

4D models of Scherk-Schwarz GUT breaking via deconstruction

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We examine new classes of GUT models where the GUT gauge group is broken by a 4D analogue of the Scherk-Schwarz mechanism. These models are inspired by “deconstructed” 5D Scherk-Schwarz orbifold models. However, no fine-tuning of parameters or assumption of higher dimensional Lorentz invariance is necessary, and the number of lattice sites can be as low as just two. These models provide simple ways to solve the doublet-triplet splitting problem, change proton decay predictions, and may provide insight into the structure of the CKM matrix. Since the number of fields in these models is finite, the corrections to the unification of gauge couplings can be reliably calculated, and as expected result only in threshold corrections to the differential running of the couplings. Our analysis also suggests new 4D models which can enjoy the benefits of orbifold models but cannot be obtained by deconstruction of a 5D model.

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I. INTRODUCTION

Unification of the gauge couplings in the supersymmetric extension of the standard model (SM) is one of the strongest experimental hints both for the existence of supersymmetry (SUSY), and for grand unified theories (GUTs). However, most SUSY GUT theories have several problematic aspects; most notably the doublet-triplet splitting problem (that is, why the color triplet Higgs fields necessary for GUTs are so much heavier than the corresponding doublets), and the non-observation of proton decay. Even though several possible resolutions to the doublet-triplet splitting problem exist in 4D [1,2], the actual implementation of these ideas usually leads to complicated models [3]. Recently, new solutions to these problems have been suggested by reviving [4] the old Scherk-Schwarz (SS) [5] idea of breaking symmetries by boundary conditions in extra dimensional models. Many of the basic ideas used in these models have been discussed within the context of string theory in the 1980s [6]. One of the main achievements of the models proposed recently is to separate the essential features of the Scherk-Schwarz mechanism from the additional constraints imposed by string theory, and to try to build minimal realistic models in an effective field theory context. In particular, Kawamura [7], Altarelli and Feruglio [8], and Hall and Nomura [9] proposed specific models with one extra dimension and an $SU(5)$ bulk gauge symmetry. (See also [10]. For earlier work on symmetry breaking by the SS mechanism see [11].) In the supersymmetric version of this model, the extra dimension is an $S^1/Z_2 \times Z_2'$ orbifold, where one of the Z_2 projections reduces

the zero mode spectrum to that of 4D $\mathcal{N}=1$ supersymmetric theories, while the second Z_2 explicitly breaks the GUT symmetry to $SU(3) \times SU(2) \times U(1)$ at the orbifold fixed-point. One of the immediate consequences of this setup is that there is no need for a field that breaks the GUT symmetry—the orbifold does that. Depending on the details of the rest of the setup, most of the other problems of SUSY GUTs can also be resolved. In one of the simplest implementations proposed by Hebecker and March-Russell [12], all SM fields live at the fixed point where $SU(5)$ is broken so there is no need to introduce a Higgs triplet, and proton decay is absent. Another possibility is to introduce the Higgs fields in the bulk with the SM matter fields at one or the other orbifold fixed points. In this case the doublet-triplet splitting problem is resolved by the triplets not having a zero mode, and proton decay is suppressed arranging that dangerous dimension five operators are absent due to the structure of the triplet masses enforced by global symmetries [9]. For more recent work on SS breaking of symmetries see [13–19].

Recently it has been realized, that many seemingly extra dimensional ideas can be implemented within a purely 4D theory by considering a “deconstructed” (or latticized) version of the extra dimension [20,21]. This construction can be used to obtain a variety of interesting 4D models [22]. The aim of this paper is to give simple 4D models that mimic the above outlined SS-type breaking of the GUT symmetry. For this we will use the supersymmetric versions of deconstructed extra dimensions obtained in [23] based on the earlier work of [24]. We will present several different GUT models. In all of them there will be $N-1$ copies (where N can be as low as just two) of the $SU(5)$ gauge group, and a single $SU(3) \times SU(2) \times U(1)$ group, which will be broken to the diagonal SM gauge group. The explicit breaking of the $SU(5)$ symmetry at the last link is the deconstructed version of the orbifold point breaking the GUT symmetry. Using this

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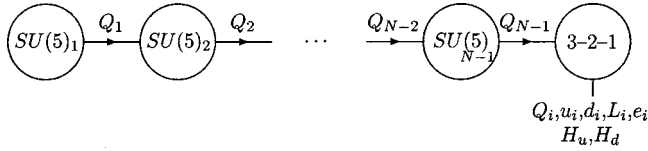


FIG. 1. The supersymmetric orbifold moose.

basic setup we will obtain different models depending on how the Higgs and SM matter fields are introduced. First we will include both the Higgs and the matter fields only into the last SM gauge group, while later we will present various modifications of the simplest model. Since there are a finite number of states in this theory, the correction to the unification of gauge couplings can be reliably estimated. This correction to the difference of the gauge couplings is of the same form as the threshold correction found in the continuum case. Note that after this work was completed, we learned that similar ideas have been pursued by Cheng, Matchev, and Wang [33].

II. GUTS WITHOUT HIGGS TRIPLET

A. The simplest model

The first model that we will consider is the simplest possible construction based on the higher dimensional example presented in [12]. In this 5D model the bulk has eight supercharges ($\mathcal{N}=2$ supersymmetry in 4D). Half of these supersymmetries are broken by the first Z_2 orbifold projection, while the second projection breaks $SU(5)$ to the SM gauge group. All minimal supersymmetric standard model (MSSM) matter and Higgs fields are included on the second orbifold fixed-point where the gauge groups is reduced to the SM group.

