Decays of the meson B_c to a P-wave charmonium state χ_c or h_c

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The decays of the meson B_c to a P-wave charmonium state χ_c or h_c (here χ_c and h_c denote $(c\bar{c}[^3P_J])$ and $(c\bar{c}[^1P_1])$, respectively), the semileptonic ones $B_c \to \chi_c(h_c)+l+\nu_l$, and the two-body nonleptonic ones $B_c \to \chi_c(h_c)+h$ (h indicates a meson) are computed. To properly deal with the recoil effects, which may be relativistic in the decays, in the computation the framework of a heavy quark model, which is based on the Bethe-Salpeter equation and QCD inspired potential, is adopted. We find that all the decay rates are quite sizable and under reasonable approximations, all of the form factors occurring in the decays can be formulated precisely by means of proper kinematics factors and two independent overlapping integrations of the wave functions. As a result, the decays will be accessible at the CERN Large Hadron Collider and in run II at the Fermilab Tevatron in the foreseeable future. In particular, the cascade decays, i.e., $B_c \to \chi_c[^3P_{1,2}]+l+\nu_l (B_c \to \chi_c[^3P_{1,2}]+h)$ and $\chi_c[^3P_{1,2}] \to J/\psi+\gamma$, being followed accordingly, may affect the observations of the B_c meson through the decays $B_c \to J/\psi+l+\nu_l$ $(B_c \to J/\psi+h)$ substantially, and the decays $B_c \to h_c+\cdots$ may be used as a fresh window experimentally to observe the h_c state potentially.

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I. INTRODUCTION

The meson $B_c(\bar{B}_c)$ has successfully observed recently. The first positive observation of it is by the Collider Detector at Fermilab (CDF) at the Tevatron, through its semileptonic decays $B_c(\bar{B}_c) \to J/\psi+\bar{l}(l)+\nu_l(\bar{\nu}_l)$ [1]. According to observations, its mass $m_{B_c}=6.40\pm 0.39\pm 0.13$ GeV and lifetime $\tau_{B_c}=0.46^{+0.18}_{-0.16}\pm 0.03$ ps, etc. are obtained. Additional experimental studies of the meson are planned at Tevatron (in run II) and at the CERN Large Hadron Collider (LHC), etc., particularly the special detectors BTeV and LHCB for B physics (including B_c), to compensate CDF, D0, ATLAS, and CMS. Numerous $B_c^{\pm}$ events (more than 10^8–10^{10} per year) in the detectors at these colliders are expected [2–4], so a lot of interesting decay channels of B_c will be able to be well studied experimentally, and more rare processes will become accessible, therefore, further extensive theoretical studies of this meson are warranted.

The meson $B_c(\bar{B}_c)$ consists of $c(\bar{c})$ and $\bar{b}(b)$ quarks, i.e., it contains two different heavy flavors explicitly, which is unique in nature. Of the double heavy mesons it is very different from $J/\psi, \eta_c, \ldots$ and $\Upsilon, \eta_b, \ldots$, and it has many interesting/distinguished properties. For instance, its production is comparatively hard so that its discovery is quite late, and it decays, through the two-flavor annihilation and decays of one of the two heavy flavors, only weakly. In particular, it happens that the decay rates of the two heavy flavors are comparable in magnitude [the b quark has a greater mass than the c quark $m_b \gg m_c$, versus a small value of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{cb} \ll V_{cs}$], so that it has much richer decay channels which have a sizable branching ratio, even more than those of the heavy mesons $B^{\pm}, B^0, B_s, D^{\pm}, D^0, D_s$, etc., hence one may study the weak decays of the two heavy flavors b, c simultaneously with the unique meson B_c.

B_c-meson production [2–5], spectroscopy [6,7], and various decays [8–12] were widely computed before the observation of CDF, and the results are consistent with the CDF observation if theoretical uncertainties and experimental errors are taken into account [1]. Whereas the semileptonic decays $B_c \to \chi_c(h_c)+l+\nu_l$ and the two-body nonleptonic decays $B_c \to \chi_c(h_c)+h$, i.e., the decays of the meson B_c to a P-wave chamonium state, are certainly interesting, and still missing in the literature, we devote this paper to reporting our computations on them, although the semileptonic decays in a letter style were reported elsewhere [12].

Why do the decays of the meson B_c to a P-wave charmonium state so interest people? Let us outline a few reasons below.

First of all, how sizable the decays will be, especially if they are accessible in run II of Tevatron and/or in LHC, are interesting problems. Especially, a specific detector has a limited efficiency to record a photon, i.e., photons may be missed in detectors, so the cascade decay $B_c \to \chi_c+\cdots$ and a radiative decay $\chi_c \to J/\psi+\gamma$ may appear to be an indication of the meson B_c through the decays $B_c \to J/\psi+\cdots$ when the photon γ is missed. Namely, the cascade decays may potentially affect the results of the B_c observation substantially, and it essentially depends on how sizable the decays are in the cascade decay. In fact, two of the radiative P-wave charmonium decays have quite a large branching ratio of about a few tenths ($Br=27.3\%$ for $\chi_c[^3P_1] \to J/\psi+\gamma$ and $Br=13.5\%$ for $\chi_c[^3P_2] \to J/\psi+\gamma$ [13]). Therefore, the precise values of the decays to P-wave charmonia are

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necessary, when one estimates quantitatively the background for the observation on the B_c meson.

