Constraining neutrino physics with big bang nucleosynthesis and cosmic microwave background radiation

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We perform a likelihood analysis of the recent results on the anisotropy of cosmic microwave background radiation from the BOOMERanG and DASI experiments to show that they single out an effective number of neutrinos in good agreement with standard big bang nucleosynthesis. We also consider degenerate big bang nucleosynthesis to provide new bounds on effective relativistic degrees of freedom N_ν and, in particular, on the neutrino chemical potential ξ_α. When including supernova type Ia data we find, at 2σ, $N_\nu \leqslant 7$ and $-0.01 \leqslant \xi_e \leqslant 0.22$, $|\xi_{\mu,\tau}| \leqslant 2.6$.

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I. INTRODUCTION

New results on cosmic microwave background radiation (CMBR) anisotropy from BOOMERanG [1,2], MAXIMA [3,4], and DASI [5] experiments represent an extraordinary confirmation of our present understanding of some of the key features of the evolution of our universe. The clean evidence for the first acoustic peak of temperature anisotropies for CMBR around $l \sim 200$ [1–6] strongly supports the scenario of a post-inflationary flat universe. On the other hand new results on the second and third peak support the adiabatic inflationary model prediction of acoustic oscillations in the primeval plasma driven by gravity, and shed new light on how energy density is distributed among several components. This is a crucial piece of information which affects many independent cosmological observables, so it is reasonable to expect that it will be possible in the next years to have a rather clear picture of which cosmological model is actually realized in our universe.

In this respect BOOMERanG and MAXIMA first data release [1,3] already stimulated a wide number of studies [3,7–15], aimed to constrain the values of the energy density parameters normalized to the critical density, Ω_b, Ω_m and Ω_Λ, due to baryons, dark matter and an effective cosmological constant, respectively. In particular many authors have addressed the issue of a tension between the determination of $\Omega_b h^2$ from CMBR data and standard big bang nucleosynthesis (SBBN) [9,12,14,16–20]. In fact, the finding of a suppressed second peak in the CMBR anisotropy resulted in a rather large value for this parameter, $\Omega_b h^2 = 0.032^{+0.005}_{-0.004}$ at 68% C.L. [7], while the experimental data on primordial ^{4}He and D abundances, prefer smaller values, $\Omega_b h^2 = 0.019^{+0.004}_{-0.002}$, Ref. [20], and $\Omega_b h^2 = 0.020 \pm 0.002$, Ref. [21] (see also [22]), at 95% C.L. These estimates are obtained assuming three standard neutrino degrees of freedom.

New experimental data from BOOMERanG have refined the data at larger multipoles, and now single out a smaller value for the baryonic fraction, $\Omega_b h^2 = 0.021^{+0.004}_{-0.003}$ [2]. This is mainly due to an increase in the analyzed dataset (roughly by a factor 8) and a better understanding of the experimental beam, calibration and pointing. The new analysis leads to a slightly increased amplitude for the second peak (but still compatible at $\sim 1\sigma$ with the previous spectrum) and hints for the presence of a third peak around $l \sim 800$, which is not as high as expected in a scenario with a large baryonic fraction. Simultaneously the DASI experiment, which also found evidence for multiple peaks in the CMBR spectrum, gave an impressive and independent confirmation of a low baryon fraction, $\Omega_b h^2 = 0.022^{+0.004}_{-0.003}$ [5], when sampling a different region of the sky and different frequencies. It is worth stressing that these high multipole data may still be affected by large systematic errors (see for example the consistency test in Table 3 in Ref. [2]), thus all conclusions relying on them should still be taken with caution. This is especially true in view of the revised spectrum at $l \geqslant 300$ from the Maxima-I experiment, which gives the wide range $\Omega_b h^2 = 0.0325 \pm 0.0125$ [6].

Nevertheless it is important, on the basis of the new data now available, to undertake a detailed study of the compatibility of these data with SBBN. For this purpose we have performed, as in [14], a likelihood analysis of BOOMERanG/DASI CMBR data and SBBN in the parameter space $(\Omega_b h^2, N_\nu)$, with N_ν the effective neutrino degrees of freedom, and indeed we find a very good agreement. In particular the SBBN 95% C.L. region, corresponding to $N_\nu = 2.8 \pm 0.3$ and $\Omega_b h^2 = 0.020 \pm 0.004$, has a large overlap with the analogous CMBR contour. This fact, if it will be confirmed by future experiments on CMBR anisotropy, can be seen as one of the greatest success, up to now, of the standard hot big bang model.