A simple construction for the S^1/Z_2 orbifold with 4 supercharges in 4D has been presented in [23]. The construction is summarized below:

	$SU(M)_1$	$SU(M)_2$	\dots	$SU(M)_{N-1}$	$SU(M)_N$
Q_1	\square	$\bar{\square}$	1	\dots	1
Q_2	1	\square	\dots	1	1
\vdots	\vdots	\vdots	\ddots	\vdots	\vdots
Q_{N-1}	1	1	\dots	\square	$\bar{\square}$
$\bar{P}_{1,\dots,M}$	$\bar{\square}$	1	\dots	1	1
$P_{1,\dots,M}$	1	1	\dots	1	\square

(2.1)

The bifundamentals Q_i 's obtain a vacuum expectation value (VEV) either due to some strong interactions or due to a superpotential that breaks the gauge group to the diagonal $SU(M)$. The fields P_i and \bar{P}_i are necessary only for anomaly cancellation in the endpoint gauge groups, obtaining a mass from the superpotential term

$$\frac{1}{M_{\text{Pl}}^{N-2}} \sum_{i=1}^M \bar{P}_i \prod_{j=1}^{N-1} Q_j P_i. \quad (2.2)$$

In order to obtain the Scherk-Schwarz-type GUT breaking in this model, we take all $SU(M)$'s to be given by $SU(5)$ (this can be easily generalized to larger GUT groups), except the last gauge group is replaced by the SM gauge group $SU(3) \times SU(2) \times U(1)$. We illustrate this model in Fig. 1. This replacement is supposed to mimic the effect of the second orbifold projection that explicitly breaks the GUT symmetry. As explained above, in this simplest version the SM matter fields are included at the orbifold fixed point, thus in our case they will only transform under the last gauge group. Therefore the matter content of the theory is thus given by

	$SU(5)_1$	$SU(5)_2$	\dots	$SU(5)_{N-1}$	$SU(3)$	$SU(2)$	$U(1)_Y$
$\bar{P}_{1,\dots,5}$	$\bar{\square}$	1	\dots	1	1	1	0
Q_1	\square	$\bar{\square}$	1	\dots	1	1	0
Q_2	1	\square	\dots	1	1	1	0
\vdots	\vdots	\vdots	\ddots	\vdots	\vdots	\vdots	\vdots
$Q_{t,N-1}$	1	1	\dots	\square	$\bar{\square}$	1	$\frac{1}{3}$
$Q_{d,N-1}$	1	1	\dots	\square	1	\square	$-\frac{1}{2}$
$P_{1,\dots,5}$	1	1	\dots	1	\square	1	$-\frac{1}{3}$
$P'_{1,\dots,5}$	1	1	\dots	1	1	\square	$\frac{1}{2}$
H_u	1	1	\dots	1	1	\square	$\frac{1}{2}$
H_d	1	1	\dots	1	1	\square	$-\frac{1}{2}$
L_i	1	1	\dots	1	1	\square	$-\frac{1}{2}$
E_i	1	1	\dots	1	1	1	1
Q_i	1	1	\dots	1	\square	\square	$\frac{1}{6}$
\bar{U}_i	1	1	\dots	1	$\bar{\square}$	1	$-\frac{2}{3}$
\bar{D}_i	1	1	\dots	1	$\bar{\square}$	1	$\frac{1}{3}$

(2.3)

Carefully inspecting this model one immediately recognizes, that for $N=2$ this theory is identical to the model proposed recently by Weiner [25], who purely from 4D considerations wrote down this model as an example of a theory that allows the unification of gauge couplings without actually unifying the representations. Thus Weiner's model is the simplest 4D realization of the Scherk-Schwarz breaking of the GUT symmetries. Therefore, all the features that Weiner's model possesses apply in the general N case here as well. The doublet-triplet splitting is resolved by not embedding the SM fields (in particular, the Higgs doublets) into unified multiplets. Proton decay is absent, since the SM fields do not interact with the X, Y gauge bosons that transform as $(3, 2)$ under $SU(3) \times SU(2)$.

One could include an $SU(5)$ on the last site instead of $SU(3) \times SU(2) \times U(1)$ if an $SU(5)$ adjoint field Σ was added such that it obtains a large ($\gg M_{GUT}$) vacuum expectation value (VEV) that breaks the $SU(5)$ to $SU(3) \times SU(2) \times U(1)$. This would then be the deconstructed version of the model presented in [13]. The advantage of this approach is that charge quantization is explained, but then the doublet-triplet splitting has to be explained with one of the conventional methods. Proton decay will be still very much suppressed due to the large value of the Σ VEV. Unification of couplings will still not be exact, due to the fact that operators involving the Σ field will give order one $SU(5)$ non-symmetric corrections to the gauge couplings at the high scale. We will not consider this possibility any further in this paper.

The fields P, \bar{P} needed to cancel the endpoint gauge anomalies must obtain a mass. In the simplest version of the model, nonrenormalizable operators of the form $(1/M_{Pl}^{N-2}) \bar{P} Q_1 \dots Q_{N-1} P$, result in mass terms of the order $v(v/M_{Pl})^{N-2}$, with v of order the GUT scale. The P, \bar{P} fields will appear as massive doublets and triplets in the theory below the unification scale. Since they form complete $SU(5)$ multiplets, they do not affect the unification of couplings, as long as their masses are reasonably large. Requiring that these masses are above the TeV scale will restrict the number of lattice sites to $N \leq 9$. However, this constraint is not very robust. If the Q fields themselves are composites, then one gets a stronger suppression of the masses and a stronger bound on N . However, since the P, \bar{P} fields are not an essential part of the construction, they are added only for anomaly cancellation, one could modify the model such that there are no P, \bar{P} fields at all, at the price of, for example, doubling the number of link fields in the theory.

