

As $r_c\to\infty$ the spectral density tends to the Dirac function, $\rho(s)\to\text{const}\cdot\delta(s)$, describing a stable massless graviton (this corresponds to the limit when the bulk kinetic term can be neglected). At the distances $r\ll r_c$ the resonance mimics the massless exchange, and therefore mediates the $1/r^2$ force. At larger distances, however, it decays into the continuum states and the force law becomes that of a five-dimensional theory, $\sim 1/r^3$.

A different but equivalent way to understand the above result is to adopt the point of view of the four-dimensional mode expansion. The analysis of the linearized equation for the small fluctuations shows (see Appendix A) that there is a continuum of 4D massive states with wave-function profiles $\phi_m(y)$ which are suppressed at the location of the brane by the following factor

$$|\phi_m(y=0)|^2\propto\frac{4}{4+m^2 r_c^2}, \quad (3.15)$$

where m denotes the continuous mass parameter of the Kaluza-Klein (KK) modes. The Newtonian potential on the brane is mediated by the exchange of all these Kaluza-Klein modes. These give rise to the expression

$$U(r)\propto\frac{M}{M_*^3}\int_0^\infty\frac{dm}{4+m^2 r_c^2}\frac{e^{-mr}}{r}. \quad (3.16)$$

At any distance r the dominant contribution comes from the modes lighter than $m=1/r$. The modes with $m<1/r_c$ have unsuppressed wave functions on the brane. Therefore, for $r>r_c$ the picture is similar to that of a five-dimensional

theory. In contrast, when $r<r_c$ the picture changes since the modes with $m>1/r_c$ have suppressed couplings on the brane. Although the number of the modes which participate in the exchange at a given distance $r<r_c$ is the same as in the five-dimensional picture, their contributions are suppressed [6,25]. Thus, the number of the light modes effectively contributing to the exchange "freezes-out" and the resulting behavior of the potential is $1/r$.

The same consideration can be applied to other spin states, for instance, to scalars [4,19,25] or to gauge fields [26–28]. In general, the picture is similar: One obtains 4D behavior for $r<r_c$ and 5D behavior at $r>r_c$.

We will investigate these properties further by adding mass terms for the bulk fields. This is in particular important for scalars the mass terms of which are not protected by any symmetries. Below, we will discuss a scalar field which has a nonzero mass terms in the bulk and on the brane (the same consideration applies to other massive fields as well). Neglecting all other fields the action takes the form

$$S=M_*^3\int d^5X\{[\partial_A\Phi]^2-M_B^2\Phi^2\}$$

$$+M_P^2\int d^4x\{[\partial_\mu\Phi(x,y=0)]^2-\mu^2\Phi^2\}. \quad (3.17)$$

We choose somewhat unconventional normalizations where the scalar field is dimensionless. This system is analyzed in detail in Appendix along the lines of what we did for the massless case. The resulting propagator has the form (in Euclidean space)

$$\widetilde{G}(p,y)=\frac{\exp\{-\sqrt{p^2+M_B^2}|y|\}}{2M_*^3\sqrt{p^2+M_B^2}+M_P^2(p^2+\mu^2)}. \quad (3.18)$$

For $p\gg 1/r_c$ and at $y=0$, the propagator resembles that of a four-dimensional field of mass μ, i.e., $\widetilde{G}(p,0)\sim(p^2+\mu^2)^{-1}$. As before, it is the four-dimensional part of the action that determines the short distance behavior.

Moreover, as in the massless case, we could study the four-dimensional mode expansion. The detailed analysis (see Appendix) reveals that we should distinguish two cases. We will discuss them in turn.

(1) For $\mu>M_B$ there is no zero mode, but rather a continuum of massive modes with mass $m\in[M_B,\infty)$. The wave-functions of the continuum modes have the following transverse space profiles at the position of the brane:

$$|\phi_m(y=0)|^2\propto\left[4+r_c^2 m^2\left\{\frac{(1-\mu^2/m^2)^2}{1-M_B^2/m^2}\right\}\right]^{-1}. \quad (3.19)$$

As shown in the Appendix, this profile results in the suppression of all the continuum modes on the brane except those in a narrow mass band of the width $\sim 1/r_c$ centered around the value of μ. In other words, to a four-dimensional observer the continuum of modes effectively appears as a single metastable mode of the mass μ.

---

(2) For $\mu < M_B$ we still have a continuum of massive modes starting at $m = M_B$ with the same profiles at the brane position given by Eq. (3.19). In addition we also find a *normalizable mode* of mass $\sim \mu$. This is to be interpreted as a truly four-dimensional localized state with the well defined 4D mass.

The existence of this localized mode can also be seen from the propagator (3.18) which has a *physical* pole at $p^2 \sim \mu^2$ if $\mu < M_B$. Since the value $m \sim \mu$ is outside the continuum band in this case all the continuum modes are suppressed on the brane. Therefore, a four dimensional observer will still effectively see a single state of mass $\sim \mu$, but this time a true 4D localized state.

It is very interesting to note that for $\mu = 0$ (i.e., no mass term on the brane) but nonzero M_B, still there is a bound state with the mass $m_{BS}^2 \sim M_B / r_c$ which is localized on the brane. This mass is very small in the regimes under consideration. Therefore, this framework provides a new mechanism for the localization of an almost-massless particles on a brane in an infinite volume flat extra dimension.

B. Tensorial structure of the propagator

We have seen in the previous section that the usual 4-dimensional Newton law for gravity is reproduced at distances $r \ll r_c$. At very short distances, however, we expect the Newton law to be modified by higher-derivative terms which we did not consider so far. Moreover, we neglected in the previous subsection the tensorial structure of the Green function, however, the predictions for the relativistic effects strongly depend on this structure. In this subsection we will address these issues.

In our model, it is not immediately obvious which scale determines the modification due to the quantum gravity corrections. This question for scalar field theory models was studied in Ref. [7] where it was concluded that the modification occurs for distances of order $1/M_*$. Here we investigate this issue for the gravitational action and in addition study an important question of the tensorial structure of the graviton propagator.

Since the field theory of gravity is non-renormalizable it should be regarded as a low-energy effective theory. The effects of quantum gravity at low energies can be encoded by adding all possible higher-derivative operators to the gravitational action. In the bulk, gravity becomes strong at the scale M_*, hence, the higher-dimensional operators in the bulk are suppressed by powers of M_*. We would like to study the effects of these terms on the propagator. For calculational convenience we choose the following form of the higher-derivative terms in the bulk:

$$S_{\text{bulk}} = \int d^5 X \sqrt{|G|}\, M_*^3 \left(\mathcal{R} + \frac{c}{M_*^2} \left(\frac{\mathcal{R}^2}{2} - \mathcal{R}_{AB} \mathcal{R}^{AB} \right) + \cdots \right),$$

$$(3.20)$$

where c is some constant and dots denote all other possible

higher-order operators.[6] As we discussed before, we also expect that additional higher-dimensional operators will be induced on the brane, in analogy to the induced brane Ricci term. The strength of these operators, however, is suppressed by M_{SM}. Below we will compute the modification of the 4D Green function on the brane due to the higher-order operators in the bulk action (3.20) and discuss the corresponding tensorial structures.

We need to calculate the gravitational perturbations created by a static source which is localized on the brane. Let us introduce the metric fluctuations:

$$G_{AB} = \eta_{AB} + h_{AB}.$$

$$(3.21)$$

We choose *harmonic gauge* in the bulk:

$$\partial^A h_{AB} = \frac{1}{2} \partial_B h_C^C.$$

$$(3.22)$$

The $\{\mu 5\}$ components of the equations of motion lead to the condition:

$$h_{\mu 5} = 0.$$

$$(3.23)$$

Thus, the surviving components of h_{AB} are $h_{\mu\nu}$ and h_{55}. In harmonic gauge the $\{55\}$ component of Einstein's equation can be solved by the substitution

$$\partial_A \partial^A h_5^5 = \partial_A \partial^A h_\mu^\mu.$$

$$(3.24)$$

The indices in all these equations are raised and lowered by a flat space metric tensor. Finally, we come to the $\{\mu\nu\}$ components of the Einstein equation. After some rearrangements these take the form:

$$M_*^3 \left(\partial_A \partial^A h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \partial_A \partial^A h_\alpha^\alpha - \frac{1}{2} \eta_{\mu\nu} \partial_A \partial^A h_5^5 \right)$$

$$+ M_*^3 \left[\frac{c}{M_*^2} \left(-\partial_A \partial^A \partial_B \partial^B h_{\mu\nu} + \frac{1}{2} \eta_{\mu\nu} \partial_A \partial^A \partial_B \partial^B h_\alpha^\alpha \right. \right.$$

$$\left. \left. + \frac{1}{2} \eta_{\mu\nu} \partial_A \partial^A \partial_B \partial^B h_5^5 \right) \right] + M_P^2 \delta(y) \left(\partial_\alpha \partial^\alpha h_{\mu\nu} \right.$$

$$\left. - \frac{1}{2} \eta_{\mu\nu} \partial_\beta \partial^\beta h_\alpha^\alpha + \frac{1}{2} \eta_{\mu\nu} \partial_\beta \partial^\beta h_5^5 - \partial_\mu \partial_\nu h_5^5 \right)$$

$$= T_{\mu\nu}(x) \delta(y).$$

$$(3.25)$$

---

[6]The truncation of the action (3.20) at any finite order in derivatives generically gives rise to unphysical poles in the propagator for the momenta of order M_* (unless the coefficients of the higher derivative terms are chosen very carefully as in the Gauss-Bonnet term for instance). However, the expansion in powers of p^2/M_*^2 breaks down in that domain so these poles are spurious and should be neglected. In the total action, if it comes from a consistent higher-dimensional theory, such as string theory, there should not be any unphysical states.

Here, we choose such a normalization that the energy-momentum tensor of a source localized on the brane is $T_{\mu\nu}(x)\delta(y)$. Multiplying both sides of these equations by $\eta_{\mu\nu}$ we obtain

$$\partial_A\partial^A h^\alpha_\alpha = -\frac{T^\alpha_\alpha\delta(y)}{3M^3_*} + \frac{c}{M^2_*}\partial_A\partial^A\partial_B\partial^B h^\alpha_\alpha. \quad (3.26)$$

Finally, using this expression we find

$$M^3_*\left(\partial_A\partial^A - \frac{c}{M^2_*}\partial_A\partial^A\partial_B\partial^B\right)h_{\mu\nu} + M^2_P\delta(y)(\partial_\alpha\partial^\alpha h_{\mu\nu}$$
$$-\partial_\mu\partial_\nu h^5_5) = \left(T_{\mu\nu} - \eta_{\mu\nu}\frac{T^\alpha_\alpha}{3}\right)\delta(y). \quad (3.27)$$

Turning to Euclidean momentum space and multiplying both sides of the equation by a (probe) conserved energy-momentum tensor we find

$$\left(M^3_*\left((p^2-\partial^2_y) + \frac{c}{M^2_*}(p^2-\partial^2_y)^2\right)\right.$$
$$\left. + M^2_P p^2\delta(y)\right)\tilde{h}_{\mu\nu}(p,y)\tilde{T}'^{\mu\nu}$$
$$= \left\{\tilde{T}_{\mu\nu}\tilde{T}'^{\mu\nu} - \frac{1}{3}\tilde{T}^\alpha_\alpha\tilde{T}'^\beta_\beta\right\}\delta(y). \quad (3.28)$$

Following [19] we look for a solution of this equation in the following form:

$$\tilde{h}_{\mu\nu}(p,y)\tilde{T}'^{\mu\nu} = A(p)B(p,y), \quad (3.29)$$

where the function B satisfies the equation

$$\left((p^2-\partial^2_y) + \frac{c}{M^2_*}(p^2-\partial^2_y)^2\right)B(p,y) = \delta(y). \quad (3.30)$$

The expression for the propagator on the brane is as follows:

$$\tilde{h}_{\mu\nu}(p,y=0)\tilde{T}'^{\mu\nu} = \frac{\tilde{T}_{\mu\nu}\tilde{T}'^{\mu\nu} - \frac{1}{3}\tilde{T}^\alpha_\alpha\tilde{T}'^\beta_\beta}{M^2_P p^2 + M^3_* B^{-1}(p,0)}. \quad (3.31)$$

Furthermore, we can calculate $B(p,0)$ from Eq. (3.30):

$$B^{-1}(p,0) \simeq 2p\left(1 + \frac{\sqrt{c}p}{M_*} + \cdots\right). \quad (3.32)$$

Using this expression we find the propagator between two points on the brane:[7]

$$\tilde{h}_{\mu\nu}(p,y=0)\tilde{T}'^{\mu\nu} = \frac{\tilde{T}_{\mu\nu}\tilde{T}'^{\mu\nu} - \frac{1}{3}\tilde{T}^\alpha_\alpha\tilde{T}'^\beta_\beta}{M^2_P p^2 + 2M^3_* p[1+\sqrt{c}pM^{-1}_* + \cdots]}, \quad (3.33)$$

---

[7]The scalar part of this propagator was obtained in [7].

where the dots denote terms which are of higher order in p/M_*. We assume that gravity above this scale becomes soft. As was emphasized in Ref. [6], and will be shown below in detail, there exists in our model a well-defined expansion of SM scattering cross sections and other SM observables in powers of the usual four-dimensional Planck mass M_P. In the leading order in this expansion the usual standard model results are reproduced for any energy scale. The gravitational corrections for energies below M_* can be calculated within the standard framework. However, at higher energies the effective gravitational action ceases to be valid and the quantum gravity corrections should be taken into account. Since we assume the "softness" of quantum gravity effects these corrections should remain negligible compared to the SM corrections.