If one would like to see CP violations in B_c decays, for example, in the decays $B_c \to h+h_1+h_2$ (h,h_1,h_2 denote various possible mesons), as emphasized in Ref. [14], the interference of the direct decays with a corresponding cascade through a resonance, e.g., $B_c \to \chi_c[^3P_0]+h$ and $\chi_c[^3P_0] \to h_1+h_2$, may enhance the visible CP-violation effects substantially. Thus to take advantage of this method quantitatively, knowledge of the decay $B_c \to \chi_c[^3P_0]+h$ is necessary.

The QCD-inspired potential model works very well for nonrelativistic double-heavy systems. The systems $(c\bar{b})$ and $(\bar{c}b)$ in forming bound states, except the reduce mass, are similar to the well-studied systems $(b\bar{b})$ and $(c\bar{c})$, so it is believed that with the potential model the static properties of the systems $(c\bar{b})$ and $(\bar{c}b)$ can be computed very well as those of bottomium $(b\bar{b})$ and charmonium $(c\bar{c})$. In general, applying the wave functions to computing the relevant decay matrix elements interests people for several reasons, one of which is the potential model will have further tests. Thus with the wave functions of B_c [the ground state of the system of $(c\bar{b})$] and $\chi_c(h_c)$ [the P-wave states of $(c\bar{c})$] obtained by the potential model, to apply the wave functions to computing the decays $B_c \to \chi_c(h_c)+\cdots$ is interesting in connection with additional tests on the model.

Since the mass of $B_c(m_{B_c})$ is much greater than that of the P-wave charmonia (m_{χ_c} and m_{h_c}), the momentum recoil in the decays concerned can be great (even relativistic), therefore we should carefully choose a suitable approach to deal properly with the momentum recoil effects in the decays.

If one tries to apply the Schrödinger wave functions of nonrelativistic binding systems to computing precise values of the decay processes, with such a great (even relativistic) recoil momentum one cannot carry out the computation, just as is done in atomic and nuclear decays by taking the wave functions of the bound states in a suitable "reference frame" and then simply "boosting" the ones accordingly, because the recoil momentum in an atomic or nuclear process is very small, always nonrelativistic, which is very different from the present decays. The great momentum recoil obviously means that the velocity between the two CMS of the B_c meson and the charmonium state is huge. The potential wave functions of the parent and the daughter states, given in each CMS, respectively, cannot be attributed just to choosing a suitable reference frame and simply boosting the wave functions (this fact will be seen clearly later on). Thus when applying the wave functions to calculation of the decays (e.g., the semileptonic decays and most two-body nonleptonic decays here) with such a great (even relativistic) momentum recoil, special handling is needed in principle. To deal with the momentum recoil properly, a so-called generalized instantaneous approximation approach (GIAA) for the decays from a nonrelativistic S-wave state to another S-wave one was proposed in Ref. [8]. The GIAA has been proved valid in the cases for the decays from a nonrelativistic S-wave state to another S-wave one in certain phases [8]. For the GIAA in the present cases for the decays from a nonrelativistic S-wave state to a P-wave one, and to compare with the other approaches, we need just a straightforward extension, hence for present purposes we adopt the GIAA here.

The key points of the GIAA may be outlined in three steps. First, to "reform" the "original" potential model, which is based on the Schrödinger equation, to the Bethe-Salpeter (BS) equation[1] even for the nonrelativistic binding systems. Then, according to the Mandelstam method [15] formulate the (weak) current matrix elements (an elementary factor for the relevant decays) sandwiched by the BS wave functions of the two bound states, so that the current matrix elements are written in a fully relativistic formulation. Finally, make the so-called "generalized instantaneous approximation" on the fully relativistic matrix elements, i.e., integrate out the "time" component of the relative momentum in the Mandelstam formulation by a contour integration, and as the final result, the current matrix elements turn out to be formulated in terms of proper operators sandwiched by the Schrödinger wave functions of the "original" potential model. Since the weak current matrices (by means of the Mandelstam method) are formulated relativistically and used as the starting "point" for the contour integration, we can be sure that the final formulation takes the recoil effects into account properly. One more advantage of the approach is that it has a more solid foundation in quantum field theory than that in the "original" potential models, because the BS wave functions and the matrix elements in the Mandelstam formulation, which are used as the starting point to make the generalized instantaneous approximation, have very solid foundations in quantum field theories.

On the other hand, the GIAA also avoids the disadvantage of directly computing the decays with the Mandelstam formulation. We know the BS equation is four-dimensional in space-time to describe the bound states, so there are difficulties, such as how to determine the QCD-inspired four-dimensional interaction kernel of the equation properly, what is the physical meaning of the excitations in the relative-time "freedom" of the two components, etc. In addition, the BS equation is harder to solve than a Schrödinger one, even when the four-dimentional kernel is fixed. Whereas under the generalized instantaneous approximation, which treats the great momentum recoil effects properly, only Schrödinger wave functions that are well-tested in the potential model appear finally, the approach avoids the difficulties of the BS equation by keeping the achievements of the potential model.

Finally we should note here that in calculating the two-body nonleptonic decays of B_c to the P-wave χ_c and h_c states, the widely adopted factorization assumption and the effective Lagrangian for four fermions in which the "short-distance" QCD corrections have been taken into account

[1]For the binding systems, B_c and $\chi_c(h_c)$, the reform is just by means of the original instantaneous approximation proposed by Salpeter to "build" the relation between the Schrödinger equations and the relevant BS ones, which can be found in many textbooks on quantum field theory; see, e.g., [16].

with the OPE (operator product expansion) and the RGM (the renormalization-group method) are applied.

