# Yukawa hierarchy transfer from the superconformal sector and degenerate sfermion masses

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We propose a new type of supersymmetric model coupled to superconformal (SC) field theories, leading simultaneously to hierarchical Yukawa couplings and completely degenerate sfermion masses. We consider models with an extra Abelian gauge symmetry to generate hierarchical structure for couplings between the standard model (SM) sector and the SC sector. Interestingly, this hierarchy is inversely transferred to the Yukawa couplings in the SM sector. In this type of model, the flavor-independent structure of the superconformal fixed point guarantees that the sfermion masses of the first and the second generations are completely degenerate at low energy.

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The origin of the hierarchy of fermion masses and mixing angles has been one of the most interesting problems in considering models beyond the standard model (SM). Especially the Yukawa texture, including the lepton sector, is attracting much attention in relation with neutrino oscillation. The most popular scenario leading to hierarchical Yukawa couplings is the Froggatt-Nielsen (FN) mechanism [1], in which flavor structure is generated by an extra $U(1)_X$ gauge interaction. The observed quark-lepton mass matrices can be explained fairly well by properly assigning the charges to quarks and leptons.

In general, a mechanism leading to Yukawa hierarchy generates also flavor-dependent soft supersymmetry breaking parameters. In the case of the extra $U(1)_X$ models, the *D*-term contribution to soft scalar masses becomes flavor dependent [2,3]. If *D*-term contributions are dominant, sfermions of the first and the second generations become very heavy at low energy, so that these sfermions decouple from flavor changing processes [4]. With such a mass spectrum of sfermions, however, one has to be careful to avoid the color-breaking problem [5].

Recently a new approach to the Yukawa hierarchy has been proposed by coupling supersymmetric standard models to superconformal (SC) gauge theories [6]. In this scenario, Yukawa couplings are exponentially suppressed thanks to large anomalous dimensions induced to quarks and leptons by an interaction with the SC sector. The resultant mass matrices are also of Froggatt-Nielsen type.

The most intriguing property of models coupled with a SC sector is the suppression of soft supersymmetry-breaking parameters [6–8]. It has been found that soft scalar masses in a softly broken superconformal gauge theory are subject to the infrared (IR) sum rules, which are given by[1]

$$\sum_i T_{\phi_i} m_{\phi_i}^2 \to 0, \quad m_{\phi_i}^2 + m_{\phi_j}^2 + m_{\phi_k}^2 \to 0. \tag{1}$$

The first equation holds for each gauge interaction with indices $T_{\phi_i}$ for chiral superfields $\phi_i$. The second one corresponds to each Yukawa interaction $y_{ijk}\phi_i\phi_j\phi_k$ present in the superconformal theory. If there exists a nonrenormalizable interaction in the superpotential at the fixed point, the sum of squared scalar masses for corresponding fields is also expected to vanish [7]. There are found special cases that suppression of the soft masses of squarks and sleptons are guaranteed by these sum rules. After the SC sector decouples, the squark and slepton masses are induced by the SM gaugino masses and appear nearly degenerate at low energy. Thus sfermion mass degeneracy can be achieved irrespectively of the origin of SUSY breaking.

Strictly speaking, however, the soft scalar masses are not completely suppressed, but converge to flavor-dependent values [7,8]. At the scale $M_c$ where the SC sector decouples from the SM sector, the soft scalar masses of squarks and sleptons are found to converge as

$$m_{q_i}^2 \to \frac{C_{ia}}{\Gamma_i} \alpha_a(M_c) M_a^2(M_c), \tag{2}$$

where $\alpha_a = g_a^2/8\pi^2$ and $M_a$ denote the gauge couplings and the gaugino masses in the SM sector, and $C_{ia}$ denote the quadratic Casimir coefficient. The important point here is

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[1] Such IR behavior was first found in Ref. [9] for supersymmetric QCD (SQCD) (and its dual), and was subsequently extended to the generic case in Ref. [7]. The off-diagonal elements of soft scalar masses were also found to be suppressed [8].

 

that $\Gamma_i$ is a numerical factor of order 1, which is obtained from the anomalous dimensions, and therefore is flavor dependent. Thus the sfermion masses are fixed to flavor-dependent values of order of $\alpha M^2$. This flavor dependence is inevitable in the Nelson-Strassler scenario, since the difference of large anomalous dimensions is responsible for the Yukawa hierarchy.

