Study of semileptonic decays $B^{\pm}\to\eta^{(\prime)}l\nu$

C. S. Kim*
Department of Physics and IPAP, Yonsei University, Seoul 120-749, Korea

Ya-Dong Yang†
Department of Physics, Technion-Israel Institute of Technology, Haifa 32000, Israel

(Received 23 July 2001; published 11 December 2001)

We study the semileptonic decays $B^{\pm}\to\eta^{(\prime)}l\nu$, which are suggested to be used to extract the hadronic form factors of B meson decays to $\eta(\eta')$ and the angle of η-η' mixing. This would be of great benefit to theoretical studies of B nonleptonic decays involving η and η', and could lead to a reliable and complementary determination of V_{ub}. The branching ratios are estimated to be $\mathcal{B}(B^{\pm}\to\eta^{(\prime)}l\nu)=4.32\pm0.83$ (2.10 ± 0.40) $\times10^{-5}$, which could be extensively studied experimentally at BaBar and Belle.

DOI: 10.1103/PhysRevD.65.017501 PACS number(s): 13.20.He, 12.15.Hh

Semileptonic B decays are a subject of considerable interests that has been extensively studied with applications of various nonperturbative theoretical frameworks. They offer the most direct method to determine the weak mixing angles and to probe the strong interaction confinement phenomenology of hadronic transitions. Recently, V_{cb} has been determined from semileptonic B decays and has become the third most accurately measured Cabibbo-Kobayashi-Maskawa (CKM) matrix element [1]. The CLEO Collaboration [2] has made measurements of the decays $B^0\to\pi^-l^+\nu$ and $\rho^-l^+\nu$ with the results

$$\mathcal{B}(B^0\to\pi^-l^+\nu)=(1.8\pm0.4\pm0.3\pm0.2)\times10^{-4},$$

$$\mathcal{B}(B^0\to\rho^-l^+\nu)=(2.57\pm0.29^{+0.33}_{-0.46}\pm0.41)\times10^{-4},$$

and

$$|V_{ub}|=(3.25\pm0.14^{+0.21}_{-0.29}\pm0.55)\times10^{-3}.$$

It is known that extracting $|V_{ub}|$ from the measured decay rates requires significant input from theoretical estimations of the hadronic form factors which involve complex strong interaction dynamics. With BaBar and Belle taking data, we are entering a new era of B physics. Prospects for an accurate measurement of these decay modes become excellent. We can foresee that the decays $B^{\pm}\to\eta l\nu$ and $\eta' l\nu$ could be also observed at B factories in the near future. In this Brief Report, we study the decays $B^{\pm}\to\eta l\nu$ and $\eta' l\nu$ to show that many interesting physical observables can be extracted from measurements of these decays.

Amplitudes of exclusive semileptonic $B\to Pl\nu$ ($l=\mu$, e and $P=\pi$, η, η') can be written as

$$\mathcal{M}(B\to Pl\nu)=\frac{G_F}{\sqrt{2}}V_{ub}\overline{l}\gamma_\mu(1-\gamma_5)\nu \times\langle P(p_P)|\overline{u}\gamma^\mu(1-\gamma_5)b|B(p_B)\rangle, \quad (1)$$

where the hadronic transition matrix can be parametrized as

$$\langle P(p_P)|\overline{u}\gamma_\mu(1-\gamma_5)b|B(p_B)\rangle =F_+^{B\to P}(q^2)(p_B+p_P)_\mu+F_-^{B\to P}(q^2)(p_B-p_P)_\mu. \quad (2)$$

Here, $q=p_B-p_P$, and $F_{+(-)}^{B\to P}(q^2)$ are the relevant form factors. Using these notations, the double differential decay width is

$$\frac{d\Gamma(B\to Pl\nu)}{dE_l dq^2}=G_F^2|V_{ub}|^2\frac{1}{16\pi^3M_B}|F_+^{B\to P}(q^2)|^2 \times[2E_l(m_B^2+q^2-m_P^2)-m_B(4E_l^2+q^2)], \quad (3)$$

where we have neglected the lepton mass.

