Density peaks and chiral peaks of fermion eigenmodes in QCD

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We show that the fermion number density of low-lying eigenfunctions of the Dirac operator in quenched QCD peaks at the same locations that the density of chirality is peaked. We use an overlap Dirac operator that has an exact chiral symmetry. The gauge connections in the Dirac operator are smeared links. We consider two different smearing procedures. Our conclusion is independent of the smearing procedure used for the gauge links.

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I. INTRODUCTION

In a recent publication [1] we have shown that the low-lying eigenmodes of a chiral lattice fermion Dirac operator, an overlap fermion operator [2] (described in Ref. [3]), has a local chiral density $\psi(x)^\dagger\gamma_5\psi(x)$ that shows a peaked structure. The positions and signs of the peaks are strongly correlated with the locations of topological objects, which would be identified as instantons and anti-instantons detected using a pure gauge operator. Zero modes correlate with only one sign of topological objects, while nonzero eigenmodes of the Dirac operator interpolate between both signs of topological object. This correlation dies away slowly as the eigenvalue of the mode rises. Spatially averaged correlation functions of hadrons made of light quarks are saturated by propagators of quarks restricted to a few low eigenmodes. The whole picture is very reminiscent of an instanton liquid model.

However, this work did not consider the possibility that the density of the fermionic modes might be large in places where the local chiral density was small. Subsequently, this point was raised by Horváth, Isgur, McCune, and Thacker [4]. They also presented evidence from lattice simulations claiming to show that the behavior of fermionic eigenmodes in the QCD vacuum is inconsistent with an instanton picture.

Why is it important to resolve this disagreement? There is an apparent inconsistency, first described by Witten in 1979 [5], between large number of colors (large-N_c) dynamics and instanton-based phenomenology. We do not wish to summarize arguments favoring one or the other approach, which are well described in a variety of publications [6]. In principle, lattice simulations might distinguish between the qualitative features of the two phenomenologies, if discrepancies between various simulations could be resolved.

The two simulations differ in several ways. Both are done in quenched approximation, using the Wilson gauge action. Reference [4] works at a lattice coupling $\beta=5.7$ and Ref. [1] works at $\beta=5.9$. The lattice spacings a are at nominal values of 0.17 and 0.11 fm, respectively, from the Sommer parameter, using the interpolating formula of Ref. [7]. The fermion action of Ref. [4] is the Wilson fermion action. This action has rather poor chiral properties, as evidenced by its large additive quark mass renormalization and the presence of exceptional configurations. Formally, it has $O(a)$ scaling violations. Its real eigenmodes are not eigenfunctions of γ_5, and the expectation value of γ_5 of a real eigenmode is a function of the eigenvalue.

The action of Ref. [1] has exact chiral symmetry. There is no additive mass renormalization of the quark mass and there are no exceptional configurations. Zero eigenvalue eigenmodes are chiral eigenstates. Nonzero eigenvalue eigenmodes have zero expectation value of γ_5. The action has only $O(a^2)$ discretization artifacts.

Our action uses "fat links." Fat link actions replace the usual one-link gauge connection with a combination of several gauge paths. This combination might or might not be projected to SU(3), but in any case it is gauge invariant and local. Locality assures that the fat link action is in the same universality class as the thin link one. Fat links improve the chiral behavior of nonchiral actions [8], and their use in our implementation of the overlap action is simply to make the calculation of the overlap operator more efficient (by a factor of roughly 20 for the action of Ref. [3], compared to the use of the thin link Wilson action in the overlap). The particular choice of fattening is not too important. In Ref. [1], we used an APE-blocked link [9] as the gauge connection. In Ref. [10] it is shown that the perturbative effect of N levels of APE smearing with a smearing parameter α is to multiply the vertices by a form factor $(1-\alpha\hat{q}^2/6)^N$ with q the gluon momentum and $\hat{q}^2=4/a^2\Sigma_\mu \sin^2 q_\mu a/2$. In coordinate space this corresponds to a Gaussian smearing with a spread $\langle x^2\rangle=(\alpha N/3)a^2$ of the quark-gluon vertex. Some readers might be concerned that several levels of APE smearing might adversely affect our results. To address that point, we describe results from another fattening, one chosen to lie rigorously within a hypercube [11]. We observe no qualitative change in our results.