In Refs. [7,8], the degeneracy in the low energy sfermion spectrum is investigated by considering the SM-gaugino mass effects. The degeneracy in the right-handed sleptons was found to be rather weak. As a result, the experimental bounds for lepton flavor violating processes, e.g., $\mu \to e + \gamma$, seem difficult to be satisfied, unless $\Gamma_i^{-1}$ is much smaller than 1 or the SM gauginos are fairly heavy.

In this article we consider a new type of model coupled to SCFTs, in which the hierarchical texture of fermion mass matrices and complete degeneracy of sfermion masses are compatible with each other. The flavor dependence of sfermion masses given by Eq. (2) arises from a difference among the anomalous dimensions of quarks and leptons. Therefore we first demand that quarks and leptons in the first and the second generations carry the identical anomalous dimension at the SC fixed point. In this case, the hierarchy of the SM-sector Yukawa couplings cannot be generated by a difference of the large anomalous dimensions like in the Nelson-Strassler models, while the sfermion masses are degenerate completely. In our scenario, the origin of the hierarchy resides in the couplings between the SM sector and the SC sector, which can be made hierarchical, e.g., by introducing an extra $U(1)_X$ gauge interaction. As we will show shortly, the SC sector plays the role not only of washing out flavor dependence in the soft scalar masses, but also of transferring the hierarchical Yukawa structure to the SM sector.

As an illustrative example, let us first consider a simple model based on $SU(5)$ grand unified theory (GUT). In the conventional FN mechanism, the flavor-dependent $U(1)_X$ charges are sometimes assigned as $n_i=(2,1,0)$ to $\psi_i=(q_L,u_R^c,e_R^c)_i$ in **10**, and $-1$ to the FN field $\chi$. In that case, quark-lepton mass matrices are qualitatively explained by assuming that $\chi$ develops the vacuum expectation value (VEV) of $\langle\chi\rangle\sim M_0/20$, where $M_0$ is the cutoff (string) scale.

Now suppose that the chiral superfields $(\Phi_i,\bar{\Phi}_i)$ in the SC sector belong to $\bar{\mathbf{5}}$ and also carry the $U(1)_X$ charges. Explicitly we assign $a_i$ to $\bar{\Phi}_i\Phi_i$ as well as $n_i$ to $\psi_i$. As is seen later on, it is the $U(1)_X$ charges $a_i$ of the SC-sector fields, not $n_i$ of SM-sector fields, that is responsible for the desired Yukawa hierarchy. Although $n_i$ does not play an important role in our model, we keep it for a moment. The invariant superpotential is given by

$$W=\tilde{\lambda}_i\left(\frac{\chi}{M_0}\right)^{n_i+a_i}\psi_i\bar{\Phi}_i\Phi_i+\tilde{y}_{ij}\left(\frac{\chi}{M_0}\right)^{n_i+n_j}\psi_i\psi_jH, \quad (3)$$

and we take the bare couplings $\tilde{\lambda}_i$ and $\tilde{y}_{ij}$ to be of order one. Also $H$ is the Higgs field in **5**, and we assume $H$ to be $U(1)_X$ neutral for simplicity. Since these operators are higher dimensional, the couplings $\tilde{\lambda}_i$ and $\tilde{y}_{ij}$ are usually thought to be suppressed towards IR.

As in the conventional FN mechanism, let us assume that the $U(1)_X$ gauge symmetry is spontaneously broken by nonzero VEV $\langle\chi\rangle$. For instance, if the $U(1)_X$ is anomalous $U(1)$ with the Green-Schwarz anomaly cancellation, the size of the VEV is determined by the loop-induced Fayet-Iliopoulos term and thus is expected to be of the desired magnitude $\langle\chi\rangle\sim M_0/20$. After redefining the dynamical FN field at the broken vacuum, the superpotential also contains the $U(1)_X$ breaking terms