The paper is organized as follows. In Sec. II, the exclusive semileptonic differential decay rates, the matrix elements, the form factors, etc., are presented. In Sec. III, the adopted approach (GIAA) and the computations of the form factors are illustrated precisely. In Sec. IV, the two-body nonleptonic decays of B_c are formulated with necessary description. Finally, in Sec. V, numerical results and discussions are presented. Under reasonable assumptions, the obtained precise dependence of the current matrix elements on the form factors, and the precise dependence of the form factors on the functions ξ_1 and ξ_2, with integrations overlapping the wave function, are put in the Appendix.

II. THE EXCLUSIVE SEMILEPTONIC DECAYS AND RELEVANT CURRENT MATRIX ELEMENTS

The T-matrix element for the semileptonic decays $B_c \to X_{(c\bar{c})} + l^+ + \nu_l$ is

$$T = \frac{G_F}{\sqrt{2}} V_{ij} \overline{u}_{\nu_l} \gamma_\mu (1-\gamma_5) v_l \langle X_{c\bar{c}}(p',\epsilon) | J^\mu_{ij} | B_c(p) \rangle, \quad (1)$$

where $X_{(c\bar{c})}$ denotes χ_c and h_c, V_{ij} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, J^μ is the charged current responsible for the decays, and p, p' are the momenta of the initial state B_c and the final state $X_{(c\bar{c})}$. Thus we have

$$\overline{\sum} |T|^2 = \frac{G_F^2}{2} |V_{ij}|^2 l^{\mu\nu} h_{\mu\nu}, \quad (2)$$

where $h_{\mu\nu}$ is the hadronic tensor and $l^{\mu\nu}$ is the leptonic tensor. $l_{\mu\nu}$ is easy to compute, whereas in general $h_{\mu\nu}$ can be written as

$$\begin{aligned} h_{\mu\nu} = & -\alpha g_{\mu\nu} + \beta_{++}(p+p')_\mu (p+p')_\nu \\ & + \beta_{+-}(p+p')_\mu (p-p')_\nu + \beta_{-+}(p-p')_\mu (p+p')_\nu \\ & + \beta_{--}(p-p')_\mu (p-p')_\nu \\ & + i\gamma \epsilon_{\mu\nu\rho\sigma}(p+p')^\rho (p-p')^\sigma, \end{aligned} \quad (3)$$

and by a straightforward calculation, the differential decay rate is obtained accordingly:

$$\begin{aligned} \frac{d^2\Gamma}{dx dy} = & |V_{ij}|^2 \frac{G_F^2 M^5}{32\pi^3} \left\{ \alpha \frac{\left(y - \frac{m_l^2}{M^2} \right)}{M^2} + 2\beta_{++} \right. \\ & \times \left[2x \left(1 - \frac{M'^2}{M^2} + y \right) - 4x^2 - y + \frac{m_l^2}{4M^2} \right. \\ & \times \left. \left(8x + \frac{4M'^2 - m_l^2}{M^2} - 3y \right) \right] \\ & + 4(\beta_{+-} + \beta_{-+}) \frac{m_l^2}{M^2} \left(2 - 4x + y - \frac{2M'^2 - m_l^2}{M^2} \right) \\ & + 4\beta_{--} \frac{m_l^2}{M^2} \left(y - \frac{m_l^2}{M^2} \right) - \gamma \left[y \left(1 - \frac{M'^2}{M^2} - 4x + y \right) \right. \\ & \left. \left. + \frac{m_l^2}{M^2} \left(1 - \frac{M'^2}{M^2} + y \right) \right] \right\}, \end{aligned} \quad (4)$$

where $x \equiv E_l/M$ and $y \equiv (p-p')^2/M^2$, M is the mass of B_c meson, and M' is the mass of the final state $X_{c\bar{c}}$. The coefficient functions α, β_{++}, and γ can be formulated in terms of form factors. Note here that we have kept the mass of the lepton m_l precisely different from those of Isgur *et al.* [9] and Grinstein *et al.* [17], so the formula here can be applied not only to the cases of e and μ semileptonic decays, but also to those of τ-semileptonic decays.

(i) If $X_{(c\bar{c})}$ is the $h_c([^1P_1])$ state, the vector current matrix element

$$\begin{aligned} \langle X_{c\bar{c}}(p',\epsilon) | V_\mu | B_c(p) \rangle \equiv & r \epsilon^*_\mu + s_+ (\epsilon^* \cdot p)(p+p')_\mu \\ & + s_- (\epsilon^* \cdot p)(p-p')_\mu, \end{aligned} \quad (5)$$

and the axial vector current matrix element

$$\langle X_{(c\bar{c})}(p',\epsilon) | A_\mu | B_c(p) \rangle \equiv i v \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} (p+p')^\rho (p-p')^\sigma, \quad (6)$$

where p and p' are the momenta of B_c and h_c, respectively, and ϵ is the polarization vector of h_c.