As we pointed out before, the bulk quantum gravity scale M_* can be smaller that 1 TeV. At distanced below $1/M_*$ the Newton law is modified. This law has only be tested at distances bigger that 0.2 mm [1]. Therefore, a model with $M_* \gtrsim 10^{-3}$ eV does not contradict the data on static force measurements. Note that for such low values of M_*, the cross-over to five-dimensional gravity only occurs for $r > 10^{63}$ cm [6].

We would like now to discuss the tensorial structure of the graviton propagator in the present model. In Eq. (3.33), the tensorial structure is similar to that of a 5D graviton (or equivalently of a 4D massive graviton) [4]. This points to the discontinuity which leads to the contradictions with observations [29,30]. However, this problem is an artifact of using the lowest tree-level approximation [31]. We discuss below two ways to avoid this problem.

In the context of infinite volume uncompactified extra dimension we note that the lowest tree-level approximation which was used to derive Eq. (3.33) breaks down at small distances [31,32]. In fact, the tensorial structure obtained in Eq. (3.33) is applicable for distances $r \gg r_c$ where the 5D behavior takes over. For short distances $r \ll r_c$ the higher corrections become dominant. Thus, one has to sum up all the tree-level graphs which are obtained by iterations of the nonlinear Einstein equations in the external source. This is equivalent of finding exact solutions to the classical equations of motion. The net result of this, as was advocated in Ref. [32], is that the coefficient 1/3 in the numerator of Eq. (3.33) is promoted to a momentum dependent form factor. For small momenta (i.e., large distances $r \gg r_c$) the form factor turns into the coefficient 1/3, however, for large momenta, i.e., small distances it returns the value 1/2, consistent with the 4D observations.

The Schwarzschild solution in this case can only be found in the approximation $r_c \to \infty$ [33,32] which by construction has no discontinuity. Moreover, some other exact cosmological solutions were found [8,34,33] which demonstrate that there is no discontinuity in the full classical theory [32]. Hence, the extra helicity ± 1 and 0 states of the 5D graviton decouple from the 4D matter fields as $r_c \to \infty$.

Note that the continuity in the graviton mass in curved (A)dS backgrounds was demonstrated recently in Refs. [35,36] (see also further considerations in Ref. [37]). We should emphasize that Refs. [35,36] as well as our works

discuss the continuity in the classical 4D gravitational interactions with 4D matter. There certainly is the discontinuity in the full theory in the sense that there are extra degrees of freedom in the model. The latter can manifest themselves at the quantum level in loop diagrams [38]. However, what is important for observations is the continuity in the tree-level couplings of gravity to matter. These couplings are continuous.

In general, the simplest way to deal with the discontinuity problem, as was suggested in Ref. [6], is to compactify the extra space on a circle of a huge radius R. This radius can be bigger that the present day horizon distance, but still smaller that r_c. For instance, if $M_* \sim 10^{-3}$ eV, then $r_c \sim 10^{63}$ cm and R can be as large as 10^{59} cm [6]. This is about thirty orders of magnitude bigger that the horizon scale; thus, this extra dimension is infinite for any practical purposes.

The convenience of such a procedure is that in this case the lowest tree-level approximation to the graviton exchange becomes applicable even at distances $r \ll r_c$ (so there is no need to sum up all the tree level graphs). The reason is that there is a zero mode which gives the correct 4D coefficient 1/2 in the tensorial structure, and moreover, all the KK modes which could, in the conventional case, turn this coefficient into 1/3 are now additionally suppressed by the ratio R/r_c. This is possible to see from the 4D expression for the 5D graviton propagator [6]:

$$G_4^{\mu\nu;\alpha\beta}(p) \simeq \left(\frac{1}{2} (\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha}) - \frac{1}{2}\eta^{\mu\nu}\eta^{\alpha\beta} \right) \frac{1}{p^2}$$

$$+ \frac{1}{\pi^2}\frac{R}{r_c}\sum_{n=1}^{\infty}\frac{1}{n^2}\left(\frac{1}{2}(\widetilde{\eta}^{\mu\alpha}\widetilde{\eta}^{\nu\beta} + \widetilde{\eta}^{\mu\beta}\widetilde{\eta}^{\nu\alpha}) \right.$$

$$\left. - \frac{1}{3}\widetilde{\eta}^{\mu\nu}\widetilde{\eta}^{\alpha\beta} \right) \frac{1}{p^2+m_n^2}, \qquad (3.34)$$

where

$$\widetilde{\eta}_{\mu\nu} = \eta_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}. \qquad (3.35)$$

The first terms in this expression corresponds to the 4D massless mode and the rest of the terms correspond to the KK modes which due to the induced kinetic term on the brane are suppressed by R/r_c [6]. Thus, the model has no discontinuity even in the lowest order tree-level approximation.

IV. GRAVITY ABOVE M_*

In this section we will try to summarize the main qualitative reasons why our framework survives all the constraints. As we shall see, the reason is the self-shielding of the SM fields from the bulk theory. The SM generates a large brane kinetic term for any bulk field coupled to the SM particles and makes it to be weakly coupled to the brane matter. In other words, the high-dimensional strongly coupled bulk theory is "projected" onto a more weakly coupled four-dimensional counterpart on the brane. This projection, as we

will see below, only takes place in an intermediate range of energies $\sqrt{M_P M_*} \gg E \gg 1/r_c$ above which brane-bulk interactions become strong. However, this window is large enough to be compatible with all the existing data. We shall present the discussion in two steps. First we will give a purely field theoretical consideration, without referring to the precise nature of quantum gravity above the scale M_*; then, in the next section, we will assume that the bulk gravitational theory above M_* possesses some generic properties of string theory, and show that in this case all the experimental constraints can be satisfied.

What is the lower bound on the scale M_*? It is impossible to answer this question without making assumptions about the nature of quantum gravity above this scale. However, the following general considerations should be valid. The usual formulation of general relativity is appropriate up to scales of the order of the fundamental Planck scale M_*. So we must think of GR as an effective field theory, valid at energy smaller than M_*; moreover, we expect it to be embedded into a more fundamental theory that regulates the ultraviolet behavior. Whatever this theory is, it is reasonable to assume that its effect is to make quantum gravity "softer" at energies above M_*, i.e., to regularize the strength of graviton self-interactions and that of the interactions of gravity with matter. The fact that this should be the case is suggested by the only known consistent theory of quantum gravity, that is string theory. This theory exhibits a well known softening of scattering amplitudes at high momenta.

As an example of the soft behavior one could consider (see Appendix) the interaction potential between two static sources in string theory. This potential has no short-distance singularity, as opposed to the case of a static potential obtained in field theory which is singular at the origin. In a field theoretical computation one could in principle also get such a smooth result if the propagator of the intermediate virtual state vanishes faster than $1/p^2$ in the ultraviolet, i.e., $p \rightarrow \infty$. This could effectively be described by introducing a certain form-factor $f(p)$ in the graviton propagator (and/or in vertices) such that $f(p) \rightarrow 1$ for small p and $f(p) \rightarrow 0$ for p larger than M_*. This would have the effect of cutting off the momentum in the graviton internal lines of any Feynman diagram above M_*.

As it was shown in [6], under these circumstances the high-energy colliders production processes of particles or the process of star cooling place essentially no constraint on the scale M_*. The reason for this is the "self-shielding" of the brane-localized standard model from the strong bulk gravity. This manifests itself in the aforementioned suppression of the heavy KK wave functions on the brane. As a result, their production in any high-energy process on the brane is extremely suppressed. For instance, consider the rate of the bulk graviton production in a SM process at energy E. The rate is given by

$$\Gamma \sim \frac{E^3}{M_*^3}\int_0^{m_{max}}|\phi_m(0)|^2 dm, \qquad (4.1)$$

where the integration is over the continuum of KK states up to the maximum possible mass which can be produced in a
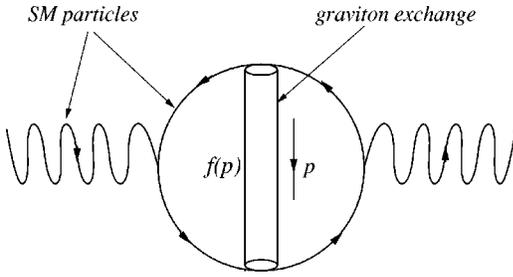
FIG. 2. Graviton loop correction to a standard model amplitude. The "tube" represents a graviton propagator above the energy M_*. The form factor $f(p)$ effectively cuts off the graviton momentum p at M_*.

given process, i.e., $m_{max} \sim E$. Since the wave functions of the heavier KK states are suppressed on the brane by a factor $1/m^2 r_c^2$ the integration domain is effectively truncated at $m \sim 1/r_c$. Thus, one obtains

$$\Gamma \sim \frac{E^3}{M_*^3 r_c} \sim \frac{E^3}{M_P^2}. \qquad (4.2)$$

If we were to neglect the induced kinetic term on the brane the rate would be given instead by the ratio E^4/M_*^3 which is unacceptably large. On the other hand, the rate (4.2) is of the order of the production rate for a single four-dimensional massless graviton and is totally negligible. Although gravity "becomes strong" at the scale M_*, the gravitational loop corrections to any standard model amplitude would be absolutely negligible even though the momenta in the internal lines are above M_*.

Consider for example the diagram in Fig. 2. The form factor effectively switches off the graviton propagator (represented by a "tube" in Fig. 2) when $p > M_*$. Thus, the diagram is dominated by the momentum running in the matter lines which could be as large as $M_{SM} \gg M_*$. Because of the smallness of the matter-gravity coupling this diagram will give a correction which is suppressed compared to the one due to the gauge boson replacing the graviton line.

In this respect we would like to point out one more advantage of the present framework. It deals with the gauge coupling unification. We think of the scenario where the SM worldvolume scale is huge, let us say of the order of the GUT scale. In such a case the gauge coupling unification is not affected by strong gravity corrections precisely because of the reasons outlined above. Thus, the prediction of 4D theory on the gauge coupling unification in supersymmetric models [10] will remain intact in this framework.

As we have shown before, the only constraint comes from the measurement of the Newton force, which implies $M_* > 10^{-3}$ eV. This can be understood by using yet another language. Consider the Newton interaction between two static sources. Without the cutoff the Newton potential between two masses m_1 and m_2 takes the form

$$V(r) = -\frac{m_1 m_2}{M_*^3} \int \frac{d^3 \vec{p}}{(2\pi)^3} \frac{\exp(i\, \vec{p} \cdot \vec{r})}{2p + r_c p^2}, \qquad (4.3)$$

which for $r \ll r_c$ reduces to the conventional 4D potential

$$V(r) = -\frac{m_1 m_2}{M_P^2} \int \frac{d^3 \vec{p}}{(2\pi)^3} \frac{\exp(i\, \vec{p} \cdot \vec{r})}{p^2}. \qquad (4.4)$$

In order to introduce the effective cutoff, however, one has to include the form factor:

$$V(r) = -\frac{m_1 m_2}{M_P^2} \int \frac{d^3 \vec{p}}{(2\pi)^3} \frac{\exp(i\, \vec{p} \cdot \vec{r}) f(p)}{p^2}, \qquad (4.5)$$

where $f(p)$ dies off for the momenta above M_*. Such a theory would predict deviations from the ordinary Newton law at distances $r < 1/M_*$. This is because in the conventional case the gravitons with momenta $\sim 1/r$ contribute at the distance r, whereas in the present case the contribution of such gravitons is suppressed by the form factor $f(p)$. Since the gravitational law has been tested down to sub-millimeter distances, we obtain the bound on M_* from these considerations, $M_* \gtrsim (0.2 \text{ mm})^{-1}$.

V. MODELING QUANTUM GRAVITY BY STRING SPECTRUM

Although we expect that the theory of quantum gravity should make graviton amplitudes "softer," nevertheless, it is unlikely that the only effect of quantum gravity can be summarized in a form factor that switches off graviton exchange at energies above M_*. In particular, quantum gravity could soften its amplitudes by introducing an enormous multiplicity of states above M_*. This expectation is certainly supported by string theory which is at present the only candidate for a self-consistent theory of quantum gravity. String theory predicts an exponentially increasing number of states which can be excited at energies above the string scale. One of the implications of this fact is the Hagedorn phenomenon.

Therefore, if the theory of quantum gravity above M_* is some version of a string theory, we have to face the existence of an exponentially large multiplicity of bulk states with the Regge recurrences governed by the scale M_*. Naively, this ruins any hope of bringing the quantum gravity scale below 1 TeV. Indeed, it seems natural that if such a multiplicity of states is coupled to the SM particles there is no way for them not to manifest themselves in all possible high-energy processes. For instance, to exclude $M_* <$ KeV it would be enough to think of the interior of the Sun where there is no sign of any exotic Hagedorn type behavior.

The purpose of this section is to show that this in fact is not the case. The absence of the Hagedorn catastrophe in the scenario with high-cutoff standard model can be completely compatible both with very low value of M_* as well as with an exponentially increasing density of states. The detailed discussion of various bounds will be given in the following subsections. Here we shall summarize the main reasons why the present framework is not excluded.

To be concrete we shall assume that the bulk spectrum is that of a closed bosonic string theory (ignoring the tachyon). The main point is to find out what is the impact of the Regge

states on the SM which is localized on the brane. To answer this question we shall use some results that will be derived in great detail in the next 3 subsections. Here we shall quote them without a proof.