$$W=\lambda_i\psi_i\bar{\Phi}_i\Phi_i+y_{ij}\psi_i\psi_jH, \quad (4)$$

where $\lambda_i=\tilde{\lambda}_i(\langle\chi\rangle/M_0)^{n_i+a_i}$ and $y_{ij}=\tilde{y}_{ij}(\langle\chi\rangle/M_0)^{n_i+n_j}$. The operators $\psi_i\bar{\Phi}_i\Phi_i$ are relevant thanks to negative anomalous dimensions of the SC-sector fields. Therefore the originally suppressed couplings $\lambda_i$ grow and approach their IR fixed points. Eventually the SM-sector fields $\psi_i$ acquire large anomalous dimensions, which lead to the suppression of the Yukawa couplings $y_{ij}$. Here we restrict ourselves to the cases where the gauge group and the representations of the SC-sector fields are identical, and therefore the anomalous dimensions at the IR fixed points are also common for all flavors. However, it should be noted that $\lambda_i$ reaches the fixed point at a much lower energy, if its initial value is suppressed. As $\lambda_i$ is smaller, the growth of the anomalous dimension of $\psi_i$ becomes slower and, therefore, Yukawa coupling $y_{ij}$ starts to decrease at a lower energy scale.

In order to see the running of the Yukawa couplings and their IR behavior explicitly, we examine the renormalization group equations (RGE's) for $\lambda_i$ and $y_{ij}$. By representing the anomalous dimension of $\phi$ by $\gamma(\phi)$, the RGE's are given by

$$\frac{d\ln\lambda_i}{d\ln\mu}=\frac{1}{2}[\gamma(\psi_i)+2\gamma(\Phi_i)],$$

$$\frac{d\ln y_{ij}}{d\ln\mu}=\frac{1}{2}[\gamma(\psi_i)+\gamma(\psi_j)+\gamma(H)]. \quad (5)$$

Here we ignore the off-diagonal elements of the anomalous dimensions for simplicity.[2] From these equations, we obtain

$$\ln\frac{y_{ij}(M_0)}{\lambda_i(M_0)\lambda_j(M_0)}=\ln\frac{y_{ij}(\mu)}{\lambda_i(\mu)\lambda_j(\mu)}+\int_\mu^{M_0}\frac{d\mu'}{\mu'}\left[\frac{1}{2}\gamma(H)(\mu')-\gamma(\Phi_i)(\mu')-\gamma(\Phi_j)(\mu')\right]. \quad (6)$$

[2]In general we should include the off-diagonal couplings $\lambda_{ik}\psi_i\bar{\Phi}_k\Phi_k$ in the superpotential (4) and the off-diagonal elements of the anomalous dimensions as well. However, it is found that there still exists an IR attractive fixed point up to the field rotation $\hat{\psi}_k\equiv\Sigma_i\psi_iU_{ik}$. Then the same result as Eq. (7) holds with $y_{ij}$ replaced by $\hat{y}_{kl}\equiv\Sigma_{i,j}U_{ki}^{-1}U_{lj}^{-1}y_{ij}$. We will present a detailed discussion on these points in a separate publication [13].

Provided that the running couplings $\lambda_i(\mu)$ approach close to the common value of the IR fixed point, the ratio of the Yukawa couplings are evaluated as

$$\ln\frac{y_{ij}(\mu)}{y_{kl}(\mu)} \sim \ln\left(\frac{\langle\chi\rangle}{M_0}\right)^{a_k+a_l-a_i-a_j} + \int_{\mu}^{M_0}\frac{d\mu'}{\mu'}[\gamma(\Phi_i\Phi_j)(\mu') - \gamma(\Phi_k\Phi_l)(\mu')], \quad (7)$$

where we used $\lambda_i(M_0)\sim(\langle\chi\rangle/M_0)^{n_i+a_i}$ as well as $y_{ij}(M_0)\sim(\langle\chi\rangle/M_0)^{n_i+n_j}$. The difference of anomalous dimensions $\gamma(\Phi_i)$ disappears as the SC-sector couplings approach towards the IR fixed point. Therefore the integral in the right-hand side of Eq. (7) is expected to be rather small in the situation that $\gamma(\Phi_i)$ are close to each other also at the cutoff scale $M_0$. Indeed the integral is found to vanish in some models as is explicitly shown later.