(ii) If $X_{(c\bar{c})}$ is the $\chi_c([^3P_0])$ state, the vector matrix element vanishes, and the axial vector current

$$\langle X_{(c\bar{c})}(p') | A_\mu | B_c(p) \rangle \equiv u_+ (p+p')_\mu + u_- (p-p')_\mu. \quad (7)$$

(iii) If $X_{(c\bar{c})}$ is the $\chi_c([^3P_1])$ state,

$$\begin{aligned} \langle X_{(c\bar{c})}(p',\epsilon) | V_\mu | B_c(p) \rangle \equiv & l \epsilon^*_\mu + c_+ (\epsilon^* \cdot p)(p+p')_\mu \\ & + c_- (\epsilon^* \cdot p)(p-p')_\mu \end{aligned} \quad (8)$$

and

$$\langle X_{(c\bar{c})}(p',\epsilon) | A_\mu | B_c(p) \rangle \equiv i q \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} (p+p')^\rho (p-p')^\sigma. \quad (9)$$

(iv) If $X_{(c\bar{c})}$ is the $\chi_c([^3P_2])$ state,

$$\begin{aligned} & \langle X_{(c\bar{c})}(p',\epsilon) | V_\mu | B_c(p) \rangle \\ & \equiv i h_{+-} \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu\alpha} p_\alpha (p+p')^\rho (p-p')^\sigma \end{aligned} \quad (10)$$

and

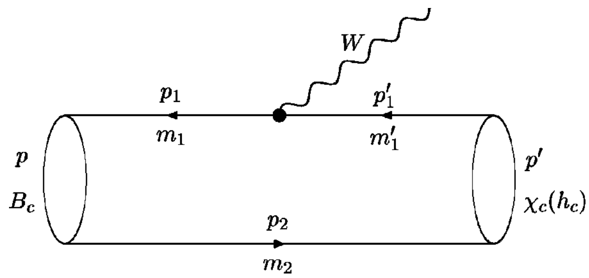

FIG. 1. The Feynman diagram for the weak current matrix element.

$$\langle X_{(c\bar{c})}(p',\epsilon)|A_\mu|B_c(p)\rangle \equiv k\epsilon^*_{\mu\nu}p^\nu + b_+(\epsilon^*_{\rho\sigma}p^\rho p^\sigma)(p+p')_\mu + b_-(\epsilon^*_{\rho\sigma}p^\rho p^\sigma)(p-p')_\mu . \quad (11)$$

The form factors r, s_+, s_-, v, u_+, u_-, l, c_+, c_-, k, b_+, b_-, and h_{+-} are functions of the momentum transfer $t=(p-p')^2$ and can be calculated precisely. In Ref. [8], we proposed an approach, the generalized instantaneous approximation, to compute those form factors for the decays of B_c to an S-wave charmonium state J/ψ or η_c. Now we are computing the form factors $r, s_+, s_-, \ldots$ appearing in the decays of B_c to a P-wave charmonium state. In fact the approach may be used directly; thus it is adopted in the present calculations.

III. THE GENERALIZED INSTANTANEOUS APPROXIMATION APPROACH TO THE WEAK CURRENT MATRIX ELEMENTS

To calculate these form factors, the GIAA developed in Ref. [8] is adopted. As outlined in the Introduction, the considered weak (electromagnetic) current matrix element is described by Fig. 1, and according to the Mandelstam formalism [15] it may be written down in terms of Bethe-Salpeter wave functions for B_c and $\chi_c(h_c)$ exactly:

$$l^\mu = i\int \frac{d^4q}{(2\pi)^4}\mathrm{Tr}[\overline{\chi}_{p'}(q')(\not{p}_2-m_2)\chi_p(q)\Gamma^\mu]. \quad (12)$$

Here $\chi_p(q)$, and $\chi_{p'}(q')$ are the BS wave functions of the initial state B_c and the final state $\chi_c(h_c)$ with the corresponding momenta p, p'. Throughout the paper we use p_1, p_2 to denote the momenta of the quarks in the initial meson B_c, and p_1', p_2' to denote the momenta of the quarks in the final meson χ_c or h_c. For convenience, let us introduce the definition of the relative momentum q (or q'):

$$p_1=\alpha_1 p+q, \quad \alpha_1=\frac{m_1}{m_1+m_2},$$

$$p_2=\alpha_2 p-q, \quad \alpha_2=\frac{m_2}{m_1+m_2}.$$

p_1, p_2, m_1, and m_2 are the momenta and masses for the antiquark and quark, respectively. The matrix element of the current Eq. (12) is now fully relativistic; thus it can be used as the starting "point" to take into account the recoil effects in the decays no matter how great the recoil moment will be in the considered decay. To apply the generalized instantaneous approach for the matrix element, we need to "reform" the potential model to that on the BS equation "ground."

A. The potential model and BS equation

In general, the BS equation for the corresponding wave function $\chi_p(q)$,

$$(\not{p}_2-m_2)\chi_p(q)(\not{p}_1+m_1)=i\int\frac{d^4k}{(2\pi)^4}V(p,k,q)\chi_p(k), \quad (13)$$

where $V(p,k,q)$ is the kernel between the quarks in the bound state, may describe the relevant quark-antiquark bound state well. Accordingly, the BS wave function $\chi_p(q)$ should satisfy the normalization condition

$$\int \frac{d^4q d^4q'}{(2\pi)^4}\mathrm{Tr}\left[\overline{\chi}_p(q)\frac{\partial}{\partial p_0}[S_1^{-1}(p_1)S_2^{-1}(p_2)\delta^4(q-q') + V(p,q,q')]\chi_p(q')\right]=2ip_0, \quad (14)$$

where $S_1(p_1)$ and $S_2(p_2)$ are the propagators of the relevant particles with masses m_1 and m_2, respectively.