To calculate the semileptonic decay width, we have to know precisely the form factors $F_+^{B\to P}(q^2)$, which challenge our poor knowledge of nonperturbative QCD. In recent years, considerable progress has been made in the calculations of $F_+^{B\to\pi}(q^2)$ with various theoretical approaches: quark models [3], QCD sum rules [4,5], and lattice QCD [6,7]. Combining the results of different approaches, say, predictions of QCD sum rules in the low q^2 region and of lattice QCD in the high q^2 region, we could possibly obtain a good theoretical description of $F_+^{B\to\pi}(q^2)$ in the whole q^2 region. However, both QCD sum-rule and lattice calculations of the form factors $F_+^{B\to\eta^{(\prime)}}(q^2)$ are not yet available in the literature. Therefore, we will use SU(3)$_F$ symmetry to relate them to $F_+^{B\to\pi}(q^2)$. For η-η' mixing, we adopt the scheme [8,9,10]

$$|\eta\rangle=\cos\phi|\eta_q\rangle-\sin\phi|\eta_s\rangle,$$

$$|\eta'\rangle=\sin\phi|\eta_q\rangle+\cos\phi|\eta_s\rangle, \quad (4)$$

where $|\eta_q\rangle=(u\overline{u}+d\overline{d})/\sqrt{2}$, $|\eta_s\rangle=s\overline{s}$, and $\phi=39.3°$ is the fitted mixing angle [9]. Assuming SU(3)$_F$ symmetry, the form factors $F_+^{B\to\eta^{(\prime)}}(q^2)$ are related to $F_+^{B\to\pi}(q^2)$ by the relations

*Email address: cskim@mail.yonsei.ac.kr
†Email address: yang@physics.technion.ac.il

$$F_+^{B\to\eta}(q^2)=\cos\phi F_+^{B\to\pi}(q^2),$$
$$F_+^{B\to\eta'}(q^2)=\sin\phi F_+^{B\to\pi}(q^2). \tag{5}$$

The form factor $F_+^{B\to\pi}(q^2)$ is known to be dominated by B^* pole in the small-recoil region $q^2\sim(m_B-m_\pi)^2$ and to scale as $F_+^{B\to\pi}(q^2\simeq m_B^2)\sim\sqrt{m_B}$ in the heavy quark limit [11]. Recent studies [7,12,13] imply the dipole behavior for $F_+(q^2)$ in the large-recoil region $q^2\sim 0$. The easiest way to extrapolate the q^2 dependence is to suppose the dipole behavior for $F_+(q^2)$ [6,14]:

$$F_+^{B\to\pi}(q^2)=\frac{F_+^{B\to\pi}(0)}{(1-q^2/m_{B^*}^2)^2}\quad(\text{dipole}), \tag{6}$$

where m_{B^*} is the pole mass of $B^*(1^-)$ associated with the weak current induced by the decay.

Becirevic and Kaidalov (BK) [15] have also proposed a numerical parametrization which satisfies the heavy quark scaling laws [11] and most of the known constraints [15],

$$F_+^{B\to\pi}(q^2)=\frac{c_B(1-\alpha_{B\pi})}{(1-q^2/m_{B^*}^2)(1-\alpha_{B\pi}q^2/m_{B^*}^2)}. \tag{7}$$

We can read from here $F_+^{B\to\pi}(0)=c_B(1-\alpha_{B\pi})$. Using BK parametrization to fit their light-cone QCD sum rule (LCSR) calculations, Khodjamirian *et al.* found $\alpha_{B\pi}=0.32^{+0.21}_{-0.07}$ and $F_+^{B\to\pi}(0)=0.28\pm0.05$ [4]. The recent results from lattice QCD are [16]

$$\alpha_{B\pi}=0.40\pm0.15,\quad F_+^{B\to\pi}(0)=0.26\pm0.05\quad(\text{lattice I}),$$
$$\alpha_{B\pi}=0.45\pm0.17,\quad F_+^{B\to\pi}(0)=0.28\pm0.06\quad(\text{lattice II}), \tag{8}$$

where the two sets of results (lattices I and II) correspond to two different methods used in Ref. [16].