It should be noted that the ratio among SM-sector Yukawa couplings $y_{ij}$ are determined solely by the ratio among initial values $\lambda_i(M_0)$, i.e., the extra $U(1)_X$ charges for the SC-sector fields $a_i$. In other words, the suppressed Yukawa couplings are obtained independently of $n_i$, once all of the couplings $\lambda_i$ have approached the IR fixed point. Therefore let us take $n_i=0$ hereafter. Interestingly also the hierarchy in the SC-sector Yukawa couplings $\lambda_i$ are transferred to the SM-sector Yukawa couplings $y_{ij}$ in the inverse order.

The SC sector must decouple[3] from the SM sector at some scale $M_c$, which we assume to be common for all SC-sector fields. This is an important condition to obtain the degenerate sfermion masses. At this scale, the desired hierarchical structure $y_{11}\ll y_{22}\ll y_{33}\sim 1$ can be realized at $M_c$, if the $U(1)_X$ charges are assigned to the SC-sector fields so that $1\sim\lambda_1\gg\lambda_2\gg\lambda_3$. Note that the coupling $\lambda_3$ of the third generation particles (or top for small $\tan\beta$) to the SC sector should be absent or kept small until the decoupling scale $M_c$, since their masses should not be suppressed. The ratio $y_{11}(M_c)/y_{22}(M_c)$ is estimated as $(\langle\chi\rangle/M_0)^{2(a_2-a_1)}$.

The soft scalar masses for $\psi_i$ are exponentially suppressed if the IR sum rule given by Eq. (1) can make them vanish. This condition is equivalent to the condition that the anomalous dimensions of $\psi_i$ can be uniquely determined from the algebraic equations for anomalous dimensions at the fixed point [7,8]. In other words, the $R$ charge of $\psi_i$ must be uniquely determined. Therefore we seek for models such that the anomalous dimensions for the quarks and leptons can be fixed and are identical for different flavors. Then the complete degeneracy of SM-sector sfermion masses as well as the Yukawa hierarchy transfer are realized. Here we shall present several types of such models.

[3]A naive way for decoupling is to give mass to the SC-sector fields. In this example, however, $\Phi_i$ and $\bar{\Phi}_i$ cannot form their mass term. A model admitting mass terms will be examined below. We should also stress that the decoupling scale $M_c$ is constrained to be around $10^{13-16}$ GeV by phenomenological requirements [7,8] that the lightest superparticle should be neutral and that SM-sector gauge couplings do not blow up until the cutoff scale $M_0\sim 10^{18}$ GeV.

(a) *Models with* $G^{N_f}\times G_{\rm SM}\times U(1)_X$. The SC-sector gauge group in the first type of models is a product of the same groups, $G_{\rm SC}=G_1\times G_2\times\cdots$ with $G_i\simeq G$. $N_f$ is the number of flavors to acquire large anomalous dimensions. Each SC-sector field $(\Phi_i,\bar{\Phi}_i)$ is charged under the $i$th factor group $G_i$, and is assumed to have the same representation for all $i$. Then the anomalous dimensions $\gamma(\Phi_i)$ and $\gamma(\bar{\Phi}_i)$ are uniquely determined at the fixed point and also are flavor independent.

When we ignore the SM gauge interactions, the IR sum rules for the soft scalar masses are given by $m_{\psi_i}^2+m_{\Phi_i}^2+m_{\bar{\Phi}_i}^2\to 0$ and $m_{\Phi_i}^2+m_{\bar{\Phi}_i}^2\to 0$ corresponding to the IR attractive behavior of the SC-sector Yukawa couplings $\lambda_i$ and the SC-sector gauge couplings, respectively. Thus the soft scalar masses of $\psi_i$ are suppressed in these types of models. In practice the scalar masses converge to nonzero values at IR by the effect of the SM gauge interaction and the gaugino masses as Eq. (2). Since these convergent values are identical, sfermions in the SM sector have a completely degenerate mass at low energy. A toy model belonging to this class will be explicitly examined later on.