As pointed out in the Introduction, the BS equation in four dimension should be reduced to a one in three dimension, i.e., the timelike component momentum should be integrated out (the instantaneous approximation) with a contour integration as proposed by Salpeter, especially when the kernel has the following property:

$$V(p,k,q)\simeq V(|\vec{k}-\vec{q}|).$$

To do this is very easy. When one makes a contour integration of the "time" component of the relative momentum on the whole BS equation, then the BS equation is deduced straightforwardly into a three-dimensional equation, which is just a Schrödinger equation in the momentum representation. Since the starting point of the common potential model is a Schrödinger equation, we may convert the potential model into a foundation based on the BS equation, as with the instantaneous approach.

To treat the possible large recoil effects in the decays, we need also to convert the instantaneous approximation to a covariant way too, i.e., to divide the relative momentum q into two parts, $q_\parallel$ and $q_\perp$, a parallel (timelike) part and an orthogonal one to p, respectively:

$$q^\mu = q^\mu_{p\parallel}+q^\mu_{p\perp},$$

where $q^\mu_{p\parallel}\equiv(p\cdot q/M_p^2)p^\mu$ and $q^\mu_{p\perp}\equiv q^\mu - q^\mu_{p\parallel}$. Correspondingly, we have two Lorentz invariant variables:

$$q_p=\frac{p\cdot q}{M_p}, \quad q_{pT}=\sqrt{q_p^2-q^2}=\sqrt{-q^2_{p\perp}}.$$

In the rest frame of the initial meson, i.e., $\vec{p}=0$, they turn back to the usual components q_0 and $|\vec{q}|$, respectively.

Now the volume element of the relative momentum k can be written in an invariant form:

$$d^4k = dk_p k_{pT}^2 dk_{pT} ds\, d\phi, \tag{15}$$

where ϕ is the azimuthal angle and $s=(k_p q_p - k\cdot q)/(k_{pT}q_{pT})$. Now the interaction kernel can be rewritten as

$$V(|\vec{k}-\vec{q}|) = V(k_{p\perp}, s, q_{p\perp}). \tag{16}$$

We define

$$\varphi_p(q_{p\perp}^{\mu}) \equiv i\int \frac{dq_p}{2\pi}\chi_p(q_{p\parallel}^{\mu}, q_{p\perp}^{\mu}),$$
$$\eta(q_{p\perp}^{\mu}) \equiv \int \frac{k_{pT}^2 dk_{pT} ds}{(2\pi)^2} V(k_{p\perp}, s, q_{p\perp})\varphi_p(k_{p\perp}^{\mu}). \tag{17}$$

The BS equation can now be rewritten as

$$\chi_p(q_{p\parallel}, q_{p\perp}) = S_1(p_1)\eta(q_{p\perp})S_2(p_2) \tag{18}$$

and the propagators can be decomposed as

$$S_i(p_i) = \frac{\Lambda_{ip}^{+}(q_{p\perp})}{J(i)q_p + \alpha_i M - \omega_{ip} + i\epsilon} + \frac{\Lambda_{ip}^{-}(q_{p\perp})}{J(i)q_p + \alpha_i M + \omega_{ip} - i\epsilon}, \tag{19}$$

with

$$\omega_{ip} = \sqrt{m_i^2 + q_{pT}^2},$$
$$\Lambda_{ip}^{\pm}(q_{p\perp}) = \frac{1}{2\omega_{ip}}\left[\frac{\not{p}}{M}\omega_{ip} \pm J(i)(m_i + \not{q}_{p\perp})\right], \tag{20}$$

where $i=1,2$ and $J(i)=(-1)^i$. Here $\Lambda_{ip}^{\pm}(q_{p\perp})$ satisfies the relations

$$\Lambda_{ip}^{+}(q_{p\perp}) + \Lambda_{ip}^{-}(q_{p\perp}) = \frac{\not{p}}{M},$$
$$\Lambda_{ip}^{\pm}(q_{p\perp})\frac{\not{p}}{M}\Lambda_{ip}^{\pm}(q_{p\perp}) = \Lambda_{ip}^{\pm}(q_{p\perp}),$$
$$\Lambda_{ip}^{\pm}(q_{p\perp})\frac{\not{p}}{M}\Lambda_{ip}^{\mp}(q_{p\perp}) = 0. \tag{21}$$

Due to these equations, $\Lambda^{\pm}$ may be referred to as the p-projection operators, while in the rest frame of the corresponding meson, they turn to the energy projection operator.