As a byproduct of our analysis we also comment on the possible primordial ^{7}Li depletion, which has already been discussed in the literature [21,23,24]. We find that a depletion factor $f_7 \sim 1/2$–$1/3$ may reconcile observations from the Spite plateau with the value of $\Omega_b h^2$.

SBBN is well known to provide strong bounds on N_ν. On the other hand, Degenerate BBN (DBBN), first analyzed in Refs. [25–28], gives very weak constraint on the effective number of massless neutrinos, since an increase in N_ν can be compensated by a change in both the chemical potential of the electron neutrino, $\mu_{\nu_e}=\xi_e T_\nu$, and $\Omega_b h^2$. However, combining this scenario with the bounds on baryonic and radiation densities allowed by CMBR data, it is possible to obtain rather strong constraints on N_ν even for DBBN. From our analysis we get the bound $N_\nu \leqslant 7$, at 95% C.L., when including Supernovae Ia (SNIa) data, which translates into a new and more stringent bound on background neutrino chemical potentials.

Some caution is naturally necessary when comparing the effective number of neutrino degrees of freedom from BBN and CMBR, since they may be related to different physics. In fact the energy density in relativistic species may change from the time of BBN ($T\sim$MeV) to last scattering ($T\sim$eV). Specifically, if a neutrino has a mass in the range eV$<m<$MeV, and decays into sterile particles, like other neutrinos, majorons etc., with lifetime $t(\mathrm{BBN})<\tau<t(\mathrm{CMBR})$, then the effective number of neutrinos at CMBR would be noticeably different than at BBN [29]. However, this possibility does not look too natural any longer, in view of the recent experimental results on neutrino oscillation [30,31], showing that all active neutrinos are likely to have masses smaller than eV. One could instead consider sterile neutrinos mixed with active ones, which could be produced in the early universe by scatterings and subsequently decay. However, for mixing angle large enough to thermalize sterile neutrinos [32], one needs a sterile to active neutrino number density ratio $n_s/n_\nu \approx 4\times 10^4 \sin^2 2\theta (m/\mathrm{keV})(10.75/g^*)^{3/2}$ of order unity [33] (θ is the mixing angle, and g^* is the number of relativistic degrees of freedom). Hence using the decay time, $\tau\approx 10^{20}(\mathrm{keV}/m)^5/\sin^2 2\theta$ sec, one finds $\tau\approx 10^{17}(\mathrm{keV}/m)^4$ yr, which is much longer than the age of the Universe, so they would certainly not have decayed at $t(\mathrm{CMBR})$. Seemingly a sterile neutrino with mass of few MeV would have the right decay time, but this is excluded by standard BBN considerations [34,35]. Let us emphasize that even though the simplest models allow to directly combine BBN and CMBR results, nevertheless one may consider more exotic scenarios [36,18], where $\Omega_b h^2$ changes between BBN and CMBR epochs, or quintessence, which would result in a change of N_ν between BBN and CMBR [37].

The paper is organized as follows: Section II is devoted to a brief review of the data used in our analysis, which is contained in Sec. III. Finally in Sec. IV we give our conclusions.

II. BBN AND CMBR DATA

A faithful estimate of primordial deuterium is provided by Ly-α features in several quasar absorption systems (QAS) at high red-shift ($z\geqslant 2$). The most recent analysis of a four QAS sample gives $D/H=(3.0\pm 0.4)\times 10^{-5}$ [38]. A new measurement has been also presented from observations of the Q-2206-199 QAS, at red-shift $z\simeq 2$, which gives $D/H=(1.65\pm 0.35)\times 10^{-5}$ [39]. We will comment on this result in our final discussion. As reference value we will use however the result quoted in [38].

For the ^{4}He mass fraction, Y_P, the key results come from the study of HII regions in blue compact galaxies. The most complete and homogeneous sample has been analyzed in Ref. [40], giving the value $Y_P=0.244\pm 0.002$. A recent study, however, has pointed out the presence of possible systematic errors in inferring the total ^{4}He abundance due to both imperfect ionization and nonuniform temperature distribution [41], leading to a typical overestimation of (2–4)% of Y_P. This issue of course deserves a deeper study to understand if uncertainties in ^{4}He measurements are actually dominated by systematic effects. Notice that in the extreme case, a value as low as $Y_P=0.234$ may represent a new problem for the very consistency of BBN scenario, in view of the low D result of [38]. In what follows we will use with caution the result of Ref. [40] quoted above.

