Precursors, black holes, and a locality bound

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We reexamine the problem of precursors in the anti–de Sitter/conformal field theory correspondence. Identification of the precursors is expected to improve our understanding of the tension between holography and bulk locality and of the resolution of the black hole information paradox. Previous arguments that the precursors are large undecorated Wilson loops are found to be flawed. We argue that the role of precursors should become evident when one saturates a certain locality bound. The spacetime uncertainty principle is a direct consequence of this bound.

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I. INTRODUCTION

The puzzles of quantum gravity become sharply focused with the black hole information paradox [1], which arises when considering the fate of quantum-mechanical information which falls into a black hole. Destruction of the information would sacrifice quantum mechanics and would apparently lead to physics that violates energy conservation, while escape of the information in Hawking radiation would appear to violate locality.

This difficult situation led to the postulated *holographic principle* [2,3], which holds that in a real sense the information can be thought of as stored in degrees of freedom at the surface of the black hole. This principle conflicts with locality as usually formulated in quantum field theory, but only in extreme circumstances; at long distances and low energies the world should remain effectively local.

The holographic principle has found a concrete realization in Maldacena's proposed anti–de Sitter (AdS)/conformal field theory (CFT) correspondence [4], which asserts that string theory in the whole of AdS spacetime has an equivalent description as dynamics of a large-N super-Yang-Mills theory on the boundary of that spacetime.

If true, this equivalence says that all information inside AdS can be equivalently described by a state of the boundary. This would include information that from the bulk perspective has not had time to casually reach the boundary. An example would be a bomb detonated at the center of AdS; from the bulk perspective the information from the bomb should not reach the boundary until a time comparable to the AdS radius R, but equivalence with the boundary theory implies that this information should be somehow encoded in the boundary state the moment the bomb goes off. Polchinski, Susskind, and Toumbas [5] formulated the important question of identifying these boundary variables in which the information is encoded and coined the name *precursors* to describe them.

Going one step further, if observation of precursors allows one to measure information that should be causally inaccessible from the bulk perspective, precursors should allow one to measure information inside a black hole in anti–de Sitter space. Indeed, according to the holographic principle, black hole formation and evaporation is a unitary process and, by AdS/CFT, should be fully encoded at all times in the boundary CFT. For this reason it would be extremely interesting to identify the precursor fields and use them to chart the internal dynamics of a black hole.

Susskind and Toumbas [6] have made the concrete proposal that the precursor fields are large Wilson loops and have presented calculations purporting to show that these Wilson loops indeed allow boundary measurements that would naïvely be forbidden by bulk locality. In particular, in the case of the explosion mentioned above, measurement of a Wilson loop of size a would allow a detection of the explosion at a time of order a before the light cone of the explosion reaches the boundary of AdS.

It should be noted that it is debatable to what extent such an observation—even if possible—constitutes observing the explosion outside its light cone. To forsee the explosion by a time a requires a Wilson loop of size a, and it would appear to take a time a to actually know that the Wilson loop has been measured—the data from the detectors along the loop would have to be sent to some central location for comparison.[1] However, as we will discuss, one could also imagine using Wilson loops to measure events inside a black hole. In this case, any measurement would be extremely interesting, since the time it would take the information to escape classically is infinite.

In this paper we investigate these claims more closely. There is a purely field theoretical calculation analogous to that of [6] that also seems to indicate that the observation of a bilinear of local operators allows one to likewise measure

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[1] We thank J. Preskill for discussions on this point.

the explosion acausally. However, we know from field theory causality that this cannot be correct. We trace the conflict to an incorrect identification of the saddle point in an integral in an analysis analogous to [6]. A closer inspection of the string theory expression of [6] shows that the saddle point has been incorrectly identified there as well, invalidating that analysis.[2] While in field theory we know that the exact calculation predicts that an event cannot be measured outside its light cone, we do not yet know how to do an analogous calculation in string theory without computing off-shell quantities. We outline a possible calculation and comment on our expectations for the result and its connection to black holes. Our results raise serious questions about the identification of large Wilson loops as precursors.

We therefore return to consider the motivations for holography and its attendant breakdown of locality. An underlying principle is that locality should fail when we attempt to make measurements in which black holes or strings are created. We propose a concrete criterion for such a locality bound and outline its possible implications for the problem of precursors in AdS/CFT. We also discuss the connection to the problem of holographically encoding the internal state of a black hole.

We close our introduction by mentioning another logical, though heretical possibility. It may be that the AdS/CFT correspondence is not a 1-1 map; it could be that the CFT does not include all the information encoding bulk physics, for example on scales less than the AdS radius scale R. Indeed, attempts [8–10] to extract such information from correlators in the CFT have run into difficulties. If there are such missing degrees of freedom they might be related to the precursors.

The outline of this paper is as follows. Section II gives a more detailed review of the problem of identifying the precursors. Section III investigates the proposal of [6] that the precursors are large Wilson loops, finds a flaw in that analysis, and proposes a refined calculation that would be necessary to demonstrate the validity of that proposal. This section can be skipped by those who do not believe that large Wilson loops are precursors. Section IV makes the alternative proposal that the precursors are related to observations at sufficiently high energies for locality to break down. We give a concrete suggestion for a criterion for such a ''locality'' bound. Section V discusses the relation of both proposals to the problem of charting the internal dynamics of a black hole, and in Sec. VI we give comments and conclusions.

II. THE PROBLEM OF PRECURSORS

We begin by describing the problem of precursors in some more detail, in the process reviewing some of the basics of AdS/CFT. We begin with 5D anti–de Sitter space in global coordinates,

$$ds^2=\frac{R^2}{\cos^2\rho}(-d\tau^2+d\rho^2+\sin^2\rho\, d\Omega_3^2). \tag{2.1}$$

[2]A related discussion has appeared in [7].

