Spectral representation and QCD sum rules for the nucleon at finite temperature

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We examine the problem of constructing spectral representations for two-point correlation functions needed to write down the QCD sum rules in a medium. We suggest constructing them from the Feynman diagrams for the correlation functions. As an example we use this procedure to write the QCD sum rules for the nucleon current at finite temperature.

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I. INTRODUCTION

It was realized quite early in their formulation that the QCD sum rules in a medium [1,2] have several new aspects not shared by the corresponding vacuum sum rules [3]. On the operator side, there appear in general new operators in the short distance expansion of the correlation functions, due to the presence of the four-velocity vector of the medium [4,5]. Even for the old operators, the evaluation of their matrix elements in a medium may need further untested assumptions. As an example, the four-quark matrix element in a nuclear medium is evaluated by ground state saturation, different from the vacuum saturation used for the vacuum matrix element [6]. Again on the spectral side of the sum rules, unlike the vacuum case, a communicating single particle (with medium dependent mass and coupling) alone does not saturate the spectral function in the low energy region; in a medium relevant two-particle states are equally important. (Throughout this work we shall be interested only in terms linear in the distribution function. For higher order terms, contributions from multiparticle states need be included.)

The present work concerns the spectral side of the QCD sum rules. We take a closer look at the contributions of the two-particle intermediate state. If we draw all the one-loop Feynman diagrams for the correlation function, we see that, in addition to the diagrams with the (single particle) pole alone and the (two-particle) branch points alone, there are other (one-particle reducible) diagrams, which appear as the products of the pole and the branch points. They have remainders after extracting contributions that modify the pole parameters. It is these remainders that are not always included in the saturation scheme, even though they are generally of the same order as those it retains.

We illustrate our point by discussing the nucleon QCD sum rules at finite temperature T. This choice is dictated by the simplicity of the nucleon current correlation function at finite T and the availability of results for comparison. For $T \leqslant \mu$, the pion mass, and zero nucleon chemical potential, pions dominate the heat bath. The leading term in the effective chiral Lagrangian involving nucleon and pion fields describes the interaction well in this case, allowing the calculation of shifts in the pole position and the residue [7]. The operator side of the sum rules is also simpler at finite T than, say, in the nuclear medium: To $O(T^2)$ only the Lorentz scalar operators entering the vacuum sum rules can contribute, the contributions of the new, nonscalar operators being at least of $O(T^3)$. Further, the thermal averages of the operators can all be reduced to their vacuum expectation values, so that no further assumptions are required to evaluate them, other than those needed in the vacuum case. What is more, these sum rules have already been worked out [8,9] and it would be interesting to see if the above-mentioned remainder terms bring in any new contribution.

For increased sensitivity, we work with the so-called subtracted sum rules, i.e., the finite T sum rules from which the corresponding vacuum sum rules have been subtracted out, displaying only the contributions of $O(T^2)$. The spectral representations for the sum rules are worked out for physical masses and couplings. Only in the evaluation of the resulting integrals do we take the chiral limit. For completeness we have derived in an Appendix the Wilson coefficients for the product of nucleon currents using a simple coordinate space method.

In Sec. II we describe the construction of the spectral representation for the correlation function. In Sec. III we evaluate these contributions in the chiral limit. Section IV displays the sum rules and their evaluation. Section V contains our concluding remarks. Appendixes A and B, respectively, give the absorptive part of the one-loop diagrams at finite T and the Wilson coefficients in the short distance expansion of two nucleon currents, along with the thermal averages of the relevant operators.

II. CONSTRUCTION OF SPECTRAL REPRESENTATION

Consider the two-point correlation function

$$\Pi(p) = i \int d^4x \, e^{ipx} \, \mathrm{Tr}[\rho T \eta(x) \overline{\eta}(0)] \tag{2.1}$$

of the nucleon current $\eta(x)_{D,i}$, composed of three quark fields, having the quantum numbers of the nucleon, the indi-

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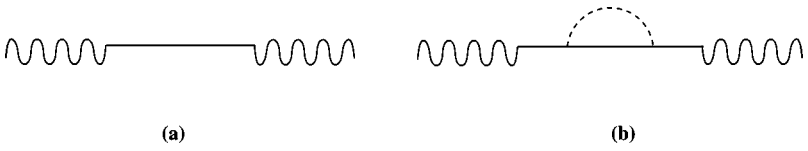


FIG. 1. Nucleon pole and self-energy insertion.

ces D,i referring to spin and isospin [10,11]. Here $\rho = e^{-\beta H}/\mathrm{Tr}\, e^{-\beta H}$ is the thermal density matrix with the Hamiltonian H of QCD. For the spectral representation we do not need to spell out the quark structure of $\eta(x)$.

A formal spectral representation for the two-point function in $E \equiv p_0$ at fixed $\vec{p}$ may be obtained immediately [12,13]. Evaluating the trace over a complete set of eigenstates of four-momentum and then again inserting the same states between the two currents to extract the x dependence, one arrives at

$$\Pi(E,\vec{p}) = i\,\mathrm{Im}\,\Pi_{11}(E,\vec{p}) + \frac{P}{\pi}\int_{-\infty}^{\infty} \frac{dE'\,\coth(\beta E'/2)\,\mathrm{Im}\,\Pi_{11}(E',\vec{p})}{E'-E}, \tag{2.2}$$

where the resulting double sum over states within the integral may be converted back to the Tr(ace) to get

$$\mathrm{Im}\,\Pi_{11}(E,\vec{p}) = \frac{1}{2}\int d^4x\, e^{ipx}\,\mathrm{Tr}\{\rho[\,\eta(x),\overline{\eta}(0)]\}. \tag{2.3}$$

Here the index 11 in $\mathrm{Im}\,\Pi$ reminds us that we are working in the real time formulation of the finite temperature field theory, where all two-point functions assume the form of a $2\otimes 2$ matrix. Thus the expression (2.1) is actually the 11-component of the corresponding matrix. However, no information is lost by considering the 11-component only, as for $p^2(=E^2-\vec{p}^{\,2})$ spacelike, where one writes the QCD sum rules, there are no other independent components. We also note that the imaginary part of the 11-component may be written as $\tanh(\beta E/2)$ times a function, the latter coinciding with the imaginary part calculated in the imaginary time formulation [14]. It is thus convenient to define

$$\mathrm{Im}\,\Pi(E,\vec{p}) = \pi^{-1}\coth(\beta E/2)\,\mathrm{Im}\,\Pi_{11}(E,\vec{p}). \tag{2.4}$$

We shall write the sum rules for $\vec{p}=\vec{0}$ and in the rest frame of the heat bath. Then we can decompose the two-point function as

$$\Pi(E,\vec{p}=\vec{0}) = \Pi_1(E^2) + \gamma_0 E\,\Pi_2(E^2), \tag{2.5}$$

where the scalar functions have, in the notation of Eq. (2.4), the spectral representations

$$\Pi_i(E^2) = \int \frac{dE'^2\,\mathrm{Im}\,\Pi_i(E'^2)}{E'^2-E^2}, \quad i=1,2, \tag{2.6}$$

the integrals receiving contributions from poles and branch cuts.

