

Solving the strong CP and the supersymmetry phase problems with parity symmetry

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We propose a simultaneous solution to the strong CP problem and the supersymmetry (SUSY) phase problem based on parity symmetry realized when the supersymmetric standard model is embedded into a left-right symmetric framework at a scale near 2×10^{16} GeV, as suggested by neutrino masses and gauge coupling unification. In this class of models, owing to parity, SUSY contributions to the muon anomalous magnetic moment can be naturally large without conflicting with the electric dipole moments (EDM's) of the electron and the neutron. The strong CP -violation parameter $\bar{\theta}$ is zero at the tree level, also due to parity (P), but is induced due to P -violating effects below the unification scale. We estimate the induced $\bar{\theta}$ to be $\leq 10^{-16}$, if we adopt a constrained supersymmetric spectrum with universal scalar masses. In the more general SUSY-breaking scenario, after imposing flavor changing constraints, we find $\bar{\theta} \sim (10^{-8} - 10^{-10})$, which is compatible with, but not much below, the present limit on the neutron EDM. We also argue that potential nonperturbative corrections to $\bar{\theta}$ from quantum gravitational effects are not excessive in these models.

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I. INTRODUCTION

One of the major problems of the standard model is a lack of understanding of the CP -violating parameter $\bar{\theta}$ characterizing the QCD sector of the Lagrangian [1]. This parameter originates from the periodic vacuum structure of QCD and leads to an electric dipole moment (EDM) of the neutron, $d_n \sim (5 \times 10^{-16}) \bar{\theta} e \text{ cm}$. The current experimental limit, $d_n \leq 6.3 \times 10^{-26} e \text{ cm}$ [2], then implies that $\bar{\theta} \leq 10^{-10}$. Why a fundamental parameter of the theory, $\bar{\theta}$, is so small compared to its natural value of order unity is the strong CP problem. A resolution of this problem is expected to provide important clues to the nature of new physics beyond the standard model.

When the standard model is embedded into its minimal supersymmetric extension (MSSM) in order to solve the quadratic divergence problem associated with the Higgs boson mass, one runs into another CP problem—the supersymmetry (SUSY) phase problem, or the SUSY CP problem. This problem owes its origin to the complex phases associated with the parameters in the soft SUSY-breaking sector of the theory—the μ term, gaugino masses, trilinear A terms, etc. Exchange of supersymmetric particles in loops will induce electric dipole moments (EDM's) for the neutron (d_n) and the electron (d_e) proportional to these SUSY phases. d_n induced through gluino and squark exchange can be estimated to be $d_n \approx (1 \times 10^{-23}) e \text{ cm}$ ($300 \text{ GeV}/M_{\text{SUSY}})^2 \sin \phi_{\text{SUSY}}$, where ϕ_{SUSY} is a typical SUSY CP phase parameter and M_{SUSY} is the gluino or squark mass. (For this estimate we used $\tan \beta = 5$ and set the μ parameter and the gluino mass equal to the squark mass.) In order to be

compatible with the experimental limit on d_n , the SUSY phase must obey $\phi_{\text{SUSY}} \leq (5 \times 10^{-3} - 5 \times 10^{-2})$ for $M_{\text{SUSY}} = 300 \text{ GeV} - 1 \text{ TeV}$, unless the gluino contribution is precisely canceled by some other diagrams [3]. Why ϕ_{SUSY} is so small, while the corresponding CP -violating phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix is of order one (in order to explain the observed CP violation in the K meson system), is the SUSY phase problem.

The SUSY phase problem becomes more acute if we attribute the recently reported 2.6 sigma discrepancy in the muon anomalous magnetic moment (a_μ) measurement [4] to supersymmetry. Exchange of charginos or neutralinos and sleptons can account for the observed discrepancy in a_μ , provided that the masses of these particles are not more than about 200–400 GeV. Now, if we replace the external muons in the diagram responsible for a_μ by electrons, a large EDM for the electron will result, if the relevant SUSY phases are of order one. For example, if the SUSY contribution to a_μ is 40×10^{-10} , d_e can be estimated by scaling of the lepton mass to be $d_e \approx (1.8 \times 10^{-24}) \sin \phi_{\text{SUSY}} e \text{ cm}$. The current experimental limit on d_e , viz., $d_e \leq 4.3 \times 10^{-27} e \text{ cm}$ [5] would require $\phi_{\text{SUSY}} \leq 2 \times 10^{-3}$. This bound will be tightened even further with the expected improvement in the limit on d_e by about a factor of 2 [6]. Solving the strong CP problem as well as the SUSY phase problem are therefore major challenges facing the (supersymmetric) standard model.

Simple solutions to the strong CP problem can be found by postulating new symmetries of nature, the most popular one being the Peccei-Quinn $U(1)_A$ symmetry [7]. Consistent implementation of this symmetry requires the existence of a new ultralight particle, the axion, which has eluded experimental searches so far. Combined laboratory, astrophysical,

and cosmological limits constrain the scale of Peccei-Quinn symmetry breaking to be in a narrow window, $f_a \sim (10^{11} - 10^{12})$ GeV [1]. On the theoretical side, global symmetries such as $U(1)_A$ have come under suspicion since it is believed that nonperturbative quantum gravitational effects will violate all global symmetries. If so, quantum gravity would destabilize the axion solution [8] unless one allows for extreme fine tuning of parameters—the very problem one sets out to avoid in postulating the new symmetry.¹ In any case, this solution has nothing to offer to the second CP problem, the SUSY phase problem.

An alternative to the axion solution to the strong CP problem is parity invariance [9] realized at a momentum scale v_R much above the weak scale.² If the standard model is embedded into a left-right symmetric gauge structure at v_R , parity (P) invariance can be consistently imposed on the Lagrangian. In this case, the θ term in the QCD Lagrangian, $g^2/(32\pi^2)\theta G\tilde{G}$, will be zero since it violates parity. The physical parameter $\bar{\theta}$ involves also the phase of the fermionic determinant, and is given by

$$\bar{\theta} = \theta + \arg\{\text{Det}(M_u M_d)\} - 3 \arg(M_{\tilde{g}}). \quad (1)$$

Here $M_{u,d}$ are the up-quark and the down-quark mass matrices and $M_{\tilde{g}}$ is the gluino mass. Owing to parity invariance, the matrices M_u and M_d become Hermitian, and the gluino mass becomes real. As a result, $\bar{\theta} = 0$ at tree level in this class of models. These models would thus have the potential to solve the strong CP problem. The fact that low energy weak interactions do not respect parity symmetry means that one must do additional work to see if $\bar{\theta}$ induced through quantum effects is sufficiently small. As we shall explicitly demonstrate in this paper, this is often the case.

The purpose of this note is to provide a simultaneous solution to the strong CP problem and the SUSY phase problem using parity symmetry. We shall demonstrate by explicit calculation that the induced $\bar{\theta}$ in these models is well within the experimental limit, if a constrained supersymmetric spectrum is adopted with universal squark masses and proportional A terms. Even in the more general scenario for supersymmetry breaking, we shall see that $\bar{\theta}$ is in the acceptable range of $\sim 10^{-10} - 10^{-8}$, after imposing flavor changing constraints. Parity invariance also makes the phases of the SUSY-breaking parameters naturally small. These phase parameters are zero at the scale v_R and their induced values at the weak scale through quantum corrections are well within the experimental limits arising from the neutron and the electron EDM. Thus this class of models can naturally explain

the observed discrepancy in the muon anomalous magnetic moment a_μ , without inducing unacceptably large EDM for the neutron and the electron.