We denote the bulk coordinates as $x=(\tau,\rho,\Omega)$ and the boundary coordinates as $b=(\tau,\Omega)$. Now, imagine that there is a source of one of the fields, say the dilaton ϕ, at the center of AdS at time $\tau=0$. For concreteness, we idealize this source as pointlike in space and time,

$$j(x)=j\,\delta(x). \tag{2.2}$$

In the bulk language, this creates a state $|j\rangle_B$. We will work in the field theory approximation and use an interaction picture with $j\phi$ treated as the interaction; the state is then

$$|j,t\rangle_B=\exp\left\{i\int^t dV_x\, j(x)\phi(x)\right\}|0\rangle_B, \tag{2.3}$$

where $|0\rangle_B$ is the bulk vacuum. For $t>0$, Eqs. (2.2) and (2.3) give

$$|j\rangle_B=\exp\{ij\phi(0)\}|0\rangle_B. \tag{2.4}$$

Fields in AdS map to operators on the boundary through the map

$$O_\Delta(b)\leftrightarrow \lim_{\rho\to\pi/2}(\cos\rho)^{-\Delta}\phi_\Delta(x), \tag{2.5}$$

where $x\to b$ is the limiting point on the boundary and Δ represents the CFT dimension of the field. This correspondence induces a map from bulk to boundary states that we spell out further in Sec. IV. In particular, corresponding to Eq. (2.3), the boundary state for times $t>0$ takes the general form

$$|j\rangle_\partial=\exp\left\{i\int db\, f(b)O(b)\right\}|0\rangle_\partial, \tag{2.6}$$

where $|0\rangle_\partial$ is the boundary vacuum, O is the operator $(1/N)\mathrm{Tr}\,F^2$ corresponding to the dilaton, and $f(b)$ is a function determined by $j(x)$.

In the context of quantum field theory in the bulk, it is clear that no information about the source reaches the boundary until time $\tau=\pi/2$, when the light cone of the source meets the boundary. On the other hand, since according to the holographic proposal the boundary theory contains all the information of the bulk theory, Eq. (2.6) should contain the information about the source before this time (see Fig. 1). For example, instead of Eq. (2.2), we might imagine the source sending a message encoded in variations of $j(x)$ over a short time around $\tau=0$, and the boundary state should contain all the information of this source. Simply put, the question of identifying the precursors is the question of understanding what degrees of freedom and observables in the boundary theory encode this information. Answering this question is an important step towards decoding the hologram and, in particular, towards understanding how approximate bulk locality is encoded and ultimately fails.

Since the boundary theory is $\mathcal{N}=4$ super-Yang-Mills theory, we know that a basis for all observables is given by the set of all Wilson loops. Equivalently, each Wilson loop can be expanded (at least formally) in terms of an infinite series of local operators at a point [11]. The question, there-

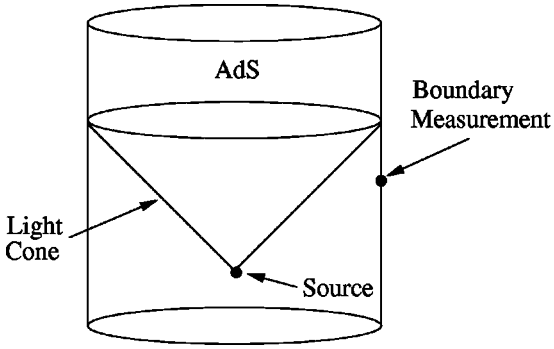

FIG. 1. Identification of local precursor fields in AdS/CFT may allow measurements outside the light cone of a source, violating naïve bulk locality.

fore, is to identify which of these Wilson loops or local operators one should measure to detect information outside the light cone of the source.

In particular, consider more closely the bulk/boundary correspondence for Wilson loops. We know that correlators of local boundary operators map to the AdS analog of the S matrix [12,13] (called the *boundary S matrix* in [13]) and would like a corresponding statement for Wilson loops. We expect that this map between correlators and S matrices also extends to a statement for Wilson loops, namely that a correlator of Wilson loops in the boundary theory corresponds to a boundary S matrix for large loops of string. Note that, of course, at least at the formal level, an arbitrary Wilson loop can be decomposed into an infinite sum of local, but arbitrarily high-dimension operators at a point [11], which we expect to correspond to representing a large string in terms of its modes.

Although we know of no complete and usable string field theory description of AdS space, we will find it useful to explain our picture in string field theory terms. At least perturbatively, the ultimate expressions we will consider can then be rewritten as first-quantized integrals over the resulting string world sheets.

The string field $\Phi[x(\sigma)]$ is a functional of string loops $x(\sigma)$, as well as ghosts and other fields which we suppress. Extending the ansatz of [14], a Wilson loop operator in the boundary theory is identified, in analogy to Eq. (2.5), as the boundary limit of the string field operator, which creates a string loop

$$W(C)\leftrightarrow \lim_{x(\sigma)\to C} Z[x(\sigma)]\Phi[x(\sigma)], \tag{2.7}$$

where $Z[x(\sigma)]$ is a (infinite) normalization factor analogous to that needed for pointlike operators. Furthermore, note that the dilaton field operator ϕ is a projection of this string field to the dilation mode.

It was proposed in [6] that *large Wilson loops* serve as the precursors: In order to measure the source at time a before its light cone reaches the boundary, one should measure the expectation value of a spatial boundary Wilson loop $W(C)$ with size of order a,

$$_\partial\langle j|W(C)|j\rangle_\partial . \tag{2.8}$$

In the next section we will examine this proposal more closely and find a flaw in the analysis of [6], reopening the question of finding the precursors.

III. LARGE WILSON LOOPS AS PRECURSORS?

A. Review and reformulation

Considering the source (2.2) of the preceding section, the authors of [6] advocate that we consider making an observation using a large Wilson loop $W(C)$,

$$_\partial\langle j|W(C)|j\rangle_\partial , \tag{3.1}$$

where the curve C lies completely outside the light cone of the source. This correlator can be calculated to linear order in j by expanding Eq. (2.6) and compared with the vacuum expectation value for the Wilson loop. Nonvanishing of the resulting difference,

$$i\int db\, f(b)\,_\partial\langle 0|[W(C),O(b)]|0\rangle_\partial \tag{3.2}$$

would be an indicator that information had been measured outside the light cone of the source.

Reference [6] infers general properties of f and uses an (approximate) calculation of

$$_\partial\langle 0|[W(C),O(b)]|0\rangle_\partial \tag{3.3}$$

given by Berenstein, Corrado, Fischler, and Maldacena [15]. Combining these answers yields a nonvanishing answer for Eq. (3.2), purporting to demonstrate that the Wilson loop $W(C)$ is indeed capable of measuring the boundary effects of the source outside its light cone.