The spectral function is generally calculated on the basis of Eq. (2.3), with the risk of omitting some contributions. Here we suggest a more definitive approach, in which we identify the set of relevant Feynman diagrams and calculate the spectral function directly from them. In their work on the propagation of a nucleon through a heat bath, Leutwyler and Smilga [7] considered the set of one-loop Feynman diagrams for the correlation function, correcting the tree amplitude, shown in Figs. 1–4. From these diagrams they calculated the amplitude in the vicinity of the nucleon pole to obtain the shifts in the nucleon mass and the residue to $O(T^2)$. The same set of diagrams also suffice to saturate the spectral functions to the same order in T in the low energy region. [The diagram with the low mass isobar $\Delta(1238)$ in the intermediate state is shown in Ref. [8] to contribute to $O(T^4)$ only.] Once the spectral representations of these diagrams are constructed, one can, of course, immediately read off the position and residue of the nucleon pole.

It is simple to find the spectral representations given by these diagrams, except for a complication with diagrams of Fig. 1(b) and Figs. 2(a) and 2(b). The latter diagrams have the structure of the *product* of a (simple or double) pole and branch cuts, due to N and πN intermediate states. We have to write these contributions as a *sum* of the pole and the branch cuts. In the rest of this section we perform this separation.

Let us begin with the diagrams of Fig. 1. Omitting the $\eta\overline{N}$ and $\overline{\eta}N$ coupling constants (to be reinstated in the next section), their sum is

$$-\frac{1}{\gamma_0 E-m} - \frac{1}{\gamma_0 E-m}\Sigma(E)\frac{1}{\gamma_0 E-m} - \cdots, \tag{2.7}$$

where the ellipsis indicates the inclusion of the series of one-particle reducible self-energy diagrams. The self-energy matrix itself may be decomposed as

$$\Sigma(E) = A_1(E^2) + \gamma_0 E A_2(E^2). \tag{2.8}$$

Given the absorptive parts, to which we turn in the next section, the scalar functions $A_i(E^2)$ have spectral representations like those for Π_i in Eq. (2.6). The second term in expression (2.7) is a product of a double pole and cuts con-

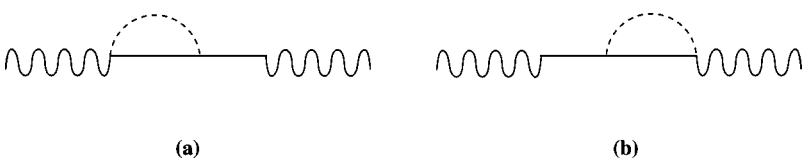


FIG. 2. Vertex correction from πN intermediate state.

FIG. 3. πN intermediate state.

tained in Σ. To write it as a sum of the pole and the cut contributions, we write $\Sigma(E)$ as

$$\Sigma(E)=A_3(E^2)+(\gamma_0 E-m)A_2(E^2),$$

$$A_3(E^2)=A_1(E^2)+mA_2(E^2), \tag{2.9}$$

and expand the scalar functions around $E^2=m^2$ to the required order to get

$$\Sigma(E)=a+(E^2-m^2)b+(\gamma_0 E-m)c+R(E), \tag{2.10}$$

where, for short,

$$a=A_3(m^2), \quad b=A_3'(m^2), \quad c=A_2(m^2), \tag{2.11}$$

the prime denoting differentiation with respect to E^2. The spectral representation for the remainder R may be obtained from those of $A_i(E^2)$ by equating the two expressions (2.9) and (2.10) for Σ. We get

$$R(E)=(E^2-m^2)^2\overline{A}_3(E^2)+(\gamma_0 E-m)(E^2-m^2)\overline{A}_2(E^2), \tag{2.12}$$

where

$$\overline{A}_3(E^2)=\int \frac{dE'^2\,\mathrm{Im}\,A_3(E'^2)}{(E'^2-m^2)^2(E'^2-E^2)}, \tag{2.13}$$

$$\overline{A}_2(E^2)=\int \frac{dE'^2\,\mathrm{Im}\,A_2(E'^2)}{(E'^2-m^2)(E'^2-E^2)}. \tag{2.14}$$

The integrals here and below run over the two branch cuts, the short cut $0\leqslant E^2\leqslant(m-\mu)^2$ and the unitarity cut $E^2\geqslant(m+\mu)^2$ (see Fig. 5 of Appendix A).

Inserting Eq. (2.10) in Eq. (2.7) and summing the series after isolating the contribution from R, we get the desired spectral representation for the diagrams of Fig. 1 as

$$-\frac{1+2mb+c}{\gamma_0 E-(m+a)}-(E^2-m^2)\overline{A}_3(E^2)-(\gamma_0 E+m)\times\{2m\overline{A}_3(E^2)+\overline{A}_2(E^2)\}. \tag{2.15}$$

Next consider the diagrams of Fig. 2, each of which is a product of a simple pole and cut contributions. They are of the form

$$-\frac{1}{\gamma_0 E-m}\Lambda(E), \tag{2.16}$$

where $\Lambda(E)$ is the vertex with a πN intermediate state. Again we decompose $\Lambda(E)$ as

$$\Lambda(E)=B_1(E^2)+\gamma_0 E B_2(E^2)$$

$$=B_3(E^2)+(\gamma_0 E-m)B_2(E^2),$$

$$B_3(E^2)=B_1(E^2)+mB_2(E^2), \tag{2.17}$$

and expand the scalar function $B_3(E^2)$ around $E^2=m^2$ to get

$$\Lambda(E)=d+S(E), \tag{2.18}$$

where $d=B_3(m^2)$ and the remainder S is

$$S(E)=(\gamma_0 E-m)\{B_2(E^2)+(\gamma_0 E+m)\overline{B}_3(E^2)\} \tag{2.19}$$

where

$$\overline{B}_3(E^2)=\int \frac{dE'^2\,\mathrm{Im}\,B_3(E'^2)}{(E'^2-m^2)(E'^2-E^2)}. \tag{2.20}$$

The expression (2.16) now separates into the pole and the cut contributions:

$$-\frac{d}{\gamma_0 E-m}-B_2(E^2)-(\gamma_0 E+m)\overline{B}_3(E^2). \tag{2.21}$$

Clearly, to find the nucleon pole position and the residue the terms with $A_i(E^2)$ and $B_i(E^2)$ in Eqs. (2.15), (2.21) are not necessary. But to write the QCD sum rules, which require the correlation function for all (large, spacelike) values of E^2, the entire spectral representation is needed.