B. Gauge coupling unification

Gauge coupling unification is a success in the MSSM, so it is natural to ask whether gauge coupling unification is maintained in this model, and at what scale the couplings are unified. Here we take the VEVs of all the bifundamental fields Q_i to be equal to v , and all the gauge couplings of the $SU(5)$ groups to be equal to g . This implies the "spacing" between lattice sites $a^{-1} = \sqrt{2}gv$ is the same across the entire lattice. We do this to simplify the calculations of the

mass matrices. Our construction does *not* require a "hopping" symmetry nor any other remnant of higher dimensional Lorentz invariance. In the MSSM, unification occurs roughly at the mass scale of the physical X, Y gauge bosons. Here, unification occurs at the scale where the diagonal subgroup is no longer a physical description of the model. This occurs once the lattice is resolved, meaning the scale of heaviest KK excitation, which is $2a^{-1}$. Once above this scale, the KK modes become the dynamical fields associated with the full product gauge theory. Thus, it is at this scale that one can perform a matching of the gauge couplings between the diagonal subgroup and the N lattice sites,

$$\begin{aligned} \frac{1}{g_{SU(3), \text{diag}}^2} &= \sum_{i=1}^{N-1} \frac{1}{g_i^2} + \frac{1}{g_{SU(3)}^2}, \\ \frac{1}{g_{SU(2), \text{diag}}^2} &= \sum_{i=1}^{N-1} \frac{1}{g_i^2} + \frac{1}{g_{SU(2)}^2}, \\ \frac{1}{g_{U(1), \text{diag}}^2} &= \sum_{i=1}^{N-1} \frac{1}{g_i^2} + \frac{1}{g_{U(1)}^2}. \end{aligned} \quad (2.4)$$

These matching relations together with the idea of using a product group broken to its diagonal subgroup is not new. In fact the idea itself dates back to the 1970s [26]. From Eq. (2.4) we can see that the same quantity $\sum_{i=1}^{N-1} (1/g_i^2)$ arises in all three diagonal subgroup gauge couplings. It is only through the explicit breaking of the gauge symmetry at the last site that one obtains a contribution which is generically not universal, and so the unification of gauge couplings in these models is not exact. These nonuniversal terms are the exact analogs of the possible brane localized kinetic terms that can be added in the continuum theory which can alter unification. However, if the number of gauge groups is significant *and* the last site couplings are not too different from the other $SU(5)$ couplings, then one expects the diagonal subgroup couplings are approximately unified. Thus for a large number of $SU(5)$ gauge groups one does not need to ensure that the $SU(3) \times SU(2) \times U(1)$ on the last site is much more strongly coupled than the $SU(5)$'s, as is the case of $N=2$. This is the analog of $SU(5)$ breaking brane localized terms in the continuum theory that are volume suppressed compared to the $SU(5)$ symmetric terms.

Below the scale $2a^{-1}$, the gauge group resulting from the diagonal breaking of $[SU(5)]^{N-1} \times \text{SM}$ group is just the SM. Between $2a^{-1}$ and a^{-1}/N , the theory appears five dimensional with a Kaluza-Klein- (KK)-like tower of nearly $SU(5)$ symmetric states. (Hereafter, we simply use "KK tower" to refer to the gauge and gaugino excitations resulting from the diagonal breaking of the gauge symmetries forming the lattice.) The $N \times N$ gauge boson mass matrix for the SM is

$$4a^{-2} \begin{pmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & & \ddots & & \\ & & & -1 & 2 & -1 \\ & & & & & -1 & 1 \end{pmatrix} \quad (2.5)$$

with eigenvalues

$$m_{3,2,1} = \frac{2}{a} \sin \frac{j\pi}{2N}, \quad (2.6)$$

where $j=0,1,2 \dots, N-1$. The gaugino and scalar adjoint masses are identical to the gauge bosons for $j>0$ [23]. The $(N-1) \times (N-1)$ mass matrix of the X, Y gauge bosons (and associated fermions and scalars) is [27]

$$4a^{-2} \begin{pmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & & \ddots & & \\ & & & -1 & 2 & -1 \\ & & & & & -1 & 2 \end{pmatrix} \quad (2.7)$$

with eigenvalues

$$m_{X,Y} = \frac{2}{a} \sin \frac{(2k+1)\pi}{4N-2}, \quad (2.8)$$

where $k=0,1, \dots, N-2$. These masses are the latticized analogues of the KK mass spectrum found in Ref. [9]. This can be seen by writing the approximate masses of the low lying modes, assuming N is large, as

$$m_{3,2,1} \sim \frac{2n+2}{R}, \quad (2.9)$$

$$m_{X,Y} \sim \frac{2n+1}{R}, \quad (2.10)$$

with the identification $R=2aN/\pi$.

We can estimate the evolution of the diagonal subgroup gauge couplings below the strong coupling scale via one-loop renormalization group equations in a manner analogous to Refs. [9,12]. The low energy couplings are

$$\begin{aligned} \alpha_i^{-1}(M_Z) &= \alpha_i^{-1}(2a^{-1}) \\ &+ \frac{1}{2\pi} \left[b_i \ln \frac{2a^{-1}}{M_Z} + c_i \sum_{n=1}^{N-1} \ln \frac{2a^{-1}}{2a^{-1} \sin \frac{n\pi}{2N}} \right. \\ &\left. + d_i \sum_{n=0}^{N-2} \ln \frac{2a^{-1}}{2a^{-1} \sin \frac{(2n+1)\pi}{4N-2}} \right], \end{aligned} \quad (2.11)$$

where $b_i=(33/5, 1, -3)$, $c_i=(0, -4, -6)$, $d_i=(-10, -6, -4)$ are the beta-function coefficients for the MSSM, one

SM KK excitation, and one X, Y KK excitation. The split $SU(5)$ multiplets are of course a direct consequence of breaking $SU(5)$ on one lattice site.

