4D constructions of supersymmetric extra dimensions and gaugino mediation

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We present 4D gauge theories which at low energies coincide with higher dimensional supersymmetric (SUSY) gauge theories on a transverse lattice. We show that in the simplest case of pure 5D SUSY Yang-Mills theory there is an enhancement of SUSY in the continuum limit without fine tuning. This result no longer holds in the presence of matter fields, in which case fine tuning is necessary to ensure higher dimensional Lorentz invariance and supersymmetry. We use this construction to generate 4D models which mimic gaugino mediation of SUSY breaking. The way supersymmetry breaking is mediated in these models to the MSSM is by assuming that the physical gauginos are a mixture of a number of gauge eigenstate gauginos: one of these couples to the SUSY breaking sector, while another couples to the MSSM matter fields. The lattice can be as coarse as just two gauge groups while still obtaining the characteristic gaugino-mediated soft breaking terms.

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I. INTRODUCTION

Models with extra dimensions provide several interesting mechanisms for supersymmetry (SUSY) breaking. These mechanisms seem to make essential use of the presence of extra dimensions, which are not obviously realizable in a simple four-dimensional setup. Recently, Arkani-Hamed, Cohen, and Georgi [1] and also Hill, Pokorski, and Wang [2] argued that it might be possible to translate many higher dimensional effects into a purely 4D construction by using a set of 4D theories which in the IR reproduce the dynamics of the extra dimensional theory.[1] These theories are also useful tools to regulate the higher dimensional theories, and even give a UV completion of them [1–3] (see also [4]).

The aim of this paper is to give a fully 4D implementation of a higher dimensional mechanism for supersymmetry breaking (gaugino mediation), using a 4D $\mathcal{N}=1$ SUSY model which at low energies is equivalent to a latticized version of these higher dimensional models. In order to do so we first show how to construct the higher dimensional supersymmetric theories from a 4D "moose" (lattice) approach. Because the minimal spinor representation of the 5D Lorentz group is twice as large as that of the 4D Lorentz group, one

might think that a fine tuning in the fermion sector is needed in order to construct a Lorentz invariant higher dimensional theory which includes fermions. In addition, 5D SUSY would then require at least 8 supercharges, which corresponds to $\mathcal{N}=2$ supersymmetry in 4D. We will demonstrate that 4D $\mathcal{N}=1$ supersymmetry plus gauge invariance (with properly chosen matter content) is enough to ensure the existence of the additional supersymmetries in the continuum limit. This phenomenon of enhanced supersymmetry generation is related to the behavior of these models at low energies in a purely 4D context, in which $\mathcal{N}=1$ SUSY is enhanced to $\mathcal{N}=2$ on the moduli space, without the fine tuning of parameters. However, in the presence of additional hypermultiplets the required superpotential does have to be tuned in the 4D theories. The analogous effect that we obtain here is that maintaining 5D Lorentz invariance will require the tuning of a superpotential coupling in the 4D lattice models. We present the explicit construction of these models which will give in the continuum limit the 5D $\mathcal{N}=1$ theory, show how to achieve the required gauge symmetry breaking dynamically, and how to add flavors. We carefully check that the mass spectrum for gauge fields, scalars, and fermions indeed matches the tower of Kaluza-Klein (KK) modes for a 5D $\mathcal{N}=1$ supersymmetric gauge theory, and that 5D Lorentz invariance and supersymmetry are indeed recovered in the continuum limit. This is not a trivial fact, because in the usual Wilson lattice action the fermions are included as adjoints living at the sites of the lattice, while for our construction they are in bifundamentals at the links.

In order to translate gaugino mediation of supersymmetry breaking into a 4D language we show how the corresponding S^1/Z_2 orbifolds are constructed. Armed with this knowledge, we present a simple 4D version of gaugino mediation, where the lattice can be as coarse as two gauge groups and still give

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[1]A similar proposal can be found in [5], where the AdS–conformal field theory (CFT) correspondence is used to construct a purely 4D model of anomaly mediation, and in [6] where a higher dimensional stabilization of the gauge hierarchy is "deconstructed." See also [7].

65 015003-1

the characteristic gaugino-mediated spectrum. One of these gauge groups contains the standard model matter fields, while another couples to the supersymmetry breaking sector. The physical gaugino is a linear combination of the gauginos for the various group factors and thus obtains a mass directly from the supersymmetry breaking sector, while the scalar mass terms for the minimal supersymmetric standard model (MSSM) matter fields will be suppressed by an additional loop factor, just like in ordinary gaugino mediation. An important difference between the 4D and 5D approach is that in the 4D approach Planck-suppressed contact terms must be subleading, because (contrary to the 5D case) they have no further exponential suppression. The reason for this is that the notion of locality from gravity's point of view is lost, if (as we will imagine) 4D gravity is minimally included into the theory. Thus from this point of view the spirit of these models more closely resembles that of gauge mediation, where the Planck-suppressed operators should also be negligible. However, the resulting mass spectrum agrees with that of gaugino mediation, and differs from the generic gauge-mediated spectrum.

The paper is organized as follows: in Sec. II we give the construction of the supersymmetric lattice models and check that the perturbative mass spectrum agrees with that of the $\mathcal{N}=1$ 5D theory. In Sec. III we show how to include flavors into the construction, and present the orbifold models. Using these results we present the 4D models for gaugino mediation in Sec. IV, and conclude in Sec. V.

II. THE CONSTRUCTION OF SUPERSYMMETRIC EXTRA DIMENSIONS

In Refs. [1–3], it has been argued that the low energy behavior of a purely 4D theory can be effectively described by the low-lying KK modes of a 5D theory compactified on a circle or an S^1/Z_2 orbifold. Here we will first present the supersymmetric versions of these theories, so that later we can use these to construct the 4D analogs of a higher dimensional mechanism for mediating supersymmetry breaking.

The theory we will consider is an asymptotically free four-dimensional $\mathcal{N}=1$ supersymmetric $SU(M)^N$ gauge theory with chiral multiplets Q_i in bifundamental representations as follows:[2]

	$SU(M)_1$	$SU(M)_2$	$SU(M)_3$	$\cdots$	$SU(M)_N$
Q_1	$\square$	$\bar{\square}$	1	$\cdots$	1
Q_2	1	$\square$	$\bar{\square}$	$\cdots$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Q_N	$\bar{\square}$	1	1	$\cdots$	$\square$

$$(2.1)$$

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[2]We will use the following conventions for our indices: $i,j,k=1,...,N$ denote the gauge group ("lattice index") and until Sec. III B, we will impose a cyclic boundary condition, i.e., i, j, k will be defined mod N; $\alpha,\beta=1,...,M$ are gauge indices in the fundamental or antifundamental representation of $SU(M)$; and $a,b=1,...,M^2-1$ are gauge indices in the adjoint representation of $SU(M)$.

The low-energy behavior of this theory was analyzed in Ref. [8]. Here we briefly summarize the relevant results from that analysis. The flat directions (moduli space) of the theory are described by the independent gauge invariant operators [9], which are given by $B_i=\det Q_i$, $i=1,...,N$; and $T_i=\text{tr}(Q_1\cdots Q_N)^i$, $i=1,...,M-1$. An expectation value of the operator B_i will break $SU(M)_i\times SU(M)_{i+1}$ to an $SU(M)$ subgroup, leaving a theory with the same structure as the original theory, but with one fewer $SU(M)$ factor in the gauge group. Once all the B_i's have expectation values the gauge group is broken to a single $SU(M)$. During this sequential breaking all the fields from the first $N-1$ Q_i's will become massive due to the supersymmetric Higgs mechanism (some scalars will be eaten by the heavy gauge bosons), except the fields corresponding to the trace of Q_i, which are then described by the composite moduli field B_i. However, giving an expectation value to the last operator B_N does not break the gauge group any further, and so one expects that the field Q_N remains massless, and forms an adjoint and scalar of the unbroken $SU(M)$ gauge group. The invariants corresponding to the remaining adjoint are given by the operators T_i above. Without further modification of the model, at a generic point in moduli space the theory will have $M-1$ unbroken $U(1)$ gauge groups and no charged fields under those $U(1)$'s. The behavior of the gauge couplings can be described by a Seiberg-Witten curve which has been exactly determined by considering various limits of the theory [8]. This theory is itself an orbifold of an $\mathcal{N}=2$ theory, and the dynamics of these two theories are closely related via the orbifold correspondence [10]. Similar constructions can be found using D3-branes distributed on a circle in type IIB string theory [11].

We will demonstrate that the field theory described above is equivalent to a latticized version of a 5D $\mathcal{N}=1$ supersymmetric gauge theory. This 5D $\mathcal{N}=1$ supersymmetric gauge theory has twice the number of supercharges as the $\mathcal{N}=1$ theory in 4D. This is an interesting phenomenon in its own right, as supersymmetries are dynamically generated at low energy. Although we do not study the case here, a similar phenomenon is expected to occur in one dimension lower, i.e., generation of a 4D SUSY gauge theory from a 3D theory with fewer supersymmetry charges.

A. Dynamical generation of the symmetry breaking

The massless matter content of the $SU(M)$ gauge theory corresponding to the theory (2.1) at low energies is that of an $\mathcal{N}=2$ 4D theory, namely, in addition to the massless $\mathcal{N}=1$ vector multiplet there is also a chiral multiplet in the adjoint representation. But in addition the singlets B_i remain massless. In order to remove these massless fields [and at the same time provide the necessary diagonal vacuum expectation values (VEV's) of the Q_i's] Arkani-Hamed, Cohen, and Georgi proposed the addition of a matching set of gauge singlet chiral superfields S_i and the superpotential

$$W_{\text{dyn}}=\frac{1}{\mu^{M-2}}\sum_i S_i(B_i-v^M),\qquad(2.2)$$

where μ is a mass scale. The question that we want to answer first is whether this superpotential can be achieved dynamically, perhaps also within a renormalizable theory. In [1] a dynamical model for the nonsupersymmetric case has been worked out. Here we show that for the case of $SU(2)$ gauge groups one can achieve this as well through supersymmetric nonperturbative dynamics within a renormalizable theory, which is understood from the works of Seiberg and others [12]. For the $SU(N)$ version of this model there will still be a branch on the moduli space of vacua that achieves the dynamical breaking of the gauge symmetry to the diagonal one. However, in order to ensure that we are on the right branch (and that the other moduli are massive) a nonrenormalizable tree-level superpotential will have to be added, at least for the example based on the simplest possible matter content. This non-renormalizable superpotential should then

be generated by some other physics at higher energies, either through nonperturbative effects or from integrating out heavy particles.