This approach proceeds via a calculation in the boundary field theory, though the boundary source function f is inferred from the bulk source j and the boundary correlator (3.3) is inferred in [15] from a bulk computation. It is equivalent, and more straightforward, to perform all calculations directly in the bulk theory, as we will now do.

Again, working to linear order in the source j, the bulk analogue to Eq. (3.2) is

$$\begin{aligned} i \lim_{x(\sigma)\to C} z[(\sigma)][x(\sigma)]\int dx\, j(x)\,_B\langle 0|[\Phi[x(\sigma)],\phi(x)]|0\rangle_B \\ = ijZ[x(\sigma)] \lim_{x(\sigma)\to C}\,_B\langle 0|[\Phi[x(\sigma)],\phi(0)]|0\rangle_B . \end{aligned} \tag{3.4}$$

By Hermiticity of the operators, we can then rewrite the expectation value of the commutator as

$$_B\langle 0|[\Phi[x(\sigma)],\phi(0)]|0\rangle_B = 2i\,\mathrm{Im}_B\langle 0|\Phi[x(\sigma)]\phi(0)|0\rangle_B . \tag{3.5}$$

We need to compute the string two-point function from the pointlike dilaton state at $x=0$ to the boundary loop $x(\sigma)\to C$. Such far off-shell calculations in string theory are notoriously difficult. However, the analysis of [14] and [15] suggests that the answer is well approximated by a saddle

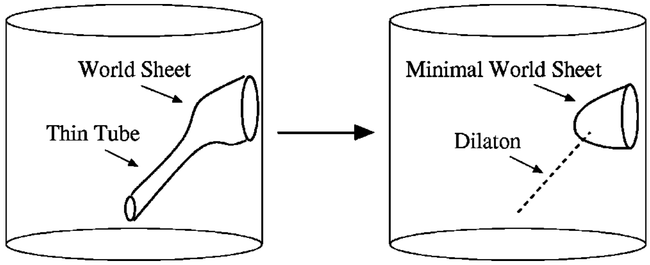

FIG. 2. The string world sheet can be approximated by a dilaton propagator attached to a minimal surface.

point. Indeed, it would seem that the obvious extremal surface corresponding to this configuration is a minimal area surface spanning the loop, which for convenience we take to be purely spacelike, and then a thin tube—or dilaton propagator—connecting the origin to a point on this surface (see Fig. 2).

Indeed, specifically considering a circular spacelike Wilson loop and directly following [15] [cf. Eq. (4.9)], with the minor modification that the pointlike operator sits in the bulk, we are led to an expression

$$\lim_{x(\sigma)\to C} Z[x(\sigma)]_B\langle 0|\Phi[x(\sigma)]\phi(0)|0\rangle_B \propto \int d\mathcal{A}'\, K_B(0,x'), \tag{3.6}$$

where the integral is over points x' on the minimal surface spanning the loop and K_B is the bulk AdS propagator.

By standard field theory causality in AdS space, the bulk propagator is purely real outside the light cone but has an imaginary piece inside the light cone. As in [6], large enough Wilson loops on the boundary, but outside the light cone, will produce spanning surfaces that enter the interior of the light cone. This leads to a nonvanishing imaginary part of Eq. (3.6) and hence the appearance that the Wilson loop is sensitive to information not accessible by usual causal observations. If one wants to measure the source at time a before its light cone reaches the boundary, a rough criterion for the relevant Wilson loops is that they should have radius $\sim a$; this condition allows the spanning minimal surface to dip into the interior of the light cone.

B. A field theory model

We now discuss a pure field theory analog of the Wilson loop analysis of Sec. III A. Suppose that instead of a Wilson loop, the boundary observer measures a bilocal operator $O(b)O(b')$. Let us consider a simple toy model of a field theory with a massless scalar ϕ coupled to a scalar ψ of mass M through a purely cubic interaction, $g\int dx\, \phi(x)\psi^2(x)$. Consider a source at $x=0$ as in Eq. (2.2), and suppose that the boundary points b and b' are spacelike separated; for concreteness take them to be at equal global AdS times, and furthermore, assume that they are both outside the light cone of the source.

With the obvious substitutions in the above steps, the result for the observation of the bilinear takes the form

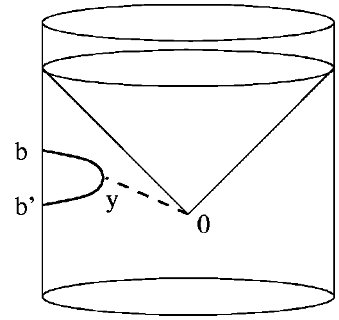

FIG. 3. The measurement of the source by a bilocal operator at the boundary can be written in terms of the imaginary part of a three-point function.

$$\begin{aligned} i\int db''\, f(b')_\partial\langle 0|[O_\psi(b)O_\psi(b'),O_\phi(b'')]|0\rangle_\partial &= ij\lim_{x\to b}\lim_{x'\to b'}(\cos\rho)^{-\Delta}(\cos\rho')^{-\Delta} \\ &\quad\times \mathrm{Im}_B\langle 0|\psi(x)\psi(x')\phi(0)|0\rangle_B, \end{aligned} \tag{3.7}$$

analogous to Eq. (3.2).

Following steps identical to those of [6,15], we approximate the expression (3.7) as follows. At tree level in the interaction parameter g, it contains

$$\begin{aligned} &\mathrm{Im}_B\langle 0|\psi(x)\psi(x')\phi(0)|0\rangle_B \\ &\quad = -\mathrm{Im}\, ig\int dy\, K_B(y,x;M)K_B(y,x';M)K_B(0,y;0) \end{aligned} \tag{3.8}$$

where we have explicitly indicated the mass in the propagator. We can represent this expression, in analogy to the sum over world sheets, as a first-quantized functional integral over world lines as shown in Fig. 3.

For $M|x-x'|\gg 1$, we expect, completely in analogy with [14,15], that this is dominated by a configuration with a minimal line connecting points $x\to b$ and $x'\to b$ and with the ϕ propagator connecting the origin to an arbitrary point along this line. So, we expect that

$$\mathrm{Im}_B\langle 0|\psi(x)\psi(x')\phi(0)|0\rangle_B \propto \int dl_y\, K_B(0,y), \tag{3.9}$$

where y is integrated along the minimal curve connecting x to x', in precise analogy with Eq. (3.6). For large enough separation of b and b', this minimal curve enters the future light cone of the source, where the bulk propagator is complex, and thus Eq. (3.9) picks up a nonvanishing imaginary part. The bilinear thus can make measurements outside the light cone.