In the class of models presented here the scale of parity restoration is in the range $v_R \sim 10^{14} - 10^{16}$ GeV. This is close to the grand unification scale where the three gauge couplings of the standard model are observed to unify in a supersymmetric context. The left-right symmetric gauge structure and the numerical value of the scale v_R are independently suggested by experimental evidence for neutrino masses: Small neutrino masses arising through the seesaw mechanism [12] are natural in the left-right framework, and the scale v_R is consistent with the inferred value of ν_τ mass from atmospheric neutrino oscillations.

Parity symmetry as a possible solution to the strong CP problem in supersymmetric contexts has been studied in earlier papers [13]. The models presented here are significantly improved versions in this regard. In earlier works [13], it was found that a consistent solution to the strong CP problem required the scale of parity restoration to be in the multi-TeV range, which would appear not to go well with neutrino masses and gauge coupling unification, unlike in the models presented here. As we shall see, potential nonperturbative corrections to $\bar{\theta}$ from quantum gravity are under control here, even with v_R near the unification scale. Parity as a solution to the SUSY phase problem has also been studied in earlier papers [14,15], where it has been shown that the EDM's of the neutron and the electron remain very small in the parameter space that fits ϵ and ϵ' in the kaon system. Unlike in these earlier works which preferred a non-Kobayashi-Maskawa (KM) mechanism for the kaon CP violation, the models here allow for the conventional KM CP violation. This is facilitated by a novel realization of the doublet-doublet splitting—the mechanism that makes one pair of Higgs(ino) doublets light and all other pairs heavy, so that at low scale the spectrum of the theory is identical to that of the MSSM. Major differences of our models compared to the MSSM are that here (i) SUSY phases are naturally small, (ii) the strong CP problem is absent, and (iii) small neutrino masses are naturally present.

II. BASIC OUTLINE OF THE MODEL

The basic framework of our model involves the embedding of the MSSM into a minimal SUSY left-right gauge structure at a scale v_R close to the grand unified theory scale. The electroweak gauge group of the model is $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with the standard assignment of quarks and leptons: left-handed quarks and leptons (Q, L) transform as doublets of $SU(2)_L$, while the conjugate right-handed ones (Q^c, L^c) are doublets of $SU(2)_R$. The quarks Q transform under the gauge group as $(2, 1, 1/3)$ and Q^c as $(1, 2, -1/3)$, while the lepton fields L and L^c transform as $(2, 1, -1)$ and $(1, 2, +1)$, respectively. The Dirac masses of fermions arise through their Yukawa couplings to one or more Higgs bidoublets $\Phi(2, 2, 0)$. The $SU(2)_R \times U(1)_{B-L}$ symmetry is broken down to $U(1)_Y$ in the supersymmetric

¹For example, quantum gravity can induce a dimension five operator in the scalar potential $g|\Phi|^4(\Phi + \Phi^*)/M_{\text{Pl}}$, where g is a dimensionless coupling, and Φ is the scalar field responsible for $U(1)_A$ symmetry breaking. Such a term will induce a nonzero $\bar{\theta}$ given by $\bar{\theta} \sim g f_a^5 / (\Lambda_{\text{QCD}}^4 M_{\text{Pl}}) \approx 10^{40} g$. The resulting constraint on the coupling g from the neutron EDM is quite severe: $g \leq 10^{-50}$.

²For models with CP invariance, see Ref. [10]. Related applications in SUSY context have been discussed in Ref. [11].

limit by $B-L = \pm 1$ doublet scalar fields, the right-handed doublet denoted by $\chi^c(1,2,-1)$ accompanied by its left-handed partner $\chi(2,1,1)$. Anomaly cancellation requires the presence of their charge conjugate fields as well, denoted by $\bar{\chi}^c(1,2,1)$ and $\bar{\chi}(2,1,-1)$. The vacuum expectation values (VEV's) $\langle \chi^c \rangle = \langle \bar{\chi}^c \rangle = v_R$ break the left-right symmetry group down to the MSSM gauge symmetry.

This embedding of the MSSM into a left-right framework provides a simple solution to the SUSY phase problem. To see this, let us note the transformation of various fields under parity symmetry: $Q \leftrightarrow Q^{c*}, L \leftrightarrow L^{c*}, \Phi \leftrightarrow \Phi^\dagger, \chi \leftrightarrow \chi^{c*}, \bar{\chi} \leftrightarrow \bar{\chi}^{c*}, G \leftrightarrow G^*, B \leftrightarrow B^*, W_L \leftrightarrow W_R^*$, and $\theta \leftrightarrow \bar{\theta}$. Here $(G, B, W_{L,R})$ are the vector superfields associated with $SU(3)_C, B-L$, and $SU(2)_{L,R}$, respectively, θ is the fermionic variable, and the transformation applies to the matter superfields as a whole. Invariance under P makes the Dirac Yukawa couplings of quarks and leptons and the associated SUSY-breaking trilinear A terms Hermitian. The gluino and the $B-L$ gaugino masses become real, the mass of the bidoublet field Φ as well as the corresponding bilinear $B\mu$ term are real, and $M_L = M_R^*$, with $M_{L,R}$ the masses of the $SU(2)_{L,R}$ gauginos. This resolves the SUSY phase problem, since all the relevant SUSY phases are zero at the scale of parity restoration [14]. Renormalization group extrapolation induces very small phases in the SUSY-breaking parameters of the MSSM, but these induced values are well consistent with experimental limits arising from d_n and d_e [15]. It is worth noting that this solution to the SUSY phase problem will be valid even in a general context of SUSY breaking, for example, without assuming universality of scalar masses and proportionality between the A terms and the Yukawa couplings. Potential contributions to the EDM's of the neutron and the electron will be proportional to the diagonal entries of the respective A matrix or the squark or slepton mass-squared matrix. Both matrices being Hermitian, these contributions vanish above the scale v_R . Since the gaugino masses are all real (assuming gaugino mass unification that occurs in various scenarios of SUSY breaking even without a unifying group, or if the left-right gauge theory is embedded into a higher symmetry group such as $[SU(3)]^3, SO(10)$, or E_6), d_n and d_e proportional to the phases of the gaugino masses also vanish above v_R .