The estimate of ^{7}Li primordial abundance using the Spite plateau can be spoiled by four possible systematic effects [23]: (a) galactic chemical evolution (GCE), which is poorly known; (b) corrections for possible depletion of initial star surface abundance; (c) the very method of how ^{7}Li is obtained from the Spite plateau; (d) presence of anomalous stars in the samples. In particular the effect due to GCE was long assumed to be negligible for metal poor stars in view of its apparent uniformity, but this has recently been questioned due to observation of some amount of Be. Furthermore, data shows a statistically significant increase with Fe/H, as shown in [23], leading to a primordial lithium abundance $^7\mathrm{Li/H}=(1.23^{+0.68}_{-0.32})\times 10^{-10}$. Evidence for this effect was instead missing in a previous analysis [42], where it was found $^7\mathrm{Li/H}=(1.73\pm 0.21)\times 10^{-10}$. The effects (b) and (c) have also recently been studied in [24], where it is pointed out that the Spite plateau can be well reproduced by models with a strong diffusion effect, and would be a factor two lower than the primordial abundance.

For these reasons, at present it is not appropriate to include ^{7}Li in a likelihood analysis of BBN. As in [21], we will rather estimate from BBN prediction the depletion factor $f_7={}^7\mathrm{Li}_{obs}/{}^7\mathrm{Li}_{prim}$, using as a reference result the one quoted in [23].

The anisotropy power spectrum from BOOMERanG experiment was estimated in 19 bins between $l=75$ and $l=1025$. Since the correlation matrix still is not public available, we will assume these data points to be independent. We have checked that the inclusion of a 10% correlation between the error bars does not affect our results. The data provide evidence for the presence of 3 peaks at $l\sim 210^{+5}_{-9}$, 550^{+8}_{-12}, 840^{+6}_{-13}, with an amplitude of $\sim$72, 49 and 45 μK respectively [43]. We include a calibration uncertainty of 25% in ΔT^2 and a Gaussian uncertainty of 1.4′ in the beam. Furthermore, since the signal at very high multipoles ($l\geqslant 850$) could be severely affected by the presence of systematic effects, we apply a jackknife test repeating the analysis without these data points, finding no significant changes in our results. For the DASI data we include the window functions and correlation matrices available on the corresponding web site [44]. We also include a 8% calibration error. There is an $\sim$20% overlap of the two regions of the sky covered by the two experiments but we do not take this effect into account in our analysis. In fact we believe that this correlation should not affect our conclusions, since our result appears stable when removing the DASI data points. We sample the parameter space of the theoretical models in a similar way as in [7], with $0.003<\Omega_b h^2<0.05$, $0.03<\Omega_c h^2<0.8$, $0<\Omega_\Lambda<1.0$, $0.5<n_S<1.5$, $0<\tau_c<0.4$, but we restrict our analysis to a *flat* universe, and we add a top-hat prior to the Hubble parameter, $h=0.65\pm0.2$.

III. LIKELIHOOD ANALYSIS

The likelihood analysis of the BBN data has been performed using the method already described in details in [16]. To constrain the values of the parameter set $(N_\nu,\Omega_b h^2)$, for SBBN, and $(\xi_e,N_\nu,\Omega_b h^2)$, for the degenerate scenario, from the data on ^{4}He and D, we define the likelihood function $\mathcal{L}_{BBN}=L_D L_{^4He}$, where each likelihood function, assuming Gaussian distribution for the errors, is given by the overlap of a theoretical and experimental distribution,

$$L_i=\frac{1}{2\pi\sigma_i^{th}\sigma_i^{ex}}\int dX\,\exp\left\{-\frac{(X-Y_i^{th})^2}{2\,\sigma_i^{th2}}\right\}\times\exp\left\{-\frac{(X-Y_i^{ex})^2}{2\,\sigma_i^{ex\,2}}\right\}. \tag{1}$$

The Y_i^{ex} and σ_i^{ex} are the experimental results and $1-\sigma$ errors for the i-th nuclide, Y_i^{th} the theoretical predictions obtained by an updated BBN code developed over the past few years [45,16]. Finally, the theoretical σ_i^{th} can be found by linear propagation of the uncertainties of the various nuclear rates entering in the nucleosynthesis reaction network [22].

For the BOOMERanG and DASI experiments we approximated the likelihood function of the CMBR signal inside the bins, C_B, as a Gaussian variable. The likelihood for a given cosmological model is then defined by $-2\ln\mathcal{L}_{CMBR}=(C_B^{th}-C_B^{ex})M_{BB'}(C_{B'}^{th}-C_{B'}^{ex})$, where C_B^{th} is the theoretical signal.