The preceding is, of course, utter nonsense. In intermediate steps, Eq. (3.7) was derived from an expression of the form

$${}_B\langle 0|[\psi(x)\psi(x'),\phi(0)]|0\rangle_B = \mathrm{Im}_B\langle 0|\psi(x)\psi(x')\phi(0)|0\rangle_B. \tag{3.10}$$

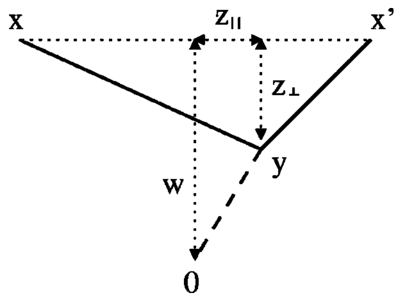

FIG. 4. The amplitude in Eq. (3.11) can be represented as a tree-level Feynman diagram.

Since x and x' are spacelike to 0, the commutator must vanish by standard field theory causality. We will derive an analogous statement in terms of flat-space Feynman diagrams in Sec. III C.

C. Searching for a pass through the mountains

What went wrong with the approximation analogous to [15], and is the same problem encountered in the string case? To answer this, consider redoing the field theory analysis in 4d Minkowski spacetime; indeed, completely analogous reasoning there leads to the conclusion that the bilinear $\psi(x)\psi(x')$ can measure information for x and x' outside the light cone of a source at the origin as long as the straight line connecting them intersects the interior of the light cone to produce a nonvanishing imaginary part.

To leading order in g, the exact expression that we should consider is

$$\mathrm{Im}\, i\int d^4y\, D(0,y;0)D(y,x;M)D(y,x';M), \quad (3.11)$$

where $D(x,y;m)$ denotes the Feynman propagator of mass m (see Fig. 4). Although Eq. (3.11) vanishes by causality as in Eq. (3.10), we can show this directly as follows. In four spacetime dimensions, the Feynman propagators can be written as

$$D(x,y;M)=\frac{i}{8\pi^2}\int_0^\infty dt\, t^{-2}e^{i(x-y)^2/2t-iM^2t/2-\epsilon t}. \quad (3.12)$$

Using this representation of the propagator, the Minkowski version of the amplitude (3.8) becomes

$$\begin{aligned}\mathrm{Im}\langle 0|\psi(x)\psi(x')\phi(0)|0\rangle &=\mathrm{Im}\frac{g}{512\pi^6}\int_0^\infty ds\, dt\, du(stu)^{-2}e^{-iM^2(s+t)/2-\epsilon((s+t+u)}\\ &\quad\times\int d^4y\, e^{(i/2)[(x-y)^2/s+(x'-y)^2/t+(y)^2/u]}. \end{aligned}\quad (3.13)$$

We can now perform the Gaussian integral over y exactly. Because the resulting expression depends only on x^2, x'^2, and $(x-x')^2$, all of which are positive, the integrals over s, t, and u can be rotated $s,t,u\to -is,-it,-iu$ without encountering poles. We now have

$$\begin{aligned}&-\mathrm{Im}\frac{g}{128\pi^4}\int_0^\infty ds\, dt\, du(st+tu+us)^{-2}\\ &\quad\times\exp\left\{-\frac{1}{2}\left[M^2(s+t)\right.\right.\\ &\quad\left.\left.+\frac{tx^2+sx'^2+u(x-x')^2}{st+tu+us}\right]\right\}. \end{aligned}\quad (3.14)$$

The integrals are manifestly real and convergent, so Eq. (3.14) has a vanishing imaginary part and Eq. (3.10) is zero to first order in g.

Now, consider approximating Eq. (3.11) in the manner of [15]. For $M|x-x'|\gg 1$, the obvious guess is that the integral is dominated by y near the line xx'. We work in the approximation $M|x-y|\gg 1$, $M|x'-y|\gg 1$. Outside the light cone, the massive Feynman propagators (in four spacetime dimensions) are

$$D(x;M)\propto\frac{M}{|x|}K_1(M|x|)\approx\sqrt{\pi/2}\frac{M}{|x|^{3/2}}e^{-M|x|}\left(1+O\,\frac{1}{M|x|}\right). \quad (3.15)$$

The massless propagator is

$$D(x,0)\propto\frac{1}{x^2}. \quad (3.16)$$

Let w be the perpendicular vector from the origin to the line xx', and decompose y into components perpendicular or parallel to this line as $y=w+z_\perp+z_\|$. Since we're working near xx', expand to leading order in $z_\perp$. From Eqs. (3.11), (3.15), and (3.16), we find

$$\begin{aligned}&\int dy\, D(0,y;0)D(y,x;M)D(y,x';M)\\ &\quad\underset{\sim}{\propto} e^{-M|x-x'|}\int d^4z\,\frac{1}{|x-y|^{3/2}|x'-y|^{3/2}}\\ &\quad\times\exp\left\{-\frac{M|x-x'|z_\perp^2}{|x-y_\||\,|y_\|-x'|}\right\}\frac{1}{(w+z)_\perp^2+z_\|^2}. \end{aligned}\quad (3.17)$$

Were it not for the last factor, the integral would clearly have a line of saddle points at $z\perp=0$ along the line xx' governed by small parameter $1/M|x-x'|$, just as reasoned above. However, the last factor in Eq. (3.17) becomes large precisely where the light cone of the source intersects this line and changes the saddle-point structure so that there are individual saddle points just off the line in the vicinity of the light cone. We have not yet completed a full treatment of the resulting (correct) saddle-point analysis, but in the field theory case we know, as discussed above, that the exact result is zero and any valid saddle-point analysis should not

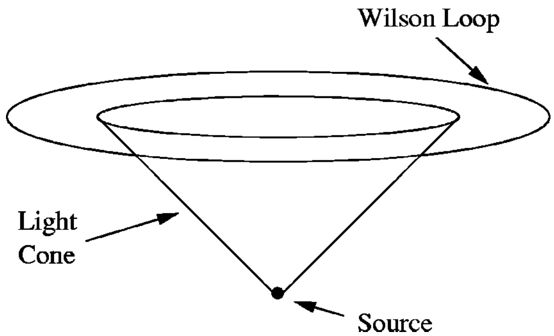

FIG. 5. The light cone of the source passes through the interior of a large Wilson loop, though the loop itself is outside the light cone.