II. ANALYSIS OF FERMION EIGENMODES

The statement of the problem is as follows: If an instanton-dominated picture of low eigenmodes is at all valid, we would expect the peaks of the wave function to closely resemble instanton zero modes. Thus, if instantons dominate, a local peak in the wave function for a low-lying eigenmode (a fermion lump) should be dominantly a lump in $\psi_L^\dagger\psi_L=\psi^\dagger(1-\gamma_5)\psi$ or $\psi_R^\dagger\psi_R=\psi^\dagger(1+\gamma_5)\psi$, but not both. However, if it happened that a local peak of $\psi^\dagger\psi$ was not also a peak of chirality, the instanton picture would not be correct.

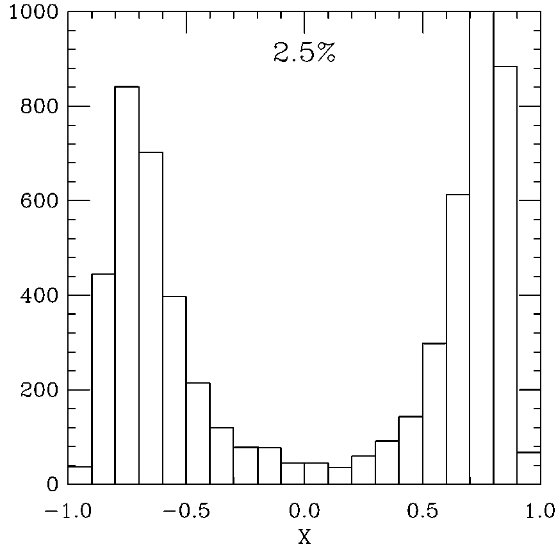

FIG. 1. Histogram of $\rho(X)$ of the lowest two nonchiral eigenmodes for the overlap action, with a cut keeping the top 2.5% of $\psi^\dagger\psi$.

In Ref. [1] we used an overlap action with $N=10$ levels of APE smearing with $\alpha=0.45$ the APE parameter. We measured the autocorrelation function of the local chirality density $\omega(x)=\psi(x)^\dagger\gamma_5\psi(x)$ and the correlator of $\omega(x)$ with a pure gauge observable sensitive to topological charge $Q(x)$. We saw that both kinds of correlator were strongly peaked at small operator separation, indicating the chiral density has spatially localized lumps and those lumps correlate with the lumps of the topological charge density. However, there is a logical possibility that a lump in $\omega(x)=\psi_L^\dagger\psi_L-\psi_R^\dagger\psi_R$ does not indicate a lump in either $\psi_L^\dagger\psi_L$ or $\psi_R^\dagger\psi_R$. To check that possibility, we follow Ref. [4] and define a "chiral order parameter" $X(x)$,

$$\tan\left(\frac{\pi}{4}(1+X(x))\right)=\frac{|\psi_L(x)|}{|\psi_R(x)|}=\left(\frac{\psi_L^\dagger(x)\psi_L(x)}{\psi_R^\dagger(x)\psi_R(x)}\right)^{1/2}, \quad (1)$$

and plot histograms of the density of the variable X, $\rho(X)$, for regions of the lattice where $\psi(x)^\dagger\psi(x)$ is large, a few percent of the points of the lattice. We distinguish two cases.

(a) Zero modes are chiral. Distributions of $\rho(X)$ for our action are just delta functions $\rho(X)\simeq\delta(X\pm1)$. Any chiral symmetry breaking inherent in a lattice fermion action would broaden this distribution (and could serve as a signal for the degree of chiral symmetry breaking in those actions).

(b) Nonzero eigenvalue eigenmodes are nonchiral, but we find that the distribution $\rho(X)$ is strongly peaked near $X=\pm1$. To show this, we take a set of six 12^4 lattices from our data set and plot histograms of $\rho(X)$. Figure 1 shows the result for the lowest two nonchiral eigenmodes for the overlap action, with a cut keeping the top 2.5% of $\psi^\dagger\psi$. The imaginary part of the eigenvalue of the Dirac operator for these modes is in the range $0.03/a$ to $0.09/a$. Keeping more lattice points in the histogram will fill in the dip. That simply implies that as we keep more and more lattice points we pick up more of the vacuum fluctuations around the chiral lumps and these eventually overwhelm the chiral structure. This happens very slowly with the two smallest nonchiral modes. Even with 30% of the lattice points the two-peak structure is clearly visible.