As can be seen from Fig. 1, the dotted line, which represents the 95% C.L. contour of SBBN, is in very good agreement with new CMBR data, and the $\Omega_b h^2$ tension between primordial nucleosynthesis and CMBR anisotropy seems to be completely solved. The constraint on $\Omega_b h^2(N_\nu)$ can be obtained by marginalizing the total likelihood function $\mathcal{L}=\mathcal{L}_{SBBN}\cdot\mathcal{L}_{CMBR}$ with respect to $N_\nu(\Omega_b h^2)$. By this procedure we get the two estimates $\Omega_b h^2=0.019\pm0.003$ and $N_\nu=2.8\pm0.4$, both at 95%.

The result on N_ν beautifully suggests the simplest scenario of three light active neutrinos. It is therefore perfectly meaningful to fix from the very beginning $N_\nu=3.034$ [46–49] (see also [50] for a recent dicussion), which leads to the same interval for $\Omega_b h^2$. In particular, for $\Omega_b h^2=0.019$ the nuclei abundances evaluate to $D/H=3.26\times10^{-5}$, $Y_P=0.2471$ and $^7Li/H=3.31\times10^{-10}$.

Although SBBN seems to be in very good agreement with the new CMBR data, providing a very strong constraint on the neutrino degrees of freedom, it relies on the theoretical assumption that background neutrinos have negligible chemical potential, as their charged lepton partners. Even though this hypothesis is perfectly justified by Occam razor, models have been proposed in the literature [19,51–57] where large neutrino chemical potentials can be generated. It is therefore an interesting issue for cosmology, as well as for our understanding of fundamental interactions, to try to constraint the neutrino–antineutrino asymmetry from cosmological observables. It is well known that degenerate BBN gives severe constraints on the electron neutrino chemical potential, $-0.06\leqslant\xi_e\leqslant1.1$, and weaker bounds on the ones of both μ and τ neutrino, $|\xi_{\mu,\tau}|\leqslant5.6$–$6.9$ [28]. This occurs since electron neutrinos are directly involved in neutron to proton conversion processes which eventually fix the total amount of ^{4}He produced in nucleosynthesis, while $\xi_{\mu,\tau}$ only enters via their contribution to the expansion rate of the universe. Combining this scenario with the bound on baryonic and radiation densities allowed by CMBR data, it is possible to obtain rather stronger constraints on all these parameters. Such an analysis was previously performed in [14,58] using BOOMERanG and MAXIMA data of Refs. [1,3]. We recall that neutrino chemical potentials contribute to the total neutrino effective degrees of freedom N_ν as

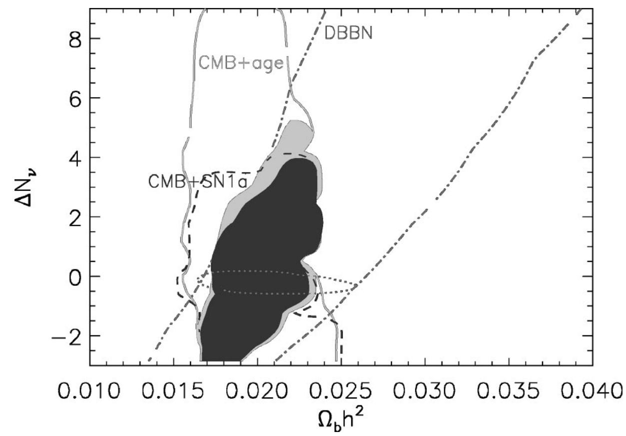

FIG. 1. The 95% C.L. contours for degenerate BBN (dot-dashed line), new CMBR results only with age prior, $t>11$ Gyr (full line), and only with SNIa prior (dashed line) are shown. The combined analyses correspond to filled areas: DBBN + CMBR + age (light region), DBBN + CMBR + SNIa (dark region). The dotted line is the 95% C.L. contour of SBBN.

$$N_\nu=3+\Sigma_\alpha\left[\frac{30}{7}\left(\frac{\xi_\alpha}{\pi}\right)^2+\frac{15}{7}\left(\frac{\xi_\alpha}{\pi}\right)^4\right]+\delta_\nu, \tag{2}$$

with δ_ν the contribution of relativistic degrees of freedom other than neutrinos and photons. Notice that, in order to get the most stringent bound on ξ_α we have to assume that all relativistic degrees of freedom, other than photons, are given by three active (possibly) degenerate massless neutrinos, i.e. $\delta_\nu=0$. Similarly the upper limit on δ_ν can be obtained from the results of our analysis in the case $\xi_{\mu,\tau}=0$. We stress that in any case a value for N_ν noticeably different than three