We begin with an $SU(M)^{2N}$ gauge theory with the periodic structure of (2.1). We further assume that the gauge coupling of every even group in the chain of $SU(N)$'s is much larger than the neighboring odd ones: $g_2 = g_{2i} \gg g_{2i-1} = g_1$, i.e., $\Lambda_2 = \Lambda_{2i} \gg \Lambda_{2i-1} = \Lambda_1$. Therefore, concerning the dynamics of any $SU(M)_{2i}$, the weaker gauge groups can be regarded as a weakly gauged global symmetry, leaving an $SU(M)_{2i}$ gauge theory with M flavors in this sector of the theory. With this particular matter content it was shown in [12] that the theory confines in the IR with chiral symmetry breaking, with the confined degrees of freedom given by $\mathcal{M}_i$, $\mathcal{B}_i$, $\tilde{\mathcal{B}}_i$ ($i = 1...N$),

	$SU(M)_{2i}$	$SU(M)_{2i-1}$	$SU(M)_{2i+1}$	$U(1)_B$	$U(1)_R$
$\tilde{\mathcal{Q}}_i$	$\bar{\square}$	$\square$	1	1	0
$\mathcal{Q}_i$	$\square$	1	$\bar{\square}$	-1	0
$\mathcal{M}_i = \mathcal{Q}_i \tilde{\mathcal{Q}}_i$		$\square$	$\bar{\square}$	0	0
$\mathcal{B}_i = \mathcal{Q}_i^M$		1	1	M	0
$\tilde{\mathcal{B}}_i = \tilde{\mathcal{Q}}_i^M$		1	1	$-M$	0

(2.3)

Analyzing the 't Hooft anomaly matching conditions one concludes that some of the global symmetries of the theory have to be broken, which is the effect of a classical constraint on the composite fields being modified by quantum dynamics. It was shown in [12] that the form of the quantum modified constraint is given by[3]

$$\det \mathcal{M}_i - \mathcal{B}_i \tilde{\mathcal{B}}_i = \Lambda_2^{2M}. \qquad (2.4)$$

Thus there is a branch on the moduli space where the global $SU(M)_{2i-1} \times SU(M)_{2i+1}$ is broken to a diagonal $SU(M)$, which is when $\det \mathcal{M}_i$ has an expectation value. Turning on the baryons $\mathcal{B}_i$ and $\tilde{\mathcal{B}}_i$ does not break the global symmetry group. Hence, in order to lift the branch of moduli space with $\det \mathcal{M}_i = 0$ (and also to get rid of unwanted massless singlets) one is forced to introduce two singlets L_i and $\tilde{L}_i$ and add a tree level superpotential,

$$W_{\text{tree}} = \frac{1}{\mu^{M-2}} (L_i \mathcal{B}_i + \tilde{L}_i \tilde{\mathcal{B}}_i), \qquad (2.5)$$

where the scale μ would have to be regarded as a cutoff scale of the theory (perhaps originating from some other strong

dynamics). The Q_i's in (2.1) should then be identified with the composite meson field $\mathcal{M}_i$, and the baryons need to be lifted from the spectrum by a superpotential term, which is generically nonrenormalizable except for the case $M = 2$ (see below). Thus for the general $SU(M)^N$ case we do not completely succeed in generating the model from a renormalizable dynamics as in the nonsupersymmetric case. For this choice of matter content an extra layer of perturbative or nonperturbative dynamics might be needed to get the non-renormalizable superpotentials as well. Once this superpotential (2.5) is added, the quantum modified constraint will ensure that the remaining $\Pi_{i=1}^{N} SU(M)_{2i-1}$ gauge groups are broken down to the diagonal $SU(M)$. In fact, as mentioned above, after confinement the meson matrix $\mathcal{M}_i$ will just play the role of the bifundamentals Q_i of (2.1) while $\det \mathcal{M}_i$ gives the invariants B_i, and so the full dynamically generated superpotential in the remaining $SU(M)^N$ theory is just

$$\frac{1}{\mu^{M-2}} \sum_i (L_i \mathcal{B}_i + \tilde{L}_i \tilde{\mathcal{B}}_i) + \frac{1}{\Lambda_2^{2M-2}} \sum_i S_i (B_i - \mathcal{B}_i \tilde{\mathcal{B}}_i - \Lambda_2^{2M}),$$

(2.6)

where S_i are nonpropagating Lagrange multiplier chiral superfields. Integrating out the fields $\mathcal{B}_i$, $\tilde{\mathcal{B}}_i$, and L_i we are exactly left with the superpotential of Eq. (2.2), except that the symmetry breaking scale v is now given by Λ_2, and because the fields $Q_i = \mathcal{M}_i$ themselves are composites one

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[3]For several other theories with a quantum modified constraint see [13]. For a connection between the existence of constraints among the gauge polynomial invariants derivable from a superpotential and the 't Hooft matching conditions see [14].

gets different powers of scales since the dimensions of these fields have not been rescaled yet.

The case of $SU(2)$ is special in that the tree level superpotential (2.6) is renormalizable. Hence, in that case the theory described above can in fact be dynamically generated. One might worry that because the representations of $SU(2)$ are pseudoreal the global symmetry in the above analysis is enlarged from $SU(2) \times SU(2)$ to $SU(4)$ and the analysis above would have to be modified. It is true that the global symmetry group is enlarged, but the analysis remains unchanged if we identify $\det \mathcal{M} - \bar{\mathcal{B}}\mathcal{B}$ with the Pfaffian of the combined meson field $\mathcal{M}'$. To be more precise, we have the following confining theory:

	$SU(2)$	$SU(4)$	$U(1)_R$	
$\mathcal{Q}$	□	□	0	
$\mathcal{M}' = \mathcal{Q}^2$	1	⊟	0	(2.7)

Note that the composite meson field $\mathcal{M}'_{ij} = \mathcal{Q}_i \mathcal{Q}_j$ contains both the mesons and baryons in the previous language. As shown in [12,15], instanton corrections will force a quantum mechanical expectation value to the composite meson field $\mathcal{M}'$:

$$\mathrm{Pf}\mathcal{M}' = \mathcal{M}'_{ij}\mathcal{M}'_{kl}\epsilon^{ijkl} = \Lambda_2^4, \qquad (2.8)$$

which will break the global $SU(4)$ to its $Sp(4)$ subgroup. In our case, below the scale Λ_2 where the first set of $SU(2)$'s confines, the global symmetry is raised to a gauge symmetry in which case the effect of confinement is to break the gauged $SU(2) \times SU(2) \subset SU(4)$ to a single $SU(2)$. In the process three scalars are eaten and three remain (of the six "mesons" $\mathcal{M}'_{ij}$). Of the three massless scalars that remain, two are given a mass by the tree level superpotential (2.5) as described above, and the Pfaffian becomes massive by the quantum modified constraint. The tree level superpotential and the fact that only a subgroup of the global symmetries is gauged explicitly breaks the $SU(4)$ "global" symmetry down to $SU(2) \times SU(2)$. Alternatively, the constraint can be incorporated in the theory as before by adding a superpotential,

$$S(\mathrm{Pf}\mathcal{M}' - \Lambda_2^4), \qquad (2.9)$$

where S is the Lagrange multiplier which enforces the constraint. Fluctuations of the Pfaffian then obtain a mass by the Lagrange multiplier. Hence, we have demonstrated that the supersymmetric version of the $SU(2)^N$ theory which is dual to the higher dimensional latticized $SU(2)$ theory can be generated dynamically, while for larger gauge groups one has to add a nonrenormalizable superpotential if one assumes the minimal matter content as we did here.

B. Matching of the perturbative mass spectra

Now that we understand how the model in (2.1) could arise from supersymmetric gauge dynamics, we analyze the various mass spectra of the model assuming that the symmetry breaking VEV's have been generated. The aim of this

analysis is to show that one indeed recovers a higher dimensional supersymmetric theory in the limit of $N \to \infty$, and that the number of supercharges is appropriately doubled. In order to be able to analyze the massive spectrum of the model (and not just the extreme infrared like in [8]) we assume that the scale v is larger than the dynamical scale of the $SU(M)$ gauge group $\Lambda < v$. This would indeed follow in the dynamically generated examples considered above, because there $v = \Lambda_2 \gg \mu \gg \Lambda_1$. In this case the gauge groups are broken before they could become strongly interacting, and a conventional perturbative analysis is possible. Then the singlet field corresponding to B_i can be identified at the lowest order in the fluctuations with $v^{N-1}\mathrm{tr}Q_i$, and therefore the mass term for the fields S_i and $\mathrm{tr}Q_i$ from Eq. (2.2) is given by $v(v/\mu)^{M-2} \gg v$.

1. Gauge boson masses

The analysis of the gauge boson mass matrix follows exactly that of the nonsupersymmetric models analyzed in [1–3], which we repeat only for completeness. The mass matrix is obtained by expanding the kinetic term $\Sigma_i (D_\mu Q_i)^\dagger D^\mu Q_i$ of the scalar components of the bifundamentals Q_i, which gives a contribution to the Lagrangian of the form [1–3],

$$\mathcal{L} \supset g^2 v^2 \sum_i (A_i^{a\mu} - A_{i-1}^{a\mu})^2, \qquad (2.10)$$

where we have used the normalization[4] $\mathrm{tr}T^a T^b = \delta^{ab}$ for the generators of the $SU(M)$ gauge groups (this normalization will ensure a canonically normalized kinetic term for gauginos, see later), and g is the gauge coupling of the $SU(M)$ groups. This gives the following mass term:

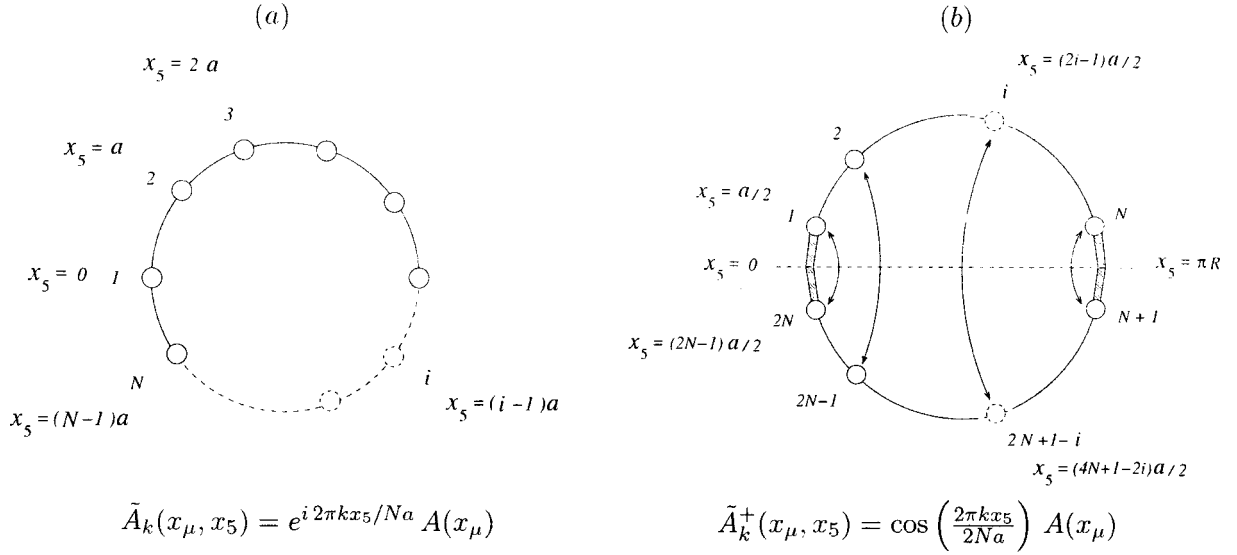$$\tfrac{1}{2} A_{i\mu}^a \mathcal{M}^2_{ijab} A_j^{b\mu}, \qquad (2.11)$$

where the mass matrix is a direct product of the identity in the gauge index space times a more involved matrix in the lattice index space

$$\mathcal{M}^2_{ijab} = 2g^2 v^2 \delta_{ab}\Omega_{ij} \quad \text{with} \quad \Omega$$

$$= \begin{pmatrix} 2 & -1 & & & -1 \\ -1 & 2 & -1 & & \\ \ddots & \ddots & \ddots & & \\ & & -1 & 2 & -1 \\ -1 & & & -1 & 2 \end{pmatrix}. \qquad (2.12)$$