A. The doublet-doublet splitting mechanism

In order to make the supersymmetric left-right gauge theory fully realistic, a mechanism should be found that keeps one pair of Higgs doublets light at the weak scale (to be identified with the H_u and H_d fields of the MSSM) and any remaining pairs of Higgs doublets superheavy at the scale v_R . The simplest possibility would appear to be to introduce just a single Higgs bidoublet $\Phi(2,2,0)$ which gets a mass of order a SUSY breaking scale. However, this is not the minimal scenario from the effective low energy point of view, since in that case the up- and down-quark mass matri-

ces become proportional, leading to vanishing quark mixings at tree level.³

In this subsection we shall present a new mechanism for doublet-doublet splitting. It involves two bidoublet Higgs fields and is achieved without any fine-tuning of parameters. The solution to the SUSY phase problem is preserved based on parity symmetry alone. As we shall elaborate further in the next subsection, in this scenario the quark mixings arise naturally at tree level, $\tan \beta$ can be smaller than m_t/m_b , symmetry breaking occurs at the renormalizable level without any pseudo Goldstone bosons, and neutrino masses are correctly reproduced with v_R near the unification scale. Furthermore, the effective low energy theory is just the MSSM, with a natural understanding of the weak scale value of the μ parameter which remains real.

Consider the following form of the superpotential involving two bidoublet fields Φ_a , $a=1,2$ and the left-handed $(\chi + \bar{\chi})$ and the right-handed $(\chi^c + \bar{\chi}^c)$ doublets of the theory:

$$W = \lambda_a \chi \Phi_a \chi^c + \lambda'_a \bar{\chi} \Phi_a \bar{\chi}^c + M \chi \bar{\chi} + M^* \chi^c \bar{\chi}^c. \quad (2)$$

Parity invariance makes the couplings λ_a and λ'_a real, since $\Phi \rightarrow \Phi^\dagger$ and $\chi \rightarrow \chi^{c*}$ under P . The mass term M and the VEV $\langle \chi^c \rangle = \langle \bar{\chi}^c \rangle = v_R$ are complex in general. After $SU(2)_R$ breaking, this superpotential leads to a mass matrix for the doublets given in Eq. (3). We use a notation in which the rows denote $(\phi_{u1}, \phi_{u2}, \chi)$ fields and the columns denote $(\phi_{d1}, \phi_{d2}, \bar{\chi})$ fields where ϕ_{u1} and ϕ_{d1} are the up and down type Higgs doublets from Φ_1 , etc.

$$M_{DD} = \begin{pmatrix} 0 & 0 & \lambda_1 v_R \\ 0 & 0 & \lambda_2 v_R \\ \lambda'_1 v_R & \lambda'_2 v_R & M \end{pmatrix}. \quad (3)$$

This mass matrix leaves one pair of Higgs doublets massless, while giving mass of order v_R to the second pair. Since λ_a and λ'_a are real, the effective μ term of the light doublets becomes real. To see this, observe that the low energy

³Realistic quark mixings can be induced through the gluino and chargino loops, provided that the trilinear A terms have a flavor structure different from that of the Yukawa couplings [15,16]. Consistency with flavor changing processes would require that $\tan \beta$ be not too large [15], and $\tan \beta \leq 10$, which excludes the simplest scenario where $\tan \beta = m_t/m_b \approx 60$. Values of $\tan \beta$ smaller than m_t/m_b may be obtained if the Φ field mixes with some other superheavy doublets of the theory in a parity-violating manner (if such mixings conserve P , $\tan \beta = m_t/m_b$ will prevail). In this case the effective μ and $B\mu$ terms are potentially complex, which would spoil the solution to the SUSY phase problem based on parity. To maintain this solution, in earlier works [16] we assumed invariance under charge conjugation symmetry C , in addition to P , which allows for reality of the effective μ and $B\mu$ terms. In such a scenario the CKM phase will be zero due to C invariance and the observed CP violation in the kaon system is explained through supersymmetric gluino or squark diagrams.

MSSM doublets are given by $H_u = \cos \alpha_u \phi_{u1} + \sin \alpha_u \phi_{u2}$ and $H_d = \cos \alpha_d \phi_{d1} + \sin \alpha_d \phi_{d2}$, where $\tan \alpha_u = \lambda_1/\lambda_2$ and $\tan \alpha_d = \lambda'_1/\lambda'_2$. Note that H_u and H_d are *real* linear combinations of Φ_a , which helps in inducing a real μ . The superpotential of Eq. (2) by itself does not lead to a μ term, which gets induced only after SUSY breaking. There are two sources that induce the μ term:

(1) Kahler potential terms of the form $\lambda''_{ab} \int d^4 \theta (Z^*/M_{\text{Pl}}) \text{Tr}(\Phi_a \Phi_b)$, where Z is a gauge singlet whose $F_Z \neq 0$ breaks supersymmetry. We assume that Z is parity even, i.e., $Z \rightarrow Z^*$ under P . The coupling matrix λ'' is therefore Hermitian. After supersymmetry breaking this term will lead to a real μ term, as desired. This also provides a reason why the μ term is of order of the electroweak scale [17]. The $B\mu$ term arises from the term $\int d^4 \theta (ZZ^*/M_{\text{Pl}}^2) \text{Tr}(\Phi_a \Phi_b)$, which is also real due to parity.

(2) A second mechanism that generates real μ and $B\mu$ terms makes use of the superpotential couplings involving a visible sector singlet S : $W \supset \kappa S (e^{i\xi} \chi^c \bar{\chi}^c + e^{-i\xi} \chi \bar{\chi} - M^2)$. Such a coupling can break the left-right gauge symmetry down to the MSSM at the renormalizable level without leaving any pseudo Goldstone bosons. Owing to parity, under which $S \rightarrow S^*$, the parameters κ, M^2 are real in this superpotential coupling. The field S also has the following coupling to the bidoublets Φ_a : $W \supset \mu_{ab} \text{Tr}(\Phi_a \Phi_b) S$. In the SUSY limit, $\langle S \rangle = 0$, $\langle \chi^c \rangle = \langle \bar{\chi}^c \rangle = v_R$, $\langle \chi \rangle = \langle \bar{\chi} \rangle = 0$, which breaks parity spontaneously. S pairs up with the neutral component $(\chi^c + \bar{\chi}^c)/\sqrt{2}$ to form a multiplet that has mass $\sqrt{2} \kappa v_R$. After SUSY breaking, the coupling $\int d^2 \theta (\chi^c \bar{\chi}^c S Z/M_{\text{Pl}})$ will induce a tadpole in the $\text{Re}(S)$ scalar of order $v_R^2 m_{3/2}$. A VEV $\langle \text{Re}(S) \rangle \sim m_{3/2}$ will result, which provides a real μ term for the bidoublet fields.⁴ It is crucial to note that by redefining the χ^c field, the coefficient of the tadpole in S can be made real without introducing any phases elsewhere [see Eq. (4) below]. If the imaginary component of S also had a tadpole, $\langle \text{Im}(S) \rangle \neq 0$, which will lead to an effective complex μ term.

B. The full Lagrangian

Now let us implement the doublet-doublet splitting mechanism just described. We shall see that there is a discrete anomaly-free $Z_4 R$ symmetry that achieves this goal within a minimal version of the left-right model [viz., using two bidoublets Φ_a , one left-handed (χ) and one right-handed (χ^c) $SU(2)$ doublet along with their conjugate ($\bar{\chi} + \bar{\chi}^c$), and the singlet S]. Their transformations under P has been given earlier, with $S \rightarrow S^*$. All the desired terms, including the Majorana mass terms for the right-handed neutrinos, are allowed by this Z_4 , and the unwanted terms that can potentially make the magnitude of μ too large or induce excessive CP phases to upset the strong CP and the SUSY phase solutions will be prevented. The Z_4 is broken at the

scale v_R , but a Z_2 remnant remains, which is identified as the usual R parity of the MSSM. This Z_2 will guarantee the stability of the proton.