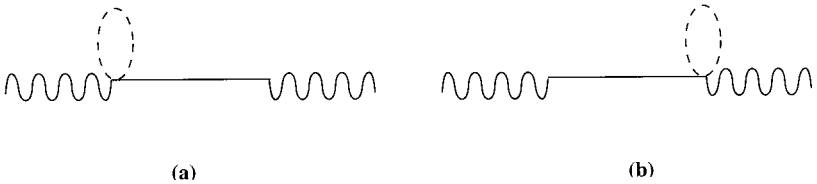
III. EVALUATION OF SPECTRAL REPRESENTATION

Having separated explicitly the contributions of the N-pole and the πN cut arising from diagrams of Figs. 1 and 2, we proceed to calculate the spectral functions of all the diagrams. We need to know the interaction vertices present in the diagrams. The πN interaction Lagrangian is given, to leading order in chiral perturbation theory, by the familiar term

$$\mathcal{L}_{\pi N}=\frac{g_A}{2F_\pi}\overline{\psi}\gamma_\mu\tau^a\psi\partial^\mu\phi^a,$$

where $g_A=1.26$, the axial-vector coupling constant of the nucleon, and $F_\pi=93$ MeV, the pion decay constant.

The coupling of $\eta(x)$ with the nucleon is defined as



(a) (b)

FIG. 4. Constant vertex correction.

$$\langle 0|\,\eta_i(x)|N_j(p)\rangle=\delta_{ij}\lambda u(p)e^{-ipx}. \tag{3.1}$$

Its couplings with a nucleon and any number of pions introduce no new constants. They may all be reduced to the coupling with the nucleon alone by using PCAC (partial conservation of axial-vector current) and current algebra [7]. We conveniently represent these couplings by writing $\eta(x)$ in terms of the relevant physical fields:

$$\eta(x)=\lambda\psi(x)+\frac{i\lambda}{2F_\pi}\vec{\tau}\cdot\vec{\phi}\gamma_5\psi(x)-\frac{\lambda}{8F_\pi^2}\vec{\phi}\cdot\vec{\phi}\psi(x). \tag{3.2}$$

We now begin with the evaluation of the self-energy integral $\Sigma(p)$. It is of the form of Eq. (A1) of Appendix A with $f(p,k)$ given by $f(p,k)=c_1\not{k}\gamma_5(\not{p}-\not{k}+m)\not{k}\gamma_5$, $c_1=-(3/4)(g_A/F_\pi)^2$. Note that any term $\sim\vec{\gamma}\cdot\vec{k}$ in f will be zero after integration over the angle of $\vec{k}$. Then for $\vec{p}=\vec{0}$ and in the chiral limit, it simplifies to $f=2c_1k_0^2E\gamma_0$. Referring to Eqs. (2.8), (A9) we get

$$\mathrm{Im}\,A_1=0,\quad \mathrm{Im}\,A_2=\frac{c_1}{4\pi^2E}n_1(|\overline{\omega}_1|)|\overline{\omega}_1|^3,\quad \overline{\omega}_1=\frac{E^2-m^2}{2E}$$

on both the branch cuts $0<E^2<m^2$ and $E^2>m^2$ in the chiral limit. The constant $2mb$ is given by

$$2mb=2m^2\int\frac{dE^2\,\mathrm{Im}\,A_2(E^2)}{(E^2-m^2)^2}=-\frac{g_A^2}{16}\frac{T^2}{F_\pi^2}. \tag{3.3}$$

The other two constants a and c are each of $O(T^3)$.

The integrals in the remainder R may be evaluated in the same way. Consider $\overline{A}_3(E^2)$ given by Eq. (2.13). Anticipating the QCD sum rules, we actually need the Borel transform of the spectral representation at large spacelike values of E^2. It is given by

$$2m\overline{A}_3(M^2)|_{Borel}=\frac{2m^2}{M^2}\int\frac{dE^2\,e^{-E^2/M^2}\,\mathrm{Im}\,A_2(E^2)}{(E^2-m^2)^2},$$

$$=-\frac{g_A^2}{16M^2}e^{-m^2/M^2}\frac{T^2}{F_\pi^2}, \tag{3.4}$$

where M is the Borel mass. The Borel transformed integrals for $(E^2-m^2)\overline{A}_3(E^2)$ and $\overline{A}_2(E^2)$ are each of $O(T^3)$.

The vertex $\Lambda(E)$ is again given by Eq. (A1) with

$$f_a(p,k)=c_2\not{k}\gamma_5(\not{p}-\not{k}+m)\gamma_5,$$

$$f_b(p,k)=c_2\gamma_5(\not{p}-\not{k}+m)\not{k}\gamma_5,$$

$$c_2=-(3/4)g_A(\lambda/F_\pi)^2,$$

for diagrams (a) and (b) of Fig. 2, respectively. With $\vec{p}=\vec{0}$ and the chiral limit, the two f's coincide and we have for the sum $f(p,k)=2c_2k_0(E-m\gamma_0)$. Comparing with Eq. (2.17) we get

$$\mathrm{Im}\,B_{1,2}(E^2)=\frac{c_2}{4\pi^2}n_1(|\overline{\omega}_1|)|\overline{\omega}_1|\overline{\omega}_1\left(1,-\frac{m}{E^2}\right). \tag{3.5}$$

It can now be seen that both d and the Borel transforms of $B_2(E^2)$ and $\overline{B}_3(E^2)$ are of $O(T^4)$. [The leading pieces from each of the two cuts are of $O(T^3)$, but they mutually cancel each other.] Thus the diagrams of Fig. 2 contribute neither to the pole nor to the cut to $O(T^2)$.