(b) *Models with nonrenormalizable interactions* $(\bar{\Phi}_i\Phi_i)^2$. When the anomalous dimensions of the SC fields $(\Phi_i,\bar{\Phi}_i)$ are negatively large enough, some nonrenormalizable operators become relevant. Then it is plausible that the SCFT with nonrenormalizable interactions is realized, although the proof has not been known. Here we assume dimension 5 operators to appear in the superpotential at the IR fixed point and consider the superpotential given by

$$W=\tilde{\lambda}_i\left(\frac{\chi}{M_0}\right)^{a_i}\psi_i\bar{\Phi}_i\Phi_i+\tilde{\zeta}_i\left(\frac{\chi}{M_0}\right)^{2a_i}(\bar{\Phi}_i\Phi_i)^2+y_{ij}\psi_i\psi_jH. \quad (8)$$

If $\bar{\Phi}_i\Phi_i$ can form a singlet under the SM gauge group, we need to forbid possible interactions $(\chi/M_0)^{a_i}\bar{\Phi}_i\Phi_i$ by $Z_2$ parity for example. Otherwise SC fields decouple at the scale of the expectation value of the FN field by obtaining masses of this order.

The breaking terms of the extra $U(1)_X$ symmetry are given by

$$\lambda_i\psi_i\bar{\Phi}_i\Phi_i+\zeta_i(\bar{\Phi}_i\Phi_i)^2, \quad (9)$$

with suppressed couplings $\lambda_i$ and $\zeta_i$. If the couplings $\zeta_i$ also grow and are attracted to a nontrivial fixed point, the anomalous dimensions of the SC-sector fields are fixed[4] as $\gamma(\Phi_i)+\gamma(\bar{\Phi}_i)=-1$. Therefore the anomalous dimensions $\gamma(\psi_i)$ of quarks and leptons are also fixed to 1 for all flavors.

[4]The condition for the gauge beta function to vanish may also be satisfied by introducing other SC-sector fields than $\Phi_i$ and $\bar{\Phi}_i$.

The IR sum rule for the soft scalar masses will be given by $m^2_{\psi_i}+m^2_{\Phi_i}+m^2_{\bar{\Phi}_i}\to 0$ and $m^2_{\Phi_i}+m^2_{\bar{\Phi}_i}\to 0$ corresponding to the IR attractive behavior of the couplings $\lambda_i$ and $\zeta_i$, respectively. Thus the soft scalar masses of $\psi_i$ are suppressed also in this type of models.

(c) *Left-right symmetric models with nonrenormalizable interactions.* We may make models of type B more flexible by assuming left-right symmetry. For example, we introduce the gauge group $G\times G_{\rm SM}\times U(1)_X$ and chiral superfields $Q:(\mathbf{N},\bar{\mathbf{R}},0)$; $\bar{Q}:(\bar{\mathbf{N}},\mathbf{R},0)$; $P_i:(\mathbf{N},\mathbf{1},a_i)$; $\bar{P}_i:(\bar{\mathbf{N}},\mathbf{1},a_i)$; $q_i:(\mathbf{1},\mathbf{R},0)$; $\bar{q}_i:(\mathbf{1},\bar{\mathbf{R}},0)$, and also the FN field $\chi:(\mathbf{1},\mathbf{1},-1)$. Then the superpotential in the SC sector is written as

$$W=\tilde{\lambda}_i\left(\frac{\chi}{M_0}\right)^{a_i}q_i\bar{P}_iQ+\tilde{\bar{\lambda}}_i\left(\frac{\chi}{M_0}\right)^{a_i}\bar{q}_i\bar{Q}P_i+\tilde{\zeta}_i\left(\frac{\chi}{M}\right)^{2a_i}(\bar{P}_iP_i)^2. \tag{10}$$

If these nonrenormalizable interactions exist at the fixed point, the IR sum rule tells $m^2_{P_i}+m^2_{\bar{P}_i}\to 0$. Since we may suppose $m^2_{P_i}=m^2_{\bar{P}_i}$ by left-right symmetry, these masses are suppressed. Note that $D$-term contributions to the soft scalar masses of $P_i$ and $\bar{P}_i$ are also the same because they carry the same $U(1)_X$ charge. Combined with other sum rules, the scalar masses of $\psi_i$ are thus found to be suppressed.