Under the $Z_4 R$ symmetry, the superpotential changes sign ($W \rightarrow -W$), as do $d^2 \theta$ and $d^2 \bar{\theta}$. The gaugino fields transform as $\lambda_a \rightarrow -\lambda_a$, and quarks and leptons are even, Φ_a : $-1, \chi: i, \bar{\chi}: -i, \chi^c: -i, \bar{\chi}^c: +i, S: -1$.

The gauge invariant superpotential consistent with this $Z_4 R$ symmetry is

$$\begin{aligned} W = & h_a Q \Phi_a Q^c + h'_a L \Phi_a L^c + \lambda_a \chi \Phi_a \chi^c + \lambda'_a \bar{\chi} \Phi_a \bar{\chi}^c \\ & + (f L L \chi \chi + f^* L^c L^c \chi^c \chi^c) / M_{\text{Pl}} \\ & + \kappa S (e^{i\xi} \chi^c \bar{\chi}^c + e^{-i\xi} \chi \bar{\chi} + a S^2 - M^2) + \mu_{ab} \text{Tr}(\Phi_a \Phi_b) S. \end{aligned} \quad (4)$$

This superpotential induces tree level CKM mixings since the light MSSM doublets $H_{u,d}$ are parity-asymmetric linear combinations of the two bidoublets. The f couplings give rise to Majorana masses for ν_R of order v_R^2/M_{Pl} . For $v_R \sim 10^{14} - 10^{16}$ GeV, the magnitude of the light neutrino masses are in the right range to explain the atmospheric and the solar neutrino oscillation data. f could have its origin in quantum gravity, but it could also arise from integrating out singlets which have masses of order M_{Pl} , e.g., through $(LN\chi + L^c N^c \chi^c)$ couplings where (N, N^c) are the singlets with Z_4 charges $(i, -i)$. Their Majorana masses $[N^2 + N^{c2}]$ preserve the Z_4 symmetry.

In the SUSY limit, we have $\langle S \rangle = 0, \langle \chi^c \rangle = \langle \bar{\chi}^c \rangle = M$ with all other fields having zero VEV's. As noted earlier, after SUSY breaking, the real component of S gets an induced VEV of order $m_{3/2}$. [Note that the phase in the $\chi^c \bar{\chi}^c S$ scalar coupling can be made real by redefining χ^c field. This redefinition does not induce new phases anywhere else. The tadpole in S that is induced after SUSY breaking is therefore real, making only $\text{Re}(S)$ nonzero.] That gives Hermitian μ_{ab} terms through the last couplings of Eq. (4), or to the real μ parameter. We can also have the coupling $\lambda''_{ab} \int d^4 \theta ([\text{Tr}(\Phi_a \Phi_b)] Z / M_{\text{Pl}})$, where Z is the spurion field that breaks SUSY. This also leads to the real μ term of the right order of magnitude. [Note that Z is parity even, and $Z \rightarrow Z^*$ under P . F_Z is then expected to be real, which would leave parity unbroken. For example, in the Polonyi model of hidden sector SUSY breaking, $W \supset \mu^2 (Z + \beta)$, where μ^2 is real due to parity. $F_Z = \mu^2$ is therefore real. We anticipate the reality to F_Z to hold even in a more general scenario for SUSY breaking.]

The superpotential of Eq. (4) reproduces the doublet-doublet mixing matrix of Eq. (3). In Eq. (3), the (3,3) entry is of order $m_{3/2}$ now, being proportional to $\langle S \rangle$. It does not correspond to any new particle having mass of order $m_{3/2}$, since χ pairs with the heavy doublet in Φ_a and has a mass of order v_R .

We shall now show that the $Z_4 R$ symmetry is an anomaly-free discrete gauge symmetry [19]. This makes it aesthetically more pleasing, as it may have its origin in a true gauge symmetry. It also protects the Lagrangian from receiv-

⁴This mechanism for generating the μ term in supergravity models has independently been suggested in Ref. [18], which appeared simultaneously with our present work.

ing uncontrollable quantum gravitational correction. To see the anomaly freedom, let us assume that the Z_4 arose from a true gauge $U(1)_R$ symmetry. The $U(1)_R$ then should be anomaly-free. If the $U(1)_R$ symmetry is broken at the Planck scale by scalar VEV's that are integer multiples of ± 4 (up to an overall normalization factor), a residual Z_4 symmetry will survive. The $U(1)_R$ anomaly cancellation will in general require introduction of additional fermionic fields. The crucial question is then if these extra fields can be removed from the low energy spectrum by giving them Z_4 invariant masses. To address this, let us embed the Z_4 into a $U(1)_R$ in the obvious fashion, by assigning $U(1)_R$ charges as follows: (We display the R charge of the superfield, which is the same for the scalar component, but the fermionic component will have its R charge shifted by -1 .) Quarks and leptons: $0, \chi: +1, \chi^c: -1, \bar{\chi}: -1, \bar{\chi}^c: +1, \Phi_a: +2, S: +2$. The superpotential W has an R charge of $+2$, and the gauginos have an R charge of $+1$.

With this assignment, one can compute all the mixed anomaly coefficients:

$$\begin{aligned} SU(3)_C^2 \times U(1)_R: & 3 - 2N_g, \\ SU(2)_L^2 \times U(1)_R: & 3 - 2N_g, \\ SU(2)_R^2 \times U(1)_R: & 3 - 2N_g, \\ U(1)_{B-L}^2 \times U(1)_R: & -3 - 2N_g. \end{aligned} \quad (5)$$

Here $N_g = 3$ is the number of generations. The 3 in the first term arises from a gluino loop. The 3 in the second term is from W -ino ($+2$) and Higgsino ($+2$ from the two bidoublets and -1 from $\bar{\chi}$). We have used the conventional $SO(10)$ $B-L$ normalization, $\sqrt{3/2}(B-L)/2$ being the normalized generator.

Since all the non-Abelian mixed anomaly coefficients are equal, we can try to cancel them by the Green-Schwarz mechanism [20]. We shall also make the Abelian mixed anomaly coefficient equal to facilitate this. This can be achieved by adding a pair of singlets which are $(2,3) + (-2,3)$ under $(B-L, R)$. Then the $U(1)_{B-L}^2 \times U(1)_R$ anomaly also becomes $(3 - 2N_g)$. A mass term for these singlet fields will have an R charge of $+6$ (scalar component), so it must be accompanied by a Higgs field with charge -4 . That breaks $U(1)_R$ to Z_4 , as desired.