We can convert the sums over the KK excitation masses into products of sines as

$$\sum_{n=1}^{N-1} \ln \frac{a^{-1}}{a^{-1} \sin \frac{n\pi}{2N}} = -\frac{1}{2} \ln \prod_{n=1}^{N-1} \sin^2 \frac{n\pi}{2N}, \quad (2.12)$$

$$\sum_{n=0}^{N-2} \ln \frac{a^{-1}}{a^{-1} \sin \frac{(2n+1)\pi}{4N-2}} = -\frac{1}{2} \ln \prod_{n=0}^{N-2} \sin^2 \frac{(2n+1)\pi}{4N-2}. \quad (2.13)$$

The products of sines take the simple forms

$$\prod_{n=1}^{N-1} \sin^2 \frac{n\pi}{2N} = \frac{4N}{2^{2N}}, \quad (2.14)$$

$$\prod_{n=0}^{N-2} \sin^2 \frac{(2n+1)\pi}{4N-2} = \frac{4}{2^{2N}}, \quad (2.15)$$

which we can use to rewrite the evolution equation (2.11) as

$$\begin{aligned} \alpha_i^{-1}(M_Z) &= \alpha_i^{-1}(2a^{-1}) + \frac{1}{2\pi} \left[b_i \ln \frac{2a^{-1}}{M_Z} + (c_i + d_i) \right. \\ &\left. \times (N-1) \ln 2 - \frac{c_i}{2} \ln N \right]. \end{aligned} \quad (2.16)$$

The sum $c_i + d_i = (-10, -10, -10)$ is independent of i , and thus the second term represents the $SU(5)$ symmetric power law running [28] between a^{-1}/N and $2a^{-1}$. The last term proportional to $\ln N$ is different for each gauge coupling, representing the logarithmic differential running [13] between a^{-1}/N and $2a^{-1}$. Thus one can see from Eq. (2.16) that each gauge coupling receives a different logarithmic dependence on the number of lattice sites. Since N sets the ratio of the unification scale to the scale where the first KK excitations appears, it is clear that this effect is entirely analogous to an ordinary 4D GUT scale threshold correction resulting from GUT fields with masses below the unification scale.

As we have discussed, unification (or at least approximate unification) is expected to occur at $2a^{-1}$. We can calculate this scale for a given number N of lattice sites by equating $\alpha_i(2a^{-1}) = \alpha_j(2a^{-1})$ for fixed i, j and solving for the unification scale. Using the one-loop evolution equations from above, we find

$$\alpha_i^{-1}(M_Z) - \alpha_j^{-1}(M_Z) = \frac{1}{2\pi} \left[(b_i - b_j) \ln \frac{2a^{-1}}{M_Z} - \frac{c_i - c_j}{2} \ln N \right]. \quad (2.17)$$

Of course any two gauge couplings intersect, leading to the one-loop MSSM result

$$\alpha_i^{-1}(M_Z) - \alpha_j^{-1}(M_Z) = \frac{1}{2\pi}(b_i - b_j) \ln \frac{M_{ij}}{M_Z}, \quad (2.18)$$

where M_{ij} is the intersection scale. We can therefore rewrite Eq. (2.17) as

$$0 = \frac{1}{2\pi} \left[(b_i - b_j) \ln \frac{2a^{-1}}{M_{ij}} - \frac{c_i - c_j}{2} \ln N \right], \quad (2.19)$$

leading to a relationship between $2a^{-1}$ and M_{ij} ,

$$2a^{-1} = N^{(c_i - c_j)/(2(b_i - b_j))} M_{ij}$$

with

$$\frac{c_i - c_j}{2(b_i - b_j)} = \begin{cases} 5/14 & \{ij\} = \{12\} \\ 5/16 & \{ij\} = \{13\} \\ 1/4 & \{ij\} = \{23\} \end{cases}. \quad (2.20)$$

If the MSSM gauge coupling unification were exact, meaning $M_{ij} = M_{\text{GUT}}$, the gauge couplings here would never unify exactly for $N > 1$. Of course the MSSM gauge coupling unification is only accurate to a few percent. In fact, unification can be more precise in this model than in the MSSM since, as we will see, the correction to α_3 is negative. One reasonable approach is to require $\alpha_1 = \alpha_2$ at the unification scale, since they are the most accurately measured couplings at the weak scale. Defining $M_{12} = M_{\text{GUT}}$, this implies

$$2a^{-1} = N^{5/14} M_{\text{GUT}}, \quad (2.21)$$

$$\frac{a^{-1}}{N} = \frac{1}{2N^{9/14}} M_{\text{GUT}}. \quad (2.22)$$

Thus the scale where the KK-like states begin to appear, roughly a^{-1}/N , is *lower* than the usual MSSM unification scale by a factor $1/(2N^{9/14})$, while the scale of gauge coupling unification is *higher* by a factor $N^{5/14}$. We can then calculate the deviation of α_3 from $\alpha_1 = \alpha_2 = \alpha_{\text{GUT}}$,

$$\left. \frac{\alpha_{\text{GUT}} - \alpha_3}{\alpha_{\text{GUT}}} \right|_{2a^{-1}} \simeq \frac{3}{14\pi} \alpha_{\text{GUT}} \ln N. \quad (2.23)$$

This is a rather small correction. For example, α_3 is unified with the other gauge couplings to $< 1\%$ accuracy for $N < 40$. However, amusingly the sign of this correction is such that $\alpha_3(M_Z)$ is closer to the experimentally measured value and so unification in this model may be more precise than in the MSSM for reasonably small values of N .

Note that the value of the unified coupling α_{GUT} is smaller than the usual GUT coupling $\alpha_{\text{GUT}} \sim 1/25$ as is expected from adding vector supermultiplets to the diagonal subgroup. Of course the 't Hooft coupling αN is increasing for larger N . Very roughly the diagonal subgroup becomes strongly coupled once $\alpha N \sim 1$, which suggests N cannot be much larger than about 25 to maintain a perturbative analysis.