To eliminate the effect of large uncertainty in V_{ub}, we relate the branching ratios $B\to\eta^{(\prime)}l\nu$ to $\mathcal{B}(B^-\to\pi^0 l\nu)$, and get

$$\mathcal{R}_1=\frac{\mathcal{B}(B^-\to\eta l\nu)}{\mathcal{B}(B^-\to\pi^0 l\nu)}=|\cos\phi|^2\frac{\int_0^{(m_B-m_\eta)^2}dq^2|F_+^{B\to\pi}(q^2)|^2[(m_B^2+m_\eta^2-q^2)^2-4m_B^2m_\eta^2]^{3/2}}{\int_0^{(m_B-m_\pi)^2}dq^2|F_+^{B\to\pi}(q^2)|^2[(m_B^2+m_\pi^2-q^2)^2-4m_B^2m_\pi^2]^{3/2}}$$
$$=|\cos\phi|^2\times\begin{cases}0.527 & (\text{dipole}),\\ 0.813 & (\text{LCSR}),\\ 0.802 & (\text{lattice I}),\\ 0.794 & (\text{lattice II}),\end{cases} \tag{9}$$

$$\mathcal{R}_2=\frac{\mathcal{B}(B^-\to\eta' l\nu)}{\mathcal{B}(B^-\to\pi^0 l\nu)}=|\sin\phi|^2\frac{\int_0^{(m_B-m_{\eta'})^2}dq^2|F_+^{B\to\pi}(q^2)|^2[(m_B^2+m_{\eta'}^2-q^2)^2-4m_B^2m_{\eta'}^2]^{3/2}}{\int_0^{(m_B-m_\pi)^2}dq^2|F_+^{B\to\pi}(q^2)|^2[(m_B^2+m_\pi^2-q^2)^2-4m_B^2m_\pi^2]^{3/2}}$$
$$=|\sin\phi|^2\times\begin{cases}0.310 & (\text{dipole}),\\ 0.599 & (\text{LCSR}),\\ 0.584 & (\text{lattice I}),\\ 0.573 & (\text{lattice II}).\end{cases} \tag{10}$$

Using the CLEO result [2], $\mathcal{B}(B^0\to\pi^+l\nu)=(1.8\pm0.6)\times10^{-4}$, and the relations $\mathcal{B}(B^0\to\pi^+l\nu)=2\mathcal{B}(B^-\to\pi^0l\nu)$, and $\phi=39.3°$, we get

$$\mathcal{B}(B^-\to\eta l\nu)=\begin{cases}(2.84\pm0.95)\times10^{-5} & (\text{dipole}),\\ (4.38\pm1.46)\times10^{-5} & (\text{LCSR}),\\ (4.32\pm1.44)\times10^{-5} & (\text{lattice I}),\\ (4.28\pm1.42)\times10^{-5} & (\text{lattice II}),\end{cases} \tag{11}$$

$$\mathcal{B}(B^-\to\eta' l\nu)=\begin{cases}(1.12\pm0.37)\times10^{-5} & (\text{dipole}),\\ (2.16\pm0.72)\times10^{-5} & (\text{LCSR}),\\ (2.10\pm0.70)\times10^{-5} & (\text{lattice I}),\\ (2.06\pm0.68)\times10^{-5} & (\text{lattice II}).\end{cases} \tag{12}$$

We can see that the predictions of LCSR form factors [4] agree very well with those of lattices I and II QCD [16]. Averaging predictions from lattice QCD and LCSR we obtain