Finally, there is the $U(1)_R^2 \times U(1)_{B-L}$ anomaly, which has a coefficient $-8\sqrt{3/2}$ in this model. We use the overall R normalization [level of $U(1)_R$] to make this equal to the other anomaly coefficients. This normalization factor is then found to be $(1/4)\sqrt{3/2}$ —very similar to $B-L$ normalization. The $[U(1)_R]^3$ anomaly also has the same coefficient (equal to -3 for $N_g=3$) if some singlet fields contribute $+24$ in the cubic anomaly. Three singlets with a fermionic R charge of $+2$ will do this job. Mass terms for these singlets will carry an R charge of $+6$ (scalar component), so to make it $+2$, we must multiply by a Higgs field with an R charge of -4 . Again, we see that the Z_4 is left unbroken by this Higgs

field. This way of canceling anomalies is not unique, but to establish that Z_4 is discrete and anomaly-free, any one example would suffice.

III. CALCULATION OF INDUCED $\bar{\theta}$

As noted earlier, parity invariance implies that the QCD Lagrangian parameter $\theta=0$, the gluino mass is real, and the quark mass matrices $M_{u,d}$ are Hermitian at the tree level. Therefore $\bar{\theta}=0$ at the tree level. Since parity is broken at v_R , a nonzero value of $\bar{\theta}$ will be induced at the weak scale through renormalization group extrapolation below v_R . We shall estimate this induced $\bar{\theta}$ in two scenarios for SUSY breaking. The first is the constrained MSSM scenario where the squark masses are degenerate at the unification scale with the trilinear A matrices and the corresponding Yukawa coupling matrices being proportional. The second scenario has a more general SUSY-breaking spectrum without universality or proportionality, but the experimental constraints arising from the flavor changing processes will be imposed. We first turn to the correction to $\bar{\theta}$ arising from the non-Hermiticity of the Yukawa coupling matrices which applies to both these scenarios.

A. $\delta\bar{\theta}$ from non-Hermiticity of the Yukawa coupling matrices

At the scale v_R , the up and down Yukawa coupling matrices are Hermitian owing to parity. The VEV's of the MSSM fields $H_{u,d}$ are real since the $B\mu$ term is real, so that the quark mass matrices M_u and M_d are Hermitian at v_R . They will develop non-Hermitian components at the weak scale, owing to renormalization group evolution below v_R . The induced $\bar{\theta}$ will have the general structure given by

$$\delta\bar{\theta} = \text{Im Tr}[\Delta M_u M_u^{-1} + \Delta M_d M_d^{-1}] - 3 \text{Im}(\Delta M_{\tilde{g}} \tilde{M}_{\tilde{g}}^{-1}), \quad (6)$$

where $M_{u,d,\tilde{g}}$ denote the tree level contribution to the up-quark matrix, down-quark matrix, and the gluino mass, respectively, and $\Delta M_{u,d,\tilde{g}}$ are the loop corrections. To estimate the corrections from ΔM_u and ΔM_d , we note that the beta function for the evolution of Y_u below v_R is given by $\beta_{Y_u} = Y_u / (16\pi^2) (3Y_u^\dagger Y_u + Y_d^\dagger Y_d + G_u)$ with the corresponding one for Y_d obtained by the interchange $Y_u \leftrightarrow Y_d$ and $G_u \rightarrow G_d$. Here G_u is a family-independent contribution arising from gauge bosons and the $\text{Tr}(Y_u^\dagger Y_u)$ term. The $3Y_u^\dagger Y_u$ term and the G_u term cannot induce non-Hermiticity in Y_u , given that Y_u is Hermitian at v_R . The interplay of Y_d with Y_u will however induce deviations from Hermiticity. Repeated iteration of the solution with $Y_u \propto Y_u Y_d^\dagger Y_d$ and $Y_d \propto Y_d Y_u^\dagger Y_u$ in these equations will generate the following structure:

$$\begin{aligned} \delta\bar{\theta} \simeq & \left(\frac{\ln(M_U/M_W)}{16\pi^2} \right)^4 [c_1 \text{Im Tr}(Y_u^2 Y_d^4 Y_u^4 Y_d^2) \\ & + c_2 \text{Im Tr}(Y_d^2 Y_u^4 Y_d^4 Y_u^2)], \end{aligned} \quad (7)$$

where M_U is the unification scale. Here c_1 and c_2 are order one coefficients which are not equal since the flavor independent parts G_u and G_d are not the same for the evolution of Y_u and Y_d (hypercharge gauge couplings and the tau lepton Yukawa couplings differentiate the two). These contributions to $\delta\bar{\theta}$ are very high order in the Yukawa couplings since the trace of products of two Hermitian matrices, having the form $\text{Im Tr}(Y_u^n Y_d^m Y_u^p Y_d^q \dots)$, contains an imaginary piece only at this order. To estimate the induced $\bar{\theta}$, we choose a basis where Y_u is diagonal, $Y_u = D$, and $Y_d = VD'V^\dagger$, where $DV_u = \text{diag}(m_u, m_c, m_t)$, and $D'v_d = \text{diag}(m_d, m_s, m_b)$ with V being the CKM matrix. The trace of the first term in Eq. (7) is then $\text{Im}(D_i^2 D_k^4 D_j^4 D_l^2 V_{ij} V_{kl} V_{il}^* V_{kj}^*)$. The leading contribution in this sum is $(m_t^4 m_c^2 m_b^4 m_s^2) / (v_u^6 v_d^6) \text{Im}(V_{cb} V_{ts} V_{cs}^* V_{tb}^*)$. The second trace in Eq. (7) is identical, except that it has an opposite sign. Numerically then,

$$\delta\bar{\theta} \sim 3 \times 10^{-27} (\tan \beta)^6 (c_1 - c_2), \quad (8)$$

where we have used the running quark masses at m_t to be $(m_t, m_c, m_b, m_s) = (166, 0.6, 2.8, 0.63)$ GeV. Clearly, $\delta\bar{\theta}$ is very small; even for $\tan \beta = 50$ its value is 10^{-16} , much below the experimental limit of 10^{-10} from neutron EDM.

Since the up(down)-quark mass matrix is a product of $Y_u(Y_d)$ and the VEV $v_u(v_d)$, the mass matrix can become complex if the VEV $v_u(v_d)$ is complex. If the bilinear soft SUSY-breaking parameter $B\mu$ becomes complex in the process of evolution below v_R , this will happen. By analyzing the renormalization group equation (RGE) for the $B\mu$ parameter, one sees that it involves traces of $(Y_u^\dagger Y_u)$ and $(Y_d^\dagger Y_d)$ or their products—in the case of universal squark masses and proportional A terms ($A_u \propto Y_u, A_d \propto Y_d$). We are again left with two Hermitian matrices (Y_u, Y_d) , with all other effective parameters being real. The imaginary component of the trace that induces a phase in $B\mu$ is then given at lowest order by an expression analogous to Eq. (7). The estimate on $\delta\bar{\theta}$ is of the same order as before, $\delta\bar{\theta} \sim 10^{-26} (\tan \beta)^6$.