C. An example of dynamical breaking to the diagonal group

Let us now give a concrete realization of the strong dynamics that can enforce the expectation values for the bifundamentals Q_i . We discuss only the $N=2$ case in detail; the cases with larger N can be trivially generalized from this. We consider an $SU(5)$ SUSY gauge theory with five flavors, and assume that the above $SU(5) \times SU(3) \times SU(2) \times U(1)$ is the weakly gauged subgroup of the global symmetries. Thus the full matter content is given by

	$SU(5)$	$SU(5)$	$SU(3)$	$SU(2)$	$U(1)$
q	\square	$\bar{\square}$	1	1	0
\bar{q}	$\bar{\square}$	1	\square	1	$-\frac{1}{3}$
\bar{q}'	$\bar{\square}$	1	1	\square	$\frac{1}{2}$
$\bar{P}_1, \dots, \bar{P}_5$	1	\square	1	1	0
P_1, \dots, P_5	1	1	$\bar{\square}$	1	$\frac{1}{3}$
P'_1, \dots, P'_5	1	1	1	\square	$-\frac{1}{2}$
H_u	1	1	1	\square	$\frac{1}{2}$
H_d	1	1	1	\square	$-\frac{1}{2}$
L_i	1	1	1	\square	$-\frac{1}{2}$
E_i	1	1	1	1	1
Q_i	1	1	\square	\square	$\frac{1}{6}$
\bar{U}_i	1	1	$\bar{\square}$	1	$-\frac{2}{3}$
\bar{D}_i	1	1	$\bar{\square}$	1	$\frac{1}{3}$

(2.24)

We assume that the first $SU(5)$ gauge group becomes strongly interacting around the GUT scale. This theory then confines with a quantum modified constraint given by

$$\det M - B\bar{B} = \Lambda^{10}, \quad (2.25)$$

where the mesons are given by $q\bar{q}$ and $q\bar{q}'$, both of which acquire a VEV, while the baryons are q^5 and $\bar{q}^3\bar{q}'^2$. In order to ensure the necessary breaking of the global symmetries to a single $SU(3) \times SU(2) \times U(1)$ subgroup which is then to be identified with the MSSM, one has to make sure that it is the mesons in Eq. (2.25) which get the expectation values and not the baryons. Note that out of only the mesons one can make just a single gauge invariant after the D terms of the $SU(5) \times SU(3) \times SU(2) \times U(1)$ are taken into account. Thus, one expects that once the baryon direction is lifted, the vacuum described above is unique. The baryon directions can be lifted by adding a Planck-suppressed superpotential term

$$\frac{1}{M_{\text{Pl}}^3} (Sq^5 + S'\bar{q}^3\bar{q}'^2) \quad (2.26)$$

to the action, where S and S' are additional singlets. As a consequence, the baryons will get a mass together with the singlets, and their mass is estimated to be $M_{\text{GUT}}(M_{\text{GUT}}/M_{\text{Pl}})^3 \sim 10^{10}$ GeV. Thus we have seen that the $N=2$ case can be easily made into a complete model where

TABLE I. A model of missing partners.

	$SU(5)_1$	$SU(5)_2$	\dots	$SU(5)_{N-1}$	$SU(3)$	$SU(2)$	$U(1)_Y$
$\bar{P}_{1,\dots,5}$	\square	1	\dots	1	1	1	0
Q_1	\square	$\bar{\square}$	1	\dots	1	1	0
Q_2	1	\square	\dots	1	1	1	0
\vdots	\vdots	\vdots	\ddots	\vdots	\vdots	\vdots	\vdots
$Q_{t,N-1}$	1	1	\dots	\square	$\bar{\square}$	1	$\frac{1}{3}$
$Q_{d,N-1}$	1	1	\dots	\square	1	\square	$-\frac{1}{2}$
$P_{1,\dots,5}$	1	1	\dots	1	\square	1	$-\frac{1}{3}$
$P'_{1,\dots,5}$	1	1	\dots	1	1	\square	$\frac{1}{2}$
$H_{u,1}$	\square	1	\dots	1	1	1	0
$H_{d,1}$	$\bar{\square}$	1	\dots	1	1	1	0
$H_{u,1}^c$	$\bar{\square}$	1	\dots	1	1	1	0
$H_{d,1}^c$	\square	1	\dots	1	1	1	0
\vdots	\vdots	\vdots	\ddots	\vdots	\vdots	\vdots	\vdots
$H_{u,N-1}$	1	1	\dots	\square	1	1	0
$H_{d,N-1}$	1	1	\dots	$\bar{\square}$	1	1	0
$H_{u,N-1}^c$	1	1	\dots	$\bar{\square}$	1	1	0
$H_{d,N-1}^c$	1	1	\dots	\square	1	1	0
h_u	1	1	\dots	1	1	\square	$\frac{1}{2}$
h_d	1	1	\dots	1	1	\square	$-\frac{1}{2}$
t_u	1	1	\dots	1	\square	1	$-\frac{1}{3}$
t_d	1	1	\dots	1	$\bar{\square}$	1	$\frac{1}{3}$
t_u^c	1	1	\dots	1	$\bar{\square}$	1	$\frac{1}{3}$
t_d^c	1	1	\dots	1	\square	1	$-\frac{1}{3}$

the GUT scale emerges dynamically. Of course for $N=2$ one is left with the question of why the gauge couplings should unify. For this, one has to assume that the couplings of $SU(3) \times SU(2) \times U(1)$ are much stronger than those of the weakly gauged $SU(5)$.

Finally, we remark that this model, just as the continuum versions of these models, is ideally set up for gaugino mediation. Gaugino mediated models in 4D have been constructed in [23,29]. Here we can simply assume that the SUSY breaking sector only couples to the endpoint $SU(5)$ site, and is transmitted through the lattice of the $SU(5)$ s to the SM fields at the other end. Since in our case the scale of breaking to the diagonal gauge group is given by M_{GUT} , the gauge mediated contributions to the scalar masses are only suppressed when the number of lattice sites is large $N \gtrsim 5$. For this case one obtains a gaugino mediated spectrum, while for small number of sites one obtains a spectrum that interpolates between gaugino and gauge mediation.