We define $\varphi_p^{\pm\pm}(q_{p\perp})$ as

$$\varphi_p^{\pm\pm}(q_{p\perp}) \equiv \Lambda_{2p}^{\pm}(q_{p\perp})\frac{\not{p}}{M}\varphi_p(q_{p\perp})\frac{\not{p}}{M}\Lambda_{1p}^{\mp C}(q_{p\perp}), \tag{22}$$

where the upper index C denotes the charge conjugation. In our notation, $\Lambda_{2p}^{\pm C}(q_{p\perp}) \equiv \Lambda_{2p}^{\mp}(q_{p\perp})$. Integrating over q_p on both sides of Eq. (18), we obtain

$$(M-\omega_{1p}-\omega_{2p})\varphi_p^{++}(q_{p\perp}) = \Lambda_{2p}^{+}(q_{p\perp})\eta_p(q_{p\perp})\Lambda_{1p}^{-C}(q_{p\perp}),$$
$$(M+\omega_{1p}+\omega_{2p})\varphi_p^{--}(q_{p\perp}) = \Lambda_{2p}^{-}(q_{p\perp})\eta_p(q_{p\perp})\Lambda_{1p}^{+C}(q_{p\perp}),$$
$$\varphi_p^{+-}(q_{p\perp}) = \varphi_p^{-+}(q_{p\perp}) = 0. \tag{23}$$

The normalization condition of Eq. (14) now becomes

$$\int \frac{q_T^2 dq_T}{(2\pi)^2}\mathrm{tr}\left[\overline{\varphi}^{++}\frac{\not{p}}{M}\varphi^{++}\frac{\not{p}}{M} - \overline{\varphi}^{--}\frac{\not{p}}{M}\varphi^{--}\frac{\not{p}}{M}\right] = 2P_0 .$$

From these equations, one may see that in the weak binding case to compare with the factor $(M-\omega_{1p}-\omega_{2p})$, the factor $(M+\omega_{1p}+\omega_{2p})$ is large, so the negative energy components of the wave functions φ^{--} are small. In the present case, for the heavy quarkonium and B_c meson this is precisely the situation, so we ignore the negative energy components of the wave functions safely at the lowest-order approximation.

Neglecting the negative energy components of the wave functions, the BS equation contains the positive component

$$\varphi_p^{++}(q_{p\perp}) \equiv \Lambda_{2p}^{+}(q_{p\perp})\frac{\not{p}}{M}\varphi_p(q_{p\perp})\frac{\not{p}}{M}\Lambda_{1p}^{-C}(q_{p\perp})$$

only, and the normalization condition becomes

$$\int \frac{q_T^2 dq_T}{(2\pi)^2}\mathrm{tr}\left[\overline{\varphi}^{++}\frac{\not{p}}{M}\varphi^{++}\frac{\not{p}}{M}\right] = 2P_0 .$$

Now let us consider the wave function φ^{++} appearing in the above equations. We know that the total angular momentum of a meson is composed from orbital angular momentum and spin, furthermore there are two ways (S-L coupling or j-j coupling) to compose the total angular momentum. Here to consider P-wave states of charmonium, we adopt the S-L coupling, i.e., we let the spins of the two quarks couple into a total spin, which can be either singlet or triplet, then we let the total spin couple to the relative orbital angular momentum, and finally we obtain the total angular momentum. In this way, the reduced BS wave function φ_P can be written approximately as

$$\varphi_{^1S_0}(\vec{q}) = \frac{\not{P}+M}{2\sqrt{2}M}\gamma_5\psi_{n00}(\vec{q}) \tag{24}$$

for the $^1S_0$ state, and

$$\varphi_{^3S_1}^{\lambda}(\vec{q}) = \frac{\not{P}+M}{2\sqrt{2}M}\not{\epsilon}^{\lambda}\psi_{n00}(\vec{q}) \tag{25}$$

for the $^3S_1$ state, where ϵ^{λ} is the polarization of this state. For the P-wave $(c\bar{c})$ wave functions,

$$\varphi_{^1P_1}(\vec{q})=\frac{\not{P}+M}{2\sqrt{2}M}\gamma_5\psi_{n1M_z}(\vec{q}) \quad (26)$$

for $^1P_1$, i.e., the h_c state, and

$$\varphi^{J_z}_{^3P_J}(\vec{q})=\frac{\not{P}+M}{2\sqrt{2}M}\not{\epsilon}^{\lambda}(S)\psi_{n1M_z}(\vec{q})\langle 1S_z,LM_z|JJ_z\rangle \quad (27)$$

for $^3P_J(J=0,1,2)$, i.e., χ_c states, where ϵ is the polarization vector of total spin and $\langle 1S_z,LM_z|JJ_z\rangle$ are the Clebsch-Gordon coefficients which couple L, S to the total angular momentum J. ψ_{n00} and ψ_{n1M_z} are the full BS wave functions.

B. The radius BS equation in momentum space

To solve the BS equation, the key problem concerns its radial component. If we ignore the negative energy contributions, the reduced BS equation (18) in the rest frame of the meson center mass system can be written as

$$\varphi_P(\vec{q})=\frac{\Lambda_2^+(\vec{q})\int\frac{d\vec{k}}{(2\pi)^3}V(\vec{k},\vec{q})\varphi_P(\vec{k})\Lambda_1^+(\vec{q})}{M-\omega_1-\omega_2}. \quad (28)$$

In the frame, the energy projection operator

$$\Lambda_1^+=\frac{1}{2\omega_1}(\omega_1\gamma_0-\vec{\gamma}\cdot\vec{q}-m_1),$$

$$\Lambda_2^+=\frac{1}{2\omega_2}(\omega_2\gamma_0+\vec{\gamma}\cdot\vec{q}+m_2),$$

where the kernel V acts on $\varphi(\vec{q})$ as

$$V(\vec{q})\varphi(\vec{q})=V_s(\vec{q})\varphi(\vec{q})+V_v(\vec{q})\gamma_\mu\varphi(\vec{q})\gamma^\mu, \quad (29)$$

i.e., to correspond to the potential model more precisely, the interaction kernel can be formally divided into the corresponding nonperturbative QCD ‘‘linear’’ one, V_s (in scalar nature), and the corresponding gluon exchange one, V_v (in vector nature).