The usual contribution of the πN intermediate state is given by the diagram of Fig. 3, for which $f(p,k)=c_3\gamma_5(\not{p}-\not{k}+m)\gamma_5$, $c_3=-(3/4)(\lambda/F_\pi)^2$. For $\vec{p}=\vec{0}$, the leading piece is given by $f=c_3(m-\gamma_0E)$. Then the Borel transformed amplitude is

$$\frac{\lambda^2}{16}\frac{T^2}{F_\pi^2}\frac{e^{-m^2/M^2}}{M^2}(-m+\gamma_0E). \tag{3.6}$$

Finally, the diagrams of Fig. 4 give a constant vertex correction to the nucleon pole,

$$\frac{\lambda^2}{16}\frac{T^2}{F_\pi^2}\frac{1}{E^2-m^2}(m+\gamma_0E). \tag{3.7}$$

We are now in a position to write the complete spectral representation for the two-point function to $O(T^2)$. Collecting results from this and earlier sections, the modified nucleon pole term is given by

$$-\frac{(\lambda^T)^2}{E^2-m^2}(m+\gamma_0E), \tag{3.8}$$

with

$$\lambda^T=\lambda\left(1-\frac{g_A^2+1}{32}\frac{T^2}{F_\pi^2}\right), \tag{3.9}$$

while the Borel transform of the spectral representation due to πN branch cuts is

$$\frac{\lambda^2}{16}\{m(g_A^2-1)+(g_A^2+1)\gamma_0E\}\frac{T^2}{F_\pi^2}\frac{e^{-m^2/M^2}}{M^2}. \tag{3.10}$$

The pole term immediately gives the results of Ref. [7]: the effective nucleon mass does not shift to $O(T^2)$, while the shift in the residue is given by Eq. (3.9). The terms proportional to g_A^2 in the branch cut contribution would have been absent had the remainder R not been taken into account. The other remainder term S does not contribute to $O(T^2)$.

IV. SUM RULES

The other element needed in writing down the QCD sum rules is the short distance expansion of the product of nucleon currents. The quark content of the nucleon current, most suitable for the nucleon sum rules, is given for the proton (i.e., $i=1$, to be omitted henceforth) by [11]

$$\eta_D(x)=\epsilon^{abc}[u^{aT}(x)C\gamma_\mu u^b(x)][\gamma_5\gamma_\mu d^c(x)]_D, \tag{4.1}$$

where C is the charge conjugation matrix. Here a,b,c are the color indices and D is a Dirac index.

As already stated, we shall find the finite T sum rules, from which the vacuum parts are subtracted out, thereby eliminating the contribution of the unit operator. Since the leading thermal contribution of the gluon operators is of $O(T^4)$ and we are working to $O(T^2)$, the contributing operators are only $\overline{uu}$, $\overline{u}\sigma_{\mu\nu}t^a u G^{\mu\nu a}$, and $\overline{u}\Gamma_1 u\overline{u}\Gamma_2 u$ up to dimension six. Of these the dimension five operator has zero coefficient in the operator product expansion of the nucleon currents [10]. Also, the thermal expectation value of the four-quark operator turns out to be T independent [8]. Thus it is only the operator $\overline{uu}$ that brings in any T dependence on the operator side of the sum rules. We derive all these results systematically in Appendix B.

Usually, in writing sum rules in a medium, one takes the parameters of the pole term as unknown, to be determined by the sum rules. Here these are λ^T and m^T, the shifted pole position. Then the sum rules are obtained by equating the Borel transforms of the spectral representation and the operator product expansion. The ''subtracted'' sum rules read

$$\overline{(\lambda^T)^2 m^T e^{-(m^T)^2/M^2}}\frac{1}{M^2}+\frac{\lambda^2 m}{16M^2}(g_A^2-1)\frac{T^2}{F_\pi^2}e^{-m^2/M^2}$$
$$=\left(\frac{M}{2\pi}\right)^2\langle 0|\overline{uu}|0\rangle K\frac{T^2}{8F_\pi^2},$$
$$\overline{(\lambda^T)^2 e^{-(m^T)^2/M^2}}\frac{1}{M^2}+\frac{\lambda^2}{16M^2}(g_A^2+1)\frac{T^2}{F_\pi^2}e^{-m^2/M^2}=0, \tag{4.2}$$

where the bar over the nucleon term denotes subtraction of its vacuum value. The factor K, given by

$$K=1-\left(\frac{W^2}{M^2}+1\right)e^{-W^2/M^2},$$

deviates from unity due to incorporation of the continuum contribution on the spectral side: the imaginary part of the leading operator contribution is assumed to saturate the spectral function from a threshold W onwards [10].

Let us evaluate the sum rules for m^T and λ^T. Write

$$m^T=m\left(1+a\frac{T^2}{F_\pi^2}\right),\quad \lambda^T=\lambda\left(1+b\frac{T^2}{F_\pi^2}\right),$$

where a and b satisfy

$$b-a\frac{m^2}{M^2}+\frac{a}{2}=-\frac{1}{32}(g_A^2-1)$$
$$+\frac{1}{16m}\frac{1}{\lambda^2}\left(\frac{M^2}{2\pi}\right)^2\langle 0|\overline{uu}|0\rangle K e^{m^2/M^2}, \tag{4.3}$$

$$b-a\frac{m^2}{M^2}=-\frac{1}{32}(g_A^2+1). \tag{4.4}$$

The value of λ may be obtained from the vacuum sum rules. Instead of using the numerical estimate [10], we may use one of the sum rules directly, which reads

$$\lambda^2=-\frac{1}{m}\left(\frac{M^2}{2\pi}\right)^2\langle 0|\overline{uu}|0\rangle K e^{m^2/M^2}. \tag{4.5}$$

Then Eq. (4.3) becomes independent of the Borel mass M and one immediately gets

$$a=0,\quad b=-\frac{1}{32}(g_A^2+1), \tag{4.6}$$

reproducing the result of Ref. [7].

It should be noticed that the prediction (4.6) is not as clean as it appears. Equation (4.3) has M dependence, although mild, within its allowed range of variation. The use of the vacuum sum rule for λ^2 compensates this variation with its own to yield the above results.