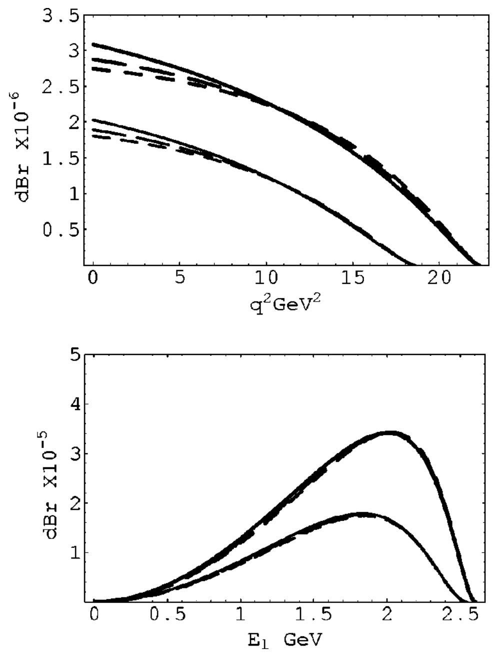

FIG. 1. The spectra $d\mathcal{B}/dq^2$ as function of q^2 and the spectra $d\mathcal{B}/dE_l$ as a function of the electron energy E_l. The thick-solid, long-dashed, and short-dashed curves are the distributions of $d\mathcal{B}(B\rightarrow\eta l v)$ with LCSR [4] and lattices (I and II) QCD [16] form factors, and the thin curves are those for $d\mathcal{B}(B\rightarrow\eta' l v)$.

$$\mathcal{B}(B^-\rightarrow\eta l\nu)=(4.32\pm0.83)\times10^{-5},$$
$$\mathcal{B}(B^-\rightarrow\eta' l\nu)=(2.10\pm0.40)\times10^{-5}. \tag{13}$$

We note that the ratios $\mathcal{R}_1$ and $\mathcal{R}_2$ are independent of the value of $F_+^{B\rightarrow\pi}(0)$, but very sensitive to the details of its q^2 dependence. We also note that to give the same numerical predictions for $B\rightarrow\pi l\nu$, the $F_+^{B\rightarrow\pi}(0)$ for dipole parametrization should be smaller than that for BK parametrization. If the same value $F_+^{B\rightarrow\pi}(0)$ is used in both BK and dipole parametrizations, one will find

$$\mathcal{R}_3=\frac{\mathcal{B}^{\text{dipole}}(B^-\rightarrow\pi^0 l\nu)}{\mathcal{B}^{\text{LCSR}}(B^-\rightarrow\pi^0 l\nu)}=3.13, \tag{14}$$

which implies that the dipole form factor will overestimate the decay rates because π meson is very light and the lepton pair invariant mass q^2 can be very near the B_u^* pole. Therefore, theoretical predictions for $B\rightarrow\pi l\nu$ (and $B\rightarrow\eta^{(\prime)}l\nu$ in turn) are very sensitive to the q^2 dependence of $F_+^{B\rightarrow\pi}(q^2)$. It is well known that the extraction of V_{ub} from decay rates of $B\rightarrow\pi(\rho)l\nu$ suffers from large theoretical uncertainties in the hadronic form factors. Testing the predictions and eventual measurements of $d\Gamma/dq^2$ can provide valuable information on the hadronic form factors governing $b\rightarrow ul\nu$ decays, and hence lead to a reliable determination of V_{ub}. With much more data to arrive soon from B factories, the q^2 and the lepton energy distributions can be precisely measured and be used to distinguish these form factor parametrizations, and to extract V_{ub}. The determination of V_{ub} from $B\rightarrow\eta^{(\prime)}l\nu$ would represent a powerful method complementary to the determination of V_{ub} from other exclusive decay modes, e.g., from $B\rightarrow\pi(\rho)l\nu$.

In Fig. 1, we plot the q^2 distributions and the lepton energy E_l distributions of the decays $B^-\rightarrow\eta^{(\prime)}l\nu$, where we have normalized the form factors to give $\mathcal{B}(B^-\rightarrow\pi^0 l\nu)=9\times10^{-5}$. We find that both LCSR and lattice QCD predict very consistent lepton energy distributions as well as consistent decay rates for the decays. Integrating out the lepton energy in Eq. (3), one obtains

$$\frac{d\Gamma(B\rightarrow Pl\nu)}{dq^2}=\frac{G_F^2|V_{ub}|^2}{192\pi^3M_B^3}|F_+^{B\rightarrow P}(q^2)|^2\times[(m_B^2+m_P^2-q^2)^2-4m_B^2m_P^2]^{3/2}. \tag{15}$$