III. A MODEL OF MISSING PARTNERS

A. Setup and doublet-triplet splitting

While models which restrict the Higgs boson to reside at the GUT breaking orbifold fixed-point give a simple resolution of the doublet-triplet splitting problem (there are no triplets) such models must sacrifice one of the most compelling theoretical motivations for GUTs: charge quantization. Since in such models the SM fields must necessarily live at the

same GUT breaking orbifold fixed point as the Higgs fields (in order to get masses from the Higgs VEV) there is no principle which restricts their $U(1)_Y$ charges to be the observed charges. For example different generations could have different hypercharges (and hence different electric charges) in this class of models. If, however, the Higgs fields are distributed among all the nodes of the model¹ then the SM matter fields can reside at any node. This opens up many possibilities; most importantly if the SM fields reside at a node with an unbroken $SU(5)$ gauge group, then the prediction of charge quantization is restored. However a different resolution of the doublet-triplet splitting problem is then required. There are several ways for this to happen, but they all share the key feature that the triplets do not have a zero mode² and thus implement a version of the missing partner mechanism [1].

To be more specific, consider the model shown in Table I (we will discuss the location of the SM generations subsequently). There are two $\mathcal{N}=2$ hypermultiplets of ‘‘bulk’’ Higgs fields represented by H_u and H_d , transforming at $\mathbf{5}$ and $\bar{\mathbf{5}}$ under $SU(5)$, as well as an independent set of fields with conjugate quantum numbers H_u^c and H_d^c , where H_u^c is in the same hypermultiplet as H_u . A latticized version of the

¹This is the analog of being a bulk field in a large N continuum limit (extra dimensional) model.

²For an early example of this idea in a string setting see Ref. [6].

“bulk” Higgs fields have a hopping term and a mass term in the superpotential:

$$W_{\text{bulk}} = \sum_{i=1}^{N-1} \lambda_i H_{u,i}^c Q_i H_{u,i+1} - \sum_{i=1}^{N-1} m_i H_{u,i}^c H_{u,i} + u \leftrightarrow d. \quad (3.1)$$

As we will discuss later it is useful to have a model with an R symmetry that forbids mass terms of the form $H_d H_u$ in order to suppress proton decay [9]. (This has the added benefit that it forces the μ term to be related to SUSY breaking.) For example Hall and Nomura [9] use an R symmetry with charge 0 for H_d , H_u and charge 2 for H_d^c , H_u^c which accomplishes this. With generic mass terms and a VEV for Q the Higgs fields mix and produce four towers of states starting at some nonzero masses. If we latticize a massless bulk hypermultiplet then the values of m_i are required to be proportional to the VEV of Q such that in the large N limit we recover just the correct kinetic terms. For a purely 4D model we however do not want to rely on the Lorentz invariance of the higher dimensional theory, and thus the bulk mass terms cannot be assumed to be equal. In the 5D models of SS GUT breaking it is exactly the 5D Lorentz invariance which enforces the structure of the mass matrices to be such that the doublets have a zero mode, while the color triplets do not. Since we do not want to rely on this symmetry, we have to slightly deviate from the strictly deconstructed version of the model in order to avoid fine tuning from the 4D point of view. The way we will ensure the existence of the doublet zero modes is by making sure that the mass matrix for the doublets is not of maximal rank, by leaving out one of the doublet fields from each hypermultiplet in the last (SM) gauge group, while still keeping all the triplet fields. This way for generic values of mass terms and couplings the triplets will all be massive, however since some of the corresponding doublets are missing on the fixed point, the corresponding mass matrix will not have maximal rank. This is an implementation of the missing partner mechanism [1,30], since some doublets are missing and cannot pair up to get a mass term. This way we are resolving the doublet-triplet splitting problem without fine-tuning, however we should emphasize that this is achieved by a modification of the deconstructed model. In particular, we are considering the case where the doublets h_d^c , h_u^c are missing on the fixed point and we have the superpotential

$$W_{\text{fp}} = -m_N t_u^c t_u + u \leftrightarrow d. \quad (3.2)$$

Let us consider the mass terms for the Higgs fermions

$$\begin{aligned} \mathcal{L} \supset & -\frac{1}{2} (\psi_{h,i\alpha} | \psi_{h,i\alpha}^c) \delta_{\alpha\alpha'} \begin{pmatrix} \Xi_h^T \\ \Xi_h \end{pmatrix}_{ij} \begin{pmatrix} \psi_{h,j\alpha'} \\ \psi_{h,j\alpha'}^c \end{pmatrix} \\ & -\frac{1}{2} (\psi_{t,i\alpha} | \psi_{t,i\alpha}^c) \delta_{\alpha\alpha'} \begin{pmatrix} \Xi_t^T \\ \Xi_t \end{pmatrix}_{ij} \begin{pmatrix} \psi_{t,j\alpha'} \\ \psi_{t,j\alpha'}^c \end{pmatrix} + \text{H.c.} \end{aligned} \quad (3.3)$$

with the $(N-1) \times N$ doublet matrix

$$\Xi_h = \begin{pmatrix} -m_1 & \lambda_1 v & & \\ & \ddots & \ddots & \\ & & & -m_{N-1} & \lambda_{N-1} v \end{pmatrix} \quad (3.4)$$

and the $N \times N$ triplet matrix

$$\Xi_t = \begin{pmatrix} -m_1 & \lambda_1 v & & \\ & \ddots & \ddots & \\ & & \ddots & \lambda_{N-1} v \\ & & & -m_N \end{pmatrix}. \quad (3.5)$$