V. CONCLUDING REMARKS

In this work we propose a method of constructing spectral representations for correlation functions in a medium. It is based on its Feynman diagrams which, to first order in the distribution function, consist of all one-loop diagrams. This construction differs from the usual saturation scheme of taking only the single particle (with medium dependent mass and coupling) and two-particle intermediate states by certain ''remainder'' terms contributed by the one-particle reducible diagrams.

We then use this spectral representation to write down the QCD sum rules for nucleons at finite temperature. They are shown to reproduce the temperature dependence of the nucleon mass and its coupling to the nucleon current, obtained earlier [7], justifying simultaneously the spectral construction and the sum rules.

These sum rules were obtained earlier by Koike [8], whose calculation of the spectral function was based essentially on Eq. (2.3). But he took into account the coupling of η to N, which then interacts also with πN, as well as to πN directly. In effect he included all the ''remainder'' terms arising from the Feynman diagrams. As a result his sum rules remain unaltered by our method of construction of the spectral representation. But there are other sum rules, like the ones for the vector mesons [15,16], which do have nonzero contributions from the ''remainder'' terms, not taken into account so far [17].

It will be observed that the Feynman diagram approach to constructing the spectral function automatically yields also the medium dependence of the mass and coupling of the single particle communicating with the current. It would thus appear that the only use of the sum rules in the medium is to rederive these results. However, the situation may not be so and one may well be able to extract new information on the matrix elements of operators. For example, consider the nucleon sum rules in a nuclear medium. In this case there arises the nucleon matrix element of the four-quark operator, mentioned already in the Introduction. Usually one relates it

to the σ term by the approximation of ground state saturation. The sum rules, on the other hand, may give the value of this matrix element without such an approximation.

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APPENDIX A

Here we derive a general formula for the discontinuity of one-loop Feynman graphs encountered in this work. Each of these graphs consists of a nucleon and a pion propagator but with different vertices. Thus they are of the form

$$F_{11}(p)=i\int\frac{d^4k}{(2\pi)^4}f(p,k)\Delta_1(k)\Delta_2(p-k) \tag{A1}$$

where $\Delta_{1,2}$ are the 11-components of the corresponding propagator matrices [18]; $\Delta_1(k)$ is for the pion,

$$\Delta_1(k)=\frac{i}{k^2-\mu^2+i\epsilon}+2\pi n_1(k_0)\,\delta(k^2-\mu^2), \tag{A2}$$

and $\Delta_2(p)$ is for the nucleon, after extracting the spinor factor $(\not{p}+m)$,

$$\Delta_2(p)=\frac{i}{p^2-m^2+i\epsilon}-2\pi n_2(p_0)\,\delta(p^2-m^2). \tag{A3}$$

Here n_1 and n_2 are the bosonic and fermionic distribution functions: $n_1(k_0)=(e^{\beta|k_0|}-1)^{-1}$, $n_2(p_0)=(e^{\beta|p_0|}+1)^{-1}$. The vertices and the spinor part of the nucleon propagator are all contained in $f(p,k)$.

Since we work at zero chemical potential and at $T<\mu$, the nucleon distribution function n_2 is negligible compared to n_1 for the pion. However, we wish to retain both n_1 and n_2 to show the cancellation of their product in the final expression for the imaginary part.

We find the imaginary part of $F_{11}(p)$ by simply integrating out the time component of k_μ. For this purpose, we write the finite T propagators as

$$\Delta_1(k)=(1+n_1)D_1(k)+n_1D_1^*(k), \tag{A4}$$

$$\Delta_2(p)=(1-n_2)D_2(p)-n_2D_2^*(p), \tag{A5}$$

where

$$D_1(k)=\frac{i}{k^2-\mu^2+i\epsilon},\quad D_2(p)=\frac{i}{p^2-m^2+i\epsilon}.$$

The k_0 integration over the integrands $f(p,k)D_1(k)D_2(p-k)$ and $f(p,k)D_1(k)D_2^*(p-k)$ is easily done by the residue theorem, getting simple poles in p_0 on the real axis defined by the appropriate $\pm i\epsilon$. Discarding the principal parts, we get the imaginary part as

$$\begin{aligned}\mathrm{Im}\,F_{11}(p)=\int\frac{d^3k}{(2\pi)^3}\frac{1}{4\omega_1\omega_2}[&\{(1+n_1)(1-n_2)-n_1n_2\}\\&\times\{f(\omega_1)\,\delta(E-\omega_1-\omega_2)+f(-\omega_1)\\&\times\delta(E+\omega_1+\omega_2)\}+\{n_1(1-n_2)\\&-(1+n_1)n_2\}\{f(-\omega_1)\,\delta(E+\omega_1-\omega_2)\\&+f(\omega_1)\,\delta(E-\omega_1+\omega_2)\}].\end{aligned} \tag{A6}$$

Here

$$\omega_1=\sqrt{\vec{k}^2+\mu^2},\quad \omega_2=\sqrt{(\vec{p}-\vec{k})^2+m^2},$$

$n_1\equiv n_1(\omega_1)$, and $n_2\equiv n_2(\omega_2)$. For brevity, the argument of f shows only the value of the integrated variable k_0.

Observe the ''wrong'' signs in the factors involving the distribution functions, because of which the product n_1n_2 does not cancel out. It is, however, possible to extract a factor of $\tanh(\beta E/2)$ from each of the terms, by virtue of the associated delta functions, which ''corrects'' the signs, leading to their cancellation. Using a notation similar to Eq. (2.4) we get

$$\begin{aligned}\mathrm{Im}\,F(p)=\int\frac{d^3k}{(2\pi)^3}\frac{1}{4\omega_1\omega_2}[&(1+n_1-n_2)\{f(\omega_1)\,\delta(E\\&-\omega_1-\omega_2)-f(-\omega_1)\,\delta(E+\omega_1+\omega_2)\}+(n_1+n_2)\\&\times\{f(-\omega_1)\,\delta(E+\omega_1-\omega_2)-f(\omega_1)\\&\times\delta(E-\omega_1+\omega_2)\}].\end{aligned} \tag{A7}$$

The positions of the branch cuts, where the imaginary parts are nonvanishing, are determined by the arguments of the δ functions.