B. $\delta\bar{\theta}$ from finite correction to the quark and the gluino masses

To compute $\delta\bar{\theta}$ arising from the finite corrections to the quark mass matrices and the gluino mass (which are not contained in the RGE evolution), we must specify the SUSY-breaking spectrum. The simplest approximation is to assume universality of scalar masses and proportionality of A terms and the respective Yukawa couplings at the Planck scale. This can be justified in models such as the ones with gauge mediated supersymmetry breaking [21]. In this case, the whole theory at the weak scale is characterized by only two Yukawa coupling matrices $Y_{u,d}$. Furthermore, all other MSSM parameters are real in the effective low energy theory below v_R . Because of this property it is very easy to estimate the lowest order contribution to nonvanishing $\bar{\theta}$ in terms of the coupling matrices.

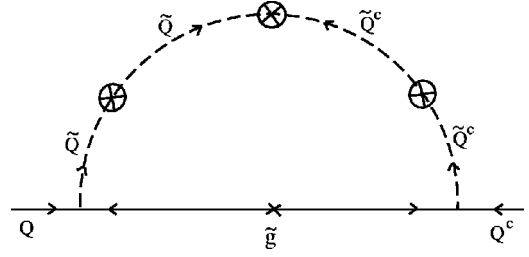


FIG. 1. One-loop gluino or squark exchange diagram contribution to the quark mass matrix. The crosses on the scalar lines correspond to mass insertions.

Consider the finite one-loop corrections to the quark mass matrices. A typical diagram involving the exchange of squarks and gluinos is shown in Fig. 1. There are analogous chargino diagrams as well. In Fig. 1, the crosses on the \tilde{Q} and \tilde{Q}^c lines represent left-left (LL) and right-right (RR) mass insertions that will be induced in the process of RGE evolution. From this figure we can estimate the form for $\Delta M_u = (2\alpha_s/3\pi)m_{\tilde{Q}}^2 A_u m_{\tilde{u}^c}^2$ where \tilde{Q} is the squark doublet and \tilde{u}^c is the right-handed singlet up squark. Without RGE effects, the trace of this term will be real, and will not contribute to $\bar{\theta}$. Looking at the RGE for $m_{\tilde{u}^c}^2$ up to two-loop order, we see that for the case of proportionality of A_u and Y_u , $m_{\tilde{u}^c}^2$ gets corrections having the form $m_0^2 Y_u^2$ or $m_0^2 Y_u^4$ or $m_0^2 Y_u Y_d^2 Y_u$. Therefore in $\Delta M_u M_u^{-1}$, the M_u^{-1} always cancels and we are left with a product of matrices of the form $Y_u^n Y_d^m Y_u^p Y_d^q \dots$. A similar comment applies when we look at the RGE corrections for $m_{\tilde{Q}}^2$ or A_u . If the product is Hermitian, then its trace is real. So to get a nonvanishing contribution to theta, we have to find the lowest order product of Y_u^2 and Y_d^2 that is non-Hermitian⁵ and we get

$$\delta\bar{\theta} = \frac{2\alpha_s}{3\pi} \left(\frac{\ln(M_U/M_W)}{16\pi^2} \right)^4 (k_1 \text{Im Tr}[Y_u^2 Y_d^4 Y_u^4 Y_d^2] + k_2 \text{Im Tr}[Y_d^2 Y_u^4 Y_d^4 Y_u^2]), \quad (9)$$

where $k_{1,2}$ are calculable constants. The numerical estimate of this contribution parallels that of the previous discussions, $\delta\bar{\theta} \sim (k_1 - k_2) \times 10^{-28} (\tan \beta)^6$. The contributions from the up-quark and down-quark matrices tend to cancel, but since the \tilde{d}^c and the \tilde{u}^c squarks are not degenerate, $k_1 \neq k_2$ and the cancellation is incomplete.

In Fig. 2 we have displayed the one-loop contribution to the gluino mass arising from the quark mass matrix. Here again one encounters the imaginary trace of two Hermitian matrices Y_u and Y_d , in the case of universality and proportionality of SUSY-breaking parameters. Our estimate for $\delta\bar{\theta}$ is similar to that of the quark mass matrix of Eq. (9).

⁵Similar reasoning was used in the standard model and supersymmetric models in earlier papers [22].

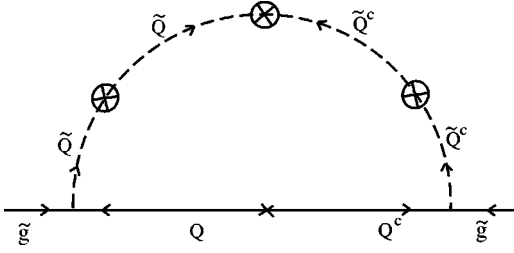


FIG. 2. One-loop diagram that induces a phase in the gluino mass.

C. Induced $\bar{\theta}$ with general SUSY-breaking terms

In this subsection, we study the more general SUSY-breaking scenario where soft SUSY-breaking terms involving squarks are given by

$$\mathcal{L}_{SSB} = \sum_{\phi = \bar{Q}, \bar{u}^c, \bar{d}^c, \dots} \phi^\dagger m_\phi^2 \phi + \bar{Q} A_u H_u \bar{u}^c + \bar{Q} A_d H_d \bar{d}^c + \text{H.c.} \quad (10)$$

For the model under study, at the v_R scale, the constraint is that $m_{\bar{Q}}^{2,T} = m_{\bar{u}^c}^2 = m_{\bar{d}^c}^2 \equiv m_{\bar{Q}^c}^2$ due to parity invariance. $A_{u,d}$ are arbitrary Hermitian matrices, and the squark mass matrices can have nontrivial flavor structure.

In this case, the lowest order correction to $\delta\bar{\theta}$ from one-loop contributions to quark masses (Fig. 1) is given by

$$\delta\bar{\theta} \simeq \frac{2\alpha_S}{3\pi m_0^5} \text{Im Tr}[m_{\bar{Q}}^2 A_f m_{\bar{Q}}^2 Y_f^{-1}] = 0 \quad (11)$$

for $f = u, d$. This contribution vanishes since the matrices $m_{\bar{Q}}^2 A_f m_{\bar{Q}}^2$ and Y_f^{-1} are both Hermitian. The next leading contribution has the form

$$\delta\bar{\theta} \simeq \frac{2\alpha_S v_{wk}}{3\pi m_0^6} \frac{\ln(M_U/M_{\text{SUSY}})}{16\pi^2} \text{Im Tr}[m_{\bar{Q}}^2 A_u m_{\bar{Q}}^2 Y_u]. \quad (12)$$

This contribution arises from Fig. 1 by inserting $m_{\bar{Q}}^2 Y_u^2$ arising from the RGE's in one of the squark lines. Since this trace involves three arbitrary Hermitian matrices, it is not real in general. To estimate this contribution, we have to make some assumption about the nonuniversality in $m_{\bar{Q}}^2$ and the nonproportionality in A and the Yukawa coupling matrix. As for the A term, the most natural choice will be to assume that it has the same hierarchical structure as the Yukawa couplings. Such a form would be suggested by flavor symmetries. Thus, we shall take $A_{23} \sim \epsilon A_{33}$, where ϵ is a small parameter, of order $V_{cb} \sim 1/30$. Such a choice will guarantee that there is no excessive flavor changing neutral current (FCNC) processes mediated by squarks. As for the squark mass matrices $m_{\bar{Q}}^2$, we take it to be approximately proportional to a unit matrix, with correction terms that are not large. This is as suggested by non-Abelian horizontal symmetries [23]. The leading contribution from Eq. (12) to $\bar{\theta}$ arises when we use index (3,2) for the first $m_{\bar{Q}}^2$, (2,3) for