Unless the m_i and λ_i are tuned to special values there are no massless triplet fermions (and by supersymmetry no massless triplet scalars). For example, the mass squared matrix of the triplet fermions is $\Xi_t^T \Xi_t$, which can be easily diagonalized for $m_i = \lambda v$ with $\lambda_1 = \dots = \lambda_{N-1} = \lambda = \sqrt{2}g$. Then the (mass)² spectrum is given by

$$m_k^2 = 4a^{-2} \sin^2 \left(\frac{(2k+1)\pi}{4N+2} \right), \quad (3.6)$$

for $k=0 \dots N-1$. In the models we are considering here there is no reason for such a special fine tuning so we expect no triplet zero modes. However the mass squared matrix for the doublet fermions $\Xi_h^T \Xi_h$ is an $N \times N$ matrix of rank $N-1$ so there must be a zero-mode fermion (and scalar). Their (mass)² spectrum is given by

$$m_{h_u, h_d}^2 = 0, \quad m_{h_u, h_d, h_u^c, h_d^c}^2 = 4a^{-2} \sin^2 \frac{m\pi}{2N}, \quad (3.7)$$

where $m=1 \dots N-1$.

The additional Higgs doublets and triplets modify the evolution of the gauge couplings up to $2a^{-1}$. The low energy couplings are now

$$\begin{aligned} \alpha_i^{-1}(M_Z) = & \alpha_i^{-1}(2a^{-1}) + \frac{1}{2\pi} \left[b_i \ln \frac{2a^{-1}}{M_Z} + (c_i + d_i + e_i + f_i) \right. \\ & \left. \times (N-1) \ln 2 + f_i \ln 2 - \frac{c_i + e_i}{2} \ln N \right], \end{aligned} \quad (3.8)$$

where $e_i = (6/5, 2, 0)$ and $f_i = (4/5, 0, 2)$ correspond to the beta function coefficients of two sets of up-type and down-type Higgs doublets and triplets, respectively. The same analysis shown in Sec. II B can be repeated with the above evolution equation. Here, we simply state the results. The inverse lattice spacing is related to M_{GUT} via

$$2a^{-1} = 2^{-1/7} N^{2/7} M_{\text{GUT}} \quad (3.9)$$

again using $\alpha_1(2a^{-1}) = \alpha_2(2a^{-1})$. We note that the $N^{2/7}$ behavior is *identical* to the continuum result found in [9], for large N . The deviation of α_3 from α_{GUT} is

$$\frac{\alpha_{\text{GUT}} - \alpha_3}{\alpha_{\text{GUT}}} = -\frac{3}{7\pi} \alpha_{\text{GUT}} \ln 8N. \quad (3.10)$$

Unlike in Sec. II B, here we find the correction is always negative, albeit at level of just a few percent.

B. Inclusion of SM matter

Having dispensed with the doublet-triplet splitting problem we can now consider the viability of different options for locating the SM quarks and leptons. As in [9], the R -symmetry has to be extended to the matter fields such that each SM matter field has R -charge 1. This is necessary to forbid proton decay from dimension 5 operators. As in the previous section we can place all the quarks and leptons on the GUT breaking orbifold fixed-point at the expense of losing the prediction of charge quantization. It is interesting to note that Yukawa coupling unification works best for the third generation (for $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle$ large), which suggests the third generation is further from the GUT breaking orbifold fixed-point than the other generations. It is also interesting to note that intergenerational Yukawa couplings can be suppressed for generations residing at different nodes, since gauge invariance will require a factor of the bifundamental Q for each link that separates the two generations. If such operators are generated by physics at a scale M_{flavor} within an order of magnitude of the GUT scale, then this leads to realistic predictions for the Cabibbo-Kobayashi-Maskawa (CKM) matrix. (Compare split-fermion models where a localized Higgs wave function [31,32] generates a hierarchy of Yukawa couplings. For our case we have taken the Higgs wave function to be uniform. The explanation of the hierarchy of the diagonal Yukawa couplings would require a varying Higgs wave function along the lattice that is also possible to implement here with appropriate choices of the coupling and mass parameters of the model.) Recall from the Wolfenstein parametrization of the CKM matrix:

$$V_{us} \sim V_{cd} = \mathcal{O}(\lambda),$$

$$V_{cb} \sim V_{ts} = \mathcal{O}(\lambda^2).$$

This structure is simply reproduced if one link separates the first and second generation, two links separate the second and third generations, and $\lambda \sim \langle Q \rangle / M_{\text{flavor}}$. Provided that the second and third generation both reside on the same side of the node with the first generation then this further implies that the first and third generations are separated by three links and we naturally predict the correct order of magnitude for the remaining CKM elements:

$$V_{ub} = \mathcal{O}(\lambda^3),$$

$$V_{td} = \mathcal{O}(\lambda^3).$$

Another alternative for introducing the SM matter in these models is to evenly distribute them among the $SU(5)$ gauge groups, corresponding to having the SM matter in the bulk in the higher dimensional language. In this case, the matter content has to be doubled, just like we saw for the Higgs fields. This is necessary in order to ensure the presence of a massless zero mode without fine-tuning. One can obtain this using the method of missing partners, where one set of SM fields is omitted at the last site, and thus the mass matrix will not be

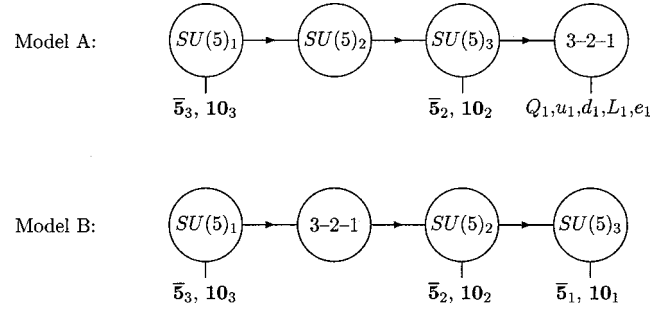


FIG. 2. Diagrammatic illustration of the two models.

of maximal rank, and a zero mode for the SM fields is necessarily present. We will not consider this possibility any further in this paper.