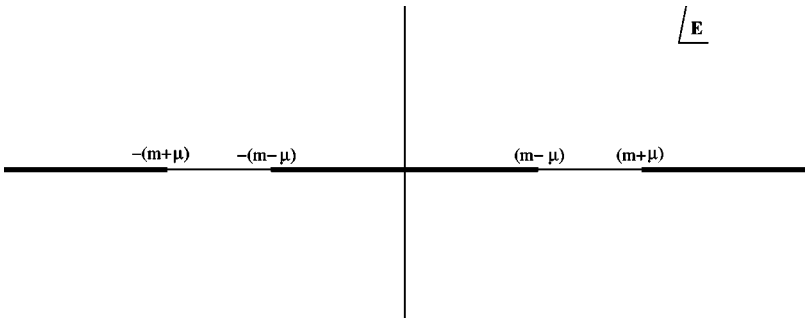

FIG. 5. The cut structure in the E plane.

We next consider the limit $\vec{p}=\vec{0}$, in which we write the sum rules. The simplified cut structure in this limit is shown in Fig. 5. It is seen that the first and the third terms in Eq. (A7) give rise to cuts for $E>0$, and the second and the fourth terms for $E<0$. The cuts in these two regions are related by symmetry under $E\to -E$. Restricting consideration to cuts for $E>0$ and setting $n_2=0$ we get

$$\begin{aligned}\mathrm{Im}\,F(E)|_{E>0}=\int \frac{d^3k}{(2\pi)^3}\frac{1}{4\omega_1\omega_2}\{&(1+n_1)f(\omega_1)\\ &\times\delta(E-\omega_1-\omega_2)+n_1 f(-\omega_1)\\ &\times\delta(E+\omega_1-\omega_2)\}.\end{aligned}\tag{A8}$$

To evaluate the integral we have, for $E>0$,

$$\frac{1}{2\omega_2}\delta(E\mp\omega_1-\omega_2)=\delta\{(E\mp\omega_1)^2-\omega_2^2\}=\frac{1}{2E}\delta(\omega_1\mp\bar{\omega}_1)$$

where $\bar{\omega}_1=(E^2-m^2+\mu^2)/2E$. Note that $\bar{\omega}_1$ is positive (negative) on the unitary (short) cut, but the value of ω_1 as given by the delta functions is always positive. We thus get

$$\mathrm{Im}\,F(E)|_{E>0}=\frac{1}{8\pi^2 E}g(|\bar{\omega}_1|)\sqrt{\bar{\omega}_1^2-\mu^2}\,f(\bar{\omega}_1),\tag{A9}$$

where $g=1+n_1$ for $E>m+\mu$ and $g=n_1$ for $0<E<m-\mu$. In writing the sum rules, we subtract the vacuum contribution (corresponding to the term "1" in g). Then $\mathrm{Im}\,F(E)$ has the same expression on both the cuts.

APPENDIX B

Here we derive systematically the known results of operator product expansion of nucleon currents [10], using the coordinate space method [19–22]. We also collect the vacuum and the thermal matrix elements of the contributing operators [8,15].

The method consists in treating the gauge field as classical and Wick expanding the operator product. The basic element is then the contraction of two, say, u-quark fields

$$\overbrace{u(x)^a_A\bar{u}}(0)^{a'}_{A'}=-iS(x)^{aa'}_{AA'}$$

where the propagator $S(x)$ satisfies

$$-i\gamma^\mu[\partial_\mu-iA_\mu(x)]S(x)=\delta^4(x),\tag{B1}$$

in the presence of the gauge fields $A_\mu(x)=gA^n_\mu(x)\lambda^n/2$, the λ^n's being the Gell-Mann matrices for $\mathrm{SU}(3)_c$. In the Fock-Schwinger gauge, defined by $x^\mu A_\mu(x)=0$, the gauge potential can be expanded in a series in the field strength $G_{\mu\nu}$ and their covariant derivatives,

$$A_\mu(x)=\frac{1}{2}x^\alpha G_{\alpha\mu}(x)+\cdots.$$

The crucial step in this method is to solve Eq. (B1) in a series at short distance [21],

$$S(x)=S_0(x)+S_2(x)+\cdots,$$

where $S_0(x)$ is the free propagator for massless quarks,

$$S_0(x)^{aa'}_{AA'}=-\frac{1}{2\pi^2}\cdot\frac{(\not{x})_{AA'}}{(x^2-i\epsilon)^2}\delta^{aa'},$$

and $S_2(x)$ is the first nonleading piece at short distance, which is proportional to the gauge field,

$$S_2(x)^{aa'}_{AA'}=(\Gamma^{\alpha\beta})_{AA'}G^{aa'}_{\alpha\beta},\quad \Gamma^{\alpha\beta}=-\frac{i}{16\pi^2}\cdot\frac{(\gamma^\alpha\not{x}\gamma^\beta)_{AA'}}{(x^2-i\epsilon)}.$$

These two terms suffice for our purpose. Note that $S_0(x)$ is diagonal in color.

With $\eta_D(x)$ given by Eq. (4.1), we have

$$\bar{\eta}_{D'}(x)=[\overline{u^a}(x)\gamma^\nu C^{-1}\overline{u}(x)^{bT}][\overline{d^c}(x)\gamma_5\gamma_\nu]_{D'}\epsilon^{abc}.$$

We are interested in the two-point correlation function

$$\begin{aligned}T\,\eta_D(x)\bar{\eta}_{D'}(0)=&\epsilon^{abc}\epsilon^{a'b'c'}(C\gamma^\mu)_{AB}\\ &\times(\gamma^\nu C^{-1})_{A'B'}(\gamma_5\gamma_\mu)_{DC}(\gamma_5\gamma_\nu)_{C'D'}\\ &\times W^{\{a\}}_{\{A\}}(x,0)\end{aligned}\tag{B2}$$

where

$$W^{\{a\}}_{\{A\}}(x,0)\equiv Tu(x)^a_A u(x)^b_B d(x)^c_C\overline{u}(0)^{a'}_{A'}\overline{u}(0)^{b'}_{B'}\overline{d}(0)^{c'}_{C'},\tag{B3}$$

is the operator product to be expanded into local operators with (singular) c-number coefficients as $x_\mu\to 0$. Its Wick expansion consists of three types of term,

$$W^{\{a\}}_{\{A\}}=I^{\{a\}}_{\{A\}}+II^{\{a\}}_{\{A\}}+III^{\{a\}}_{\{A\}},$$

corresponding, respectively, to one, two, and three contractions of the quark fields.