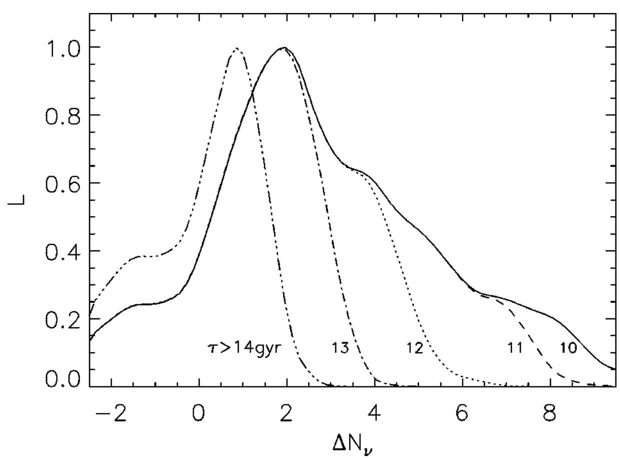

FIG. 2. CMBR likelihoods for age priors of $\tau > 10,11,\ldots,14$ Gyr.

does require a nonvanishing chemical potential for electron neutrinos, or more generally a nonthermal spectrum.

Figure 1 summarizes our main results for the DBBN scenario. Defining $\Delta N_\nu = N_\nu - 3$, we plot in the plane $(\Delta N_\nu, \Omega_b h^2)$ the 95% C.L. contour allowed by DBBN (dot-dashed line), together with the analogous 95% C.L. region coming from the CMBR data analysis, with only weak age prior, $\tau > 11$ Gyr (full line). Finally, the light filled region is the 95% C.L. region of the joint product distribution $\mathcal{L} \equiv \mathcal{L}_{DBBN} \cdot \mathcal{L}_{CMBR}$. The main new feature, with respect to the results of Ref. [14] is that the resolution of the third peak shifts the CMBR likelihood contour towards smaller values for $\Omega_b h^2$, so, when combined with DBBN results, it singles out smaller values for N_ν. In fact from our analysis we get the bound $N_\nu \leqslant 8$, at 95% C.L., which translates into the new bounds $-0.01 \leqslant \xi_e \leqslant 0.25$, and, for $\delta_\nu = 0$, $|\xi_{\mu,\tau}| \leqslant 2.9$, noticeably more stringent than what can be found from DBBN alone.

A similar analysis can be also performed combining CMBR and DBBN data with the constraints on the $\Omega_m - \Omega_\Lambda$ plane from the Supernova Ia data [59], which strongly reduce the degeneracy between Ω_m and Ω_{rad}. At 95% C.L. we find (dark filled region in Fig. 1) $N_\nu \leqslant 7$, corresponding to $-0.01 \leqslant \xi_e \leqslant 0.22$ and $|\xi_{\mu,\tau}| \leqslant 2.6$, for $\delta_\nu = 0$. In the other extreme scenario, where basically all extra contributions to Hubble parameter are given by extra relativistic species, we get $\delta_\nu \leqslant 4$.

Another possibility to break the degeneracy between Ω_m and Ω_Λ, is to put priors on the age of the universe τ, as pointed out in Ref. [58]. In Fig. 2 we show the normalized likelihood functions for age priors of $\tau > 10,11,\ldots,14$ Gyr, using only CMBR data. It is clear that one needs the slightly unrealistic prior of $\tau > 13$ Gyr to get bounds stronger than $\Delta N \leqslant 4$, as obtained by the inclusion of SNIa data.

Recently Ref. [60] stressed the point that the inclusion of large scale structure data can provide a lower bound on ΔN_ν. In Ref. [60] SNIa data are not considered, neither a DBBN scenario; however, where comparison is possible, the results on upper bounds are in fair agreement with our results.[1] It is worth noticing once again that our rather stringent bound on N_ν is the outcome of the combined analysis of CMBR *and* DBBN. Each of the two corresponding likelihood contours in fact, taken separately, give a much weaker bound (see Fig. 1).