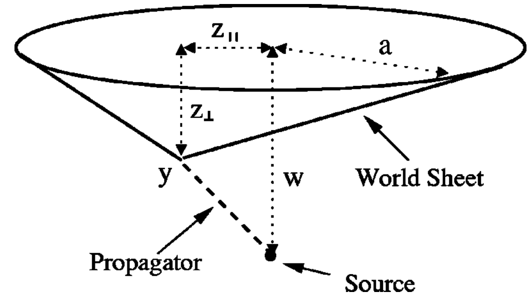

FIG. 6. The integral over all world sheets can be approximated by integrating over y where the dilaton propagator is attached to a conical world sheet.

contradict this. The main point of this discussion was to show how the saddle points are not of the form assumed in [15].

D. Wilson loops, reconsidered

The reasoning of Sec. II could equally be applied in Minkowski space to argue that measurement of large loops of string allow us to see events at spacelike separation. Consider a circular Wilson loop near a pointlike source such that the future light cone of the source passes through the interior of the loop, intersecting the disk spanning the circle. (See Fig. 5.) The loop, however, is large enough that it is fully outside the light cone.

As emphasized above, to check whether the Wilson loop measures the effects of the source, we need to compute the off-shell two-point function of Eq. (3.5). Though we do not know how to do this properly, we expect it to be represented in the form of an integral over world sheets with the topology of the disk, with a pointlike source vertex operator $V(0)$ at the origin, with the boundary on the curve $x(\sigma)$, and weighted by the Polyakov action S_P:

$$\int^{x(\sigma)} \mathcal{D}X\, \mathcal{D}g\, e^{-S_P[X,g]}V(0). \tag{3.18}$$

Computing this in $AdS_b \times S^5$ is even more problematic, given the lack of technology for Ramond-Ramond backgrounds.

Since the exact calculation is difficult, we will try an approximation in the spirit of [14,15]. We assume the integral over all world sheets can be rewritten, as in Eq. (3.6), as an integral over minimal world sheets, but with the constraint that they are attached at an *arbitrary* point y to a dilaton propagator (see Fig. 6). The resulting expression is

$$\int dy\, e^{-TA}D(0,y), \tag{3.19}$$

where T is the string tension, A is the world sheet area, and D is the dilaton propagator.

For simplicity, we take the source to lie on the axis of the loop. For a circular Wilson loop of radius a, the minimal world sheet is a tilted cone whose base is the Wilson loop and whose apex is at y. As before, let $y = w + z_\perp + z_\parallel$, where w is the vector from the origin to the center of the loop, $z_\perp$ is the distance from the plane of the loop, and $z_\parallel$ is the radial distance from the center of the loop. The area of the tilted cone is given by the integral

$$A = \int_0^{2\pi} d\theta \frac{a}{2} \sqrt{(a - z_\parallel \cos\theta)^2 + z_\perp^2}. \tag{3.20}$$

In the limit where $Ta^2 \gg 1$, the integral (3.19) appears to be dominated by world sheets with $z_\perp^2 \ll a^2$. We can therefore expand (3.20) to leading order in $z_\perp^2$ and integrate term by term, obtaining

$$A \approx \pi a^2 \left(1 + \frac{z_\perp^2}{2a\sqrt{a^2 - z_\parallel^2}} \right). \tag{3.21}$$

This reduces Eq. (3.19) to

$$e^{-T\pi a^2} \int d^4 z\, e^{-Tz_\perp^2/(2a\sqrt{a^2 - z_\parallel^2})} \frac{1}{(w+z)_\perp^2 + z_\parallel^2}. \tag{3.22}$$

Notice the close analogy to the approximate expression for the particle, Eq. (3.17).

Clearly, as in the case of the particle, there is no longer a surface of saddle points along the minimal disk spanning the Wilson loop, but rather there are saddle points shifted off this disk near the light cone of the source. Although we have again not performed a systematic saddle-point approximation about these, the strong analogy to the particle case suggests that once correctly computed, the resulting expression vanishes.

Of course, it would be instructive to attempt to complete a more accurate calculation of the correlator of a large Wilson loop—or loop of string in bulk language—with an approximately pointlike source. It is conceivable that study of the exact off-shell string amplitude (3.18) will produce a nonvanishing result. This faces difficulties, but may be tractable. One approach is a careful treatment by an intermediate semiclassical approximation working about the correct saddle points, as sketched above. Another alternative would be to work directly in AdS space. In AdS space, taking a state to the boundary in effect corresponds to working on shell, and this statement may hold equally well for macroscopic string loops. In this case, if the initial dilaton can be arranged to be on shell, the issue might be settled by a completely on-shell calculation in AdS space. Of course, a full calculation would require confronting the difficult problem of Ramond-Ramond backgrounds, but this approach is worth exploration.

IV. TOWARDS A THEORY OF PRECURSORS

According to the above analysis, calculations to date have not demonstrated that the large Wilson loops of [6] serve as precursors, and this reopens the problem of their correct identification. We therefore turn to an investigation of this problem.

A. Field theory locality

We begin by discussing the problem in the context of a field theory of a scalar field ϕ (e.g., the dilaton) in AdS space. Let us start by examining more closely the form of the boundary state $|j\rangle\partial$ created by the source (2.2).

First, consider the map between bulk and boundary in more detail. Begin with the bulk theory in the supergravity limit, at weak coupling. At zeroth order in the coupling, the field ϕ has an expansion in terms of canonically normalized annihilation and creation operators and mode functions:

$$\phi(x)=\sum_{nl\vec{m}} a_{nl\vec{m}}\phi_{nl\vec{m}}(\vec{x},\tau)+a^{\dagger}_{nl\vec{m}}\phi^{*}_{nl\vec{m}}(\vec{x},\tau). \tag{4.1}$$

Single-particle states are of the form

$$|nl\vec{m}\rangle_B=a^{\dagger}_{nl\vec{m}}|0\rangle_B . \tag{4.2}$$

Likewise, as emphasized in [16], the corresponding boundary operator should have an expansion of the form

$$O(\hat{e},\tau)=\sum_{nl\vec{m}} \frac{\alpha_{nl\vec{m}}}{\sqrt{2\omega_{nl}}}Y_{l\vec{m}}(\hat{e})e^{-i\omega_{nl}\tau}+\text{H.c.}, \tag{4.3}$$

where the unit vector $\hat{e}$ labels a point on the S^3 boundary.