The mass eigenvalues will then be given by those of the Ω matrix with a multiplicity given by $M^2 - 1$, the dimension of the gauge index space. The diagonalization of Ω follows by writing Ω as $2 - C - C^\dagger$, where C is the matrix of cyclic permutations

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[4]This normalization differs from the one used in [1,2]. The results of [1,2] can be obtained by replacing $\sqrt{2}g \to g$ everywhere in our paper.

(a)

$$\tilde{A}_k(x_\mu, x_5) = e^{i\,2\pi k x_5/Na}\, A(x_\mu)$$

$$\langle \tilde{A}_k(x_\mu) | A_j(x_\mu) \rangle = \frac{1}{\sqrt{N}}\, e^{i\,2\pi k(j-1)/N}$$

(b)

$$\tilde{A}_k^+(x_\mu, x_5) = \cos\left(\frac{2\pi k x_5}{2Na}\right)\, A(x_\mu)$$

$$\langle \tilde{A}_k^+(x_\mu) | A_j(x_\mu) \rangle = \sqrt{\frac{2}{2^{\delta_{k0}} N}}\, \cos\left(\frac{2\pi k(j-1/2)}{2N}\right)$$

FIG. 1. Mode decomposition for the (a) periodic and (b) orbifold "moose" diagrams. The mass eigenvector expansion is the discrete/latticized analogue of the continuous Fourier expansion. The orbifold $SU(M)^N$ moose diagram is constructed from the $SU(M)^{2N}$ periodic diagram by removing two diametrically opposite links and identifying the sites with their reflection about the reflecting axis.

$$C = \begin{pmatrix} & 1 & & \\ & & \ddots & \\ & & & 1 \\ 1 & & & \end{pmatrix}, \qquad (2.13)$$

whose eigenvectors are given by $(1, \omega_k, \omega_k^2, \ldots, \omega_k^{N-1})$, and eigenvalues by ω_k, where $\omega_k = e^{i\,2\pi k/N}$, $k = 0, \ldots, (N-1)$, are the Nth roots of unity. From this the mass eigenvalues are [1–3]

$$m_k = 2\sqrt{2}\, gv\, \sin\frac{k\pi}{N}, \qquad 0 \le k \le N-1 \qquad (2.14)$$

corresponding to the normalized eigenvectors:

$$\bar{A}_{k\mu}^a = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \omega_k^{j-1} A_{j\mu}^a \qquad 0 \le k \le N-1. \qquad (2.15)$$

This mode decomposition is just the discrete analogue of the usual continuous Fourier expansion [see Fig. 1(a)]. Each mass level is $M^2 - 1$ degenerate, forming an adjoint representation of the unbroken diagonal $SU(M)$ gauge group (on top of this gauge degeneracy, there is also an accidental lattice degeneracy since $m_k = m_{N-k}$). For small enough k the spectrum approximates the Kaluza-Klein tower of states corresponding to the compactification of the 5D theory on a circle. In order to find the lattice spacing of the corresponding 5D theory on a circle with circumference Na, we identify as in [1–3] the low-lying mass spectra by

$$\frac{2\pi k}{Na} = 2\sqrt{2}\, gv\, \frac{k\pi}{N},$$

from which the lattice spacing is found to be $a = 1/(\sqrt{2}\, gv)$.

2. Scalar masses

The scalar fields in the bifundamental chiral multiplets of (2.1) receive masses from the D-term contributions to the action. In particular, the Lagrangian contains a contribution $\mathcal{L} \supset -\frac{1}{2} \Sigma_a D_i^a D_i^a$ for each $SU(M)_i$ factor in the gauge group, with

$$D_i^a = g(Q_i^{*\,\alpha\beta} T_{\alpha\gamma}^a Q_i^{\alpha,3} - Q_{i-1}^{*\,\alpha,3} T_{\gamma\beta}^a Q_{i-1}^{\alpha\gamma}) = g\,\mathrm{tr}(Q_i^\dagger T^a Q_i$$
$$- Q_{i-1} T^a Q_{i-1}^\dagger), \qquad (2.16)$$

where T^a are the generators of $SU(M)$ in the fundamental representation (thus $-T^{a*} = -T^{at}$ are the generators in the antifundamental) and it is understood that we impose cyclic boundary conditions, i.e., $Q_0 \equiv Q_N$. When the Q's develop a VEV, the fluctuations around this VEV get a mass. Decomposing $Q_i^{\alpha\beta} = v\,\delta^{\alpha\beta} + \phi_i^{\alpha\beta}$, we obtain the following mass term:

$$\mathcal{L} \supset -g^2 v^2 \sum_{i,a} \left[(T^a \phi_i)(T^a \phi_i) + (T^a \phi_i^\dagger)(T^a \phi_i^\dagger) + 2(T^a \phi_i) \right.$$
$$\times (T^a \phi_i^\dagger) + (T^a \phi_i^\dagger)(T^a \phi_{i-1}^\dagger) + (T^a \phi_i)(T^a \phi_{i-1})$$
$$\left. + (T^a \phi_i^\dagger)(T^a \phi_{i-1}) + (T^a \phi_i)(T^a \phi_{i-1}^\dagger) \right], \qquad (2.17)$$

$$\Theta = \begin{pmatrix} 1 & -1 & & \\ & \ddots & \ddots & \\ & & \ddots & -1 \\ -1 & & & 1 \end{pmatrix}. \qquad (2.30)$$

It is then easy to derive the fermionic spectrum. The square of the gauge matrix $B^\dagger B = BB^\dagger$ has one zero eigenvalue and $M^2 - 1$ degenerate eigenvalues equal to 1. On the other hand, $\Theta^\dagger \Theta = \Theta \Theta^\dagger = \Omega$, showing that the fermionic mass levels agree with those computed before for the vectors and the scalars. Concerning the zero modes, as a result of the trace-lessness of the generators of $SU(M)_i$ the trace of q_i does not acquire a mass from the Kähler potential. Instead it combines with the fermionic component of the singlet S_i to form a Dirac spinor that gets a mass $v(v/\mu)^{M-2}$ from the superpotential (2.2). The fermionic spectrum is thus (i) $2(M^2-1)$ massless Weyl spinors, (ii) (M^2-1) Dirac spinors with mass $m_k = 2\sqrt{2}gv\,\sin(k\pi/N)$, $0 \le k \le N-1$, showing the supersymmetric nature of the low-energy spectrum. Indeed, the massless 5D $\mathcal{N}=1$ vector multiplet includes a gauge boson (3 components on-shell), a Dirac fermion (4 components), and a real scalar (1 component). Upon Kaluza-Klein reduction, we get a 4D $\mathcal{N}=2$ massless vector multiplet: a gauge boson (2 components), two Weyl fermions (2×2 components) and a complex scalar (2 components); and massive vector multiplets: massive gauge boson (3 components), fermion (4 components), and real scalar (1 component). This decomposition agrees exactly with the spectrum we have found. Moreover each mass level transforms in the adjoint of the unbroken diagonal $SU(M)$ gauge group.

C. 5D Lorentz invariance and supersymmetry

Up to now we have shown that the mass spectrum of the theory indeed matches that of the higher dimensional supersymmetric theory. As we have mentioned above, the question of the existence of the full 5D Lorentz invariance is tightly related to the question of whether the full 5D $\mathcal{N}=1$ supersymmetry is present or not. The reason is that the global "hopping" symmetry of the 4D theory, which is closely related to the enhanced SUSY at low energies, becomes a spacetime symmetry of the 5D theory. The Lorentz symmetry generators are part of the full SUSY algebra. Hence, 5D supersymmetry requires an enhanced Lorentz symmetry. This manifests itself in a doubling in the number of supersymmetry generators, which then form an irreducible representation of the 5D Lorentz group. If supersymmetry is indeed enhanced, then the theory must automatically be 5D Lorentz invariant for consistency (meaning that the speed of light should not differ in the fifth direction). The converse is also true: if one can show that 5D Lorentz invariance is maintained, then the existence of the four supercharges implies that there must be another set of four supercharges present in the theory. We will pursue this latter route. We will calculate the kinetic term along the fifth dimension for the fermionic fields, and show that 5D Lorentz invariance is automatically obtained; that is, the speed of light along the fifth direction automatically matches the speed along the nonlat-ticized four dimensions, just as has been suggested in [1]. Then by 4D supersymmetry one also obtains a similar conclusion for the scalars, from which it follows that the full 5D supersymmetry must be present.