Consider a bulk field A of some high integer spin which has a five-dimensional mass M_B and a brane mass $\mu \sim M_B$. In addition, it has both the brane and bulk kinetic terms, just like our graviton. The Lagrangian of this field can be schematically written as follows:

$$M_*^3\{[\partial_{(5)}A]^2 - M_B^2 A^2\} + \bar{M}^2\delta(y)$$
$$\times \{[\partial_{(4)}A(x,y=0)]^2 - \mu^2 A^2\}, \qquad (5.1)$$

where $\bar{M}$ is some scale to be specified below. In these notations A is dimensionless. Assume that the field A couples derivatively to the localized matter on the brane and the corresponding coupling is defined by inverse powers of M_*. As it will be shown, in all the 4D processes the effect of such a bulk field is reduced to that of a single 4D state (of the same spin) which has the mass $\sim \mu$, and the coupling square proportional to $1/\bar{M}^2$. Depending on the spin of the state A, this has to be multiplied by an appropriate power of p/M_* arising from the derivatives in the original coupling. The crucial point is that the scale $\bar{M}$ which is induced by the localized matter loops, depends on the number of derivatives in the coupling of the bulk field with the SM. For a field A which is coupled with $n-1$ derivatives to the SM fermions the scale is $\bar{M}^2 \sim M_P^{2(n-1)}/M_*^{2(n-2)}$. As a result, the coupling of this bulk field to the localized SM states takes the form

$$\text{effective coupling} \sim \frac{p^{2(n-1)}}{M_P^{2(n-1)}}. \qquad (5.2)$$

This is the very same mechanism by which the standard model weakens its coupling to the strong bulk gravity. Moreover, we note that the higher is the power of the derivative interaction the larger is the induced 4D kinetic term on the brane, and, consequently, the weaker is the coupling of A to the SM.

Therefore, the SM shields itself not only from strong bulk gravity but also from other bulk fields. Furthermore, the higher is the spin of the bulk state the more efficient is the shielding. Given this fact, it is easy to understand how the Hagedorn catastrophe is avoided: it is true that the number of modes available at higher energies grows exponentially, however, most of them are coupled with higher derivatives to the SM and thus their effective 4D coupling becomes very weak, Eq. (5.2).

These arguments show that the dominant contribution comes from spin-2 states that couple via a single derivative to the SM fermions (note that there are also spin-2 states that couple via higher derivatives and therefore are less important). Since the scale $\bar{M}$ depends only on the number of derivatives in the coupling, then they all couple to the SM by the M_P-suppressed interactions, just like an ordinary graviton. Therefore, their emission rate in high energy processes scales as follows:

$$\Gamma \sim \frac{E^3}{M_P^2} n_{max}. \qquad (5.3)$$

Due to the assumed Regge behavior $n_{max} = (E/M_*)^2$. As a result, Eq. (5.3) constrains M_* to be above 10^{-3} eV. Remarkably, this bound coincides with that coming from sub-millimeter gravity measurements.

A. String spectrum

In the following we consider a field theoretical model the spectrum of which mimics that of closed bosonic string theory in critical dimension (neglecting tachyons). We will show that under certain assumptions the enormous multiplicity of states accessible at low energies can be totally compatible with observation. Before doing so, we will briefly recall what the main features of the string spectrum are.

A generic closed string state can be described by two copies (left and right moving) of an infinite set of creation operators[8] $\alpha_n^{\mu\dagger}$, $\tilde{\alpha}_n^{\mu\dagger}$, where μ is a Lorentz index and n labels the "oscillator level," $n=0 \ldots \infty$. The generic state is given by the action of this operators on the Fock vacuum state $|0\rangle|\tilde{0}\rangle$:

$$|a^{\mu_1 \cdots \mu_n \mu_{n+1} \cdots \mu_{n+k}}, p\rangle$$
$$= \alpha_{m_1}^{\mu_1\dagger} \cdots \alpha_{m_n}^{\mu_n\dagger} \tilde{\alpha}_{\tilde{m}_1}^{\mu_{n+1}\dagger} \cdots \tilde{\alpha}_{\tilde{m}_k}^{\mu_{n+k}\dagger}|0,p\rangle|\tilde{0},p\rangle, \qquad (5.4)$$

with the constraint that the total level N of left and right oscillators be equal:

$$N = \sum_i m_i = \sum_i \tilde{m}_i. \qquad (5.5)$$

In Eq. (5.4) p is the momentum of the state and must obey the mass shell condition $p^2 = M^2$, where M is determined by the string scale M_{st} according to the Regge behavior

$$M^2 = 4(N-1)M_{st}^2. \qquad (5.6)$$

Moreover, the state (5.4) must obey the transversality condition

$$p_\mu|a^{\mu \cdots \mu_n \mu_{n+1} \cdots \mu_{n+k}}, p\rangle = 0. \qquad (5.7)$$

Taking all possible Lorentz-irreducible combinations of indices the expression (5.4) gives rise to the states with different spins which can range from 0 (trace on all indexes) to $n+k$ (totally symmetric, transverse, traceless combination). For example, the states at the level N are of the form $|a\rangle_{left} \times |a\rangle_{right}$ with, say $|a\rangle_{left}$, defined as follows:

---

[8]One usually does this construction in the light-cone gauge where all the obtained stringy states are physical [39]. Since we are dealing only with the kinematical features of the string spectrum, we will not discriminate between the light cone-gauge construction and that of the Lorentz covariant formalism.

$$\alpha_N^{\mu\dagger}|0\rangle$$

$$\alpha_{N-k}^{\mu\dagger}\alpha_k^{\rho\dagger}|0\rangle \quad (k=1\ldots[N/2])$$

$$\cdots$$

$$\alpha_1^{\mu_1\dagger}\ldots\alpha_1^{\mu_N\dagger}|0\rangle, \qquad (5.8)$$

and respectively for $|a\rangle_{right}$ with the substitution $\alpha\to\tilde{\alpha}$. Thus, at each mass level there are states of any spin n with $n\leq 2N$. These are given by all possible combination of creation operators whose individual level numbers add up to N. One is naturally led to the question: what is the total number of states with a given mass M? If we forget about the Lorentz structure for a moment, this is equivalent of counting the total number $p(N)$ of *partitions* of the integer N, i.e., the number of sets of the form $\{n_1,\ldots,n_k\}$[9] such that $\Sigma n_i=N$. This is a well known problem in the number theory and the solution, for large N, scales as follows:

$$p(N)\sim\exp(\sqrt{b\,N}), \quad N\to\infty, \qquad (5.9)$$

where b is a constant of order 1. Taking into account that each oscillator can come in $d=(D-2)$ varieties (D being the dimensionality of space) the constant b is replaced by $b\,d$. Thus, the density of states of a given mass M grows *exponentially* for $M>M_{st}$:

$$\rho(M)\sim\exp\left(\sqrt{b\,d}\,\frac{M}{M_{st}}\right). \qquad (5.10)$$

One possible objection against very low quantum gravity scale is that in string theory there is a very large number of states with masses growing as $\sqrt{N}M_{st}$. Moreover, the number of states at each mass level grows with N as $(\exp\sqrt{N})$. Therefore, if M_* is the scale where classical gravity breaks down we should expect to deal with the exponentially large number of accessible states in contradiction with experiment. In particular, a system with the density of states such as Eq. (5.10) exhibits very peculiar thermodynamic properties above $T=M_{st}$. The partition function for this system is roughly

$$\mathcal{Z}=\sum_E \rho(E)\exp\left(-\frac{E}{T}\right)\sim\sum_E \exp\left(\frac{E}{M_{st}}-\frac{E}{T}\right). \qquad (5.11)$$

The latter diverges badly when $T\geq T_H\sim M_{st}$ ("Hagedorn transition," see, e.g., Ref. [40] and citations therein).

However, we will see below that what really matters in our model is the number of states of a given mass produced by a *given number of creation operators* acting on the vacuum. For instance, it is clear from Eq. (5.8) that at any level N there is only *one* (modulo Lorentz permutations) state created by two $\alpha^\dagger$'s (one left and one right); in fact, it

---

[9]k is called the *length* of the partition.

turns out [41] that the number $p_n(N)$ of partitions of an integer N *of fixed length* n scales for large N but fixed n ($n\ll N$) as follows:

$$p_n(N)\sim\frac{(N-n)^{n-1}}{n!(n-1)!}. \qquad (5.12)$$

Thus, for fixed n there is only a *polynomial* dependence on N. Therefore, the number of states $p_{n+k}^{(d)}(N)$ created by n left oscillators and k right oscillators which have a total of $n+k$ Lorentz indexes grows with N at most as

$$p_{n+k}^{(d)}(N)\sim N^{n+k-2}d^{n+k}. \qquad (5.13)$$

This fact will be crucial for phenomenological estimates.

B. Modeling couplings to 4D matter

We now consider a five dimensional *field theoretical* model with the spectrum of closed bosonic string, namely a tower of massive tensor fields $A^{C_1\cdots C_j}$, with masses $M=\sqrt{4(N-1)}M_*$, and $j\leq 2N$ for each $N=1,2,\ldots\infty$. In particular, we associate to each string state given in Eq. (5.4) a tensor field with the same number of Lorentz indexes defined by $j=n+k$ and the corresponding Regge masses. The symmetric traceless part of this tensor contains a spin-j field, the maximal spin state in the multiplet. In addition, A gives rise to lower spin states corresponding to its traces and/or antisymmetric components.

From the four-dimensional point of view each high-dimensional j-th rank tensor decomposes into various 4D fields with spins up to j. The couplings of these higher-spin fields to 4D matter on a brane depends on a concrete string theory realization of the model. Since we do not really have a precise stringy model we take these couplings to have the following minimal form in terms of A but a generic form in terms of the world volume fields:

$$\mathcal{L}_{int}=\frac{A^{C_1\cdots C_j}(x,y=0)}{M_*^{j-2}}\Sigma *\hat{O}_{C_1\ldots C_j}\Sigma$$

$$\equiv\frac{A^{\mu_1\cdots\mu_j}(x,y=0)}{M_*^{j-2}}J_{\mu_1\ldots\mu_j}(x), \qquad (5.14)$$

where Σ collectively denotes the SM fields which are confined to the brane and thus do not depend on y, $\hat{O}_{C_1\ldots C_j}$ is some tensor operator of dimension j which contains derivatives and could also contain the mass M of the field Σ. In Eq. (5.14) and below we will not be distinguishing between symmetric and antisymmetric parts. The consideration will apply universally to all fields with multiple indices and scalars.

To make parallels with the case of a massless graviton we choose to work with the bulk field A which is dimensionless. In this case, the bulk kinetic term for A is multiplied by M_*^3. Moreover, we assumed that the localized field Σ is a scalar which has canonical 4D dimensionality, $[\Sigma]=[\text{mass}]$. In the case of a spin-1/2 field, the operator $\hat{O}$ will also contain gamma matrices and will have dimensionality $j-1$.

Therefore, we write the action for the field A in the following form:

$$S_A = M_*^3 \int d^4x \, dy [(\partial_{(5)} A)^2 - M_B^2 A^2]$$

$$+ \frac{1}{M_*^{j-2}} \int d^4x \, A(x, y=0) \cdot J(x). \quad (5.15)$$

Here, M_B denotes the bulk mass for the field A. Below we are going to study how these high-spin fields affect the phenomenology on the brane.

C. Induced kinetic terms

Just as it happens for a graviton, the interaction of the tensor field A with the localized matter fields will modify its kinetic term on the brane. In particular, the vacuum polarization diagram with the internal SM lines localized on the brane (see Fig. 1) will in general give rise to the induced brane kinetic term for A. Although the mechanism is very generic and could originate from perturbative as well as non-perturbative worldvolume effects, for simplicity we will discuss below a one-loop effect. The expression for the corresponding diagram is

$$\Gamma^{(2)}_{\mu_1 \ldots \mu_n \nu_1 \ldots \nu_n}(p, y)$$

$$= \frac{\delta(y)}{M_*^{2(n-2)}} \int d^4k \frac{O_{\mu_1 \ldots \mu_n}(p, k, M) O_{\nu_1 \ldots \nu_n}(p, k, M)}{(k^2 + M^2)[(p+k)^2 + M^2]}, \quad (5.16)$$

where M stands for the mass of the particle in the loop and the numerator in the integrand is a tensor of rank $2n$. This tensor is constructed out of the loop- and external momenta and the tensor $\eta_{\mu\nu}$. The result of the integration has the following generic form (ignoring the tensor structure)

$$\Gamma^{(2)}(p, y) \sim \frac{\delta(y)}{M_*^{2(n-2)}} [c_1 M_{SM}^{2n} + c_2 p^2 M_{SM}^{2n-2} + \cdots$$

$$+ c_n p^{2n}] \ln M_{SM}, \quad (5.17)$$

where c_n's are some coefficients and M_{SM} denotes, as before, the ultraviolet cutoff of the world-volume theory. Here for simplicity we took a particle in the loop the mass of which is much smaller than M_{SM}. In general this does not need to be the case and additional mass corrections should be included; however, for heaviest states, i.e., $M \sim M_{SM}$, the form of Eq. (5.17) will remain the same.[10]

As a result of this diagram, the loop-corrected effective action for the field A will contain additional kinetic and higher derivative terms which are localized on the brane. These terms arise respectively from the momentum dependent parts in Eq. (5.17), while the first term in Eq. (5.17) represents a brane mass term for A; we will discuss it momentarily.