Now let us examine a model of type $A$ explicitly by performing numerical analysis of approximated RG equations. Consider a model[5] with $G_{\rm SC}=SU(5)_1\times SU(5)_2$, $G_{\rm SM}=SU(3)^3\times \mathbf{Z}_3$ and $U(1)_X$. We simply assume that SM-sector fields of the third generation decouple from the SC sector and examine Yukawa couplings and sfermion masses of the first and the second generations. The following chiral superfields are introduced:

| | $SU(3)\times SU(3)\times SU(3)$ | $SU(5)_1$ | $SU(5)_2$ | $U(1)_X$ | $\mathbf{Z}_2$ |
|---|---|---|---|---|---|
| $\Phi_1$ | $(\mathbf{3},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{3})$ | $\mathbf{5}$ | $\mathbf{1}$ | 0 | − |
| $\bar{\Phi}_1$ | $(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})+(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})+(\mathbf{1},\mathbf{1},\bar{\mathbf{3}})$ | $\bar{\mathbf{5}}$ | $\mathbf{1}$ | 0 | − |
| $\Phi_2$ | $(\mathbf{3},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{3})$ | $\mathbf{1}$ | $\mathbf{5}$ | 1/2 | − |
| $\bar{\Phi}_2$ | $(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})+(\mathbf{1},\bar{\mathbf{3}},\mathbf{1})+(\mathbf{1},\mathbf{1},\bar{\mathbf{3}})$ | $\mathbf{1}$ | $\bar{\mathbf{5}}$ | 1/2 | − |
| $\psi_i$ | $(\mathbf{3},\bar{\mathbf{3}},\mathbf{1})+(\mathbf{1},\mathbf{3},\bar{\mathbf{3}})+(\bar{\mathbf{3}},\mathbf{1},\mathbf{3})$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | + |
| $H$ | $(\mathbf{3},\bar{\mathbf{3}},\mathbf{1})+(\mathbf{1},\mathbf{3},\bar{\mathbf{3}})+(\bar{\mathbf{3}},\mathbf{1},\mathbf{3})$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | + |
| $\chi$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | −1 | − |

Here $\mathbf{Z}_2$-parity is needed to forbid interactions of $\chi\bar{\Phi}_i\Phi_i$. We consider the superpotential

$$W=\sum_{i=1,2}\lambda_i\psi_i\bar{\Phi}_i\Phi_i+\sum_{i=1,2,3}y_i\psi_i\psi_iH, \tag{11}$$

where SM Yukawa matrix is restricted to the diagonal one $y_i$ for simplicity. At the cutoff scale $M_0$, the SC-sector Yukawa couplings are given by $\lambda_i(M_0)\sim(\langle\chi\rangle/M_0)^{a_i}$ with $a_i=(0,1)$. The SM Yukawa couplings $y_i$ are also assumed to be of order 1. In the following numerical analysis, we fix the expectation value to $(\langle\chi\rangle/M_0)^2=1/300$.

The explicit form of the anomalous dimensions are known only in perturbative expansion. Here we use the anomalous dimensions evaluated at one-loop order,

$$\gamma(\Phi_i)=\gamma(\bar{\Phi}_i)=-\frac{24}{5}\alpha'_i+3\alpha_{\lambda_i}-\frac{8}{3}\alpha,$$

$$\gamma(\psi_i)=5\alpha_{\lambda_i}-\frac{16}{3}\alpha+6\alpha_{y_i}. \tag{12}$$

Here $\alpha'_i=g'^2/8\pi^2$ and $\alpha=g^2/8\pi^2$ are the gauge couplings of $SU(5)_i$ and $SU(3)^3$, respectively, and $\alpha_{\lambda_i}=|\lambda_i|^2/8\pi^2$, $\alpha_{y_i}=|y_i|^2/8\pi^2$. We expect that the qualitative aspect of running couplings is captured by the perturbative renormalization group, though the beta functions are not justified near the fixed point.

First let us examine how the Yukawa hierarchy is transferred from the SC-sector couplings $\lambda_i$ to the SM Yukawa couplings $y_i(i=1,2)$. The beta functions for the couplings appearing in the superpotential may be written down as[6]

$$\mu\frac{d\alpha'_i}{d\mu}=-\alpha'^2_i[6+9\gamma(\Phi_i)],$$

$$\mu\frac{d\alpha_{\lambda_i}}{d\mu}=\alpha_{\lambda_i}[\gamma(\psi_i)+2\gamma(\Phi_i)], \tag{13}$$

[5]A prototype of this model was studied in Ref. [6]. $G_{\rm SM}$ is regarded as a subgroup of $E_6$, and the quark and leptons are contained in three copies of the 27-dimensional representation. We may consider a model with $G_{\rm SM}=SU(5)$ as well.