A_u , and (3,3) for the rest. Using $v_{wk}/m_0 \sim 1/5$, $A_{33}/m_0 \sim 1/10$, $A_{23} \sim 10^{-2} A_{33}$, $(m_{\bar{Q}}^2)_{23} \sim 10^{-2} m_0^2$, we find $\delta\bar{\theta} \sim 10^{-8}$. This is a conservative estimate and yet it is encouraging that we are close to the present upper limit on $\delta\bar{\theta}$ of 10^{-9} to 10^{-10} . To be completely consistent with the neutron EDM limit, we should have the relevant phase to be of order 0.1, or the off-diagonal entries somewhat smaller than allowed by FCNC constraints. Since such departures from natural values need be only mild, we feel that this scenario is also quite viable. It is interesting that in this scheme, d_n is not much below the present experimental limit, while d_e is well below the current limit.

Let us now address the contribution to $\bar{\theta}$ arising from the induced phase of the gluino mass. The leading contribution (see Fig. 2) in this case is given by $\delta\bar{\theta} \simeq (2\alpha_S/3\pi) \text{Im Tr}(A_u Y_u) (v_{wk}^2/m_0^2 M_{\tilde{g}}^-)$. Without RGE running, this trace is real since Y_u and A_u are Hermitian. Allowing for RGE running, we estimate $\delta\bar{\theta} \simeq (2\alpha_S/3\pi) [\ln(M_U/M_W)/(16\pi^2)] \text{Im Tr}(M_{\bar{Q}}^2 Y_u A_u) (v_{wk}^2/m_0^4 M_{\tilde{g}}^-)$. Taking the (2,3) entries of $M_{\bar{Q}}^2$ and A_u to be 10^{-2} times that of the respective (3,3) entries, and with $A_{33} = m_0/10$, we arrive at $\delta\bar{\theta} \simeq 10^{-8} - 10^{-10}$ for $v_{wk}/m_0 \sim 1/5$. This is again not far from the present upper limit and with a mild fine-tuning of parameters, of order 10%, one gets the desired solution to the strong CP problem.

IV. PLANCK SCALE CORRECTIONS

One interesting aspect of the model presented here is that it is quite safe from potentially large corrections to $\bar{\theta}$ induced by quantum gravity. If it is assumed that the high scale parity conserving theory originates from a more fundamental theory, one can expect nonrenormalizable operators in the theory suppressed by the mass scale associated with the fundamental theory. Such corrections to $\bar{\theta}$ will respect the gauge symmetry as well as the anomaly-free Z_4 discrete gauge symmetry. We should ensure two things: (i) The effective μ term induced by quantum gravity is not more than the weak scale, and (ii) the quantum gravity induced phases which may not respect parity do not upset the solution to the strong CP problem. All other constraints, such as the solution of the SUSY phase problem, will be automatically satisfied once these two are taken care of.

As for the magnitude and the phase of the effective μ term, the most relevant higher dimensional operator suppressed by the Planck mass that is invariant under the gauge symmetries and the Z_4 symmetry is $W \supset \kappa_{ab} \text{Tr}(\Phi_a \Phi_b) \chi^c \bar{\chi}^c S / M_{\text{Pl}}^2$. The magnitude of the resulting μ term is $\kappa_{ab} v_R^2 M_{\text{SUSY}} / M_{\text{Pl}}^2 \sim 10^{-8} M_{\text{SUSY}}$. Clearly, this is very small correction to the magnitude of μ . Suppose that quantum gravity does not respect parity symmetry. The coefficients κ_{ab} will then be non-Hermitian. The phase of the μ term will then be $\arg(\mu) \sim 10^{-8}$. Through the gluino diagram this will lead to $\bar{\theta} \sim 10^{-10}$, which is consistent with the d_n limit. This shows that the complex couplings κ_{ab} can be of order one.

The quark mass matrices can also have corrections from Planck scale physics. The most relevant term is $W \supset QQ^c \Phi \chi^c \bar{\chi}^c / M_{\text{Pl}}^2$, which will induce corrections of order $10^{-8} v_{wk}$ for some of the quark masses. The matrix structure need not be Hermitian if quantum gravity violates parity. We suspect that gauged flavor symmetries (discrete or continuous) must exist in the underlying theory, or else the light fermion masses can become too large from quantum gravity. Most likely the estimate of $10^{-8} v_{wk}$ will apply for the third generation. If the coefficient of this nonrenormalizable operator is of order $10^{-1} - 10^{-2}$, the solution to the strong CP problem via parity will be preserved. Note that the superpotential coupling $W \supset Q \chi Q^c \chi^c / M_{\text{Pl}}$ is not invariant under the discrete Z_4 , unless accompanied by another factor S / M_{Pl} . The correction to the quark mass matrix from this term is extremely small $\Delta M_u \sim (M_{\text{SUSY}} / M_{\text{Pl}})^2 v_{wk}$. We have verified that all other Planck induced corrections are much below the experimental limits on $\bar{\theta}$.

A question can be raised as to the form of SUSY-breaking parameters and whether they indeed will respect parity symmetry. A complete answer to this will have to await a full understanding of nonperturbative SUSY breaking which is lacking at the moment. We note that perturbative gravity, which is utilized in conventional supergravity models of SUSY breaking may well respect parity; we have given an example in the Polonyi model. A second example is gauge mediated SUSY breaking. If SUSY is broken at a scale of $10^4 - 10^5$ GeV, quantum gravity corrections for the μ term and the A term, which will be of order gravitino mass, will be of order $10^{-10} - 10^{-8}$ GeV. Even if they are complex and non-Hermitian, the strong CP problem will be solved, as the induced $\bar{\theta}$ will be of order $10^{-10} - 10^{-12}$. We may use one of the other proposed solutions to generate a μ term of the weak scale in this case [24]. If the messenger fields do not couple to the fields $\chi^c, \bar{\chi}^c$, they will not feel the effects of parity breaking, although parity is broken at $v_R \sim 10^{16}$ GeV. The effective SUSY-breaking parameters will then obey the constraints of parity.

In order to make the cosmological constant vanishingly small, a Z_4 odd constant term of magnitude $m_{3/2} M_{\text{Pl}}^2$ might be added in the hidden sector superpotential W_0 . In this case, a higher dimensional term of the type $(W_0 \Phi_a \Phi_b / M_{\text{Pl}}^2)$ will be induced, where Φ_a stands for any of the visible sector superfield. This can lead to a complex μ term of order $m_{3/2}$ [25]. If the supersymmetry-breaking scale is low as in gauge mediated models, $m_{3/2} \sim \text{eV}$, this complex contribution to the μ parameter will be negligible. In conventional supergravity models, one will have to rely on realizing parity as a discrete gauge symmetry [26] so that such an induced μ term will

remain real, or one will have to find an alternative explanation for the smallness of the cosmological constant.