The second term in Eq. (5.17) is equivalent to a four dimensional kinetic term in the action. The largest contribution to it comes from either the cutoff and/or the heaviest particle in the world-volume theory the mass of which is of order M_{SM}.

Therefore, as in the previous sections, this leads to the coefficient in front of the induced 4D kinetic term which is of the order of M_P. Indeed, the induced 4D kinetic term for the higher spin field can take the form[11]

$$S_n = \frac{M_P^{2(n-1)}}{M_*^{2(n-2)}} \int d^4x [\partial_{(4)} A(x, y=0)]^2. \quad (5.18)$$

This expression sets the "crossover scale" $r_c^{(n)}$ for the field A to be

$$r_c^{(n)} = \frac{M_P^{2n-2}}{M_*^{2n-1}} = \frac{1}{M_*} \left(\frac{M_P}{M_*} \right)^{2n-2}. \quad (5.19)$$

Moreover, the coupling of this field to the localized matter (analog of the Newton coupling) is

$$G_{(n)} = \frac{1}{M_*^{2n-1} r_c^{(n)}} = \frac{1}{M_P^{2(n-1)}}. \quad (5.20)$$

Since $M_* r_c^{(n)} \gg \gg 1$, this coupling is tremendously suppressed compared to what it would have been if we were to neglect the induced kinetic term on the brane. Thus, we see the same phenomenon: the SM fields shield themselves from the strong bulk dynamics.

Let us note that the parameter $r_c^{(n)}$ depends on the rank of the tensor field. In particular, it is determined by the dimensionality of the operator $\hat{O}_{\mu_1 \ldots \mu_n}$ to which this tensor field is coupled in Eq. (5.14). The latter is related to the number of derivatives by which the field A couples to 4D matter. Above we have assumed that a given number n in Eq. (5.19) corresponds to n oscillators acting on the Fock vacuum in the oscillator picture. This fact will be important in counting these fields with the right multiplicity.

We turn now to the first term in Eq. (5.17). This is just a four-dimensional induced mass term for the field A. Depending on the scenario at hand, this term can take two significantly different values. We will study below both of these possibilities.

For a generic interaction the first term in Eq. (5.17) will take the form

$$S_m = \frac{M_{SM}^{2n}}{M_*^{2(n-2)}} \int d^4x [A(x, y=0)]^2. \quad (5.21)$$

After rescaling the field appropriately to bring it to a canonical dimension we obtain that the 4D mass of this field is

---

[10]Hereafter, for simplicity we will not discriminate between M_{SM} and the induced scale M_{ind}, i.e., we put $M_{SM} \sim M_{ind} = M_P$.

[11]Here we discuss the higher spin states with $n \geq 2$ which give rise to the dangerous exponential multiplicity of states. The case with $n = 0, 1$ will be discussed briefly below.

M_{SM}. Therefore, it is not likely that such a heavy state could play any role in the low-energy 4D dynamics on the brane. Indeed, the total action for the field A now consists of three parts, $S_A + S_n + S_m$. The latter has the form of the action which we discussed in Sec. III for a scalar field with the bulk mass M_B and the brane mass $\mu \sim M_{SM}$. According to the results obtained in Sec. III, this state will effectively appear to four-dimensional observers as a 4D field of mass M_{SM}; thus, it will have no effect on the 4D physics at accessible energies. Therefore, in this case there is no additional constraint on the value of M_*.

However, one could expected that for some particular choices of the interaction terms (5.15) the induced mass on the brane could be much smaller, e.g., of order M_B. To come to this point let us recall that for a massless spin-two field which couples gravitationally to 4D matter the 4D reparametrization invariance prevents the generation of any type of mass term. Moreover, in the case of a *massive* spin-two state the bulk reparametrization invariance is explicitly broken by the mass term. Thus, one could expect that the brane mass term will be induced by radiative corrections. The latter, however, has to vanish in the limit of zero tree-level bulk mass. Therefore, the induced mass could in principle be determined by the bulk mass.

It is reasonable to expect that this will happen also in the higher spin cases for some specific choice of the interaction current in Eq. (5.15). Indeed, in the massless limit the higher form symmetric and antisymmetric fields have the corresponding well-known gauge invariant actions. This invariance is explicitly broken in the bulk by the mass term. In particular, if the matter current on the brane J is conserved up to the terms which are proportional to the tree level bulk mass of the field A, i.e., $\partial \cdot J \sim \mathcal{O}(M_B^2)$, then the induced mass term could be of the order of the bulk mass, $\mu \sim M_B$. In this case these light fields will have an interesting impact on the 4D phenomenology on the brane.[12] The content of the next section is devoted to the analysis of the phenomenological, astrophysical and cosmological data which might be affected by the presence of these states.

Before we turn to these studies let us summarize briefly the main properties of these light modes. The total action for the field A takes the form

$$
\begin{aligned}
S = M_*^3 \int \mathrm{d}^4x\, dy [(\partial_{(5)}A)^2 - M_B^2 A^2] + \frac{1}{M_*^{n-2}} \\
\times \int \mathrm{d}^4x\, A(x, y=0) \cdot J(x) + \frac{M_P^{2(n-1)}}{M_*^{2(n-2)}} \\
\times \int \mathrm{d}^4x [(\partial_{(4)}A(x, y=0))^2 - \mu^2 A(x, y=0)^2].
\end{aligned}
$$

(5.22)

Here, as we discussed above, $\mu \sim M_B = \sqrt{4(N-1)} M_*$, $N = 1, 2, \ldots$. This action is of the form discussed in Sec. III and analyzed in detail in Appendix. According to these results we can distinguish two cases: if $\mu > M_B$ and $\mu < M_B$. In the first case, for each bulk field A we find a continuum of modes with masses starting at M_B. Only a small portion of the continuum with the mass around μ and width around $1/r_c^{(n)}$ is unsuppressed on the brane: the brane induced kinetic term converts their strong bulk couplings into a significantly weaker coupling $1/\sqrt{M_*^3 r_c^{(n)}}$, similar to what happens for the graviton [4,6].

To give an example, consider a state from the continuum with the mass m. It can be produced in a process involving brane-fields. The amplitude, F, for this process is proportional to the bulk-brane coupling which in its turn is specified by the square of the wave-function at the position of the brane. Thus, we write

$$
|F|^2 \sim \frac{|\phi_{m,\mu}^{(n)}(y=0)|^2}{M_*^{2n-1}} |F_m^{(n)}|^2,
$$

(5.23)

where $F_m^{(n)}$ is a kinematical factor. In order to obtain the total cross section this must be integrated over m from M_B to ∞. However, the function $|\phi_m(y=0)|^2$ is sharply peaked around the value of μ (see Appendix) which is nothing but the brane-induced mass. The width of this peak is of the order of $1/r_c^{(n)}$. The result of the integration is as follows:

$$
\int_{M_B}^{\infty} \mathrm{d}m\, \frac{|\phi_{m,\mu}^{(n)}(y=0)|^2}{M_*^{2n-1}} |F_m|^2 \sim \frac{1}{M_*^{2n-1} r_c^{(n)}} |F_{m=\mu}^{(n)}|^2.
$$

(5.24)

Therefore, the integration procedure has effectively converted the *large* coupling constant $1/M_*^{2n-1}$ into a significantly suppressed constant $1/(M_*^{2n-1} r_c^{(n)})$, as advertised before.

Furthermore, we consider the case $\mu < M_B$. All the states in the continuum are significantly suppressed on the brane. However, as discussed before, there is in addition a localized 4D state of mass $\sim \mu$. The coupling of this state is also suppressed by the parameter $1/r_c^{(n)}$ (see Appendix).

Hence, in both cases considered above the situation is identical for all the practical purposes: the relevant contribution to any 4D process comes only from the states with "effective" 4D mass around $\mu \sim M_B$. These states are coupled to the 4D matter by the weak coupling $1/(M_*^{2n-1} r_c^{(n)})$. In what follows, we will discuss for simplicity only the case $\mu > M_B$, keeping in mind that the physics is similar even if $\mu < M_B$.

So far we were dealing with the massive Regge modes for which the exponential multiplicity is present. However, in addition we expect to have a few 5D massless modes in the bulk. For instance, in the bosonic sector of a close string a graviton will be accompanied by a dilaton and a two-form antisymmetric field. Although these massless modes are not important for the problem of the exponentially growing number of states, nevertheless, they could mediate gravity com-

---

[12]This consideration and the phenomenological discussions below also apply to the case of a pure four-dimensional theory where the Regge modes would have masses of order M_*.

peting forces in 4D world volume. Therefore, we should deal with them. The dilaton can acquire a potential on a brane; in this case it cannot mediate gravity competing 4D force [42]. The higher dimensional two-form field will give rise to a pseudoscalar and a vector particle on the brane. The pseudoscalar can be dealt in analogy with the dilaton, the brane-induced potential will suppress its interactions. However, the massless vector field should be dealt separately. One possibility is to give to it a small mass by the Higgs mechanism. After these discussions we turn to the constraints.

VI. PHENOMENOLOGICAL CONSTRAINTS

In this section we study the production of the five-dimensional higher spin fields introduced in the previous section, in processes taking place on the brane. The resulting estimates for rates and cross sections will be subsequently used to put bounds on M_* through the analysis of astrophysical, cosmological and collider observations.

1. Annihilation

A five dimensional high spin field coupled to brane fields can be produced in the processes when the localized charged matter annihilate on the brane.

Consider the production of a "stringy" n-th rank tensor mode $A_m^{(n)}$ of "4-dimensional mass" m (i.e., belonging to the continuum of states) in the annihilation process involving a massless brane fermion-antifermion or brane scalar-antiscaler, and a brane-photon, $\Phi\bar{\Phi}\to\gamma A_m^{(n)}$. The amplitude has the form

$$F_m^n = \frac{e}{M_*^{3/2+n-2}}\phi_m(0)\epsilon^{*\mu}\epsilon_A^{*\mu_1\cdots\mu_n}\frac{O_{\mu\mu_1\cdots\mu_n}(p,p\,',q)}{q^2},$$
(6.1)

where p,p' are the incoming momenta of the Φ particles, q is the momentum transfer, ϵ^μ, $\epsilon_A^{\mu_1\cdots\mu_n}$ are the polarizations of the photon and of the A field, respectively, and $O_{\mu\mu_1\cdots\mu_n}(p,p',q)$ is a tensor of dimension $n+1$ whose form depends on the specific choice of the interaction term (5.14). Summing over initial and final polarizations we get an expression for the amplitude squared which can be used to calculate the density of the differential cross section,

$$\frac{d^2\sigma_n}{d\,t d\,m} \sim \frac{e^2|\phi_m(0)|^2}{M_*^{2n-1}}s^{n-3}f_n(t/s,m^2/s),$$
(6.2)

where $f_n(x,y)$ is a dimensionless function whose precise form depends on the result of the sum over polarizations and on the form of the interaction term, and s and t are the Mandelstam variables. From the kinematics in the center of mass (c.m.) frame we get $s=4E^2$, $t=1/2(s-m^2)(\cos\Theta_{c.m.}-1)$ $\in[m^2-s,0]$, therefore

$$\frac{d\sigma_n}{d\,m}\sim e^2\frac{|\phi_m(0)|^2}{M_*^{2n-1}}s^{n-3}\int_0^{s-m^2}d(-t)f_n(t/s,m^2/s)$$

$$=e^2|\phi_m(0)|^2\frac{s^{n-2}}{M_*^{2n-1}}\int_0^{1-z}f_n(x,z)dx,$$
(6.3)

where $z\equiv m^2/s$. Now we must integrate over m. As we discussed above, due to the form of $|\phi_m(0)|^2$, this effectively amounts of replacing m by μ and M_*^{2n-1} by $r_c^{(n)}M_*^{2n-1}$. The result is suppressed by powers of M_P:

$$\sigma_n\sim e^2\frac{s^{n-2}}{r_c^{(n)}M_*^{2n-1}}\int_0^{1-z}f_n(x,z)dx$$

$$=e^2\frac{s^{n-2}}{M_P^{2n-2}}\int_0^{1-z}f_n(x,z)dx.$$
(6.4)

2. Photoproduction

Another type of process which contributes to the production of five dimensional higher spin modes is the photoproduction reaction $\Phi\gamma\to\Phi A_m^n$ with the bulk field A in the final state.

The photoproduction rate into the bulk modes can be calculated in the same way as it was done for the annihilation process. The cross section is the same as in Eq. (6.2), with s and $(-t)$ interchanged in the amplitude. Thus the density of the differential cross section will take the form

$$\frac{d^2\sigma_n}{d\,t d\,m}\sim e^2\frac{|\phi_m(0)|^2}{M_*^{2n-1}}(-t)^{n-3}g_n(t/s,m^2/s),$$
(6.5)

where $g(x,z)$ is another dimensionless function. Integrating this expression over the continuum with the approximation $|\phi_m(0)|^2\sim\delta(m-\mu)/r_c^{(n)}$, we get

$$\frac{d\sigma_n}{d\,t}\sim\frac{e^2}{r_c^{(n)}M_*^{2n-1}}s^{n-3}\left(\frac{(-t)}{s}\right)^{n-3}g_n(t/s,\mu^2/s)$$

$$=\frac{e^2}{M_P^{2n-2}}s^{n-3}\tilde{g}_n(t/s,\mu^2/s),$$
(6.6)

where we have introduced $\tilde{g}_n\equiv((-t)/s)^{n-3}g_n$. This expression for the differential cross section is again suppressed by powers of M_P.