When substituting Eqs. (24) and (26) (the wave functions in the meson center mass system) into the reduced BS equation (28), the equation for a spin singlet state $S=0$ becomes

$$\phi_{S=0}(\vec{q})=\frac{1}{4\omega_1\omega_2(M-\omega_1-\omega_2)}\times\left\{m_1m_2\int[4V_v(\vec{q},\vec{k})-4V_s(\vec{q},\vec{k})]\times\phi_{S=0}(\vec{k})d\vec{k}\right\}, \quad (30)$$

where the $\phi_{S=0}(\vec{q})$ is $\phi_{n00}(^1S_0)$ or $\phi_{n1M_z}(^1P_1)$. Since the square of the relative momentum $\vec{q}^2$ is small compared with the quark mass squared in the ‘‘double heavy’’ meson as a lowest-order approximation, we have ignored such higher terms and use $\omega_1=m_1$, $\omega_2=m_2$ in the numerator.

Now let us factorize out the radial component of the wave function and its relevant BS equation in momentum space from the angular ones:

$$\psi_{nLM_z}(\vec{q})=\phi_{nL}(|\vec{q}|)Y_{LM_z}(\theta,\varphi),$$

where n is the principal quantum number, L is the orbital angular momentum, M_z is the projection of the third component of L, $\phi_{nL}(|\vec{q}|)$ is the radial wave function, and $Y_{LM_z}(\theta,\phi)$ is the spherical harmonic function. For the spin singlet states, multiplying $Y^*_{LM_z}(\hat{q})$ to two sides of the reduced BS equation and summing over M_z by using the formula

$$\frac{4\pi}{2L+1}\sum_{M_z}Y_{LM_z}(\hat{q})Y^*_{LM_z}(\hat{k})=P_L(\cos\theta),$$

where θ is the angle between the unit vector $\hat{q}$ and $\hat{k}$, the radial reduced BS equation for the $^1S_0$ state is obtained:

$$\phi_{n0}(|\vec{q}|)=\frac{1}{4\omega_1\omega_2(M-\omega_1-\omega_2)}\times\left\{m_1m_2\int[4V_v(\vec{q},\vec{k})-4V_s(\vec{q},\vec{k})]\phi_{n0}(|\vec{k}|)d\vec{k}\right\}, \quad (31)$$

whereas for the $^1P_1$ state,

$$\phi_{n1}(|\vec{q}|)=\frac{1}{4\omega_1\omega_2(M-\omega_1-\omega_2)}\times\left\{m_1m_2\int[4V_v(\vec{q},\vec{k})-4V_s(\vec{q},\vec{k})]\times\phi_{n1}(|\vec{k}|)\cos\theta d\vec{k}\right\}, \quad (32)$$

where $\phi_{n0}(|\vec{q}|)$ and $\phi_{n1}(|\vec{q}|)$ are the radial parts of the wave functions.

Similarly, for the spin triplet states $S=1$ we have

$$\sum_{lm}\langle 1S_z,LM_z|JJ_z\rangle\phi_{S=1}(\vec{q})=\sum_{lm}\langle 1S_z,LM_z|JJ_z\rangle\frac{1}{4\omega_1\omega_2(M-\omega_1-\omega_2)}\times\left\{m_1m_2\int[4V_v(\vec{q},\vec{k})-4V_s(\vec{q},\vec{k})]\phi_{S=1}(\vec{k})d\vec{k}\right\}, \quad (33)$$

where the $\phi_{S=1}(\vec{q})$ is $\phi_{n00}(^3S_1)$ or $\phi_{n1M_z}(^3P_J)$. Then the equation for $^3S_1$ is

$$\phi_{n0}(|\vec{q}|)=\frac{1}{4\omega_1\omega_2(M-\omega_1-\omega_2)} \times\left\{m_1m_2\int [4V_v(\vec{q},\vec{k})-4V_s(\vec{q},\vec{k})]\phi_{n0}(|\vec{k}|)d\vec{k}\right\}, \tag{34}$$

and for ^3P_J,

$$\phi_{n1}(|\vec{q}|)=\frac{1}{4\omega_1\omega_2(M-\omega_1-\omega_2)} \times\left\{m_1m_2\int [4V_v(\vec{q},\vec{k})-4V_s(\vec{q},\vec{k})] \times\phi_{n1}(|\vec{k}|)\cos\theta d\vec{k}\right\}. \tag{35}$$

The normalization of ϕ_{nL} now reads

$$\int \frac{q_T^2dq_T}{(2\pi)^2}\left[\frac{m_1m_2}{\omega_1\omega_2}\phi_{nL}^2(|q_T|)\right]=2M.$$

To ignore all the spin-orbital coupling interactions, under the additional approximation the three triplet P-wave states ^3P_J and also the singlet $^1P_1$ are degenerated.