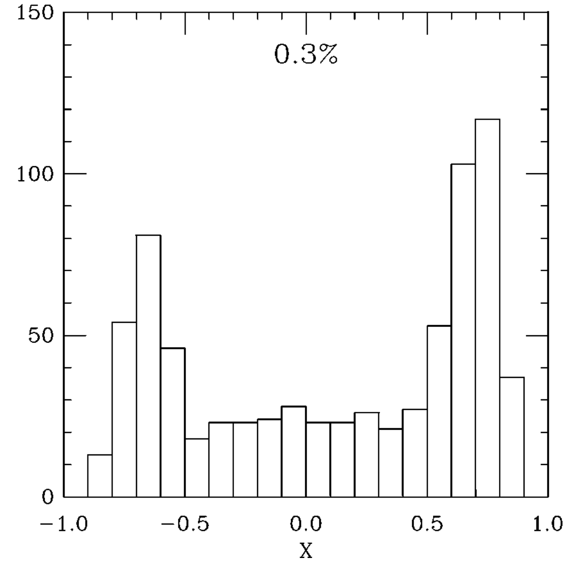

FIG. 2. Histogram of $\rho(X)$ of the highest two eigenmodes (out of ten) for the overlap action, with a cut keeping the top 0.3% of $\psi^\dagger\psi$.

Figures 2 and 3 show the results for the largest modes we have recorded, the ninth and tenth eigenvalue modes of the squared Dirac operator. The imaginary part of the eigenvalue of the Dirac operator for these modes is around $0.20/a$–$0.26/a$. Figure 2, where about the top 0.3% of $\psi^\dagger\psi$ is kept in the histogram, shows strong peaks around $X=\pm1$. The peaks disappear as we add more points to the distribution. With 2.5% of the lattice points kept the histogram is almost flat, as Fig. 3 shows.

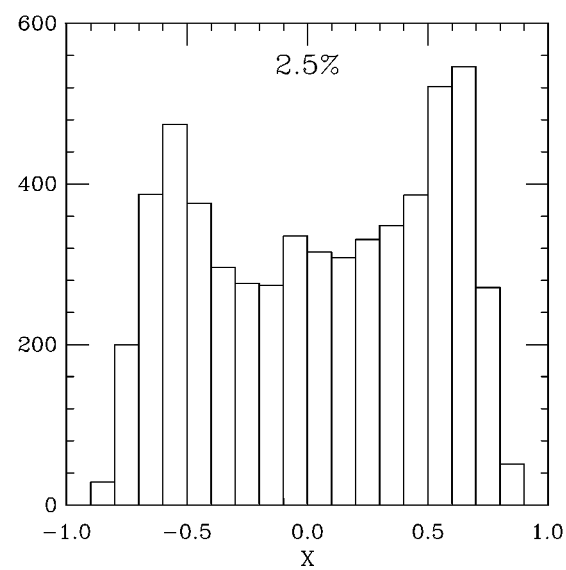

FIG. 3. Histogram of $\rho(X)$ of the highest two eigenmodes (out of ten) for the overlap action, with a cut keeping the top 2.5% of $\psi^\dagger\psi$.

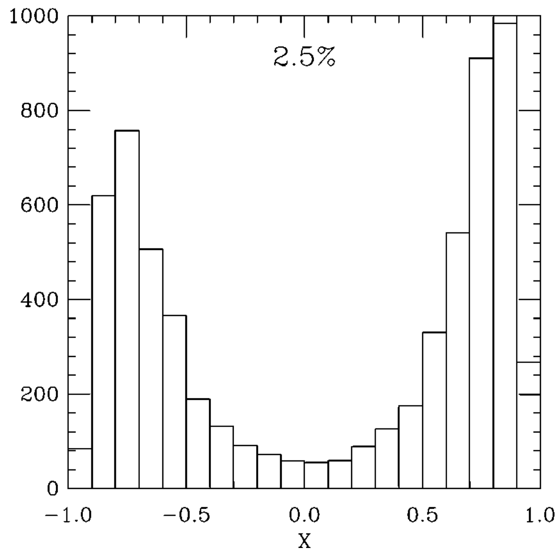

FIG. 4. Histogram of $\rho(X)$ of the lowest two nonchiral eigenmodes (out of ten) for the overlap action, with a hypercubic fat link, with a cut keeping the top 2.5% of $\psi^\dagger\psi$.

Lower-eigenvalue eigenmodes show stronger peaking than higher-eigenvalue modes, but all the modes we examined show peaking. We remind the reader that we showed in Ref. [1] that hadronic correlators for light quark masses are well approximated by quark propagators containing only a few eigenmodes.