A. Star cooling

The possibility of producing an exponentially large number of Regge states at very low energy could in principle affect the cooling rate of stars and supernovae. Requiring that this effect be smaller than the observed energy produced by these objects (in particular SN1987) was indeed the strongest constraint on the model introduced in [2], as was also shown in [43]. In this case the states in question were KK modes of the graviton, with mass spacing of the order of 1 mm^{-1}. Let us consider the total production of bulk modes

by a stellar object of temperature T. For example, we can estimate the emission rate due to the "photoproduction" or "bremsstrahlung" processes. As far as the order of magnitude is concerned, the other processes considered in [2,43] will give roughly the same contribution.

The rate is given by the cross section (6.4) multiplied by the number of particles per unit volume ($\sim T^3$), times the relative velocity of the initial particles, everything averaged over the thermal bath of the star. This gives

$$\Gamma_n \sim E\left(\frac{E}{M_P}\right)^{2n-2},\qquad (6.7)$$

in this expression $E \equiv \langle E \rangle \sim T$. This must be multiplied by the number of states with given n that contribute to the rate. As discussed in the previous section, this is equal to the number of states which, in string language, are created by n oscillators. On the other hand, the number of such states at each level N goes at most as in Eq. (5.13), with n in place of $n+k$.

Thus, the total rate of the production of all the states specified by a given n but belonging to an arbitrary mass level is then bounded as follows (neglecting the factor of d^n):

$$\Gamma \leqslant \sum_{N=1}^{N_{max}} E\left(\frac{E}{M_P}\right)^{2n-2} N^{n-2} \sim E\left(\frac{E}{M_P}\right)^{2n-2} N_{max}^{n-1}, \quad (6.8)$$

where N_{max} is determined by the mass of the heaviest state that can be produced at energy E, i.e. $\mu_{max} \simeq \sqrt{N_{max}} M_* = E$. Thus we get the estimate

$$\Gamma \sim E\left(\frac{E}{M_P}\right)^{2n-2}\left(\frac{E}{M_*}\right)^{2n-2} \sim E\left(\frac{E^2}{M_P M_*}\right)^{2n-2}. \quad (6.9)$$

Even for $M_* \sim \text{mm}^{-1}$, the "natural" expansion parameter, which is $E^2/(M_P M_*)$, becomes of order 1 at energies above 1 TeV. Thus the rates for the different n's start being of the same order at a scale much higher than that at which all these states are accessible by kinematics. Therefore, in a star where $T \ll 1$ TeV, all contributions other than Γ_2 are negligible:

$$\Gamma_2 \sim \sum_N \frac{E^3}{M_P^2} \sim \frac{E^3}{M_P^2} N_{max} \sim E\frac{E^4}{M_*^2 M_P^2}. \quad (6.10)$$

This rate is strikingly similar to that of the model of Ref. [2] for the case of *two* large extra-dimensions, where M_* plays the role of $1/R$. So the lower bound on M_* from this kind of processes is precisely of the order of an inverse millimeter or so.

B. Collider signatures

The similar considerations apply to collider experiments. Although a huge number of states can be produced starting at a very low energy ($\sim M_*$), the cross sections for production of states labeled by different n do not become significant until energies of order $\sqrt{M_P M_*}$ (~ 1 TeV if M_*

$\sim 10^{-3}$ eV). Thus, the dominant contribution at smaller center of mass energies comes from $n=2$, and according to Eq. (6.6) it scales as follows:

$$\frac{d\sigma_2}{dt} \sim \frac{1}{sM_P^2}. \qquad (6.11)$$

As before, we have to sum over the $n=2$ states of each mass level up to $N_{max} \sim s/M_*^2$. The result is

$$\frac{d\sigma_2}{dt} \sim \frac{1}{M_P^2 M_*^2}. \qquad (6.12)$$

For the comparison, the contribution of the states with a generic n is given (at most) by [see Eq. (6.6)]

$$\frac{d\sigma_n}{dt} \sim \sum_{N=1}^{N_{max}} \frac{s^{n-3}}{M_P^{2n-2}} N^{n-2} \sim \frac{1}{s^2}\left(\frac{s}{M_P M_*}\right)^{2n-2}, \qquad (6.13)$$

in which we find again that $s/(M_P M_*)$ is a natural expansion parameter. Since the denominator in this ratio is at least $(1 \text{ TeV})^2$, therefore, at the energies accessible in present day colliders only the contribution from $n=2$ will be important.

For $n=2$ the cross section (6.6) is of the same form as the one calculated in Ref. [44], where the production of massive spin 2 KK modes in models with large extra dimensions of size R was considered. The expression found in [44] is

$$\frac{d\sigma}{dt} \sim \frac{1}{sM_P^2} F_1(t/s, \mu^2/s), \qquad (6.14)$$

where the precise form of F_1 is given in [44]. In this case, one must sum over the tower of KK modes corresponding to n compact extra dimensions, with masses given by

$$m^2 = \frac{1}{R^2}(k_1^2 + \cdots + k_n^2), \qquad (6.15)$$

for integers k_i, $i=1 \ldots n$, so the number of available states up to energy s is roughly $(\sqrt{s} R)^n$. In particular, for the case of two extra dimensions, we get from Eq. (6.14)

$$\sum_{KK\,modes} \frac{d\sigma}{dt} \sim \frac{R^2}{M_P^2}. \qquad (6.16)$$

Remarkably, this is the same result we found in Eq. (6.12) provided that we exchange $M_* \leftrightarrow 1/R$ (although the two frameworks are totally different). In particular, the bounds obtained in [44] on R by comparing their results with present collider data, can be directly translated into bounds on our M_*, which therefore is again only constrained to be larger than $\sim 1/\text{mm}$.

Despite of this similarity, the predictions of our model and the one considered in [44] begin to differ drastically when the energy is high enough: indeed, when the bound is saturated, the framework with two large extra dimensions pre-

dicts the existence of the density of states which grows roughly linearly with the energy

$$\rho(m) \sim mR^2, \qquad (6.17)$$

while in the present context all states with $n \geq 2$ will become equally important [the dimensionless ratio in Eq. (6.13) becomes of order 1], and the total density of states is *exponentially* increasing

$$\rho(m) \sim e^{m/M} * d(m). \qquad (6.18)$$

[Here $d(m)$ is some function whch contains powers of m and depends on dimensionality of space.] Therefore, for instance, the spectrum of missing energy signatures will be very different in these two cases.

C. Cosmology

In this section we will consider cosmological constraints coming from the overproduction of bulk states. In order to be as model independent as possible, we shall discuss the following initial conditions for the hot big bang:

(1) The bulk is virtually empty;

(2) The brane states are in thermal equilibrium at some temperature T_{brane}.

Our goal is to find out what is the normalcy temperature T_* defined in [2] as the temperature below which universe expands as a normal 4D Friedmann-Robertson-Walker (FRW) universe. Then, by requiring that T_* be at least higher than the nucleosynthesis temperature, we can derive bounds on M_*. The reason why we expect that this requirement may restrict M_* is the fact that the brane can cool by "evaporation" into the bulk string states. If this rate is higher than the cooling rate due to the expansion, the FRW scenario will be affected. We do not want this to happen below the nucleosynthesis temperatures. This may impose some constraints on M_*.

Cooling by evaporation into bulk string states

In order to estimate the rate of bulk state production at temperature T we shall use the star cooling rate (6.10) where we shall substitute E by T. This tells us that at each mass level the dominant contribution to the cooling rate comes from the production of 2-index fields, provided the natural "expansion parameter" $T/\sqrt{M_P M_*}$ is smaller than one. For $M_* \sim 10^{-3}$ eV this requires T to be below TeV. Then the cooling rate is given by

$$\Gamma_2(T) \sim \frac{T^5}{M_P^2 M_*^2}. \qquad (6.19)$$

The resulting change of the matter energy density on the brane due to evaporation is

$$\left.\frac{d\rho}{dt}\right|_{evaporation} \sim -T^4 \Gamma_2(T) \sim -\frac{T^9}{M_P^2 M_*^2}. \qquad (6.20)$$

This has to be compared with the cooling rate caused by the cosmological expansion
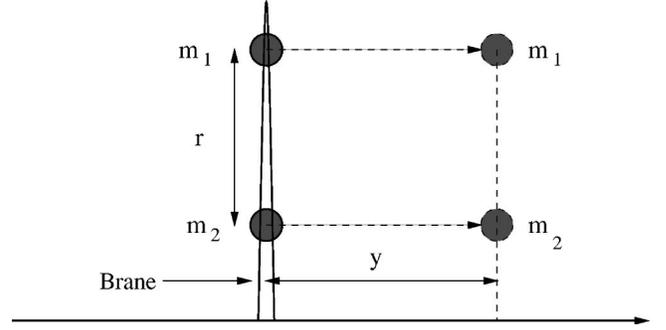

FIG. 3. Removing the test masses from the brane into the bulk "switches on" the five-dimensional potential. For $r < r_c$ the five-dimensional potential is stronger then the four dimensional one.

$$\left.\frac{d\rho}{dt}\right|_{expansion} \sim -3H\rho \sim -3\frac{T^2}{M_P}\rho, \qquad (6.21)$$

where H is the Hubble parameter. In the radiation dominated epoch of standard FRW cosmology ($H \sim T^2/M_P, \rho \sim T^4$), the ratio of the two rates is

$$\frac{\left.\frac{d\rho}{dt}\right|_{evaporation}}{\left.\frac{d\rho}{dt}\right|_{expansion}} \sim \frac{T^3}{M_P M_*^2}. \qquad (6.22)$$

Requiring this ratio to be $\ll 1$ we find that, for $M_* \sim 10^{-3}$ eV, $T_* \sim 20$ MeV or so. Thus these considerations put approximately the same bound on M_* as colliders experiments and astrophysics: for lower M_* the normalcy temperature is not high enough for standard nucleosynthesis to proceed unaffected. On the other hand, the cosmological evolution above this scale is dramatically modified and requires independent study.

VII. BLACK HOLES

The sources localized on the brane at distances $r < r_c$ interact via the weak four-dimensional gravity. On the other hand, the sources in the bulk interact via strong five-dimensional gravity. This fact will have interesting implications for the black hole physics.

Let us consider an elementary particle of mass M such that $M_P \gg M \gg M_*$. In a crude approximation we can think of it as a gravitating source of uniform density localized within its Compton wavelength $\sim 1/M$. From the point of view of the brane observer this particle is not a black hole. However, the very same particle in the bulk would appear as a black hole since its 5D Schwarzschild radius is bigger thank its Compton wavelength (see below). Thus, if such a particle is gradually removed from the brane it turns into a bulk black hole.

We shall investigate how the transition between the brane gravity to the bulk gravity takes place. For this purpose, we will study first the bulk gravitational potential between two object of masses m_1 and m_2 (see Fig. 3).

A. Interactions on the brane and in the bulk

The full 5D static potential can be obtained by summing over the continuum of KK modes. Each of these can be viewed as a four-dimensional massive particle which generates a Yukawa-type force between the sources. The superposition of these contributions gives the following potential:

$$V(r,y) = -\frac{m_1 m_2}{16\pi^2 M_*^3} \int_0^\infty |\phi_m(y)|^2 \frac{\exp(-mr)}{r} dm.$$
(7.1)

Inserting the explicit form of the wave functions ϕ_m for the KK modes (A3), (A6) we find the expression for the potential to be

$$V(r,y) = -\frac{m_1 m_2}{M_*^3} \int_0^\infty dm \left(\frac{[2\cos(my) + mr_c\sin(my)]^2}{4 + m^2 r_c^2} \right)$$
$$\times \frac{\exp(-mr)}{r}.$$
(7.2)

We can approximately evaluate this integral by dividing the range of integration into the two regions, $m < 1/r_c$ and $m > 1/r_c$. Since r_c is the largest scale, we can take both r/r_c and y/r_c to be much smaller than unity. Let us look at the value of the integrand in the first region. The exponential can be replaced with unity. As a result, the contribution to the integral equals to $1/(rr_c)$. For $m > 1/r_c$ one can again evaluate the integral approximately which in this case is equal to $2y^2/(r^4 + 4r^2 y^2)$. Thus, the approximate expression for the potential is

$$V(r,y) \approx -\frac{m_1 m_2}{16\pi^2 M_*^3} \left(\frac{1}{rr_c} + \frac{2y^2}{r^4 + 4r^2 y^2} \right).$$
(7.3)

We will see below that this agrees with an exact expression for the potential which we will obtain from the propagator.

It is not difficult to interpret this expression. For $y = 0$ we see the ordinary Newton potential governed by $G_N = 1/16\pi M_P^2 \sim 1/(r_c M_*^3)$ (note that we look at distances $r < r_c$). After the sources are moved off the brane the strong potential which is not suppressed by r_c is switched on. For $y < r$ the correction to the potential is $(y/r)^2(1/r^2)$, while for $y > r$ a "full-strength" five-dimensional potential, $1/M_*^3 r^2$, is recovered.