The $d=5$ baryon number violating operator $QQQL / M_{\text{Pl}}$ in the superpotential is forbidden in this model by Z_4 symmetry, but the operator $QQQLS / M_{\text{Pl}}^2$ is allowed. If the associated couplings are order one for the light generations, we estimate proton lifetime induced by these operators to be $\tau_p \sim 10^{60}$ yr.

V. CONCLUSIONS

We have shown in this paper that it is possible to embed the supersymmetric standard model into a parity-symmetric framework at a unification scale of 2×10^{16} GeV in a simple way. Such an extension is well motivated by the data on neutrino oscillations as well as gauge coupling unification. We have demonstrated that this embedding can naturally solve the strong CP problem and the SUSY phase problem simultaneously. The effective low energy theory is the MSSM, but with naturally small phases for the SUSY-breaking parameters along with an order one phase in the CKM matrix. Thus it allows for large SUSY contributions to the muon $g-2$, as indicated by experiment, without violating the bounds on the electron EDM. The induced $\bar{\theta}$ in these models depends strongly on the way SUSY breaking is communicated. With universality of squark masses and proportionality of the A terms, we found $\bar{\theta} \leq 10^{-16}$, while with maximal deviation from universality and proportionality is consistent with FCNC constraints $\bar{\theta} \sim 10^{-10} - 10^{-8}$. In the latter case, the neutron EDM should be soon accessible, while d_e will be much smaller than the present experimental limit. We have also shown that potential corrections induced by quantum gravity are under control in this class of models. Since left-right gauge symmetry is realized at a scale $v_R \sim 10^{16}$ GeV, evolution of couplings between v_R and M_{Pl} can induce flavor changing neutral current processes which are in the interesting range for current and future experiments. We plan to study this issue in detail in a forthcoming publication.

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- [1] For reviews, see J.E. Kim, Phys. Rep. **150**, 1 (1987); R.D. Peccei, in *CP Violation*, edited by C. Jarlskog (World Scientific, Singapore, 1989).
- [2] P.G. Harris *et al.*, Phys. Rev. Lett. **82**, 904 (1999).
- [3] T. Falk and K. Olive, Phys. Lett. B **439**, 71 (1996); T. Ibrahim

- and P. Nath, *ibid.* **418**, 98 (1998); M. Brhlik, G. Good, and G. Kane, Phys. Rev. D **59**, 115004 (1999).
- [4] Muon $g-2$ Collaboration, H.N. Brown *et al.*, Phys. Rev. Lett. **86**, 2227 (2001).
- [5] E.D. Commins *et al.*, Phys. Rev. A **50**, 2960 (1994).

- [6] D. DeMille, Talk at EDM 2001 Workshop, Brookhaven, New York (2001).
- [7] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. D **16**, 1791 (1977); S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, *ibid.* **40**, 279 (1978).
- [8] S. Giddings and A. Strominger, Nucl. Phys. **B307**, 854 (1988); R. Holman *et al.*, Phys. Lett. B **282**, 132 (1992); M. Kamionkowski and J. March-Russell, *ibid.* **282**, 137 (1992); S.M. Barr and D. Seckel, Phys. Rev. D **46**, 539 (1992); R. Kallosh *et al.*, *ibid.* **52**, 912 (1995).
- [9] M.A.B. Beg and H.S. Tsao, Phys. Rev. Lett. **41**, 278 (1978); R.N. Mohapatra and G. Senjanović, Phys. Lett. **79B**, 283 (1978); K.S. Babu and R.N. Mohapatra, Phys. Lett. D **41**, 1286 (1990); S. Barr, D. Chang, and G. Senjanović, Phys. Rev. Lett. **67**, 2765 (1991).
- [10] H. Georgi, Hadronic J. **1**, 155 (1978); S. Barr and P. Langacker, Phys. Rev. Lett. **42**, 1654 (1979); A. Nelson, Phys. Lett. **136B**, 387 (1984); S.M. Barr, Phys. Rev. Lett. **53**, 329 (1984); P. Frampton, S. Glashow, and T. Yoshikawa, *ibid.* **87**, 011801 (2001).
- [11] M. Dine, R.G. Leigh, and A. Kagan, Phys. Rev. D **48**, 2214 (1993); S.M. Barr, *ibid.* **56**, 1475 (1997); G. Hiller and M. Schmaltz, Phys. Lett. B **514**, 263 (2001); M. Pospelov, *ibid.* **391**, 324 (1997); C. Hamzaoui and M. Pospelov, hep-ph/0105270.
- [12] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D.Z. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, in *Proceedings of Workshop on Unified Theory and Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Report No. KEK-79-18, Tsukuba, 1979); R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).
- [13] R.N. Mohapatra and A. Rasin, Phys. Rev. Lett. **76**, 3490 (1996); Phys. Rev. D **54**, 5835 (1996); R. Kuchimanchi, *ibid.* **76**, 3486 (1996); R.N. Mohapatra, A. Rasin, and G. Senjanović, Phys. Rev. Lett. **79**, 4744 (1997).
- [14] R.N. Mohapatra and A. Rasin, Phys. Rev. Lett. **76**, 3490 (1996); Phys. Rev. D **54**, 5835 (1996).
- [15] K.S. Babu, B. Dutta, and R.N. Mohapatra, Phys. Rev. D **60**, 095004 (1999); **61**, 091701(R) (2000).
- [16] K.S. Babu, B. Dutta, and R.N. Mohapatra, Phys. Rev. Lett. **85**, 5064 (2000).
- [17] G. Giudice and A. Masiero, Phys. Lett. B **206**, 480 (1988).
- [18] R. Kitano and N. Okada, hep-ph/0107084.
- [19] L. Krauss and F. Wilczek, Phys. Rev. Lett. **62**, 1221 (1989); L.E. Ibanez and G.G. Ross, Phys. Lett. B **260**, 291 (1991); T. Banks and M. Dine, Phys. Rev. D **45**, 1424 (1992).
- [20] M. Green and J. Schwarz, Phys. Lett. **149B**, 117 (1985); Nucl. Phys. **B255**, 93 (1985); M. Green, J. Schwarz, and P. West, *ibid.* **B254**, 327 (1985).
- [21] For a review, see S. Martin, *Perspectives in Supersymmetry*, edited by G. Kane (World Scientific, Singapore, 1999); G. Giudice and R. Rattazzi, Phys. Rep. **322**, 419 (1999).
- [22] J. Ellis and M.K. Gaillard, Nucl. Phys. **B150**, 141 (1979); R.N. Mohapatra, S. Ouvry, and G. Senjanovic, Phys. Lett. **126B**, 329 (1983); M. Dugan, B. Grinstein, and L. Hall, Nucl. Phys. **B255**, 413 (1985).
- [23] See, e.g., M. Dine, A. Kagan, and R. Leigh, Phys. Rev. D **48**, 4269 (1993); K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. **83**, 2522 (1999).
- [24] See, e.g., K.S. Babu and Y. Mimura, hep-ph/0101046.
- [25] A. Casas and C. Munoz, Phys. Lett. B **306**, 288 (1993).
- [26] K. Choi, D. Kaplan, and A. Nelson, Nucl. Phys. **B391**, 515 (1993).