In comparing states, the bulk and boundary vacua should correspond,

$$|0\rangle_{\partial}\leftrightarrow|0\rangle_B . \tag{4.4}$$

To get the relation between excited states, use the operator correspondence (2.5). This becomes

$$\alpha_{nl\vec{m}}\leftrightarrow k_{nl}a_{nl\vec{m}} , \tag{4.5}$$

where k_{nl} are constants given by the asymptotics of the mode functions $\phi_{nl\vec{m}}$ (see, e.g., the appendix to [10]). So the correspondence between single-particle states takes the form

$$\alpha^{\dagger}_{nl\vec{m}}|0\rangle_{\partial}\leftrightarrow k_{nl}|nl\vec{m}\rangle_B . \tag{4.6}$$

Indeed, Eq. (4.5) relates an arbitrary multiparticle bulk state to a boundary state.

We can also read off the relation between the bulk and boundary states created by an arbitrary source. This is most easily accomplished by inverting the relationship (2.5) to determine the bulk field corresponding to a given boundary operator. This is done using the transfer matrix $M(x,b)$ given in [12,16], and we find the relation

$$\phi(x)\leftrightarrow\int db\, M(x,b)O(b). \tag{4.7}$$

One immediately deduces that the boundary state corresponding to the source (2.3) is given by the formula (2.6), with the identification

$$f(b)=\int dV_x\, j(x)M(x,b). \tag{4.8}$$

Now, the question is what kinds of boundary operators with support only outside the light cone of the source can detect the state $|j\rangle_{\partial}$. Clearly neither O nor any of its derivatives do, since O corresponds to the bulk field through Eq. (2.5), and the bulk fields commute outside the light cone. Within the context of this simple field theory model, the only way to get operators that play the role of precursors is to identify other local operators on the boundary that are sensitive to the data in the state $|j\rangle_{\partial}$.

An example that has the appearance of a cheat is if there is another set of local observeables that can be written in terms of the $\alpha_{nl\vec{m}}$'s in the form

$$O'(\hat{e},\tau)=\sum_{nl\vec{m}} c_{nl\vec{m}}\frac{\alpha_{nl\vec{m}}}{\sqrt{2\omega_{nl}}}Y_{l\vec{m}}(\hat{e})e^{-i\omega_{nl}\tau}+\text{H.c.} \tag{4.9}$$

For generically chosen coefficients $c_{nl\vec{m}}$ these operators will not commute with the field operators outside the light cone. The reason this looks like a cheat is that the relationship between O and O' is of course highly nonlocal.

It should be recalled, however, that in the context of AdS/CFT, operators like O are composites of the fundamental Yang-Mills boundary fields. This leads us to the question of whether there are other gauge-invariant operator combinations of these fields that are able to measure the state (2.6) outside the light cone.

A toy model for such a possibility was given in [5]. Polchinski, Susskind, and Toumbas considered modeling the boundary theory as a theory of $N\times N$ matrix scalar fields ψ_{mn}. In terms of these fields, the boundary state (2.6) can be thought of as a squeezed state. Indeed, for free scalar fields, an obvious analog to TrF^2 of Yang-Mills boundary fields is an operator of the form

$$O=\frac{1}{N}\text{Tr}[(\nabla\psi)^2]. \tag{4.10}$$

The state (2.6) then takes the form of a squeezed state in terms of the annihilation and creation operators

$$b_{mn}(k),\ \ b^{\dagger}_{mn}(k) \tag{4.11}$$

for the ψ_{mn} fields

$$|j\rangle_\partial = \exp\left\{ \tfrac{1}{2}\int d^3k\, d^3k'\, F(k,k') b^\dagger_{mn}(k) b^\dagger_{nm}(k') \right\} |0\rangle_\partial \tag{4.12}$$

as in [5].

The authors of [5] investigate the problem of detecting such a state using a bilocal bilinear in the fields ψ_{mn} and suggest that this is possible. In our language, one should investigate expressions of the form

$$\left[\psi_{mn}(b)\psi_{nm}(b'), \int db'' M(0,b'') O(b'') \right] \tag{4.13}$$

(or analogous expressions with the source coupled to the stress tensor) for b,b' outside the light cone.[3]

The bilinear in Eq. (4.13) is the analogue of a certain kind of decorated Wilson loop in the limit of zero coupling, and a nonvanishing result for Eq. (4.13) would be a potentially interesting indication that such decorated loops play a role as the precursors. This possibility is under investigation. Another interesting question is to better understand the relationship of such decorated loops to the AdS boundary S matrix [12,13]. Assuming these loops correspond to elements of the boundary S matrix, they should not exhibit any bulk acausality that should not be evident in that S matrix. Certainly, for generic low-energy scattering experiments in the string theory of the bulk, we do not expect to be able to explicitly exhibit this nonlocality. In Sec. IV B we turn to a discussion of physical situations where we expect that nonlocality *should* be manifest.

B. Saturation of a string/gravity locality bound

Another place to look for clues regarding the precursors is to return to the motivations for holography. It is believed that, in contexts where strong gravitational effects are relevant, the number of fundamental degrees of freedom are drastically reduced in a fashion conflicting with naïve locality. One situation where this is thought to occur is black hole formation. Therefore, in searching for origins of the nonlocal precursors, we should consider situations where locality breaks down due to black hole formation.

A likely connected statement (through black hole/string correspondence [17]) is the belief that when string effects are important, naïve locality is again violated, as has been seen, for example, in string modifications of the uncertainty principle [18].

In order to understand in what situations holographic bounds begin to affect causality, recall that in field theory causality is formulated as the statement that fields commute at spacelike separations:

$$[\phi(x), \phi(x')] = 0, \quad (x-x')^2 > 0. \tag{4.14}$$

[3]It would also be interesting, though not convincing because of gauge noninvariance, to exhibit a nonvanishing commutator between the source and a single ψ_{mn}.