Let us study the more general case when the two sources are placed at different positions y and y_0 in the extra coordinate. Instead of using the KK picture we will directly solve for the five dimensional (Euclidean) propagator

$$(\Box_4 [1 + r_c \delta(y)] - \partial_y^2) G(x - x_0, y, y_0) = \delta^4(x - x_0) \delta(y - y_0).$$
(7.4)

The Green's function depends separately on y and y_0, since five dimensional translational invariance is broken by the presence of the brane. By Fourier-transforming this expres-

sion with respect to $x - x_0$ and going to the Euclidean momentum space $E \rightarrow ip_4$ (i.e., $p^2 \rightarrow -p^2$ and $p \equiv \sqrt{p^2}$) we find the following equation:

$$((p^2 - \partial_y^2) + p^2 r_c \delta(y)) \tilde{G}(p, y, y_0) = \delta(y - y_0). \quad (7.5)$$

This equation can be solved with the ansatz

$$\tilde{G}(p, y, y_0) = A(p, y_0) e^{-p|y|} + B(p, y_0) e^{-p|y - y_0|}, \quad (7.6)$$

where $A(p, y_0)$ and $B(p, y_0)$ are the functions to be determined. Inserting Eq. (7.6) into Eq. (7.5) we find

$$A(p, y_0) = \frac{-e^{-p|y_0|}}{p + 1/r_c}, \quad B(p) = \frac{1}{p}, \quad (7.7)$$

where we have used the identities $\partial_y |y| \equiv \epsilon(y)$, $\partial_y \epsilon(y) = 2\delta(y)$, $\epsilon(y)^2 = 1$. The momentum space Euclidean Green function can be written as follows:

$$\tilde{G}(p, y, y_0) = \frac{1}{p} e^{-p|y - y_0|} - \frac{1}{p} e^{-p(|y| + |y_0|)} \frac{1}{1 + 1/r_c p}.$$
(7.8)

Since the quantity pr_c is large (we are considering interaction at distances $r \ll r_c$) we can expand the denominator of the second term to get the expression

$$\tilde{G}(p, y, y_0) \simeq \frac{1}{p} e^{-p|y - y_0|} - \frac{1}{p} e^{-p(|y| + |y_0|)} + \frac{1}{p^2 r_c} e^{-p(|y| + |y_0|)}.$$
(7.9)

The Fourier transform of this Green function is the potential between two static sources of mass m_1 and m_2 at positions y and y_0 in the fifth dimension and separated by the distance r along the brane world volume. By straightforward integration we find the expression for the potential

$$V(r, y, y_0) \simeq -\frac{m_1 m_2}{16\pi^2 M_*^3} \left(\frac{1}{r^2 + |y - y_0|^2} - \frac{1}{r^2 + (|y| + |y_0|)^2} \right.$$
$$\left. + \frac{1}{rr_c} \arctan \frac{r}{(|y| + |y_0|)} \right).$$
(7.10)

The potential (7.10) reveals some interesting properties. For instance, if the masses are placed on different sides of the brane, or if one of the masses is located on the brane, the first two terms cancel exactly. Since the first two terms correspond to the strong five-dimensional potential (coupled with $1/M_*^3$), objects on opposite sides of the brane (Fig. 4) interact only via the third term in Eq. (7.10), which corresponds to the weak 4D gravity.

This term results from the exchange of KK modes with masses $\lesssim 1/r_c$. Modes with $m \lesssim 1/r_c$ can be thought to form a resonance state which mimics the exchange of a single zero-mode graviton coupled via the four-dimensional Newton constant. We conclude that the brane in some sense "screens" the five-dimensional force, i.e., the force due to the exchange of KK excitation of mass larger than $1/r_c$ is
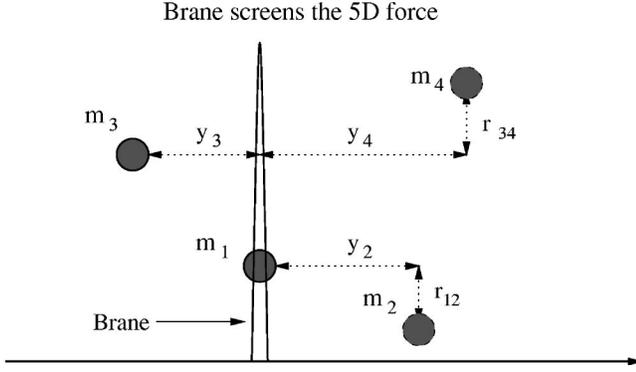
Brane screens the 5D force

FIG. 4. The brane suppresses the exchange of higher KK modes. Object placed on different sides of the brane (for example m_3 $-m_4$) interact only with the third term in Eq. (7.10). The same is true if one of the objects is on the brane while the other one is in the bulk (for example $m_1 - m_2$).

suppressed. This fact is clear from the mode expansion picture. Heavy modes are suppressed on the brane; the Schrödinger equation (A2) that determines the wave-function profiles in the extra dimension is just the equation for a particle in a one-dimensional delta function type potential with strength proportional to the mass of that particle. Therefore the contribution of heavy modes in the exchange is suppressed and the force is mostly due to the exchange of the modes with the small mass.

B. Emission of black holes in the bulk

As mentioned above, a particle of mass $M > M_*$ becomes a black hole in the bulk. This can be understood from the expression for the Schwarzschild radius of the black hole in five dimensions

$$r_{S5} \sim \frac{1}{M_*} \sqrt{\frac{M}{M_*}}. \qquad (7.11)$$

For $M > M_*$ the Schwarzschild radius becomes larger than the characteristic size of the particle $1/M$ (the Compton wavelength) and the particle becomes a black hole. For instance, for $M_* \sim 10^{-3}$ eV and black holes of the masses 1 eV, 1 TeV and 10^{19} GeV, the Schwarzschild radii would equal to 3 cm, 10^4 m, and 10^{12} m, respectively.

The lifetime of a five dimensional black hole, which decays via the Hawking radiation, is given by the relation

$$\tau_5 \sim \frac{1}{M_*} \left(\frac{M}{M_*} \right)^2, \qquad (7.12)$$

and it is substantially larger than the lifetime of a four-dimensional black hole with the same mass ($\tau_4 \sim M^3/M_P^4$). For $M_* \sim 10^{-3}$ eV, the lifetimes of a black hole the with masses of 1 eV, 1 TeV and M_P would be 10^{-4} s, 10^{19} s and 10^{50} s, respectively.

One may wonder if there is a possibility that a heavy bulk-particle is produced on the brane (e.g. in an accelerator) and is emitted in the bulk and after becoming a long-lived black hole, is attracted back to the brane where it decays.

Such an event could produce an interesting signature of a displaced vertex.[13] Unfortunately the probability of such an event is very low as we shall briefly discuss.

To determine the relative rate for the events in which a particle emitted into the bulk returns back to the brane within the size of the detector we have to evaluate the relevant fraction of the phase space. Let us assume that a particle of bulk mass M is produced on the brane in a process of energy E. If we denote the magnitude of the momentum along the brane by p and the momentum in the transverse direction by p_y then

$$p^2 + p_y^2 \leqslant E^2 - M^2. \qquad (7.13)$$

The constraint that the particle comes back to the brane can be expressed in terms of its escape velocity from the brane (which in our case can be estimated as $v_{esc} \simeq 10^4$ m/s):

$$|p_y| \leqslant M v_{esc}. \qquad (7.14)$$

We also require the particle to be within the detector when it hits the brane. This constrains the maximum value of the momentum p along the brane. During the motion in the y direction the particle experiences an approximately constant force due to Earth's gravity, with the acceleration $g = 10$ m/s^2

$$F(r = R_E, y) = -\frac{\partial V}{\partial y}$$

$$= -\frac{M m_E}{M_*^3 R_E r_c} \partial_y \arctan(r/y)$$

$$= -\frac{M m_E}{M_*^3 r_c} \frac{1}{y^2 + R_E^2} \simeq -Mg. \qquad (7.15)$$

The time needed for the particle to return back is $t = 2p_y/Mg$. If we take the radius of the detector to be l, this translates into the condition

$$|p| \leqslant \frac{M^2 l g}{2 p_y}. \qquad (7.16)$$

The fraction of phase space for which the black holes come back to the brane within the distance l from the place of production can easily be visualized from the plot in the p $-p_y$ plane (Fig. 5).

Let us first estimate the branching ratio for the black hole produced at TeV energies to come back to the brane. We have to estimate what will be the fraction of the "Regge" states that will not have significant phase space suppression for falling back to the brane due to attraction by the Earth's gravity. Those are particles that satisfy $M^2 v_{esc}^2 \geqslant (E^2 - M^2)$, i.e.,

---

[13]Here we are discussing the particle which is not necessarily localized on the brane.
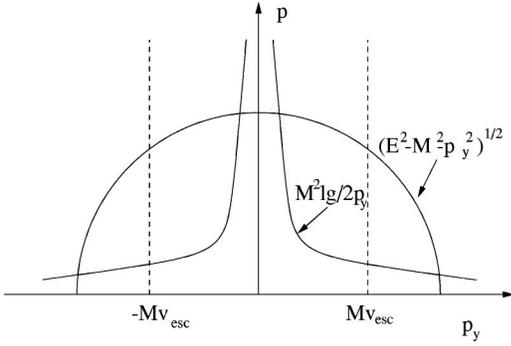
FIG. 5. The phase space for the production of black holes. E is the energy available in the experiment. Conservation of energy requires that $p^2 + p_y^2 \leq E^2 - M^2$ (semi-circle). For a black hole to hit the brane, p_y must satisfy $p_y \leq Mv_{esc}$ (vertical line). In order to hit the brane within a detector of radius l, the momentum p must be less than $M^2lg/2p_y$. The ratio of the area satisfying these constraints to the area of the semi-circle is the phase space suppression factor.

$$0 < \frac{1}{n} \left(\frac{1 \text{ TeV}}{10^{-3} \text{ eV}} \right)^2 - 1 \lesssim 10^{-9}. \qquad (7.17)$$

Solving Eq. (7.17) for n, we find that for $n \in [10^{30} - 10^{21}, 10^{30}]$ (when $M_* = 10^{-3}$ eV) there is no phase space suppression.[14] The ratio of the total rate Γ_{tot} of black hole production, to the rate Γ_{back} for production of those that will return to the brane is

$$\frac{\Gamma_{back}}{\Gamma_{tot}} = 10^{-9}. \qquad (7.18)$$

Some of the black holes that come back to the brane would be attracted toward the center of the earth and would not be detected. Let us now estimate the number of black holes that will hit the brane within a detector of radius l, which can be taken to be of order one meter or so. Solving for the intersections of Eqs. (7.13) and (7.16) one finds that for

$$0 < \left(\frac{E^2}{M^2} - 1 \right) < \frac{lg}{c^2}, \qquad (7.19)$$

the two curves do not intersect, i.e., the semi circle is fully contained within the curve (7.16) (c is the speed of light). When the condition (7.19) is satisfied, there is no phase space suppression for the black holes to come back to the brane within the detector size. Let us translate the bound (7.19) into the bound on the number of Regge states at $E = 1$ TeV that have no phase space suppression. One finds that the ratio of the total number of kinematicaly accessible modes to the number of modes that do not have phase space suppression is just lg/c^2. Therefore, the ratio of the rate for a

---

[14]Note that the particles with such a large n can be black holes from the standpoint of the world volume theory as well.

black hole entering back into the detector, Γ_{det}, to the total rate of black hole production Γ_{tot} is

$$\frac{\Gamma_{det}}{\Gamma_{tot}} = \frac{lg/c^2}{1 + lg/c^2} \approx lg/c^2 = 10^{-16}. \qquad (7.20)$$

This ratio is practically unobservable.

VIII. BARYON NUMBER VIOLATION BY VIRTUAL BLACK HOLES

The potential danger for any theory with a low quantum gravity scale M_* are the high-dimensional operators that may violate exact or approximate global symmetries of SM [such as flavor or the baryon (B) and lepton (L) numbers]. In this section we shall argue that in our framework the strength of such dangerous operators is suppressed by the scale M_P, and not by M_*, and, therefore, they are harmless.

In order to see this let us first discuss the possible origin of such operators. It is believed usually that non-perturbative quantum gravity effects such as the virtual black holes (VBH) violate the global symmetries of the theory. Such a non-conservation should be seen in an effective low energy theory as a variety of B- or L-violating effective operators, e.g., such as

$$qqql, \qquad (8.1)$$

where q and l are quark and lepton fields respectively. The question is the strength of these operators. This issue is impossible to address without the knowledge of the microscopic quantum gravity theory. Nevertheless, in certain cases one can estimate the maximal strength in a quasi-classical approximation. The main reason for an expectation that VBH violate global charges is the no-hair theorem [45], which implies that BH are characterized by "charges" that are coupled to the massless fields. Conservation of such charges cannot be violated by BH, since an outside observer can measure the conserved flux. Such a measurement is impossible for a global charge, which renders it uncontrollable.