The single contraction terms $I^{\{a\}}_{\{A\}}$ may be obtained by contracting two d's or two u's. In the latter case, there arise four terms, all of which are equal, as may be verified by interchanging the color indices and the Dirac indices in the terms and noting that $C\gamma_\mu$ and $\gamma_\nu C^{-1}$ are symmetric matrices. Thus

$$\begin{aligned}I^{\{a\}}_{\{A\}}(x)=&-iS(x)^{cc'}_{CC'}u(x)^a_A u(x)^b_B\overline{u}(0)^{a'}_{A'}\overline{u}(0)^{b'}_{B'}\\ &-4iS(x)^{aa'}_{AA'}u(x)^b_B d(x)^c_C\overline{u}(0)^{b'}_{B'}\overline{d}(0)^{c'}_{C'}.\end{aligned}\tag{B4}$$

Similarly, terms with two and three contractions give rise to

$$\begin{aligned}II^{\{a\}}_{\{A\}}(x)=&-2S(x)^{aa'}_{AA'}S(x)^{bb'}_{BB'}\overline{d}(0)^{c'}_{C'}d(x)^c_C\\ &-4S(x)^{aa'}_{AA'}S(x)^{cc'}_{CC'}\overline{u}(0)^{b'}_{B'}u(x)^b_B\end{aligned}\tag{B5}$$

and

$$III^{\{a\}}_{\{A\}}(x) = -2iS(x)^{aa'}_{AA'}S(x)^{bb'}_{BB'}S(x)^{cc'}_{CC'}. \tag{B6}$$

Since we do not need pure gluon operators, we do not work with Eq. (B6). The quark fields in Eqs. (B4) and (B5) may now be expanded as

$$u(x) = u(0) + x^\mu D_\mu u(0) + \frac{x^\mu x^\nu}{2} D_\mu D_\nu u(0) + \cdots, \tag{B7}$$

to get the series of local operators. Our task is now to project out the Lorentz scalar operators $\overline{u}u$, $O_5 \equiv \overline{u}\sigma^{\mu\nu}G_{\mu\nu}u$, and $\overline{u}\Gamma_1 u \overline{u}\Gamma_2 u$. We consider the vacuum matrix element for this purpose.

Let us begin with $I^{\{a\}}_{\{A\}}(x)$. As we look for operators of dimension no higher than six, we replace $S(x)$, $u(x)$, and $d(x)$ by the first terms in their expansions:

$$\begin{aligned} I^{\{a\}}_{\{A\}}(x) = &-iS_0(x)^{cc'}_{CC'}\, u^a_A\, u^b_B\, \overline{u}^{a'}_{A'}\, \overline{u}^{b'}_{B'} \\ &-4iS_0(x)^{aa'}_{AA'}\, u^b_B\, d^c_C\, \overline{u}^{b'}_{B'}\, \overline{d}^{c'}_{C'} \end{aligned} \tag{B8}$$

where $u^a_A \equiv u^a_A(0)$, etc. While projecting out the scalar part of the four-quark operator, one simultaneously uses the approximation of vacuum saturation to relate it to the two-quark operator. For the first term we have $(N=12)$ [3]

$$\begin{aligned} \langle 0|u^a_A u^b_B \overline{u}^{a'}_{A'} \overline{u}^{b'}_{B'}|0\rangle = (&-\delta^{aa'}\delta^{bb'}\delta_{AA'}\delta_{BB'} \\ &+\delta^{ab'}\delta^{ba'}\delta_{AB'}\delta_{BA'}) \cdot \frac{\langle 0|\overline{u}u|0\rangle^2}{N^2}, \end{aligned} \tag{B9}$$

which gives rise to traces over γ matrices and sums over ϵ^{abc}'s. It is simple to evaluate the traces after removing the C matrix. We get

$$\langle 0|T\eta_D(x)\overline{\eta}_{D'}(0)|0\rangle_{I,\ 1st\ term} = \frac{i}{3\pi^2}\langle 0|\overline{u}u|0\rangle^2 \cdot \frac{\not{x}_{DD'}}{(x^2)^2}. \tag{B10}$$

A similar treatment for the second term results in a trace over an odd number of γ matrices, giving

$$\langle 0|T\ \eta_D(x)\overline{\eta}_{D'}(0)|0\rangle_{I,\ 2nd\ term} = 0. \tag{B11}$$

Next consider the twice contracted piece $II^{\{a\}}_{\{A\}}$. The product of the two $S(x)$'s may be expanded up to dimension two in the gauge fields to get

$$\begin{aligned} \epsilon^{abc}\epsilon^{a'b'c'}S(x)^{aa'}_{AA'}S(x)^{bb'}_{BB'} = &\, 2\,\delta^{cc'}S_{0\,AA'}S_{0\,BB'} \\ &- S_{0\,AA'}\Gamma^{\alpha\beta}_{BB'}G^{c'c}_{\alpha\beta} \\ &- S_{0\,BB'}\Gamma^{\alpha\beta}_{AA'}G^{c'c}_{\alpha\beta}. \end{aligned} \tag{B12}$$

In association with the matrices $C\gamma^\mu$ and $\gamma^\nu C^{-1}$ in the first term of $II^{\{a\}}_{\{A\}}$, the last two terms above are equal and we get its contribution to the operator product as

$$\begin{aligned} T\ \eta_D(x)\overline{\eta}_{D'}(0)|_{II,\ 1st\ term} &= -4(\gamma_5\gamma^\mu)_{DC}(\gamma_5\gamma^\nu)_{C'D'} \\ &\times\Big[\mathrm{tr}(C\gamma_\mu S_0\gamma_\nu C^{-1}S_0^T)\Big(\overline{d}_{C'}d_C + x_\lambda \overline{d}_{C'}D_\lambda d_C \\ &+ \frac{x^\lambda x^\sigma}{2}\overline{d}_{C'}D_\lambda D_\sigma d_C\Big) \\ &- \mathrm{tr}(C\gamma_\mu S_0\gamma_\nu C^{-1}\Gamma^{\alpha\beta\,T})\overline{d}_{C'}G_{\alpha\beta}d_C\Big]. \end{aligned} \tag{B13}$$

Note that the color indices in the operators are now summed. To project out the operator $\overline{d}d$ from Eq. (B13), we use

$$\langle 0|\overline{d}_{C'}d_C|0\rangle = \frac{\delta_{CC'}}{4}\langle 0|\overline{d}d|0\rangle, \tag{B14}$$

to get

$$\langle 0|T\,\eta_D(x)\overline{\eta}_{D'}(0)|0\rangle_{II,\ 1st\ term,\ \overline{u}u} = \frac{2}{\pi^4 x^6}\langle 0|\overline{d}d|0\rangle\,\delta_{DD'}. \tag{B15}$$