At maximum recoil point ($q^2=0$), we have

$$\mathcal{R}_4=\left.\frac{d\Gamma(B\rightarrow\eta^{(\prime)}l\nu)/dq^2}{d\Gamma(B^-\rightarrow\pi^0 l\nu)/dq^2}\right|_{q^2=0}=\frac{(m_B^2-m_{\eta^{(\prime)}}^2)^3}{(m_B^2-m_\pi^2)^3}\left|\frac{F_+^{B\rightarrow\eta^{(\prime)}}(0)}{F_+^{B\rightarrow\pi^0}(0)}\right|^2, \tag{16}$$

$$\mathcal{R}_5=\left.\frac{d\Gamma(B\rightarrow\eta' l\nu)/dq^2}{d\Gamma(B^-\rightarrow\eta l\nu)/dq^2}\right|_{q^2=0}=\frac{(m_B^2-m_{\eta'}^2)^3}{(m_B^2-m_\eta^2)^3}\left|\frac{F_+^{B\rightarrow\eta^{(\prime)}}(0)}{F_+^{B\rightarrow\eta}(0)}\right|^2=\frac{(m_B^2-m_{\eta'}^2)^3}{(m_B^2-m_\eta^2)^3}|\cot\phi|^2. \tag{17}$$

As indicated by QCD sum-rule calculations [4,5], the value of $F_+^{B\rightarrow\eta^{(\prime)}}(q^2)$ is rather stable under the variation of q^2 when the value of q^2 is small. So the ratios $\mathcal{R}_4$ and $\mathcal{R}_5$ can be safely extrapolated to a few GeV^2 to make the phase spaces sizable. Once the ratios are measured, they can be used to extract the form factor $F_+^{B\rightarrow\eta^{(\prime)}}(0)$ and the mixing angle ϕ from the above relations.

In the literature, semileptonic decays $D_s\rightarrow\eta(\eta')l\nu$ have been taken as sources of extracting η-η' mixing angle and testing the mixing schemes [9,17]. We note that the decays $D_s\rightarrow\eta(\eta')l\nu$ involve strange contents $|\eta_s\rangle$ of $\eta(\eta')$, and $B^-\rightarrow\eta(\eta')l\nu$ involve nonstrange contents $|\eta_q\rangle$ of $\eta(\eta')$, therefore $B^-\rightarrow\eta(\eta')l\nu$ and $D_s\rightarrow\eta(\eta')l\nu$ could provide combined testing of a η-η' mixing scheme.

As it is well known that η and η' are too complicated objects to be reliably described within QCD yet, it may be very hard to calculate the transition form factors $F_+^{B\rightarrow\eta^{(\prime)}}(q^2)$ within the frameworks of lattice QCD and QCD sum rules. The experimental extraction of those form factors will improve our theoretical understanding of many interesting nonleptonic B decay modes involving η and η', and might shed light on the problem, currently under discussion [18], of the puzzling large branching ratios of $B \to K\eta'$ observed by CLEO [19].

Finally, we note a few experimental comments: The background for $B \to \eta^{(\prime)} l \nu$ would be much smaller than that of $B \to \pi l \nu$, due to much lower multiplicity, since the random background caused by $B \to \eta X$ is about an order of magnitude smaller than that by $B \to \pi X$. The reconstruction of $\eta \to \gamma\gamma$ in experimental analyses may be much easier than $\pi^0 \to \gamma\gamma$, even though the signal/noise ratio is worse, because the mass of η is much bigger than that of π^0. And we could even require the momentum of η to be larger than 1 GeV to remove combinatorial backgrounds substantially.

To conclude, we studied semileptonic decays $B^{\pm} \to \eta(\eta') l^{\pm} \nu$, which can be used to extract the hadronic form factors of B meson decays to $\eta(\eta')$ and the angle of η-η' mixing. The branching ratios are found to be $\mathcal{B}(B^- \to \eta(\eta') l \nu) = 4.32 \pm 0.83 (2.10 \pm 0.40) \times 10^{-5}$.