In order to show this we have to show that the action at leading order is given by terms that are the discretized versions of the 5D $\mathcal{N}=1$ SUSY Yang-Mills theory. However, that theory has only adjoint fermions living at lattice sites, as opposed to our bifundamental fermions. So it is not *a priori* obvious that one indeed gets the required action. In order to show this we first identify the scalar component of the bifundamental fields with the link variable $U_{i,i+1}$ via $Q_i = vU_{i,i+1}$, where the $U_{i,i+1}$'s are unitary matrices transforming as bifundamentals under that $SU(M)_i \times SU(M)_{i+1}$ (as is appropriate for link variables). Now we define fermions transforming as adjoints only under single gauge groups (thus living at the sites of the lattice in the 5D language) by defining $\psi_i = q_i U_{i,i+1}^\dagger$. With this identification of the fermion fields living on the sites we can write the interaction part of the action in (2.26) as

$$i\sqrt{2}g \sum_i \lambda_i (q_i Q_i^\dagger - Q_{i-1}^\dagger q_{i-1}) + \text{H.c.} = i\sqrt{2}gv \sum_i \lambda_i (\psi_i$$
$$- U_{i-1,i}^\dagger \psi_{i-1} U_{i-1,i}) + \text{H.c.} \qquad (2.31)$$

In 5D the Dirac fermion is irreducible, so one expects λ and ψ to form a Dirac fermion, and the above term to correspond to the discretized version of the kinetic term of the 5D action along the fifth dimension. This is indeed the case, since one can define the 5D Dirac spinor by

$$\Psi_D = \begin{pmatrix} i\lambda \\ \bar{\psi} \end{pmatrix},$$

and then the discretized version of $i\bar{\Psi}_D \slashed{D}_5 \Psi_D$ is indeed reproduced by Eq. (2.31), where the lattice spacing is identified with $a = 1/(\sqrt{2}gv)$, and the relevant gamma matrices in Weyl representation are[5]

$$\gamma^5 = i\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} & -1 \\ -1 & \end{pmatrix}, \quad \gamma^j = \begin{pmatrix} & \sigma^j \\ -\sigma^j & \end{pmatrix}.$$

This shows that 5D Lorentz invariance is automatic in these models, and in turn that the full 5D supersymmetry must be present in the continuum limit. Thus we have shown that the kinetic terms of the fermions automatically have the same speed of light along the 5th dimension as for the other four. By 4D $\mathcal{N}=1$ SUSY the scalar kinetic terms also have the right continuum limit, and it has already been shown in [1–3] that the same applies to the gauge bosons. In fact, as explained before, since the 5D theory must have at least eight supercharges, there are only two possibilities: either one obtains the 5D $\mathcal{N}=1$ theory and then Lorentz invariance

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[5]Note that, in order to satisfy the 5D Clifford-Dirac algebra, the Dirac matrix in the fifth direction picks up a factor i compared to the usual γ^5 defined in 4D.

is automatically implied, or the theory is not Lorentz invariant. The reason for this is that the 4D construction already guarantees the presence of four supercharges. Thus in a Lorentz invariant theory the other four must also be present.

D. Comments on nonperturbative matching

Until now we have concentrated on the region where $v \gg \Lambda$, when nonperturbative effects are not important, because the gauge group is broken before it could become strongly interacting. Another important check would be to match the nonperturbative effects for the case when $v \sim \Lambda$. In this case nonperturbative effects will be important, and can be described by an auxiliary Seiberg-Witten curve. These curves have in fact been analyzed for the 4D lattice theory in [8], and for the 5D $\mathcal{N}=1$ theories on a circle in [17]. The degrees of both curves match, as do the number of moduli appearing in the theory. This suggests that there is at least a chance that these two curves could become equivalent in the continuum limit. It would be very interesting to actually find a detailed mapping of the two curves, which is, however, beyond the scope of this paper.

III. ADDING FLAVORS AND ORBIFOLDING

A. Adding flavors

Adding extra flavors to the theory is straightforward. However, there is one important difference compared to the case without flavors. Until now one did not need to tune any parameter of the theory to recover the higher dimensional supersymmetric model. This is not surprising, because a 4D $\mathcal{N}=1$ SUSY theory with only a vector and chiral multiplet (and no superpotential for the chiral multiplet) already has $\mathcal{N}=2$ supersymmetry. However, this is no longer true in the presence of hypermultiplets. In this case the $\mathcal{N}=1$ Lagrangian needs to contain a superpotential coupling of the form $\sqrt{2}g\bar{\Phi}\Sigma\Phi$, where g has to be equal to the gauge coupling, $\bar{\Phi}$, Φ form the hypermultiplet, and Σ is the chiral superfield in the adjoint. Thus we expect that a similar tuning has to occur in this case as well. A fundamental flavor in the 5D theory will have to be included as a flavor into every gauge group. Thus the matter content will be modified to

	$SU(M)_1$	$SU(M)_2$	$SU(M)_3$	$\cdots$	$SU(M)_N$
Q_1	□	□̄	1	$\cdots$	1
Q_2	1	□	□̄	$\cdots$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Q_N	□̄	1	1	$\cdots$	□
P_1	□	1	1	$\cdots$	1
$\tilde{P}_1$	□̄	1	1	$\cdots$	1
P_2	1	□	1	$\cdots$	1
$\tilde{P}_2$	1	□̄	1	$\cdots$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
P_N	1	1	1	$\cdots$	□
$\tilde{P}_N$	1	1	1	$\cdots$	□̄

(3.1)

The superpotential needed for this model to be $\mathcal{N}=2$ supersymmetric is given by

$$W_{\text{flavor}} = \sqrt{2}g \sum_i \text{tr}(\tilde{P}_i Q_i P_{i+1}) + m_0 \sum_i P_i \tilde{P}_i. \quad (3.2)$$

This is the most general renormalizable superpotential that one can add to the theory, but, as explained above the coefficient in the cubic term has to equal the gauge coupling g. This superpotential generates a mass term for the fermionic components of the flavor fields that looks like

$$\mathcal{L} \supset -\frac{1}{2}(p_{i\alpha}|\tilde{p}_{i\alpha})\delta_{\alpha\alpha'}\left(\frac{\Xi^t |}{\Xi}\right)_{ij}\left(\frac{p_{j\alpha'}}{\tilde{p}_{j\alpha'}}\right) + \text{H.c.} \quad (3.3)$$

with

$$\Xi = \begin{pmatrix} m_0 & \sqrt{2}gv & & \\ & \ddots & \ddots & \\ & & \ddots & \sqrt{2}gv \\ \sqrt{2}gv & & & m_0 \end{pmatrix}. \quad (3.4)$$

The matrix Ξ can be easily diagonalized by noting that it is written in the form $m_0 + \sqrt{2}gvC$ where C is the cyclic permutation matrix whose eigenvalues are the Nth roots of unity $\omega_k = e^{2\pi i(k/N)}$. Thus the fermionic mass spectrum is given by M degenerate Dirac spinors with (mass)^2, $m_k^2 = 2g^2v^2 + m_0^2 + 2\sqrt{2}gvm_0\cos 2\pi(k/N)$, for $k=0...N-1$.

Each mass level transforms as a fundamental of the unbroken $SU(M)$ gauge group. For an even number of lattice sites, the lowest mass level is $m_0 + \sqrt{2}gv$, and thus only if we tune this parameter to zero will we obtain a massless flavor in the bulk. Clearly, by supersymmetry or direct calculation the mass spectrum of the complex scalars will match that of the fermions, and we do not repeat the calculation here.

This bulk mass term can also be recovered by considering the interaction terms and how they would arise from the latticized version of a higher dimensional Lagrangian. For example, the Yukawa coupling and mass term from the superpotential will have to reproduce the kinetic and mass terms of the higher dimensional Lagrangian. Again writing $Q_i = vU_{i,i+1}$, these terms can be written as

$$\mathcal{L} \supset -\sqrt{2}gv\sum_i \tilde{p}_i(U_{i,i+1}p_{i+1} - p_i) - (m_0 + \sqrt{2}gv)\sum_i \tilde{p}_i p_i + \text{H.c.}, \quad (3.5)$$

which is simply the lattice discretization of the kinetic term $\bar{P}_D\slashed{D}_5 P_D$ and of a bulk mass term $(m_0 + \sqrt{2}gv)\bar{P}_D P_D$ for the 5D Dirac spinors $P_D = \binom{p_i}{\tilde{p}_i}$. We can see now that from this point of view the fine-tuned value of the coupling in the superpotential was necessary in order to recover the correctly normalized kinetic term in the 5D theory with the lattice spacing $a = 1/(\sqrt{2}gv)$.

Other multiplets can be introduced similarly, except that the superpotential will in general be nonrenormalizable.

B. Orbifolding

Until now we have exclusively considered a periodic lattice with link fields connecting all of the gauge groups. In this model the full 4D $\mathcal{N}=2$ supersymmetry is unbroken, including in the zero mode sector. One interesting modification is to explicitly break (at least some of) the supersymmetry via the latticized analog of an orbifold. The construction is remarkably simple: identify the fields related by "reflection" about the Z_2 symmetry of the moose circle by cutting the moose in half and removing the link supermultiplets between adjacent sites corresponding to the orbifold fixed points [as in Fig. 1(b)].[6]

We begin with a cycle 4D $SU(M)^{2N}$ theory as described above, and orbifold the moose circle (the extra dimension) by a Z_2 reflection symmetry. After removing two diametrically opposite links, this gives an $SU(M)^N$ theory corresponding to a 5D theory on an interval, similar to the "aliphatic" models considered by Cheng, Hill, Pokorski, and Wang [2,3]. The opening of the moose diagram also explicitly breaks the "hopping" symmetry of the lattice at the end points of the interval. In particular, this would cause the endpoint gauge groups to be anomalous, so we add M chiral

multiplets $\tilde{P}$ and M chiral multiplets P in the antifundamental and fundamental representation of $SU(M)_1$ and $SU(M)_N$, respectively. These fields would correspond to fields stuck to the orbifold fixed points ("the branes") in the higher dimensional language, and are also reminiscent of the Horǎva-Witten compactification of 11D supergravity on an interval, in which case E_8 gauge multiplets are forced to live on the end points of the interval to cancel anomalies. The difference here is that after breaking of the gauge symmetries these fields can get a mass term from the superpotential

$$\frac{1}{\tilde{\mu}^{N-2}}\sum_{i=1}^{M}\tilde{P}_i\prod_{j=1}^{N-1}Q_jP_i. \tag{3.6}$$

In this superpotential, gauge indices on the superfields are contracted so as to make a gauge singlet under the full $SU(M)^N$, and $\tilde{\mu}$ is a mass scale. Once the scalar components of the link multiplets Q_1 and Q_{N-1} acquire VEV's, the P's get a mass given by $v(v/\tilde{\mu})^{N-2}$. Thus they will have no VEV's for their scalar components. The orbifold theory is summarized in the table below:

	$SU(M)_1$	$SU(M)_2$	$\cdots$	$SU(M)_{N-1}$	$SU(M)_N$
$\tilde{P}_1,\tilde{P}_2,...,\tilde{P}_M$	$\bar{\square}$	1	$\cdots$	1	1
Q_1	$\square$	$\bar{\square}$	1	$\cdots$	1
Q_2	1	$\square$	$\cdots$	1	1
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
Q_{N-1}	1	1	$\cdots$	$\square$	$\bar{\square}$
$P_1,P_2,...,P_M$	1	1	$\cdots$	1	$\square$

$$\tag{3.7}$$

The resulting gauge boson, fermion, and scalar mass matrices can be easily calculated. The mass term for the gauge boson is [2,3]

$$\mathcal{L}\supset\frac{1}{2}A_{i\mu}^a\mathcal{M}_{ijab}^2A_j^{b\mu}, \tag{3.8}$$

with

$$\mathcal{M}_{ijab}^2=2g^2v^2\delta_{ab}\Omega_{ij},$$

$$\Omega=\begin{pmatrix}1 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 1\end{pmatrix}. \tag{3.9}$$

The mass spectrum is [2,3]

$$m_k^2=8g^2v^2\sin^2\frac{k\pi}{2N}, \quad 0\le k\le N-1. \tag{3.10}$$

The zero mode remains, as well as half the massive modes, corresponding to the symmetric modes about the orbifold action. We can see that these are in fact the symmetric modes by diagonalizing the mass matrix (3.9). The resulting eigenvectors corresponding to the modes of the 5D gauge boson are [2,3],

$$\tilde{A}_k=\sqrt{\frac{2}{2^{\delta_{k0}}N}}\sum_{j=1}^{N}\cos\frac{(2j-1)k\pi}{2N}A_j, \quad k=0,...,N-1. \tag{3.11}$$

Note also that the wave functions of the periodic theory with $2N$ sites are given by Eq. (2.15), $\tilde{A}_k^{\text{periodic}}=\Sigma_{j=1}^{2N}\omega_k^{j-1}A_j$, where ω_k is $e^{i2\pi k/2N}$. The modes specified by k and $N-k$ are degenerate and correspond to right- and left-moving modes. The orbifold has then picked out modes with definite parity

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[6]We thank Nima Arkani-Hamed for discussions on this point.

under the Z_2 orbifold symmetry, in this case even parity. The mode decomposition is again the discrete analogue of the continuous orbifold expansion [see Fig. 1(b)].