In the literature one may find a number of estimates for VBH-mediated p-decay first discussed by Zel'dovich [46]. The main idea is that an elementary particle, carrying a global charge in question, may quantum mechanically collapse into a VBH, or be captured by one. VBH can later decay into a final state of an arbitrary global charge leading to its non-conservation. To estimate the rate of such a process Zel'dovich used a "geometric" cross section of the gravitational capture of a particle inside a VBH, which is simply given by its Schwarzschild radius squared. Thus

$$\sigma \sim \left(\frac{M_{BH}^2}{M_P^4} \right). \qquad (8.2)$$

The resulting estimate for the proton lifetime was

$$\tau_p \sim \frac{1}{m_{\text{proton}}} \left(\frac{M_P}{m_{\text{proton}}} \right)^4. \qquad (8.3)$$

Somewhat different estimate can be obtained [47] if the amplitude of the proton collapse into a VBH is evaluated from the BH-proton wave-function overlap integral

$$\int d^3 r \Psi_{\text{proton}} \Psi_{BH}. \tag{8.4}$$

The main point in all these studies, most important for the present discussion, is that either proton or some of its constituent quarks must be trapped inside a VBH . Once captured by a VBH the memory about the baryon charge is erased and VBH can decay into an arbitrary kinematicaly allowed set of particles with the same color and the electric charge, but different baryon number. The resulting p-decay rate is suppressed by the powers of (m_{proton}/M_P). Therefore, the dangerous operators (8.1) appear suppressed by powers of M_P

$$\frac{E^n}{M_P^{n+2}} qqql, \tag{8.5}$$

where E is the energy in the process. Thus, in conventional 4D theories, were $M_* = M_P$, the corresponding rate is very much suppressed even for $n = 1$. Naively, one may think that in theories with low quantum gravity scale the relevant scale to be used in the above equation instead of M_P is M_*. This would be an obvious phenomenological disaster for our framework. Fortunately, this naive expectation is wrong as we shall now explain. Let us again consider a B-violating process induced by VBH in which a proton, or some of its constituent quarks, collapses into a VBH. The relevant suppression in such a process is M_P, not M_*, due to the fact that the strong bulk gravity is shielded from proton. Recall, that in the limit $M_P \rightarrow \infty$ gravity switches off regardless of the value of M_*.

An alternative simple way to see this is to remember that proton is localized on the brane, where gravity is week and the scale of a microscopic VBH is M_P just as in ordinary 4D gravity. As a result, the collapse of a proton into a VBH will go as in the ordinary case. The same would be true regarding any other process that break the global symmetries of the SM. Thus, we conclude that in our framework the VBH-mediated processes are harmless.

So far our analysis was done in the minimal case, in which there are no new exotic states that can carry baryon number into the bulk. If such states are introduced, some experimentally interesting possibilities may open up; such are neutron-anti-neutron oscillation (without observable proton decay). The existing bounds on such processes are much milder than that for the proton decay, and they can be a subject of an independent experimental search [48].

For instance, a neutron may mix to a heavy bulk fermion X, to which we can prescribe a baryon number $B = 1$ (but zero lepton number). However, since in the bulk the B-number is not conserved due to very low mass of VBH, this mixing can lead to a process

$$n \rightarrow X \rightarrow \text{VBH} \rightarrow \bar{n}. \tag{8.6}$$

Note that if the corresponding mixing operator on the brane is induced by gravity, it will be suppressed by powers of M_P

$$\frac{udd\bar{X}}{M_P^{5/2}}, \tag{8.7}$$

and will be practically unobservable. Of experimental interest is the case when it is induced by non-gravitational effects, for instance, by integrating out some *perturbative* heavy brane states with masses $M \ll M_P$, in which case the strength may be controlled by their mass

$$\frac{udd\bar{X}}{M^{5/2}}, \tag{8.8}$$

and can be experimentally observable. Note that the source of the baryon number non-conservation, is again gravitational, since the above operator *per se* does not violate baryon number. B-violation can only occur, if the virtual X-fermion collapses into a bulk BH. Note that the analogous high-dimensional operator for proton will also require more SM particles in the final state as well as violation of the lepton number and thus will be suppressed by additional powers of M_P.

IX. CONCLUSIONS

In this paper we proposed a framework in which the "rigid" SM is coupled to gravity which becomes "soft" above the scale $M_* \ll M_{\text{SM}}$. It was assumed that the quantum theory of gravity above M_* has some generic properties of the closed string theory. We showed that the bound $M_* > 10^{-3}$ eV is compatible with all the present day observations, despite the exponentially increasing density of string states. The key phenomenon is "shielding" by which the rigid SM makes gravity weak without affecting its softness. This is due to the renormalization of the kinetic term of a graviton and other string states by SM loops. As a result, the 4D gravitational coupling is set by *non-gravitational* physics, while the scale of the softness is still determined by the gravitation.

We discussed an explicit model in which the SM lives on a 3-brane embedded in infinite-volume flat 5D space. The spectrum of 5D bulk gravity above the scale M_* is that of a closed string theory. Despite this fact, a brane observer sees at the distances $M_*^{-1} \ll r \ll M_P^2/M_*^3$ the 4D gravity with the Newton constant set by the SM physics.

In high energy processes on the brane the production of the string states becomes significant only at energies above $E \sim \sqrt{M_P M_*}$. As a result, collider experiments, astrophysics and early cosmology *independently* put the same lower bound, $M_* \sim 10^{-3}$ eV. The same bound is obtained from sub-millimeter gravitational measurements [1]. For this value, the model has experimental signatures both for colliders as well as for sub-millimeter gravity measurements.

We have discussed some unusual properties of the black holes in the present framework. Despite the low quantum gravity scale, the virtual black hole mediated baryon number

violating operators are suppressed by powers of M_P and are harmless.

If supplemented by low-energy supersymmetry, our framework maintains the successful prediction of the gauge coupling unification [10], despite the very low value of the quantum gravity scale.

APPENDIX

Let us consider a five-dimensional scalar field Φ, with an induced kinetic term on a four-dimensional brane, placed in the origin of the fifth dimension. We denote the coordinates with $x^A \equiv (x^\mu, y)$. The range of the fifth coordinate is $y \in [-\infty, \infty]$. Equally well this can be thought of as having the compact fifth dimension of very large radius R. We will consider two separate cases, in which the field Φ is massless or massive, respectively.

1. Massless field

The model Lagrangian is

$$\mathcal{L} = \partial^A \Phi \partial_A \Phi + r_c \delta(y) \partial^\mu \Phi \partial_\mu \Phi. \tag{A1}$$

The kinetic term on the brane is induced with strength r_c. We look for solutions of the form $\Phi = \phi_m(y) \sigma_m(x^\mu)$, where $\sigma_m(x^\mu)$ satisfy the four dimensional Klein-Gordon equation $(\partial^\mu \partial_\mu + m^2) \sigma_m = 0$. Then the profiles $\phi_m(y)$ are determined by the equation

$$(\partial_y^2 + m^2 + r_c \delta(y) m^2) \phi_n(y) = 0. \tag{A2}$$

Outside the origin the solutions are plane waves of frequency m. The wavefunctions have definite parity and we will concentrate on those that have non-zero value at the origin. Let us divide the space in two regions $I \equiv [-\infty, 0]$, $II \equiv [0, \infty]$. We take the wave functions to be

$$(\text{I}) \quad \phi_I(y) = A \cos(my) - B \sin(my),$$

$$(\text{II}) \quad \phi_{II}(y) = A \cos(my) + B \sin(my). \tag{A3}$$

Integrating Eq. (A2) from $y = -\epsilon$ to $y = +\epsilon$, we find

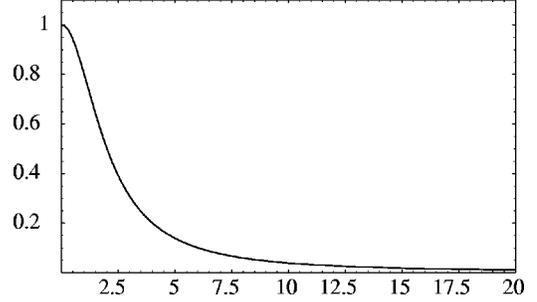$$B^2 = \frac{r_c^2 m^2}{4} A^2. \tag{A4}$$

FIG. 6. Modulus squared of the wave function at the origin (A6) as a function of the mass. The modulus squared is plotted on the y axis (in units of $1/2\pi$), while the x axis is the mass of the KK modes in units of $1/r_c$.

Since we are dealing with plane-wave-normalizable wave functions, we can choose

$$A^2 + B^2 = 1/2\pi. \tag{A5}$$

[This choice reproduces the correct normalization of the propagator if we use $|\phi_m(0)|^2$ as the spectral density in Eq. (3.13).] The resulting value of the modulus squared of the wave function on the brane is

$$|\phi_m(y=0)|^2 = \frac{1}{2\pi} \frac{4}{4 + r_c^2 m^2}. \tag{A6}$$

The suppression of the squared modulus of the wave function is shown in Fig. 6.

To summarize, a massless five-dimensional field with an induced kinetic term on the brane gives rise to a continuous KK spectrum, starting from the zero mass. Higher KK states are suppressed on the brane according to Eq. (A6).

The Euclidean propagator for this model, in the case of a source located at the origin, is easily found as in [4]. The defining equation is

$$[\Box_5 + r_c \delta(y) \Box_4] G(x,y) = \delta^4(x) \delta(y). \tag{A7}$$

Fourier transforming in the x^μ variables we get

$$[p^2 - \partial_y^2 + r_c p^2 \delta(y)] \tilde{G}(p,y) = \delta(y), \tag{A8}$$

where p^2 is the Euclidean four-momentum. With the ansatz $\tilde{G}(p,y) = D(p,y) B(p)$, with $D(p,y)$ satisfying $(p^2 - \partial_y^2) D(p,y) = \delta(y)$, it is straightforward to obtain the solution

$$\tilde{G}(p,y) = \frac{1}{2p + r_c p^2} \exp\{-p|y|\}. \tag{A9}$$

2. Massive field

Let us now consider a field of bulk-mass M and brane-mass μ. The Lagrangian is

$$\mathcal{L} = \partial^A \Phi \partial_A \Phi + r_c \delta(y) \partial^\mu \Phi \partial_\mu \Phi - M^2 \Phi^2 - \mu^2 r_c \delta(y) \Phi^2, \tag{A10}$$

where we have suppressed an overall factor of M_*^3. The equation of motion for the field Φ is

$$(\partial^A \partial_A + r_c \delta(y) \partial^\mu \partial_\mu + M^2 + r_c \mu^2 \delta(y)) \Phi = 0. \quad (A11)$$

Decomposing the field Φ in KK modes we end up with the equation

$$(\partial_y^2 + r_c \delta(y)(m^2 - \mu^2)) \phi_m(y) = (M^2 - m^2) \phi_m(y). \quad (A12)$$

This is the equation for a particle of energy $m^2 - M^2$ in a delta-function type potential. For $m > M$ there will be a continuum of scattering states; moreover, in the case $\mu < M$, for $m < M$ there are also two bound states in the spectrum. Let us first look at the case $m < M$. The solutions in regions (I) and (II) are

(I) $\quad \phi_I^{BS}(y) = A \exp(\sqrt{M^2 - m^2} y),$

(II) $\quad \phi_{II}^{BS}(y) = A \exp(-\sqrt{M^2 - m^2} y). \quad (A13)$

Integrating Eq. (A12) from $y = -\epsilon$ to $y = +\epsilon$ gives the condition

$$2 \sqrt{M^2 - m^2} = r_c(m^2 - \mu^2). \quad (A14)$$

This has no solution for $\mu > M$, while for $\mu < M$ it is satisfied for

$$m_{BS}^2 = \mu^2 - \frac{2}{r_c^2} \pm \sqrt{\frac{1}{r_c^4} + \frac{M^2 - \mu^2}{r_c^2}}. \quad (A15)$$

The modulus squared of the bound state wavefunction at the origin is easily evaluated from the normalization condition

$$A^2 \int_{-\infty}^{\infty} \exp(-\sqrt{M^2 - m^2} |y|) dy = 1 = A^2 \frac{2}{\sqrt{M^2 - m^2}}. \quad (A16)$$

We can derive an effective four dimensional action for the localized mode by integrating the Lagrangian (A10) over the fifth dimension, writing $\Phi = \phi_b(y) \sigma_b(x)$, with σ_b dimensionless and $\phi_b = \exp(-\sqrt{M^2 - m^2} |y|)$:

$$\mathcal{L}_b = M_*^3 \left\{ \sigma_b \Box \sigma_b \left[\int dy\, \phi_b^2 (1 + r_c \delta(y)) \right] \right.$$

$$\left. + \sigma_b^2 \left[\int dy\, \phi_b (-\partial_y + M^2 + \mu^2 r_c \delta(y)) \phi_b \right] \right\}, \quad (A17)$$

where we have restored the factor M_*^3. Using Eq. (A12) for ϕ_b, we get

$$\mathcal{L}_b = M_*^3 \left(r_c + \frac{1}{\sqrt{M^2 - m_{BS}^2}} \right) [\sigma_b \Box \sigma_b + m_{BS}^2 \sigma_b^2]. \quad (A18)$$
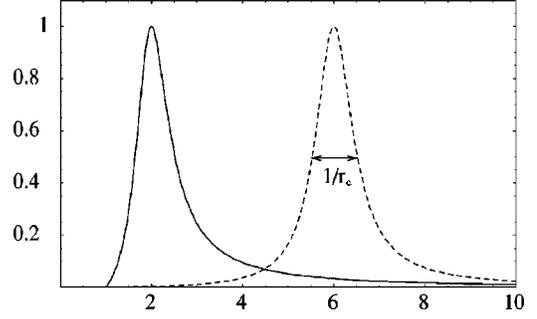

FIG. 7. Modulus squared of the wave function on the brane, for the continuum modes and two choices $\mu = 2/r_c$ (solid), $\mu = 6/r_c$ (dashed), and $M = 1/r_c$. The modulus squared is plotted on the y axis (units are $1/2\pi$). On the x axis is the mass m of the KK modes in units of $1/r_c$.