We project out the other operator O_5 by

$$\langle 0|\overline{d}_A D_\mu D_\nu d_B|0\rangle = \frac{1}{32}\left\{\delta_{BA}\gamma_{\mu\nu} - \frac{i}{3}(\sigma_{\mu\nu})_{BA}\right\}\langle 0|O_5|0\rangle, \tag{B16}$$

from which we also get

$$\langle \overline{d}_A G_{\mu\nu} d_B\rangle = \frac{1}{48}(\sigma_{\mu\nu})_{BA}\langle 0|O_5|0\rangle. \tag{B17}$$

It turns out that the two terms in Eq. (B13) contributing to O_5 mutually cancel each other,

$$\langle 0|T\ \eta_D(x)\overline{\eta}_{D'}(0)|0\rangle_{II,\ 1st\ term,\ O_5} = 0. \tag{B18}$$

We have yet to find the contribution of the second term of Eq. (B5). On using Eq. (B12), it can be seen to consist of terms containing the BB' element of a matrix with an odd number of γ matrices times operators like $\overline{u}_{B'}u_B$, $\overline{u}_{B'}D_\lambda D_\sigma u_B$, and $\overline{u}_{B'}G_{\alpha\beta}u_B$. Also, Eqs. (B14), (B16), and (B17) show that their projection on to the scalar operators produces a BB' element of an even number of γ matrices, so we finally get a trace over an odd number of γ matrices. We thus get [10]

$$\langle 0|T\,\eta_D(x)\overline{\eta}_{D'}(0)|0\rangle_{II,\ 2nd\ term,\ \overline{u}u,\ O_5} = 0. \tag{B19}$$

Thus, as far as the operators $\overline{u}u$, O_5, and the four-quark operator are concerned, we have for the vacuum correlation function

$$i\int d^4x\, e^{ipx}\langle 0|T\,\eta_D(x)\,\overline{\eta}_{D'}(0)|0\rangle$$

$$=\left(\frac{1}{2\pi}\right)^2 p^2\ln(-p^2)\langle 0|\overline{d}d|0\rangle\delta_{DD'}-\frac{2}{3}\langle 0|\overline{d}d|0\rangle^2\frac{\not{p}_{DD'}}{p^2}. \tag{B20}$$

Since the operators concerned are Lorentz scalars, one may think of getting the result for the thermal correlation function by simply replacing the vacuum expectation values in the above equation by their ensemble averages. However, this would not be true for the four-quark operator, for, in writing Eq. (B9) we have already used the isospin (and spin) structure of the vacuum matrix element, which differs from that for the ensemble average. A new calculation is thus necessary for the latter.

To first order in the pion distribution function, the thermal average of an operator $\mathcal{O}$ is given by [15]

$$\mathrm{Tr}(\rho\mathcal{O})=\langle 0|\mathcal{O}|0\rangle+\sum_{i=1,2,3}\int\frac{d^3k\,n_1(\omega_1)}{(2\pi)^3\,2\omega_1}\times\langle\pi^i(k)|\mathcal{O}|\pi^i(k)\rangle. \tag{B21}$$

On using the soft pion methods, the pion matrix element may be reduced to the vacuum expectation value of a double commutator,

$$\mathrm{Tr}(\rho\mathcal{O})=\langle 0|\mathcal{O}|0\rangle-\frac{1}{F_\pi^2}\int\frac{d^3k\,n_1(\omega_1)}{(2\pi)^3\,2\omega_1}\times\sum_{i=1,2,3}\langle 0|[Q_5^i,[Q_5^i,\mathcal{O}]]|0\rangle \tag{B22}$$

where Q_5^i is the axial-vector charge, $Q_5^i=\int d^3x A_0^i(x)$. An elementary way to evaluate these commutators is to write Q_5^i in terms of quark fields, express the commutators in terms of anticommutators of quark fields, and replace the latter by their canonical values. They may then be vacuum saturated as before.

We record first the well-known contribution of $\overline{uu}$ to the thermal nucleon correlation function,

$$\Pi(p)\overset{\overline{uu}}{\to}\left(\frac{1}{2\pi}\right)^2\left(1-\frac{T^2}{8F_\pi^2}\right)\langle 0|\overline{uu}|0\rangle p^2\ln(-p^2). \tag{B23}$$

For the four-quark operators in Eq. (B8), we have

$$\sum_{i=1,2,3}\langle 0|[Q_5^i,[Q_5^i,u_A^a u_B^b u^{\dagger a'}_{A'},u^{\dagger b'}_{B'}]]|0\rangle=\frac{\langle 0|\overline{uu}|0\rangle^2}{N^2}\cdot[-2\{3\gamma^0_{AA'}\gamma^0_{BB'}+(\gamma^0\gamma_5)_{AA'}(\gamma^0\gamma_5)_{BB'}\}\delta^{aa'}\delta^{bb'}-(A',a'\leftrightarrow B',b')] \tag{B24}$$

and

$$\sum_{i=1,2,3}\langle 0|[Q_5^i,[Q_5^i,u_B^b d_C^c u^{\dagger b'}_{B'},d^{\dagger c'}_{C'}]]|0\rangle=\frac{\langle 0|\overline{uu}|0\rangle^2}{N^2}\cdot[-2\{3\gamma^0_{BB'}\gamma^0_{CC'}-(\gamma^0\gamma_5)_{BB'}(\gamma^0\gamma_5)_{CC'}\}\delta^{bb'}\delta^{cc'}+4(\gamma^0\gamma_5)_{BC'}(\gamma^0\gamma_5)_{CB'}\delta^{bc'}\delta^{cb'}]. \tag{B25}$$

When these evaluations are inserted in the two-point function, it turns out that the two operators bring contributions equal in magnitude but opposite in sign. As a result, there is no thermal contribution to $O(T^2)$ from the four-quark operators,

$$\Pi(p)\overset{4\text{-}quark}{\to}-\frac{2}{3}\left(1+0\cdot\frac{T^2}{F_\pi^2}\right)\langle 0|\overline{uu}|0\rangle^2\frac{\not{p}}{p^2}. \tag{B26}$$

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