However, we expect that the corresponding equation in string theory, schematically

$$[\Phi[x(\sigma)], \Phi[x'(\sigma)]] = 0, \tag{4.15}$$

does not hold when gravitational or string effects become strong. Consider the limit where the curves $x(\sigma)$ and $x'(\sigma)$ are nearly pointlike; we expect commutativity to fail when when we consider modes of the operators that are sufficiently high energy to create a string or black hole (or other M-theoretic object) occluding the points x and x'. Of course, the operators $[x(\sigma)]$ include all possible momenta, but to apply this criterion we can work in a wave-packet basis [19,20] in which states have nearly definite momenta and positions satisfying the Heisenberg uncertainty relation $\Delta x \Delta p \gtrsim 1$. These ideas lead us to the following.

Criterion for a locality bound

Consider two particles (or strings) of momenta p_1 and p_2 colliding with impact parameter (measured in the center-of-mass frame) b. These will be said to saturate the locality bound if the collision is sufficiently energetic to create either a string or black hole with size larger than b.

A rough condition for this is that the energy simply be large enough to form a black hole or string larger than b—which effect is most important depends on the string coupling. So in spacetime dimension D this condition becomes[4]

$$E_{cm} > \min(b/l_{st}^2, b^{D-3}/g_s^2 l_{st}^{D-2}), \tag{4.16}$$

where l_{st} is the string length. Of course, interaction, form factor, etc. effects are expected to modify this bound, particularly at large energy or impact parameter; we might expect the correct bound from string production to be somewhere between the two values in Eq. (4.16).

Note that the spacetime uncertainty relation [21] follows as a consequence of our estimate (4.16) and the statement that a process confined to a time interval Δt must have energy $E \gtrsim 1/\Delta t$. Combining these implies

$$\Delta t > \max[l_{st}^2/\Delta x, g_s^2 l_{st}^8/(\Delta x)^7] \tag{4.17}$$

for $D=10$, as in [21]. Note also that, as in [21], at least according to these estimates, the crossover between string dominance and black hole dominance occurs at scales

$$b \sim g_s^{1/3} l_{st}, \tag{4.18}$$

the Planck length of eleven-dimensional M theory.

For simplicity (4.16) has been given in terms of flat space kinematics, but the same basic physical principle should determine where locality bounds are saturated in AdS space, and indeed in the limit of a large AdS radius R the statements should correspond. Because of the complications of AdS kinematics, let us investigate the bound in the simple picture in which large-radius AdS space is represented as a cavity of

[4]With appropriate modifications in case of creation of other fundamental extended objects.

radius R, with a flat internal metric. In this situation it is straightforward to get a feeling for which configurations saturate our bound.

Indeed, consider a lightlike particle with rectangular momentum $(E_1, E_1, \vec{0})$ emitted from the center of the cavity at time $T=0$. Suppose a second particle is traveling in the opposite direction, with momentum $(E_2, -E_2, \vec{0})$, and is located in the vicinity of the boundary at R, with separation transverse to the momenta, also at time $T=0$. Let us ask what energy E_2 is required to saturate the estimate (4.17). In this case the center-of-mass (CM) impact parameter is $\approx R$, and the CM energy is $2\sqrt{E_1 E_2}$, so our estimate states

$$E_2 \gtrsim \frac{1}{E_1} \min\left(\frac{R^2}{l_{st}^4}, \frac{R^{14}}{g_s^4 l_{st}^{16}}\right) \tag{4.19}$$

(for black holes of radius $\lesssim R$, we take $D=10$). Similar statements can be readily derived for other configurations. Note that for large R and small g_s, this suggests that the relevant bound is from string creation.

In short, while it is not clear *how* in detail the information is holographically encoded and exhibited, the above physical criterion serves as a guide to when locality should fail and holographic effects are expected to become important. If there is indeed an underlying unitary and holographic theory (such as M theory), this criterion indicates where it should cease to appear local and start to appear holographic. The estimate (4.17) clearly neglects important effects, but gives a rough idea as to the nature of such a locality bound.

Turning to the boundary theory, we can now use the correspondence between the AdS boundary S matrix and the boundary correlators to infer which correlators in the boundary theory we expect to exhibit effects that violate naïve bulk locality. As discussed earlier, the Wilson loops, or equivalently, via [11], the set of all local operators, form a basis for the boundary observables, but the question is what combinations of these operators are most sensitive to effects that begin to saturate our holographic bound.

From the above discussion, we expect these to be projections onto operators that correspond to the creation of large, high-energy intermediate states, for example black holes or large strings. Two obvious possibilities exist. One is to consider the high-energy components of local operators $O(x)$ (or equivalently the high-energy components of Wilson loops). This corresponds to resolving variations of boundary correlators on very short time and distance scales. Alternatively, one might consider correlators with a very large number of softer operators that combine to give a large energy.

It is not clear that such nonlocalities would be manifest at string tree level. The authors of [22] attempted to exhibit such effects in a three-point string tree-level calculation but could not conclude that what they saw was not a gauge artifact. These effects may require higher loop or nonperturbative calculations, which would certainly make sense if intermediate black holes or large strings play a role.

It is also not clear that a large Wilson loop is sufficient to probe these nonlocalities. A Wilson loop is not intrinsically high energy any more than the field operator $\phi(x)$ is in field theory; rather it involves a sum over all energies. One expects to be sensitive to nonlocalities by projecting onto certain high-energy components of these operators.

Of course, concrete calculations that exhibit such nonlocal results, particularly from very high-energy operators, or large collections of soft operators, may well be rather difficult if indeed the nonlocality results from higher-loop or nonperturbative effects. We leave this problem for the future.

V. BLACK HOLE INFORMATION

In light of the above, we now revisit the original motivation of using the precursor fields to ''see'' inside a black hole.