As usual, the overall coefficient is the inverse square of the coupling constant of this four dimensional mode to matter. Using Eq. (A15) we can write it as

$$G_b^{-1} = M_*^3 r_c \left[1 - \frac{1}{1 \pm \sqrt{1 + r_c^2(M^2 - \mu^2)}} \right]$$

$$\simeq M_P^2 (1 + O(1/Mr_c)). \quad (A19)$$

Therefore, the localized modes are coupled with strength M_P. Now let us look at the continuum of states with $m > M$. The wave function for these modes is

(I) $\quad \phi_I(y) = A \cos(\sqrt{m^2 - M^2} y) - B \sin(\sqrt{m^2 - M^2} y),$

(II) $\quad \phi_{II}(y) = A \cos(\sqrt{m^2 - M^2} y) + B \sin(\sqrt{m^2 - M^2} y). \quad (A20)$

Once again, matching the derivatives at the origin gives the condition

$$2 \sqrt{m^2 - M^2} B = r_c A(m^2 - M^2). \quad (A21)$$

From Eq. (A21) and the condition (A5) we can find the modulus squared of wave function at zero

$$|\phi_m(y=0)|^2 = \frac{1}{2\pi} \left[1 + \frac{r_c^2 m^2}{4} \left(\frac{(1 - \mu^2/m^2)^2}{1 - M^2/m^2} \right) \right]^{-1}. \quad (A22)$$

The term $1/(1 - M^2/m^2)$ in the denominator ensures that the wavefunction at the origin vanishes for $m = M$. For $\mu < M$ the suppression is much like the suppression in Eq. (A6). When μ becomes bigger than M, the bound state disappears and the continuum states with mass $m \approx \mu$ are enhanced on the brane. The enhancement of continuum modes with mass close to μ is shown in Fig. 7.

The suppression (A22) can be rewritten in terms of the dimensionless variable $x \equiv mr_c$ like

$$|\phi_m(0)|^2 = \frac{1}{2\pi} \left[1 + \frac{1}{4} \left(\frac{(x^2 - \mu^2 r_c^2)^2}{x^2 - M^2 r_c^2} \right) \right]^{-1}. \quad (A23)$$

The dimensionless function in Eq. (A23) (also shown on Fig. 7 for a specific choice of values M and μ) has extremal value one at $x = \mu r_c$. The width of the peak, defined as the distance between points at which the function drops to one half of its maximal value, is $\delta x \approx 1 + M^2/4\mu^2$. For any function $f(m)$ which is slowly varying over a range of the order $1/r_c$, we can then make the approximation

$$\int dm |\phi_m(0)|^2 f(m) \simeq \frac{1}{r_c} f(\mu). \tag{A24}$$

This approximation is extensively used in the text. Each bulk mode of mass $M < \mu$ produces a continuum of KK states with masses $m > M$. Due to the specific resonance form of the wavefunction on the brane for those modes, we can approximate their effect with a single mode of mass μ. This "mode" is coupled to brane fields with strength proportional to $1/r_c M_*^3$. The Euclidean propagator can be easily found also in the massive case: Eq. (A8) is replaced by

$$[p^2 - \partial_y^2 + M^2 + r_c \delta(y)(p^2 + \mu^2)]\tilde{G}(p,y) = \delta(y). \tag{A25}$$

Again, write $\tilde{G}(p,y) = D(p,M,y)B(p,M,\mu)$ with $D(p,M,y)$ satisfying $(p^2 + M^2 - \partial_y^2)D(p,M,y) = \delta(y)$. It is then straightforward to obtain

$$\tilde{G}(p,y) = \frac{1}{r_c} \frac{1}{2\sqrt{p^2 + M^2}/r_c + (p^2 + \mu^2)}$$
$$\times \exp\{-\sqrt{p^2 + M^2}|y|\}. \tag{A26}$$

Notice that the above expression has a pole corresponding to the bound state found above. Also, notice that for large momenta compared to $1/r_c$, and for $y = 0$, it becomes approximately

$$\tilde{G}(p,y) \simeq \frac{1}{M_*^3 r_c} \frac{1}{p^2 + \mu^2}, \tag{A27}$$

where we have reinserted the appropriate overall factor of $1/M_*^3$ that should multiply the Lagrangian (A10). This describes a four dimensional state with mass μ and coupled with strength $1/M_P^2$, in agreement with the previous discussion.

3. Short-distance potential in string theory

In this appendix we show how string theory softens the behavior of the gravitational interaction between two static sources, preventing the potential from blowing up in the short-distance limit.

For simplicity, consider closed bosonic string theory in the critical dimension $D = 26$. The interaction potential between two static point-like sources of masses m_1 and m_2

separated by a distance r can be found[15] in [49]:

$$V(r) = m_1 m_2 \int_0^\infty \frac{dt}{t} t^{-1/2} \exp\left(\frac{-tr^2}{2\pi\alpha'}\right)(\eta(it))^{-24}, \tag{A28}$$

where $\eta(\tau)$ is the Dedekind η-function:

$$\eta(\tau) = (\exp\{2\pi i\tau\})^{1/24} \prod_{n=1}^\infty (1 - \exp\{2\pi in\tau\}). \tag{A29}$$

This function has the property $\eta(-1/\tau) = (-i\tau)^{1/2}\eta(\tau)$ and has the following expansion:

$$\eta(i\tau)^{-24} = \exp(2\pi\tau) + 24 + O(\exp(-2\pi\tau)), \quad \tau \to \infty \tag{A30}$$

$$\eta(i\tau)^{-24} = \tau^{12}[\exp(2\pi/\tau) + 24 + O(\exp(-2\pi/\tau))], \quad \tau \to 0. \tag{A31}$$

The behavior of Eq. (A28) for large separation compared to the string scale $\sqrt{\alpha'}$ is readily obtained noting that the integral is dominated by the small-t region: using Eq. (A31) (and dropping the first term in the expansion, which corresponds to tachyon exchange and is unphysical) we find that at large distances the potential behaves as $m_1 m_2/r^{23}$, reproducing Newton's law in 26 space-time dimensions. However, we are interested in the short distance behavior ($r^2 \ll \alpha'$). In this regime the whole integration domain contributes, and we cannot just insert one of the expansions (A30),(A31) in place of $\eta(it)$. Nevertheless, we can reason as follows: suppose the potential blows up as $r \to 0$, as does any potential mediated by point-particle exchange; then we should expect the integral to diverge when we put $r = 0$ in Eq. (A28). This however is not the case: the integral can only diverge at the extrema, and by Eqs. (A30) and (A31)

$$\frac{1}{t^{3/2}}(\eta(it))^{-24} \sim 24 t^{21/2}, \quad t \to 0,$$

$$\frac{1}{t^{3/2}}(\eta(it))^{-24} \sim 24 t^{-3/2}, \quad t \to \infty, \tag{A32}$$

where we have again dropped the first term in the expansion of $(\eta(it))^{-24}$. We see that the integration is finite at both extrema, meaning that the potential does not diverge at $r = 0$, and has therefore an expansion of the form

---

[15]We take these sources to couple to string states similar to D0-brane couplings.

$$V(r) = a_0 + a_1 r + O(r^2), \quad r \to 0. \tag{A33}$$

In other words, the stringy behavior of gravity, which becomes relevant for distances below the string length, smooth out the divergences characteristic of interactions mediated by point particles, and "softens" the short-distance behavior of gravity. This can be expressed by saying that the graviton propagator has a form factor that becomes effective above the fundamental Planck scale and makes the ultraviolet behavior milder.

---

[1] C.D. Hoyle, U. Schmidt, B.R. Heckel, E.G. Adelberger, J.H. Gundlach, D.J. Kapner, and H.E. Swanson, Phys. Rev. Lett. **86**, 1418 (2001); J.C. Price, in *Proceedings of International Symposium on Experimental Gravitational Physics*, edited by P. Michelson, Guangzhou, China (World Scientific, Singapore, 1988); J. Long, "Laboratory Search for Extra-Dimensional Effects in Sub-Millimeter Regime," Talk given at the International Conference on Physics Beyond Four Dimensions, ICTP, Trieste, Italy (2000); A. Kapitulnik, Talk given at the International Conference on Physics Beyond Four Dimensions, ICTP, Trieste, Italy (2000).

[2] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *ibid.* **436**, 257 (1998); N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D **59**, 086004 (1999).

[3] For earlier discussions on lowering the string scale, see E. Witten, Nucl. Phys. **B471**, 135 (1996); J. Lykken, Phys. Rev. D **54**, 3693 (1996).

[4] G. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B **485**, 208 (2000).

[5] R. Sundrum, J. High Energy Phys. **07**, 001 (1999).

[6] G. Dvali, G. Gabadadze, M. Kolanović, and F. Nitti, Phys. Rev. D **64**, 084004 (2001).

[7] N. Arkani-Hamed, G. Dvali, G. Gabadadze, and M. Perelstein (work in preparation).

[8] C. Deffayet, Phys. Lett. B **502**, 199 (2001).

[9] C. Deffayet, G. Dvali, and G. Gabadadze, Phys. Rev. D (to be published), astro-ph/0105068.

[10] S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D **24**, 1681 (1981); M.B. Einhorn and D.R.T. Jones, Nucl. Phys. **B198**, 475 (1982); W. Marciano and G. Senjanovic, Phys. Rev. D **25**, 3092 (1982).

[11] D.M. Capper, Nuovo Cimento A **25**, 29 (1975).

[12] S.L. Adler, Phys. Rev. Lett. **44**, 1567 (1980); Phys. Lett. **95B**, 241 (1980); Rev. Mod. Phys. **54**, 729 (1982); **55**, 837(E) (1983).

[13] A. Zee, Phys. Rev. Lett. **48**, 295 (1982).

[14] G. Dvali and M. Shifman, Phys. Lett. B **396**, 64 (1997); **407**, 452 (1997).

[15] E. Witten, Nucl. Phys. **B507**, 658 (1997).

[16] G. Dvali, G. Gabadadze, and Z. Kakushadze, Nucl. Phys. **B562**, 158 (1999); G. Dvali and Z. Kakushadze, *ibid.* **B537**, 297 (1999).

[17] G. Gabadadze and M. Shifman, Phys. Rev. D **61**, 075014 (2000); A. Gorsky and K. Selivanov, *ibid.* **62**, 071702 (2000).

[18] S.H. Shenker, hep-th/9509132.

[19] G. Dvali and G. Gabadadze, Phys. Rev. D **63**, 065007 (2001).

[20] A. Vilenkin, Phys. Lett. **133B**, 177 (1983).

[21] J. Ipser and P. Sikivie, Phys. Rev. D **30**, 712 (1984).

[22] N. Kaloper and A. Linde, Phys. Rev. D **59**, 101303 (1999).

[23] R. Gregory, V. Rubakov, and S.M. Sibiryakov, Phys. Rev. Lett. **84**, 5928 (2000).

[24] I.I. Kogan, S. Mouslopoulos, A. Papazoglou, G.G. Ross, and J. Santiago, Nucl. Phys. **B584**, 313 (2000).

[25] M. Carena, A. Delgado, J. Lykken, S. Pokorski, M. Quiros, and C.E.M. Wagner, Nucl. Phys. **B609**, 499 (2001).

[26] G. Dvali, G. Gabadadze, and M. Shifman, Phys. Lett. B **497**, 271 (2001).

[27] M. Chaichian and A.B. Kobakhidze, Phys. Rev. Lett. **87**, 171601 (2001).

[28] S.L. Dubovsky and V.A. Rubakov, hep-th/0105243.

[29] H. van Dam and M. Veltman, Nucl. Phys. **B22**, 397 (1970).

[30] V.I. Zakharov, JETP Lett. **12**, 312 (1970).

[31] A.I. Vainshtein, Phys. Lett. **39B**, 393 (1972).

[32] C. Deffayet, G. Dvali, G. Gabadadze, and A. Vainshtein, hep-th/0106001.

[33] R. Dick, Class. Quantum Grav. **18**, R1 (2001).

[34] C. Deffayet, G. Dvali, G. Gabadadze, and A. Lue, Phys. Rev. D **64**, 104002 (2001).

[35] I.I. Kogan, S. Mouslopoulos, and A. Papazoglou, Phys. Lett. B **503**, 173 (2001).

[36] M. Porrati, Phys. Lett. B **498**, 92 (2001).

[37] I.I. Kogan, S. Mouslopoulos, A. Papazoglou, and L. Pilo, hep-th/0105255.

[38] F.A. Dilkes, M.J. Duff, J.T. Liu, and H. Sati, Phys. Rev. Lett. **87**, 041301 (2001); M.J. Duff, J.T. Liu, and H. Sati, Phys. Lett. B **516**, 156 (2001).

[39] M.B. Green, J.H. Schwarz, and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, England, 1987).

[40] M.J. Bowick, hep-th/9210016.

[41] G.E. Andrews, in *Handbook of Discrete and Combinatorial Mathematics*, edited by K.H. Rosen *et al.* (CRC Press, Boca Raton, FL, 2000).

[42] G. Dvali, G. Gabadadze, and M. Porrati, Mod. Phys. Lett. A **15**, 1717 (2000).

[43] S. Cullen and M. Perelstein, Phys. Rev. Lett. **83**, 268 (1999).

[44] G.F. Giudice, R. Rattazzi, and G.D. Wells, Nucl. Phys. **B544**, 3 (1999).

[45] J.D. Bekenstein, Phys. Rev. D **5**, 1239 (1972).

[46] Ya.B. Zel'dovich, JETP Lett. **24**, 25 (1976).

[47] A.D. Dolgov, hep-ph/9910532.

[48] E.g., see, Yu. Kamyshkov, in *Next Generation Nucleon Decay and Neutrino Detector, Stony Brook, 1999*, pp. 84-8.

[49] J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, England, 1998), Vol. I.