C. Suppression of proton decay

The amount of suppression of proton decay crucially depends on which generations are charged under $SU(5)$. When all three generations reside at the GUT breaking orbifold fixed point (Fig. 1) and there are no dangerous Higgs triplets, as in the Weiner model [25], proton decay is completely absent. This is because the heavy $SU(5)$ gauge bosons (X, Y) do not couple to quarks and leptons. In models where only the first generation resides at the GUT breaking orbifold fixed point (Fig. 2, model A), proton decay will proceed due to the mixing of the first generation with the other generations that do couple to heavy $SU(5)$ gauge bosons. In these models there will be an additional suppression of these proton decay amplitudes by CKM factors relative to standard heavy gauge boson processes. However the leading contribution to proton decay in the minimal $SU(5)$ SUSY GUT came from the Higgs triplet fermions, even assuming that they could be somewhat heavier than the heavy gauge bosons. But in these models there is nothing that requires the Higgs triplet to have a direct coupling to the first generation quarks, so these models are not challenged by the current bounds on the proton lifetime. Alternatively if the second and third generations reside on the GUT breaking orbifold fixed point while the first generation resides elsewhere, then proton decay can still be suppressed since the dangerous operator involves at least two different generations. If the triplet couplings to the second and third generations vanishes, there is no contribution to proton decay.

As in the Hall and Nomura model, if the Higgs triplet fermion mass is between H_a and H_a^c , but only H_a couples to quarks and leptons due to the global R -symmetry, then there is no dimension 5 operator generated. Since the mass of the lightest X, Y gauge bosons is lower than the usual MSSM GUT scale, we must check, model by model, that proton decay mediated by dimension-6 operators does not violate current experimental bounds.

D. Two realistic models

In this section we will discuss two simple realistic models. The field content of bifundamentals (Q), spectators (P),

and Higgs bosons (H) is along the lines discussed in the previous sections. The distributions of quarks and leptons (as well as squarks and sleptons) is depicted schematically in Fig. 2.

Both models implement the missing doublet scenario described earlier. In model A the first generation resides at the GUT breaking orbifold fixed point, while the second generation resides on the adjacent GUT symmetric site, and the third generation resides on the site two links away from the second generation. Note that the spacing of the generations qualitatively reproduces the CKM matrix as discussed above. Also the third generation is the furthest away from the source of GUT breaking so this model can naturally incorporate a more accurate Yukawa unification in the third generation, while there is no reason to expect Yukawa unification in the first generation. As discussed above an R symmetry prevents the coupling of SM fields to triplets necessary to generate dimension 5 proton decay operators, so the leading contribution to proton decay is through X , Y gauge boson exchange. Since the first generation does not directly couple to $SU(5)$ gauge bosons, these couplings are only generated through mixing with the other generations. Thus there is an additional CKM suppression (changing flavor to the second generation) of these decays relative to usual GUTs, so this model is phenomenologically viable. This model also has the signature that proton decays with electrons in the final state are highly suppressed (they arise only through lepton flavor violating effects). Experimentally the limit on proton decays to second generation particles is considerably weaker than that for first generation decays.

The main theoretical blemish of model A is that charge quantization is not guaranteed. However we can simply overcome this problem by carrying over the virtues of this model to a similar model which cannot be obtained by latticizing a 5D orbifold. Model B is just such a model: in this model all three generations reside at GUT symmetric sites, while the GUT breaking site sits in the middle. Thus charge quantization is a prediction of model B. Furthermore, the missing partner mechanism can be implemented in a straightforward manner (following the discussion above) and the spacing between generations is maintained so the CKM structure is maintained. Dimension 6 proton decay operators no longer have an additional CKM suppression, but the model is not in conflict with the experimental limit since the number of lattice sites, $N=4$, is small. Since in model B proton decay can proceed directly to first generation particles, the bound on

proton decay via $\tau_{p \rightarrow e^+ \pi^0} > 1.6 \times 10^{33}$ yr is the most stringent. In particular, the decay through a heavy X gauge boson gives

$$\tau_{p \rightarrow e^+ \pi^0} \sim 10^{33} \text{ yr} \left(\frac{M_X}{5 \times 10^{15} \text{ GeV}} \right)^4 \quad (3.11)$$

and so these operators are safe so long as $M_X \gtrsim 5 \times 10^{15}$ GeV, which requires $N \lesssim 8$.

IV. CONCLUSIONS

We have considered 4D supersymmetric models that predict the unification of couplings at a high scale. These models were inspired by the deconstruction of 5D $SU(5)$ gauge theories, where the GUT group is broken by the Scherk-Schwarz mechanism on an $S^1/Z_2 \times Z'_2$ orbifold. The 4D models consist of a chain of $SU(5)$ gauge groups, with one site being replaced by the SM gauge group, and bifundamental fields that break the full gauge symmetry to the diagonal SM group. In the simplest model, the SM matter is included at the last site where the gauge symmetry is reduced, and thus the doublet-triplet splitting problem is automatically resolved, while proton decay is exactly zero. However, this model does not explain the quantum numbers of the SM fields. A slightly modified version has the SM fields at $SU(5)$ sites, and the Higgs fields are distributed among all the sites. These models do explain charge quantization, and the doublet-triplet splitting can be resolved by a very simple version of the missing partner mechanism. Proton decay is now reintroduced, but only through the dimension 6 operators, which can be sufficiently suppressed as long as the number of sites is not too large. By putting the SM fields at different sites, a realistic hierarchy of the CKM matrix elements can be obtained.

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