The lattice spacing of the corresponding 5D theory on an S_1/Z_2 orbifold of length Na can be obtained in the same way as before, by identifying the low-lying mass spectra

$$\frac{\pi k}{Na}=2\sqrt{2}gv\frac{k\pi}{2N}. \tag{3.12}$$

We obtain $a=1/(\sqrt{2}gv)$, the same spacing as the periodic lattice.

We also compute the fermion masses in order to check that the spectrum corresponds to that of the orbifolded theory. Following the discussion of Sec. II B 3, the complex fermion mass matrix in two-component Weyl notation is

$$\mathcal{L}\supset\frac{1}{2}(\lambda_i q_i)\mathcal{M}_{ij}\begin{pmatrix}\lambda_j\\q_j\end{pmatrix}+\text{H.c.}, \tag{3.13}$$

with $i,j=1,...,N$, $\hat{i},\hat{j}=1,...,N$ and

$$\mathcal{M}_{ij}=i\sqrt{2}gv\left(\begin{array}{c|c} & \hat{\Theta}^t \\ \hline \hat{\Theta} & \end{array}\right)_{ij} \tag{3.14}$$

(we have not written the gauge structure which is identical to the cyclic case) and the lattice substructure, $\hat{\Theta}$, is given by the $(N-1)\times N$ matrix

$$\hat{\Theta}=\begin{pmatrix}1 & -1 & & \\ & \ddots & \ddots & \\ & & \ddots & -1 \\ & & 1 & -1\end{pmatrix}. \tag{3.15}$$

Although not identical to the gauge boson (mass)^2 matrix, the (mass)^2 matrix for the fermion is closely related

$$\mathcal{M}^\dagger\mathcal{M}=2g^2v^2\left(\begin{array}{c|c}\hat{\Theta}^t\hat{\Theta} & \\ \hline & \hat{\Theta}\hat{\Theta}^t\end{array}\right) \tag{3.16}$$

where $\hat{\Theta}^t\hat{\Theta}$ and $\hat{\Theta}\hat{\Theta}^t$, respectively an $N\times N$ and $(N-1)\times(N-1)$ matrix, are equal to

$$\hat{\Theta}^t\hat{\Theta}=\hat{\Omega},\quad \hat{\Theta}\hat{\Theta}^t=\begin{pmatrix}2 & -1 & & \\ -1 & \ddots & \ddots & \\ & \ddots & \ddots & -1 \\ & & -1 & 2\end{pmatrix}. \tag{3.17}$$

The eigenvalues of $\hat{\Omega}$ are given in Eq. (3.10). It can be immediately checked that $\hat{\Theta}\hat{\Theta}^t$ does not have a zero mode by evaluating its determinant

$$\det\hat{\Theta}\hat{\Theta}^t=N. \tag{3.18}$$

In fact, the eigenvalues can be readily determined by first diagonalizing the mass matrix with diagonal components missing, and then adding back the term proportional to the identity matrix. The result is,

$$m_k^2=8g^2v^2\sin^2\left(\frac{k\pi}{2N}\right),\quad 1\le k\le N-1. \tag{3.19}$$

Therefore the massive eigenvalues pair up to give a Dirac mass term while a Weyl fermion remains massless. As we will see in Sec. IV the eigenvectors of $\hat{\Theta}\hat{\Theta}^t$ are odd about the Z_2 symmetry, which specifies the orbifold action on the link field fermions in the 5D language.

The same arguments can be applied to the scalars from the link multiplets. From the D terms in the Lagrangian we find that the $2(N-1)M^2$ real scalars in Q_i consist of one set of would-be Goldstone bosons that are eaten by the $(N-1)(M^2-1)$ massive vector fields, $2(N-1)$ singlets that are given mass by the tree level superpotential S_iB_i, and a set of $(N-1)(M^2-1)$ massive scalars with masses identical to the gauge bosons. So, we obtain a massless vector and massless Weyl fermion, corresponding to an unbroken 4D $\mathcal{N}=1$ vector supermultiplet, plus a massive tower of states that fall precisely into $\mathcal{N}=2$ vector supermultiplets. We see again that the diagrammatic picture of a linear set of gauge groups connected by link fields physically and intuitively becomes a latticization of the line segment obtained from an S_1/Z_2 orbifold. In this particular construction only 4D $\mathcal{N}=1$ supersymmetry is preserved in the zero mode sector.

IV. GAUGINO MEDIATION IN 4D

One application of our construction of supersymmetric extra dimensions is to explore ways to communicate supersymmetry breaking to the supersymmetrized standard model. The central problem is to generate a supersymmetry-breaking spectrum with no highly fine-tuned mass hierarchies, while simultaneously avoiding current bounds from experiment. Generally this requires that the supersymmetry-breaking sector is well separated from the MSSM. For example, flavor nondiagonal contributions to squark and slepton mass matrices are severely constrained from experimental bounds on flavor changing neutral current processes. One way to avoid these constraints is to generate soft supersymmetry breaking scalar masses dominantly through gauge interactions. This happens in ordinary four-dimensional gauge mediation where both gaugino and scalar masses are generated through one- and two-loop diagrams with "messenger" fields [18]. An alternative proposal, called "gaugino-mediation" [19–21], physically separates the supersymmetry-breaking sector across an extra dimension on S_1/Z_2 similarly to the "anomaly-mediated" models of [22]. Direct couplings between the supersymmetry breaking fields and the chiral matter fields are exponentially suppressed by the small wave-function overlap of one on the other. In this model (contrary to anomaly mediation) the gauge supermultiplets of the MSSM are placed in the 5D bulk, coupling directly with the supersymmetry-breaking fields that are assumed to be localized at one orbifold fixed point. The MSSM

chiral matter lives on the other orbifold fixed point. Once supersymmetry is broken, a large supersymmetry-breaking mass is endowed to the gauginos while a loop-suppressed (flavor-diagonal) contribution is generated for the scalar masses at the compactification scale. Large supersymmetry-breaking scalar masses are induced by ordinary 4D renormalization-group (RG) evolution to the weak scale,[7] generating a spectrum that is similar to a "no-scale" supergravity model.

Here we will use the construction of the supersymmetric extra dimensions presented in the previous sections to "translate" the mechanism of gaugino mediation into a purely 4D model, that will result in a perturbative SUSY-breaking soft mass spectrum identical to that of gaugino mediation. However, since gravity in this construction is not made higher dimensional, one has to ensure that the flavor-changing Planck-suppressed contact terms are subdominant. This can be done by requiring that the scale of mediation of SUSY breaking Λ is smaller than the Planck scale, $\Lambda \ll M_{Pl}$, just like in gauge mediation models. This will imply that the gravitino is the lightest supersymmetric particle (LSP), which avoids the possibility of a cosmologically troubling stau LSP that can occur in continuum gaugino media-

tion when the size of the extra dimension is of order or smaller than the inverse GUT scale. In fact, lowering the SUSY mediation scale in our case is analogous to increasing the size of the extra dimension in continuum gaugino mediation.

A. Gaugino masses

We start with the setup in (3.7), where the $SU(M)_i$ gauge groups are all identified with the gauge groups of the MSSM. This generates a tower of $N-1$ states in massive $\mathcal{N}=2$ vector supermultiplet representations, which for small $k \ll N$ are indistinguishable from the KK tower generated in gaugino mediation. On the $i=0$ end point of the lattice, we place the sector of fields needed to break supersymmetry dynamically. Rather than specifying this in detail, we follow Refs. [19], [20] and simply assume that the result of dynamical supersymmetry breaking is that the auxiliary component of a gauge singlet chiral superfield located on the $i=0$ lattice site acquires a VEV, $\langle S \rangle = F_S \theta^2$. The chiral matter multiplets of the MSSM are placed on the $i=N-1$ lattice site. The resulting matter content is given by

	$SU(5)_0$	$SU(5)_1$	$\cdots$	$SU(5)_{N-2}$	$SU(5)_{N-1}$
$\tilde{P}_1,\ldots,\tilde{P}_5$	$\bar{\square}$	1	$\cdots$	1	1
Q_1	$\square$	$\bar{\square}$	$\cdots$	1	1
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
Q_{N-1}	1	1	$\cdots$	$\square$	$\bar{\square}$
$P_1,\ldots,P_5$	1	1	$\cdots$	1	$\square$
$\bar{\mathbf{5}}_{1,2,3}$	1	1	$\cdots$	1	$\bar{\square}$
$\mathbf{10}_{1,2,3}$	1	1	$\cdots$	1	$\boxminus$
H_d	1	1	$\cdots$	1	$\bar{\square}$
H_u	1	1	$\cdots$	1	$\square$

$$(4.1)$$

We have written the interactions in $SU(5)$ language for compactness, although we could also have simply latticized the SM gauge group. The action on the $i=0$ point is assumed to have the superpotential terms

$$\mathcal{L} = \int d^2\theta \, \frac{S}{\Lambda} W_\alpha W^\alpha + \text{H.c.}, \qquad (4.2)$$

where W_α is the field strength chiral superfield for $SU(5)_0$, and Λ is the SUSY mediation scale. We will assume that the scale Λ arises from supersymmetry breaking and is larger than the inverse lattice spacing. We have assumed for sim-

plicity that there is a chiral multiplet S with a SUSY-breaking VEV, but of course in a more complete model the SUSY breaking has to be specified. For example, one could imagine that the operator (4.2) is generated by a gauge mediation from the SUSY-breaking sector to the gaugino, and in this case there would be an additional loop factor $g^2/(16\pi^2)$ appearing in the gaugino mass. For more on this possibility see Sec. IV E. Once S acquires a supersymmetry-breaking VEV, a gaugino mass is generated for the gaugino fields of the $SU(5)_0$ gauge group

$$\mathcal{L} \supset \frac{1}{2} \frac{F_S}{\Lambda} \lambda\lambda + \text{H.c.} \qquad (4.3)$$