The authors thank G. Cvetic, Hongjoo Kim, and Y. J. Kwon for a careful reading of the manuscript and their valuable comments. The work of C.S.K. was supported by Grant No. 2000-015-DP0077 of the KRF. Y.Y. is supported by the U.S.–Israel Binational Science Foundation and the Israel Science Foundation.

[1] Partical Data Group, D. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
[2] CLEO Collaboration, J. P. Alexander *et al.*, Phys. Rev. Lett. **77**, 5000 (1996); Phys. Rev. D **58**, 052004 (2000).
[3] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C **29**, 637 (1985); N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D **39**, 799 (1989); D. Scora and N. Isgur, *ibid.* **52**, 2783 (1995); I. L. Grach, I. M. Narodetskii, and S. Simula, Phys. Lett. B **385**, 317 (1996); M. Bayer and D. Melikhov, *ibid.* **436**, 344 (1998); P. Colangelo and P. Santorelli, *ibid.* **327**, 123 (1994); M. Ladisa, G. Nardulli, and P. Santorelli, *ibid.* **455**, 283 (1999); A. Deandrea, R. Gatto, G. Nardulli, and A. D. Polosa, Phys. Rev. D **61**, 017502 (2000).
[4] A. Khodjamirian, R. Rückl, S. Weinzierl, C. Winhart, and O. Yakovlev, Phys. Rev. D **62**, 114002 (2000).
[5] E. Bagan, P. Ball, and V. M. Braun, Phys. Lett. B **417**, 154 (1998); P. Ball, Phys. Rev. D **48**, 3190 (1993); J. High Energy Phys. **09**, 005 (1998).
[6] UKQCD Collaboration, K. C. Bowler *et al.*, Phys. Lett. B **486**, 111 (2000).
[7] A. Abada *et al.*, Nucl. Phys. **B416**, 675 (1994); C. R. Allton *et al.*, Phys. Lett. B **345**, 513 (1995).
[8] J. L. Rosner, Phys. Rev. D **27**, 1101 (1983).
[9] T. Feldmann, P. Kroll, and B. Stech, Phys. Rev. D **58**, 114006 (1998); Phys. Lett. B **449**, 339 (1999); T. Feldmann, Int. J. Mod. Phys. A **15**, 159 (2000), and references therein.
[10] A. Bramon, R. Escribano, and M. D. Scadron, Eur. Phys. J. C **7**, 271 (1998).
[11] N. Isgur and M. B. Wise, Phys. Rev. D **41**, 151 (1990); **42**, 2388 (1990).
[12] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Phys. Rev. D **60**, 014001 (1999).
[13] H. Y. Cheng, C. Y. Cheung, and C. W. Hwang, Phys. Rev. D **55**, 1559 (1997).
[14] R. Alexan *et al.*, Phys. Rev. D **51**, 6235 (1995); UKQCD Collaboration, D. R. Burford *et al.*, Nucl. Phys. **B447**, 425 (1995).
[15] D. Becirevic and A. Kaidalov, Phys. Lett. B **478**, 417 (2000).
[16] A. Abada, D. Becirevic, Ph. Boucaud, J. P. Leroy, V. Lubicz, and F. Mescia, hep-ph/0011065.
[17] D. Melikhov, Phys. Lett. B **394**, 385 (1997).
[18] Dong-sheng Du, C. S. Kim, and Ya-dong Yang, Phys. Lett. B **426**, 133 (1998); A. Ali, J. Chay, C. Greub, and P. Ko, *ibid.* **424**, 161 (1998); A. Datta, S. Pakvasa, and X. G. He, *ibid.* **419**, 369 (1998); A. S. Dighe, M. Gronau, and J. L. Rosner, Phys. Rev. Lett. **79**, 4333 (1997); Maozhi Yang and Ya-dong Yang, Nucl. Phys. **B609**, 469 (2001).
[19] CLEO Collaboration, B. H. Behrens *et al.*, Phys. Rev. Lett. **80**, 3710 (1998); S. J. Richichi *et al.*, *ibid.* **85**, 520 (2000).