[6]In the Novikov-Shifman-Vainstein-Zakharov (NSVZ) scheme [10], the gauge beta function becomes singular at some strong coupling. We simply omit this pole singularity as an approximation.

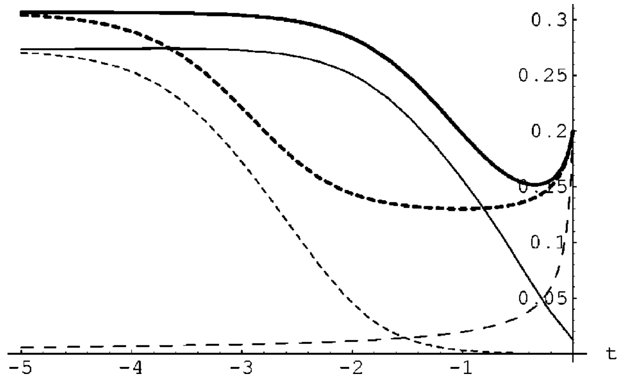


FIG. 1. Running of gauge couplings, Yukawa couplings in the SC-sector, and the SM gauge coupling.

$$\mu\frac{d\alpha}{d\mu}=-\alpha^2[-13+5\gamma(\Phi_1)+5\gamma(\Phi_2)+3\gamma(\psi_1)+3\gamma(\psi_2)],$$

$$\mu\frac{d\alpha_{y_i}}{d\mu}=\alpha_{y_i}[2\gamma(\psi_i)+\gamma(H)],$$

where we neglected the anomalous dimensions for the third generation and Higgs fields, which do not couple with the SC sector.

The ratio of Yukawa couplings $y_i$ between the first and the second generations is estimated for this model as

$$\ln\frac{y_1(\mu)}{y_2(\mu)}=2\ln\frac{\langle\chi\rangle}{M_0}+2\int_{\mu}^{M_0}\frac{d\mu'}{\mu'}[\gamma(\Phi_1)(\mu')-\gamma(\Phi_2)(\mu')]. \tag{14}$$

On the other hand, the SC-sector gauge couplings are found to be

$$\frac{1}{\alpha_i'(\mu)}=\frac{1}{\alpha_i'(M_0)}-6\ln\frac{M_0}{\mu}-9\int_{\mu}^{M_0}\frac{d\mu'}{\mu'}\gamma(\Phi_i)(\mu'), \tag{15}$$

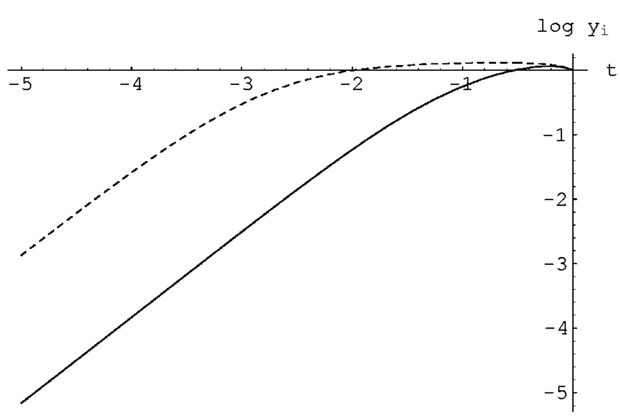


FIG. 2. Power law running of Yukawa couplings and hierarchy transfer.