Below the scale $a^{-1} = \sqrt{2} g v$, the full $(2N-1) \times (2N-1)$

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[7] One or two orders of magnitude of RG evolution is sufficient [19].

gaugino mass term becomes

$$i\frac{gv}{\sqrt{2}}(\lambda_0,\lambda_1,\cdots\lambda_{N-1}|q_1,\cdots q_{N-1})$$

$$\times \begin{pmatrix} \begin{array}{ccc|ccc} 2\epsilon_F & 0 & \cdots & & & \\ 0 & 0 & & & \hat{\Theta}^t & \\ \vdots & & \ddots & & & \\ \hline & \hat{\Theta} & & 0 & \cdots & \\ & & & \vdots & \ddots & \end{array} \end{pmatrix} \begin{pmatrix} \lambda_0 \\ \lambda_1 \\ \vdots \\ \lambda_{N-1} \\ q_1 \\ \vdots \\ q_{N-1} \end{pmatrix} + \text{H.c.} \quad (4.4)$$

where $\epsilon_F \equiv -iaF_S/(2\Lambda)$, and $\hat{\Theta}$ is given in Eq. (3.15). For $|\epsilon_F| \ll 1$, the mass matrix can be approximately diagonalized using perturbation theory. The perturbation for the square of the mass matrix has the following form:

$$\delta(M^\dagger M) = 2g^2v^2 \begin{pmatrix} \begin{array}{ccc|ccc} 2|\epsilon_F|^2 & 0 & \cdots & \epsilon_F^* & 0 & \cdots \\ 0 & & & 0 & & \\ \vdots & & \ddots & \vdots & & \\ \hline \epsilon_F & 0 & \cdots & & & \\ 0 & & & & & \\ \vdots & & & & & \end{array} \end{pmatrix}$$

$$(4.5)$$

Since the first-order perturbation for the zero mass eigenvalue gives a result of order $|\epsilon_F|^2$, one is forced to look at the second-order perturbations as those will also involve terms of order $|\epsilon_F|^2$. For this, we need the eigenvectors of the unperturbed (mass)^2 matrix

$$2g^2v^2 \left(\begin{array}{c|c} \hat{\Theta}^t\hat{\Theta} & \\ \hline & \hat{\Theta}\hat{\Theta}^t \end{array} \right). \quad (4.6)$$

The $N \times N \hat{\Theta}^t\hat{\Theta}$ block has the following eigenvectors:

$$\tilde{\lambda}_k^+ = \sqrt{\frac{2}{2^{\delta_{k0}}N}} \sum_{m=0}^{N-1} \cos\frac{(2m+1)k\pi}{2N}\lambda_m, \quad k=0,...,N-1$$

$$(4.7)$$

with eigenvalues

$$m_k^2 = 8g^2v^2\sin^2\frac{k\pi}{2N}, \quad k=0,...,N-1. \quad (4.8)$$

The $(N-1)\times(N-1)\hat{\Theta}\hat{\Theta}^t$ block has eigenvectors

$$\tilde{\lambda}_k^- = \sqrt{\frac{2}{N}} \sum_{m=1}^{N-1} \sin\frac{mk\pi}{N}q_m, \quad k=1,...,N-1, \quad (4.9)$$

with eigenvalues

$$m_k^2 = 8g^2v^2\sin^2\frac{k\pi}{2N}, \quad k=1,...,N-1. \quad (4.10)$$

The first-order perturbation for the zero mode gives a shift in the (mass)^2 of $8|\epsilon_F|^2g^2v^2/N$, while the second-order perturbation gives

$$-\frac{4}{N^2}|\epsilon_F|^2g^2v^2\sum_{j=1}^{N-1}\frac{\sin^2\frac{j\pi}{N}}{\sin^2\frac{j\pi}{2N}}. \quad (4.11)$$

Using the relation

$$\sum_{j=1}^{N-1}\frac{\sin^2\frac{j\pi}{N}}{\sin^2\frac{j\pi}{2N}} = 2(N-1),$$

we obtain for the full perturbation in the mass of the zero mode,

$$m_0 = 2\sqrt{2}gv\frac{|\epsilon_F|}{N}. \quad (4.12)$$

One can also calculate the mass splittings of the higher mass fermionic modes; however, due to the degeneracy of the mass eigenvalues one has to use degenerate perturbation theory. The result we obtain for the splittings is

$$m_k^2 = 8g^2v^2\left(\sin\frac{k\pi}{2N} \pm 2\frac{|\epsilon_F|}{N}\cos^2\frac{k\pi}{2N}\right)\sin\frac{k\pi}{2N},$$

$$k=1,...,N-1. \quad (4.13)$$

All of the gaugino masses are shifted relative to the gauge boson masses—supersymmetry is broken.

The zero-mode gaugino mass can be written in a somewhat more suggestive form

$$m_0 = \frac{1}{N}\frac{F_S}{\Lambda}. \quad (4.14)$$

The gaugino mass appears to vanish in the large-N limit. In fact, a similar phenomenon is also present in continuum gaugino mediation, where the corresponding gaugino mass was given by $m_0 = F_S/M^2L$. There M represented both the scale suppressing the higher dimensional SUSY-breaking operator as well as the scale where the 5D theory was becoming strongly coupled. These scales were taken to be equal for simplicity [19]. We can relate this result to the gaugino mass found in our construction by first identifying one power of $1/M$ as the scale $1/\Lambda$ suppressing the gaugino mass operator. The other factor of $1/M$ should be identified with the inverse lattice spacing. The reason is that even though the full latticized theory is never strongly coupled (which was one of the main motivations for this construction), below the scale of the lattice spacing the unbroken diagonal subgroup is as strongly coupled as the continuum theory for the same number N of massive "KK" modes. This suggests the identification of the 5D strong-coupling scale (the other M) with the
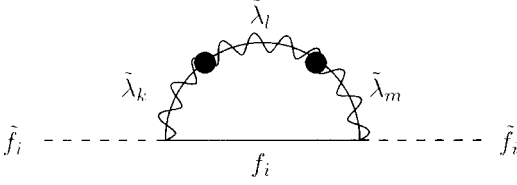
FIG. 2. One-loop contribution to the MSSM matter scalar masses. The ●'s on the gaugino line represent the SUSY breaking mass insertion F/Λ.

inverse lattice spacing a. Then, continuum gaugino mediation [19,20] predicts a gaugino mass identical to Eq. (4.14),

$$\frac{1}{ML}\frac{F_S}{M} \sim \frac{1}{a^{-1}Na}\frac{F_S}{\Lambda}. \tag{4.15}$$

In 5D it is easy to see that N cannot be arbitrarily large before the gauge couplings blow up, so no consistent "large-N" limit can be taken. In our latticized theory we see the same effect for diagonal subgroup, except that in this construction the strong coupling physics is resolved at the scale of the lattice spacing—meaning it is really an artifact of considering just the diagonal subgroup. Above the lattice spacing the full asymptotically free product gauge theory is resolved, leading to a fully perturbative theory.

B. Scalar masses at one loop

The leading contributions to the MSSM matter scalar masses arise through loop diagrams of gauginos interacting with the supersymmetry-breaking operators on the $i=0$ lattice site, shown in Fig. 2. The contributions are flavor diagonal, since they only involve some mixed combination of gauginos running in the loop. Here we will carry out the

calculation in a way that is completely analogous to the continuum gaugino mediation result given in Ref. [19]. In particular, we will calculate the scalar mass contribution from a one-loop diagram with a gaugino running in the loop, and two insertions of the nonrenormalizable supersymmetry-breaking operator Eq. (4.2). (Hence, the gaugino mass matrix has *not* been shifted by the supersymmetry-breaking contributions since we have not yet integrated them out.) In the interaction eigenstate basis, the MSSM matter multiplets interact only with the $SU(5)_{N-1}$ gaugino. However, once the gaugino mass matrix is diagonalized, chiral multiplets interact with the entire tower of Majorana gaugino mass eigenstates $\widetilde{\lambda}_k$.

The gaugino wave functions correspond to the eigenvectors (4.7) and (4.9) of the unperturbed $(2N-1)\times(2N-1)$ matrix (4.6). Hence, the $2N-2$ massive gauginos that are paired with equal but opposite in sign masses can be split into two sets of $N-1$ gauginos with cosine and sine wave-function expansions. In the large-N limit the set of gaugino fields $\widetilde{\lambda}_k^-$ with sine expansions do not directly couple to either the supersymmetry-breaking fields or the MSSM matter fields, and so they will not be needed in the calculations below.

The scalar mass loop calculation involves a gaugino propagator extending between two different lattice sites. In the mass eigenstate basis the full gaugino propagator is a sum over the N gauginos. We find it convenient to incorporate the "end-point" lattice site couplings

$$\langle \widetilde{\lambda}_j^+ | \lambda_k \rangle = \sqrt{\frac{2}{2^{\delta_{j0}}N}}\cos\frac{(2k+1)j\pi}{2N} \tag{4.16}$$

into the sum over the gaugino propagators. The result is

$$\mathcal{P}(q;k,l)=\frac{2}{N}\slashed{q}\sum_{j=0}^{N-1}\frac{1}{2^{\delta_{j0}}}\cos\frac{(2k+1)j\pi}{2N}\cos\frac{(2l+1)j\pi}{2N}\frac{1}{q^2+\left(\dfrac{2}{a}\right)^2\sin^2(j\pi/2N)}, \tag{4.17}$$

which represents the summed gaugino propagator with Euclidean momentum q extending between the kth to lth lattice sites. Note that we have not written the mass term since it will drop out of the scalar mass calculation below. We only need the propagator extending from the $k=0$ to $l=N-1$ lattice site. With suitable rearrangements, this is

$$\mathcal{P}(q;0,N-1)=\frac{2}{N}\slashed{q}\sum_{j=0}^{N-1}\frac{1}{2^{\delta_{j0}}}\frac{(-1)^j\cos^2(j\pi/2N)}{q^2+\left(\dfrac{2}{a}\right)^2\sin^2(j\pi/2N)}. \tag{4.18}$$

We note that in the large-N limit, this reproduces the continuum gaugino propagator found in Ref. [19]. The finite sum can be done, and we find

$$\mathcal{P}(q;0,N-1)=a^2\slashed{q}\prod_{j=0}^{N-1}\frac{1}{(aq)^2+4\sin^2(j\pi/2N)}. \tag{4.19}$$

Given this relatively simple expression for the gaugino propagator extending between the end points of the lattice, we can now carry out the scalar mass calculation.