A. Large Wilson loops and flossing black holes

First, consider the possibility that an improved version of the tree-level calculation of [6] indeed reveals acausal effects; then we should obviously consider applying it to black holes. Consider, for example, a black hole of radius $r_h \ll R$ sitting at the center of AdS space, and suppose that we wish to measure whether a bomb dropped into the black hole has detonated or not. According to [6], we could hope to do so by measuring a large Wilson loop at the boundary. By the criterion of Sec. III, this Wilson loop would be able to measure a source inside the black hole if its spanning minimal surface crosses the horizon and intersects the future light cone of the source. Clearly, a Wilson loop that is a great circle on the S^3 boundary of AdS space will, by symmetry, have a spanning surface that cuts through the center of the black hole. If we move this circle off the equator of the sphere, then eventually it will not enter the horizon; we expect this to happen when the circle reaches a latitude of order

$$\Delta\theta \sim r_h / R. \tag{5.1}$$

Although our preceding analysis demonstrates that this surface is not the correct saddle point for the functional integral, the correct saddle point is a deformation of this surface. *If* this saddle point yields a nonvanishing result for such measurements, then we would expect that to occur for Wilson loops in the range (5.1) about great circles.

However, in addition to the preceding arguments, there are physical reasons to be suspicious of such claims. Consider the picture of a world sheet instantaneously slicing a black hole, as in Fig. 7. This process involves a virtual string, but is dual to another (idealized) process involving a real string state. This is a process in which an observer near the boundary creates a piece of string, then stretches it to macroscopic scales, slices it through the black hole, and then shrinks it back down at the opposite side of AdS space, as shown in Fig. 8.

So an obvious question is whether one expects to be able to mine information from a black hole by this process of flossing it with a string. If the answer is negative, it seems even more unlikely that the information is manifested in the far off-shell version of this process.

We are skeptical that such an effect can be seen in a tree-level calculation. Indeed, as discussed in [23], which

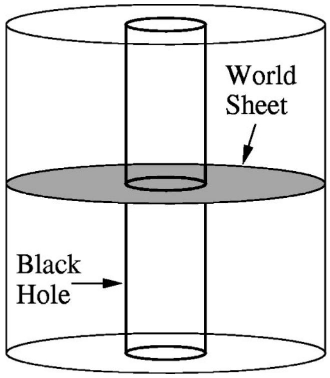

FIG. 7. Measuring an instantaneous Wilson loop corresponds to a virtual world sheet which goes through the black hole.

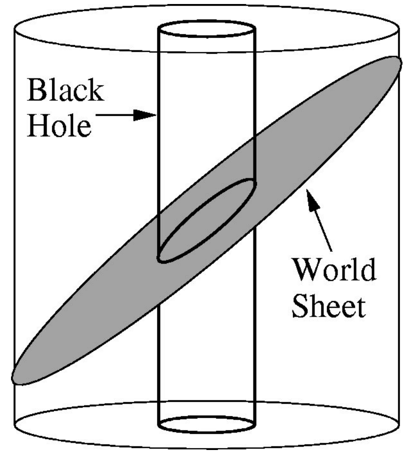

FIG. 8. A timelike world sheet corresponds to a string flossing a black hole.

considers a configuration with a stationary string threading a black hole, the string is expected to inherit the casual structure of the spacetime. In this situation, the only string excitations that make it out to an observer at infinity are the Hawking radiation of oscillation modes on the string, and these do not contain information about the state inside the horizon or any perturbations of it by classical sources inside the black hole. Trying to pull the string back out of the black hole adds another layer of difficulty; this should not be possible without the string breaking off a closed loop that remains inside the black hole.[5] We would expect this closed loop to contain any information from inside the black hole, and the remaining external string state to be insensitive to the internal state of the black hole, at least at string tree level.

Again, at a higher level in g_s one certainly might imagine seeing interesting effects, if the basic ideas of holography are correct and realized through stringy corrections. If so, it is plausible that Wilson loops in the range (5.1) are indeed sensitive to those effects, although we will advocate an alternative viewpoint.

B. Black holes and holography

In parallel to the discussion of the preceding section, we could ask in greater generality where in the boundary theory we might expect to see the information contained in the interior of a black hole. To address this, we recall two facts. The first is the AdS/CFT correspondence between the AdS boundary S matrix and the CFT correlators, outlined above. Secondly, for a large black hole in a much larger AdS space, the bulk dynamics should be closely approximated by a black hole in flat space. We expect intermediate states with large black holes to arise in specific blocks of the S matrix. One example is a matrix element with sufficient energy in the initial state focused into a region of order is its Schwarzchild radius; in this case, the final state is expected to include a very large number of outgoing soft quanta—the Hawking radiation—and if the information is contained therein, it should be in subtle correlations between this large number of quanta. This number should be $O(A)$ for an intermediate black hole of area A.

Mapping these statements to the boundary theory, we might investigate black holes through correlators that have operators corresponding to energetic and narrowly focused incoming states, with center of mass energy E, and a large number $[O(E^{8/7})$ in ten dimensions$]$ of operators corresponding to the soft outgoing quanta. The subtle relative phase information in these would describe the black hole information, which may be difficult to see otherwise. It could also be, in line with our earlier arguments, that certain other correlators with few but very high-energy operators, arranged so that they start to saturate locality bounds, as in the preceding section, would be sensitive to this information.

Unfortunately, with the present state of our knowledge this proposal does not give futher details about how to escape the black hole information paradox; in a sense it is simply mapping our earlier attempts at a holographic explanation of its resolution into the AdS/CFT arena.

VI. CONCLUSION

The question of identifying the precursor variables in AdS/CFT is an important one, both because of its relevence to understanding the detailed relation between approximately local bulk physics and boundary physics and because of its promise to finally explain how holography resolves the black hole information paradox. In this paper we have investigated the proposal of [6] that large Wilson loops are the precursors and found a difficulty; specifically, the analysis of [15] that was used misidentified the saddle point dominating the functional integral over world sheets. We gave an alternative proposal, in which the precursors are related to observables that saturate a certain locality bound. The physical idea underly-

[5]The string trajectory that corresponds to pulling the string completely back out of the black hole without leaving a loop clearly cannot satisfy the classical string equations of motion, and we would expect the neighboring trajectories to contribute rapidly varying and cancelling phases to the corresponding quantum-mechanical amplitude.

ing this bound is that one should not be able to make observations that involve concentrating an amount of energy within a region smaller than the corresponding Schwarzschild radius or in a region small enough such that other large objects, such as strings, with size comparable to the region, will be created. This bound can be saturated either by individual high-energy operators or by collections of soft operators with large total energy. This suggests to us where to look in order to understand how AdS/CFT resolves the black hole information paradox, but unfortunately does not yet tell us how to make detailed calculations exhibiting the solution.

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