Next we consider results obtained with an overlap action that uses more local fat links. The fat links of the hypercubic blocking [11] mix links only within a hypercube but achieve almost the same level of smoothness as the APE smearing considered previously.

The analogue of Fig. 1 from eigenmodes of the overlap action with hypercubic fat links is shown in Fig. 4. We used the same 12^4 configurations and kept the same fraction of lattice points as in Fig. 1. The shape of the curve is qualitatively similar to that from APE-blocked links. The peaks of fermion density are again chiral. For this action, keeping the sites of the lattice where $\psi^\dagger\psi$ is in its top 2.5% corresponds to capturing about 17% of the volume integral of $\psi^\dagger\psi$ in an eigenmode.

III. CONCLUSIONS

Our studies with a chiral fermion action at a lattice spacing near 0.11 fm showed that low-eigenvalue fermionic eigenmodes have a structure strongly correlated with the locations of instantons and anti-instantons. This note demonstrates that in the locations where the fermionic modes are largest the modes are also chiral. It seems to us that the simplest description of what we see is in terms of an instanton liquid model of the QCD vacuum. We do not doubt that the numerical results of Ref. [4] are correct, but we are concerned that the combination of a larger lattice spacing and the use of a nonchiral fermion action introduces lattice artifacts which compromise extrapolation to the continuum limit.

Since this paper appeared, other groups have addressed the same question. Similar results to those shown here have been presented using an alternative chiral lattice action (the Wilson overlap action) [12], a lattice action with improved but inexact chiral symmetry (domain wall fermions) [13], and a lattice action with inexact chiral symmetry but smaller lattice spacing (the clover action) [14]. None of these last three works compared the distribution of chirality with topological charge density measured with a gauge observable.

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[1] T. DeGrand and A. Hasenfratz, Phys. Rev. D **64**, 034512 (2001).
[2] H. Neuberger, Phys. Lett. B **417**, 141 (1998); Phys. Rev. Lett. **81**, 4060 (1998).
[3] MILC Collaboration, T. DeGrand, Phys. Rev. D **63**, 034503 (2001).
[4] I. Horváth, N. Isgur, J. McCune, and H.B. Thacker, preceding paper, Phys. Rev. D **65**, 014502 (2002).
[5] E. Witten, Nucl. Phys. **B149**, 285 (1979); **B156**, 269 (1979).
[6] A selection of arguments about the connection between the large-N_c limit of QCD and instanton models includes M. Shifman, Prog. Theor. Phys. Suppl. **131**, 1 (1998); T. Schafer and E.V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998); N. Isgur and H.B. Thacker, Phys. Rev. D **64**, 094507 (2001); T. Schafer and E. Shuryak, hep-lat/0005025.
[7] ALPHA Collaboration, M. Guagnelli, R. Sommer, and H. Wittig, Nucl. Phys. **B535**, 389 (1998).
[8] Compare MILC Collaboration, T. DeGrand, Phys. Rev. D **58**, 094503 (1998); **60**, 094501 (1999); T. DeGrand, A. Hasenfratz, and T.G. Kovacs, Nucl. Phys. **B547**, 259 (1999). Fat links also have an extensive use in staggered fermion simulations; see J.F. Lagae and D.K. Sinclair, Phys. Rev. D **59**, 014511 (1999); MILC Collaboration, K. Orginos, D. Toussaint, and R.L. Sugar, *ibid.* **60**, 054503 (1999); F. Knechtli and A. Hasenfratz, *ibid.* **63**, 114502 (2001).
[9] APE Collaboration, M. Albanese *et al.*, Phys. Lett. B **192**, 163 (1987); M. Falcioni, M.L. Paciello, G. Parisi, and B. Taglienti, Nucl. Phys. **B251**, 624 (1985).
[10] C. Bernard and T. DeGrand, Nucl. Phys. B (Proc. Suppl.) **83**, 845 (2000).
[11] A. Hasenfratz and F. Knechtli, Phys. Rev. D **64**, 034504 (2001).
[12] R.G. Edwards and U.M. Heller, this issue, Phys. Rev. D **65**, 014505 (2002).
[13] T. Blum *et al.*, following paper, Phys. Rev. D **65**, 014504 (2002).
[14] I. Hip, T. Lippert, H. Neff, K. Schilling, and W. Schroers, this issue, Phys. Rev. D **65**, 014506 (2002).