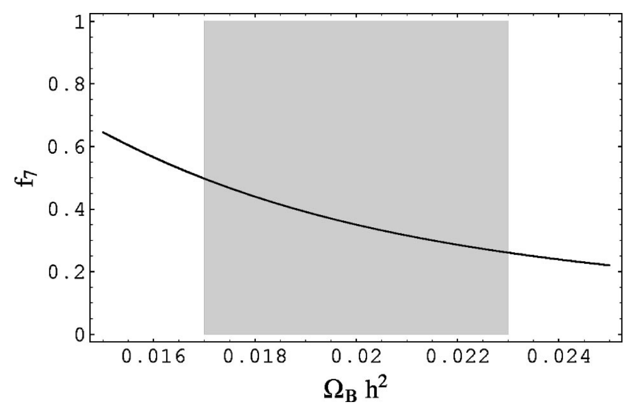

FIG. 3. The ^{7}Li depletion factor, defined as the ratio between the experimental and theoretical values.

As we mentioned in Sec. II, it has recently been stressed that depletion effects on ^{7}Li may be efficient in reducing the primordial abundance down to the value observed in the Spite plateau. In Fig. 3 we plot the ^{7}Li depletion factor f_7, defined as the ratio of the experimental value of Ref. [23] and the theoretical estimate from our BBN code. Values for f_7 of the order of 1/2–1/3 cannot be ascribed to a statistical fluctuation in the star sample considered in [23], but should rather be understood by a careful analysis of all systematic effects which we briefly reviewed in Sec. II.

There are some points we would like to address as final remarks. First of all we stress once again that further data on the third peak in the CMBR anisotropy spectrum are needed to check for possible systematics. This is a crucial point for a clean determination of the baryonic fraction, since discrimination between SBBN and DBBN, or SBBN and other theoretical framework for light nuclei production, relies on both second *and* third peak heights. In this respect we note the good agreement between the BOOMERanG and DASI results.

As a second observation, we recall that we already pointed out that a new measurement of primordial D has been reported recently, leading to a weighted average $D/H = (2.2 \pm 0.2) \times 10^{-5}$ [39]. Notice however that this new result is rather incompatible with the previous estimations, suggesting that a naive average of all results may be meaningless. This fact motivates our choice of using the data reported in Ref. [38]. The data spread may be a signal of unclear systematics in Deuterium abundance and further studies are needed in order to clarify this issue.

[1] A study of the effects of large neutrino asymmetries on CMBR + large scale structures has also been performed in Ref. [61], where the compatibility with DBBN for $m_\nu \lesssim 1$ eV has been analyzed. Their results refer to a critical universe with no cosmological constant.

IV. CONCLUSIONS

It is a great success of cosmology and astrophysical observations that severe constraints can be put on the number of neutrino degrees of freedom and, more generally, of light particle species which were relativistic at the epoch of recombination. Of course this is a fundamental piece of information for the whole microscopic theory of fundamental interactions. The increasing precision in measurements of primordial abundances of light nuclei, and the impressive progress in measuring the CMBR anisotropy, are conspiring to give us a very precise determination of N_ν. Despite of the conservative expectation of three, light, active neutrinos, largely nondegenerate, it should be stressed that many other scenarios have been considered in the literature, based on theoretical ideas which, going beyond the standard model, try to grasp possible extension of our knowledge of fundamental interactions at higher energy. It is really exciting that, along with customary accelerator physics, we have at hand a severe way to scrutinize these models by cosmological measurements.

In this paper we have studied in details the implications of the new BOOMERanG and DASI data on CMBR anisotropy for the estimation of the baryonic energy density fraction, compared with the predictions of standard BBN, in the parameter space $\Omega_b h^2 - N_\nu$. Observation of the third peak at multipole $l \sim 850$ turned into a sensible improvement of the compatibility of the two independent ways of constraining Ω_b, and single out the values $\Omega_b h^2 = 0.019 \pm 0.003$, and $N_\nu = 2.8 \pm 0.4$, both at 2σ.

We have also considered the scenario of a degenerate neutrino background, which strongly affects primordial nuclei production. The new CMBR BOOMERanG and DASI data lead to a new and stronger constraint on the effective relativistic degrees of freedom, $N_\nu \leqslant 8$ (only weak age prior), or $N_\nu \leqslant 7$ (with only SNIa prior), both at 95% C.L., which bounds more severely the neutrino chemical potentials, $|\xi_{\mu,\tau}| \leqslant 2.9$, and $-0.01 \leqslant \xi_e \leqslant 0.25$, and $|\xi_{\mu,\tau}| \leqslant 2.6$, and $-0.01 \leqslant \xi_e \leqslant 0.22$, respectively.

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