The one-loop diagram for scalar masses can be written in terms of the above summed gaugino propagator as

$$\widetilde{m}^2=\frac{g^2}{16\pi^2}\left|\frac{F_S}{\Lambda}\right|^2\int d^4q\,\mathrm{tr}\bigg[P_R\frac{1}{\slashed{q}}P_L\mathcal{P}(q;N$$
$$-1,0)P_R\mathcal{P}(q;0,0)P_L\mathcal{P}(q;0;N-1)\bigg]. \tag{4.20}$$

The propagator for the zeroth to zeroth lattice site can be obtained from Eq. (4.17),

$$\mathcal{P}(q;0,0) = \frac{\not{q}}{Nq^2}\left[1 + \sum_{k=1}^{N-1} \frac{2(aq)^2\cos^2\frac{k\pi}{2N}}{(aq)^2 + 4\sin^2\frac{k\pi}{2N}} \right] \equiv \frac{\not{q}}{Nq^2}[1+\Sigma]. \tag{4.21}$$

Notice that, unlike $\mathcal{P}(q;0,N-1)$, there are no delicate cancellations between the zero mode and the massive tower states. The scalar mass calculation therefore reduces to performing the integral

$$\widetilde{m}^2 = \frac{g^2}{16\pi^2 N}\left|\frac{F_S}{\Lambda}\right|^2 a^4 \int d^4q \prod_{n=0}^{N-1} \frac{1}{((aq)^2 + 4\sin^2(n\pi/2N))^2}[1+\Sigma] \tag{4.22}$$

$$= \frac{g^2}{16\pi^2 N}\left|\frac{F_S}{\Lambda}\right|^2 \int_{2\sin(\pi/2N)}^{\infty} d(aq)\frac{2\pi^2}{(aq)\prod_{n=1}^{N-1}((aq)^2 + 4\sin^2(n\pi/2N))^2}[1+\Sigma]. \tag{4.23}$$

One approximation is to neglect the sum over the massive tower of gauginos for $\mathcal{P}(q;0,0)$, i.e., $[1+\Sigma]\to 1$. This gives a reasonable estimate to within a factor of 4 or so; however, we will retain the full sum in our calculations below. Notice that the integral is logarithmically IR divergent but UV finite, and so we start the momentum integration at the scale of the lightest massive gaugino. The IR divergence is handled by the usual 4D logarithmic evolution of the zero-mode gaugino mass for energy scales $q < 2a^{-1}\sin(\pi/2N)$.

The momentum integral (including the sum) evaluates to c/N^2 with a numerical coefficient c that asymptotes to 1 to very good accuracy for large N ($|c-1|<0.1$ for $N>4$). The gauge coupling g for the $SU(5)_{N-1}$ group on the $N-1$ lattice site must also be converted to the gauge coupling of the unbroken diagonal subgroup $SU(5)_{\text{diag}}$ via

$$g_{\text{SM}} = \frac{g}{\sqrt{N}}. \tag{4.24}$$

If we ignore other factors of 2 and quadratic Casimirs, we obtain

$$\widetilde{m}^2 = \frac{g_{\text{SM}}^2}{16\pi^2}\left|\frac{1}{N}\frac{F_S}{\Lambda}\right|^2 \tag{4.25}$$

$$= g_{\text{SM}}^2\left(\frac{m_0}{4\pi}\right)^2. \tag{4.26}$$

This result agrees exactly with continuum gaugino mediation. Hence, our 4D latticized supersymmetric theory generates a gaugino and scalar mass spectrum that is identical to the 5D continuum gaugino mediation result.

C. Other nongravitational contributions to scalar masses

Here we consider whether the induced scalar masses are really flavor diagonal. In continuum gaugino mediation, it was argued that direct couplings between MSSM matter scalar masses and supersymmetry-breaking fields are forbidden by 5D locality. However, the wave functions of the fields localized to the orbifold fixed points are not truly delta functions, but instead have some width extending into the fifth direction. The overlap of fields located on one fixed point with the other is therefore anticipated to be exponentially suppressed by roughly e^{-ML}. What is the analog in our construction? Naively 4D effective theory suggests we should be able to write dangerous operators such as

$$\int d^4\theta \frac{S^\dagger S}{\Lambda^2}L_i^\dagger L_j, \tag{4.27}$$

$$\int d^4\theta \frac{Q_k^\dagger Q_k}{\Lambda^2}L_i^\dagger L_j, \tag{4.28}$$

where L_i can be any matter superfield of the MSSM. We assume that the same scale Λ suppressing the gaugino mass operator, Eq. (4.2), also enters these operators. However, the coefficients are undetermined and could be order 1 or (in a gauge mediation model) could be loop suppressed. However, these operators would only be generated if the MSSM fields coupled directly to the SUSY-breaking sector. Our assumption is that it is only the $i=0$ gauge group that couples to SUSY breaking; thus these operators which directly couple the MSSM to the SUSY-breaking sector are absent, due to this version of "locality on the lattice." There are, however, "local" operators that contribute to flavor nondiagonal scalar masses. These have the form

$$\int d^4\theta \frac{S^\dagger S}{\Lambda^{N+1}}Q_1 Q_2 \cdots Q_{N-1} L_i^\dagger L_j. \tag{4.29}$$

After the link fields acquire VEV's, this gives rise to an operator of the form

$$\frac{v^{N-1}}{\Lambda^{N-1}} \int d^4\theta \frac{S^\dagger S}{\Lambda^2} L_i^\dagger L_j. \tag{4.30}$$

Clearly once supersymmetry is broken this gives rise to a flavor-nondiagonal scalar mass

$$\frac{v^{N-1}}{\Lambda^{N-1}} \left|\frac{F_S}{\Lambda}\right|^2 \phi_{L_i}^* \phi_{L_j}. \tag{4.31}$$

If we write $\epsilon_v \equiv v/\Lambda$, then this contribution becomes

$$\epsilon_v^{N+1} \left|\frac{F_S}{\Lambda}\right|^2 \phi_{L_i}^* \phi_{L_j}. \tag{4.32}$$

For $\epsilon_v < 1$, this contribution is power suppressed by the number of lattice sites N. We have already found that the relationship between the latticized theory and the continuum is $N \sim ML$, and so we see that the latticized theory has an analog of the exponential suppression expected in continuum gaugino mediation. For this to be the case, it is crucial that the scale suppressing this higher dimensional interaction Λ is larger than the induced link VEV (or inverse lattice spacing).

In addition, the link field scalars also acquire supersymmetry-breaking masses. This arises because we can write the operator

$$\int d^4\theta \frac{S^\dagger S}{\Lambda^2} Q_1^\dagger Q_1. \tag{4.33}$$

for the first link field, which leads to the mass term,

$$\left|\frac{F_S}{\Lambda}\right|^2 \phi_1^* \phi_1. \tag{4.34}$$

Since the link field already has a (large) VEV, this supersymmetry-breaking mass simply shifts the scalar mass by $|F_S/\Lambda|^2$ which is of order the gaugino mass. The other link fields do not have a direct coupling to the SUSY-breaking sector, and so acquire loop-suppressed SUSY-breaking contributions for their scalar components.

D. Planck-suppressed contributions to scalar masses

We argued that the operators in Eqs. (4.27), (4.28) are absent due to the assumed "locality on the lattice." However, in our construction gravity is assumed to be ordinary 4D Einstein gravity, and thus we have every reason to expect ordinary 4D Planck-suppressed operators will violate "locality on the lattice." In particular, the usual Planck-suppressed operators resulting from replacing the dynamical scale Λ with $M_{\rm Pl}$ in Eq. (4.27) are present here

$$\int d^4\theta \frac{S^\dagger S}{M_{\rm Pl}^2} L_i^\dagger L_j, \tag{4.35}$$

where again L_i can be any matter superfield of the MSSM. These give rise to flavor off-diagonal contributions to scalar masses of order $|F_S/M_{\rm Pl}|^2$. The limits on the size of these contributions are strongly model dependent, but roughly one
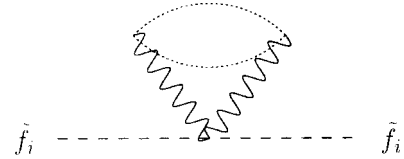

FIG. 3. Two-loop gauge-mediated contribution to the MSSM matter scalar masses (only one example diagram shown). The internal loop of dotted lines corresponds to the messenger scalars.

finds the ratio of off-diagonal to diagonal scalar (mass)^2 s to be $m_{ij}^2/m_{ii}^2 < 10^{-4}$ for at least some choices of $i \neq j$. This means that we must require

$$\left|\frac{F_S}{M_{\rm Pl}}\right|^2 < 10^{-4} |m_0|^2, \tag{4.36}$$

or that

$$N\Lambda < 10^{-2} M_{\rm Pl}. \tag{4.37}$$

This is a separate requirement that must be imposed on our latticized theory (that does not appear in continuum gaugino mediation). Interestingly, this also implies that the gravitino is the lightest supersymmetric particle since

$$m_{3/2} = \frac{F_S}{M_{\rm Pl}} < 10^{-2} m_0. \tag{4.38}$$

E. Gauge-mediated contributions to scalar masses

Up to now we have assumed that the only source of supersymmetry breaking in the model is the operator in Eq. (4.2). If that is indeed the case, then the relevant contributions to the soft breaking mass terms are the ones listed in the previous sections. However, in more realistic models the operator (4.2) appears through gauge mediation from the messenger fields, and therefore the gaugino mass itself has a loop suppression factor $g^2/(16\pi^2)$. The expression for the gaugino mass is given by

$$m_{\rm gaugino} = \frac{g^2}{16\pi^2 N} \frac{F_S}{\Lambda} = \frac{g_{\rm SM}^2}{16\pi^2} \frac{F_S}{\Lambda}, \tag{4.39}$$

which is the ordinary 4D gauge mediation result. In this case, however, the MSSM scalars will also pick up a soft breaking (mass)^2 term from gauge mediation, which is no longer loop suppressed compared to the gaugino mass.[8] An example of a two-loop diagram of this sort is given in Fig. 3. These diagrams have been explicitly evaluated for an extra dimension on S^1/Z_2 with gauge fields in the bulk by Mirabelli and Peskin [23]. There they assumed the messenger sector and the MSSM matter fields were separated on the two orbifold fixed points separated by a distance L in the fifth dimension. They found that the gauge-mediated contribution to the scalar mass is suppressed by an additional factor of $1/(ML)^2$ where M is the cutoff scale. In the latticized case we there-

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[8] We thank Yuri Shirman for reminding us of these operators.

fore expect that the scalar mass will be suppressed by an additional factor of $1/(\Lambda N a)^2$ compared to the gaugino. In order to estimate this diagram, we use the intuitive derivation given in [23] in which the extra loop of messengers is shrunk to a point. It was shown in [23] that the effect of the messenger loop can be represented as a two derivative effective operator that results in an extra factor of q^2 in the loop. Thus we can estimate the size of the scalar masses to be of order[9]

$$\tilde{m}^2 \sim \left(\frac{g^2}{16\pi^2}\right)^2 \left|\frac{F_S}{\Lambda}\right|^2 \frac{1}{(a\Lambda)^2} \int_{2\sin(\pi/2N)}^{\infty} d(aq)$$

$$\times \frac{2\pi^2(aq)}{\Pi_{n=1}^{N-1}((aq)^2 + 4\sin^2(n\pi/2N))^2}. \quad (4.40)$$

We have numerically verified that the integral is well approximated by c/N^4 with $c \sim 4$ at large N, and so the expression we find for the scalar masses is given by

$$\tilde{m}^2 \sim \frac{1}{(\Lambda N a)^2} m_{\text{gaugino}}^2. \quad (4.41)$$

We can see that if $\Lambda a \sim 1$, then by decreasing N the scalar spectrum interpolates between gaugino mediation for $N \gtrsim 5$ and ordinary gauge mediation for $N = 1$. If, however, $1/(\Lambda a) \ll 1$, then a gaugino-mediated spectrum is obtained even for $N = 2$ [25].