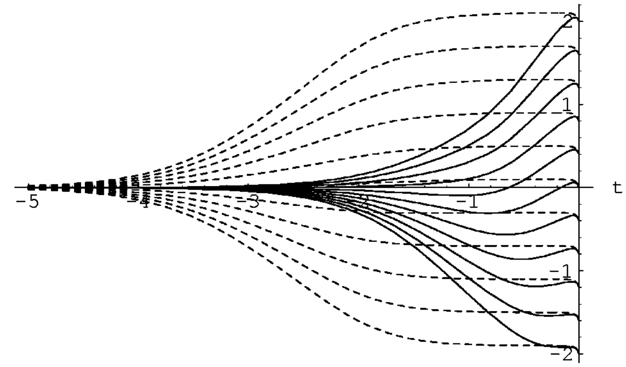


FIG. 3. Convergence behavior of the soft scalar masses, $m^2_{\psi 1}$ and $m^2_{\psi 2}$.

by integrating their RG equations given in Eqs. (13). Here note that $\alpha_1'(\mu)$ and $\alpha_2'(\mu)$ become equal as approaching the fixed point. If the initial gauge couplings are equal, $\alpha_1'(M_0)=\alpha_2'(M_0)$, Eq. (15) tells us that the integral in Eq. (14) vanishes as announced before. Therefore the Yukawa transfer is precisely realized; $y_1(\mu)/y_2(\mu)=(\langle\chi\rangle/M_0)^2$. Even when the initial gauge couplings are different, we find that this formula holds fairly well by examining it numerically.

In Fig. 1 the RG running behavior of the SC-sector couplings, $\alpha_i'$ and $\lambda_i$, are shown in the case of $\alpha_1'(M_0)=\alpha_2'(M_0)=0.2$. The running couplings of the first and the second generations are shown by solid and dashed lines, respectively. The bold lines stand for the SC-sector gauge couplings, and $t=\log_{10}(\mu/M)$ is the renormalization scale parameter. Both SC-sector Yukawa couplings $\lambda_i$ approach the same value in the IR region. The long-dashed line shows the SM-sector gauge coupling $\alpha$, whose initial value is set to 0.2. An aspect of the Yukawa suppression is shown in Fig. 2 with bare Yukawa couplings $y_i(M_0)=1.0$. The solid and dashed lines stand for $\log_{10}y_1(\mu)$ and $\log_{10}y_2(\mu)$, respectively.

Next let us examine convergence behavior and IR degeneracy of the soft scalar masses. The beta functions for the soft supersymmetry-breaking parameters are immediately derived from rigid ones by using Grasmannian expansion, whose explicit forms are given, for example,[7] in Refs. [7,8]. In Fig. 3 the convergence behavior of soft scalar masses is demonstrated. The bold lines and the dashed lines show the running behavior of soft scalar masses $m^2_{\psi_1}$ and $m^2_{\psi_2}$, respectively, with varying their bare parameters in $[-2.0\,m_0^2, 2.0\,m_0^2]$. These running couplings are obtained by analyzing the coupled beta functions for the soft supersymmetry-breaking parameters deduced from Eqs. (13). The other soft parameters, gaugino masses, $A$ parameters, and scalar masses of $(\Phi_i,\bar{\Phi}_i)$ are all set to $m_0$ at $M_0$. It is seen that both soft scalar masses $m^2_{\psi 1}$ and $m^2_{\psi 2}$ converge to the same value as the SC-sector couplings approach their fixed point.

To summarize, it is possible to construct models with a

[7] See also Refs. [11].

SC sector such that the Yukawa hierarchy and sfermion mass degeneracy are realized simultaneously. The origin of hierarchy is attributed to the hierarchical couplings $\lambda_i$ of quarks and leptons to SC-sector matters, which is inversely transferred to the SM-sector Yukawa couplings by superconformal dynamics. Here we have considered models with an extra $U(1)_X$ gauge symmetry to make these couplings $\lambda_i$ hierarchical, although the origin of the hierarchy may be introduced in some other ways. The flavor independent structure of the superconformal fixed point ensures the degenerate sfermion masses at low energy. The detailed analysis of the present scenario will be reported in a separate publication [13], including a proper treatment of flavor mixing as well as phenomenological constraints on the decoupling scale.

We may consider other scenarios using superconformal field theories to wash out the flavor dependence in the sfermion masses. For example, it is possible to eliminate the *D*-term contributions in the sfermion masses. It has been known that the *D*-term contributions are proportional to the scalar mass of the FN field [2]. Therefore, if this scalar mass of the FN field is suppressed by proper coupling with an SC sector, then the *D*-term contributions are washed out. Such scenarios will be reported elsewhere [12].


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