F. Realistic models

We can now use these results to construct realistic models of mediating supersymmetry breaking to the MSSM. If a messenger sector does live on the SUSY-breaking site, we recover a variation of the gauge-mediated spectrum. The important difference between ordinary gauge mediation and our latticized version is that there is an additional $1/(\Lambda N a)^2$ suppression of the scalar (mass)^2 s relative to the gaugino (mass)^2. Raising $\Lambda N a$ therefore has a similar effect on the sparticle mass spectrum as raising the number of messenger fields in ordinary gauge mediation, in which the scalar (mass)^2 are suppressed by a factor $1/n_{\text{mess}}$ relative to the gaugino (mass)^2.

For larger $\Lambda N a \gtrsim 5$, or for any N if SUSY breaking is communicated exclusively by the operator Eq. (4.2), the soft mass spectrum is identical to gaugino mediation. One interesting possibility is to consider how small N can be and yet also obtain a viable soft mass spectrum. If $1/(\Lambda a) \ll 1$, then one obtains the gaugino mediation spectrum even for $N = 2$. For the remainder of this section, we wish to consider this case with just two gauge groups.

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[9]Note that the factor $1/(\Lambda a)^2$ was erroneously omitted in (4.40) and (4.41) in the first version of this paper. This was corrected after the appearance of Ref. [25].

Two gauge groups

The dominant contribution to the scalar masses comes from the one-loop diagram given by Eq. (4.26). In this case, the integration over momentum can be done exactly

$$\int_{\sqrt{2}/2}^{\infty} d(aq) \frac{2\pi^2}{(aq)[(aq)^2+2]^2} \left[1 + \frac{(aq)^2}{(aq)^2+2}\right]$$

$$= \pi^2 \frac{-3+8\ln 2}{8N^2} \sim 0.785. \quad (4.42)$$

We see that this integral evaluates to c/N^2 with $c \sim 3$, and therefore the scalar masses are slightly larger than what would be expected for a large number of lattice sites. They are, however, still well suppressed compared with the size of the gaugino mass, and so the usual gaugino mediation spectrum results even for this two-lattice-site example.

One concern is that the link field might communicate SUSY breaking to the MSSM matter scalars, since it is charged under all gauge groups in this two-site example. We have already shown in Eq. (4.34) that the first link field acquires a SUSY-breaking mass of order the gaugino (mass)^2. However, there are no superpotential couplings between the link field and the matter scalars, so at most this field gives a (flavor diagonal) two-loop suppressed contribution to the MSSM matter scalars through loops of the gauge and gaugino field. This is suppressed by one more loop than the contribution found above, and so can be neglected.

Some fine tuning of $a^{-1} \ll \Lambda$ is needed in this case, however, to suppress the two-loop gauge-mediated contribution (4.41). In addition, operators like (4.29) could also lead to additional scalar mass contributions

$$\int d^4\theta \frac{S^\dagger S}{\Lambda^3} Q_1 L_i^\dagger L_j = \frac{v}{\Lambda} \left|\frac{F_S}{\Lambda}\right|^2 \phi_{L_i}^* \phi_{L_j} \quad (4.43)$$

but they are suppressed by the same factor $v/\Lambda \sim 1/(\Lambda a)$ and so can be similarly suppressed. In addition, we expect additional loop suppression of the coefficient of this operator when the mediation of supersymmetry breaking occurs through messengers. The hierarchy of scales that remains is $a^{-1} \sim v \ll \Lambda < 10^{-2} M_{\text{Pl}}$ so that the contribution from the Planck-suppressed operators is small. This means new physics is appearing at scales below the usual gauge coupling unification scale.

G. Gauge coupling unification

Finally, we discuss the issue of unification of the gauge couplings in this scenario. There are two limiting cases that we will consider, namely two gauge groups, and alternatively, large N.

For a larger number of lattice sites N one does not need to separate the scale of SUSY breaking and the lattice spacing anymore. One can take $\Lambda \sim v \sim 10^{16}$ GeV, since the gauge-mediated contributions to the scalar masses can now be suppressed by the $1/N^2$ factor in (4.41), assuming that the operators in (4.29) also come with loop suppression factors. For

N gauge groups, one will have N KK modes appearing, with the first massive mode roughly at the scale v/N. For large N, this scale is well below v, and thus one expects a phase of rapid power-law running of the gauge couplings to start at around this scale. If we assume that, in addition to the gauge bosons the Higgs fields also live at every lattice site (that is, the Higgs boson is in the bulk in the continuum limit), then one can use the results of [24] to show that despite the power-law nature of the running the gauge couplings still unify, but at a scale below M_{GUT}, between M_{GUT}/N and M_{GUT}, which for moderately large $5 \leqslant N \leqslant 50$ is still a relatively high scale close to M_{GUT}. Thus the unification of couplings can still be maintained for the case of a large number of gauge groups as well, but it will happen at a scale below M_{GUT}, and with nonperturbative values for the couplings due to the period of power-law running. Once one gets above the scale v, the running switches back to a logarithmic running, since at this scale the gauge groups are not broken to the diagonal subgroup anymore.

For just two gauge groups we have seen that $v \ll \Lambda \ll M_{\mathrm{Pl}}$, and so the gauge couplings are not unified at the energy scale where the lattice opens up. To see what happens in this case, we take the unbroken gauge group on each lattice site to be just $SU(3) \times SU(2) \times U(1)$ and assume the three gauge couplings of *each* lattice site are the same for a given gauge group. Above the scale v, the full 4D theory is the SM×SM, in which the MSSM matter is charged under one of them, while the link fields are charged under both. We take three link fields, one transforming as a bifundamental under $SU(3) \times SU(3)$, one transforming as a bifundamental under $SU(2) \times SU(2)$ and one field charged under $U(1) \times U(1)$. These fields acquire scalar component VEV's that break the SM×SM group structure down to just the SM.

The ordinary MSSM gauge couplings α_a for $a = [U(1)_{\sqrt{3/5}Y}, SU(2), SU(3)]$ are related to the gauge couplings $\bar{\alpha}_a$ of the end-point lattice group through $\alpha_a = \bar{\alpha}_a/2$, evaluated near the scale v. The gauge couplings therefore appear to undergo a discontinuous jump at this scale, if we follow the gauge couplings of the diagonal subgroup up to v and then the gauge couplings of the MSSM lattice site for energies beyond v. We can evolve the individual gauge couplings g_a above the scale v by the usual renormalization-group procedure. Of course we must also include the link fields (and the anomaly cancellation fields) in the beta functions. At one loop the running of the gauge couplings from $\bar{\Lambda}_h$ down to v is given by the usual formula

$$\frac{1}{\bar{\alpha}_a(\bar{\Lambda}_h)} = \frac{1}{\bar{\alpha}_a(v)} - \frac{\bar{b}_a}{4\pi} \ln \frac{\bar{\Lambda}_h}{v} \qquad (4.44)$$

written entirely in terms of the parameters of the lattice site gauge groups (barred quantities), where $\bar{\alpha}_a(\bar{\Lambda}_h, v)$ are the gauge couplings for the two scales $\bar{\Lambda}_h, v$. Let us now decompose this expression in terms of the usual MSSM gauge couplings α_a and beta function coefficients b_a. The diagonal

subgroup will feel one KK mode of gauge and gaugino fields, but since these fields have masses of order $v/2$, we can approximately treat this scenario as going from the 4D MSSM lattice site directly to the diagonal subgroup with just the 4D MSSM matter content. That is, $\bar{\alpha}_a(v) \to N\alpha_a(v)$. The beta function coefficient above the scale v is

$$\bar{b}_a = b_a + n_a = (\tfrac{66}{5}, 2, -6) + (\tfrac{6}{5}, 4, 6). \qquad (4.45)$$

With the ratio $n_3/n_2 = 3/2$, we can choose the hypercharge of the link field connecting $U(1) \times U(1)$ such that the above shift in the beta function coefficients satisfies the conditions for one-loop unification given by Ref. [24]

$$\frac{B_{12}}{B_{23}} = \frac{B_{13}}{B_{23}} = 1, \quad B_{ij} = \frac{n_i - n_j}{b_i - b_j}. \qquad (4.46)$$

Hence, unification of gauge couplings can be maintained for the $N=2$ case, although at a slightly lower scale than in the usual 4D MSSM. However, in order to have a fully unified theory one would also have to embed these link fields into the GUT group.

V. CONCLUSIONS

We have presented 4D constructions for supersymmetric models with extra dimensions. We have found that in the simplest model (5D $\mathcal{N}=1$ SUSY YM) the necessary enhancement of 4D $\mathcal{N}=1$ supersymmetry automatically takes place without any fine tuning, and thus 5D Lorentz invariance is also recovered. For a theory with more complicated matter content this result no longer holds, and a fine tuning in the interaction terms is necessary. We have used these models to translate the *a priori* five-dimensional mechanism of gaugino mediation of supersymmetry breaking into a simple 4D model. In these 4D versions of gaugino mediation supersymmetry breaking is transmitted to the MSSM because the physical gaugino is a mixture of gauge eigenstate gauginos, one of which couples to the supersymmetry breaking sector, while another to the SM matter fields. We find that a lattice as coarse as two gauge groups is sufficient to ensure the appearance of the soft breaking mass spectrum characteristic of gaugino mediation so long as the inverse lattice spacing is much smaller than the SUSY mediation scale. With more than about five gauge groups one also obtains the gaugino-mediated spectrum even if the SUSY mediation scale is of order the inverse lattice spacing.

ACKNOWLEDGMENTS

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