C. The generalized instantaneous approximation

After neglecting the negative energy component and the ‘‘treatment’’ above, the weak current matrix elements are as follows:

$$l^{\mu}=i\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}\left[\overline{\eta}(q'_{p'\perp})\frac{\Lambda_2^+(q_{p\perp})}{q_p+\alpha_2M-\omega_2+i\epsilon}\eta(q_{p\perp}) \times\frac{\Lambda'^+_1(q'_{p'\perp})}{-q'_{p'}+\alpha'_1M'-\omega'_1+i\epsilon}\Gamma^{\mu}\frac{\Lambda_1^+(q_{p\perp})}{-q_p+\alpha_1M-\omega_1+i\epsilon}\right]. \tag{36}$$

The generalized instantaneous approximation approach is to perform a contour integration with Cauchy’s theorem about the timelike component q_P in a complex plan on the whole current matrix elements precisely, just as was done by Salpeter for the original one on the BS equations. As the final result of the approach, the matrix elements turn out to be a three-dimensional integration about the spacelike components $q_\perp$, and the Schrödinger wave functions for B_c and $\chi_c(h_c)$ emerge in the integration properly.

If we choose the contour along the lower half-plane, after completing the contour integration, the current matrix elements become

$$l^{\mu}=\int\frac{d^3q_\perp}{(2\pi)^3}\mathrm{Tr}\left[\frac{\overline{\eta}(q'_{p'\perp})\Lambda'^+_1(q'_{p'\perp})}{M'-\omega'_1-\omega'_2} \times\Gamma^{\mu}\frac{\Lambda_1^+(q_{p\perp})\eta(q_{p\perp})\Lambda_2^+(q_{p\perp})}{M-\omega_1-\omega_2}\right].$$

This matrix element can also be written in the framework in which the momentum $q'_\perp$ is the integral argument by means of a suitable Jacobi transformation, i.e.,

$$l_{\mu}(r)=\int\frac{{q'}^2_{p'T}dq'_{p'T}ds}{(2\pi)^2}\mathrm{tr}\left[\overline{\varphi}^{++}_{p'}(q'_{p'\perp})\Gamma_{\mu}\varphi_p^{++}(q_{p\perp})\frac{\not{P}'}{M'}\right]. \tag{37}$$

The above formula with the argument $q'_\perp$ as the integral argument is more convenient, especially in the cases in which we calculate the matrix elements involving a P-wave state in the final state.

After performing the contour integration on the matrix elements l_μ precisely, the dependence of the matrix elements on the overlapping integrations of the Schrödinger wave functions for the initial state and the final state becomes transparent, as do all the form factors.

Since B_c and $\chi_c(h_c)$ are weak-binding states, we may reasonably make the approximation[2]

$$\omega_{20}\equiv\omega'_2\frac{p\cdot p'}{MM'}\simeq m_2\frac{p\cdot p'}{MM'}, \tag{38}$$

so as to see the main factors.

With the approximation, the form factors can be formulated by two independent and ‘‘universal’’ functions with certain kinematics factors. For convenience, we denote the two functions as ξ_1 and ξ_2, and in fact they are just two overlapping integrations of the wave functions of the initial and final bound states,

$$[\epsilon^{\lambda}(p')\cdot\epsilon_0]\xi_1\equiv\int\frac{d^3q'_{p'\perp}}{(2\pi)^3}\psi'^*_{n1M_z(\lambda)}(q'_{p'T})\psi_{n00}(q_{pT}), \tag{39}$$

$$\epsilon^{\alpha}_{\lambda}(p')\cdot\xi_2\equiv\int\frac{d^3q'_{p'\perp}}{(2\pi)^3}\psi'^*_{n1M_z(\lambda)}(q'_{p'T})\psi_{n00}(q_{pT})q'^{\alpha}_{p'\perp},$$

where

[2]The approximation $\omega'_2\simeq m_2$ may be taken here, since at the present stage the paper is about the lowest-order calculations. In fact, one may estimate the uncertainties due to the approximation precisely. It is one of the relativistic corrections and in order of $O(v^2/c^2)$ for the weak-binding system.

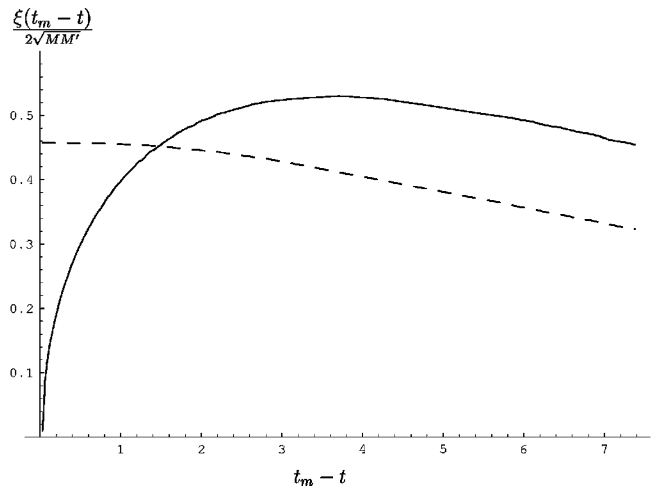

FIG. 2. The universal functions ξ_1 and ξ_2 vs t_m-t. They are the overlapping integrations of the wave functions for $\chi_c(h_c)$ and B_c with the definition as in Eq. (39). The solid line is of ξ_1, the dashed one is of ξ_2.

$$\epsilon_{0\mu}\equiv\frac{p_\mu-\dfrac{p\cdot p'}{M'^2}p'_\mu}{\sqrt{\dfrac{(p\cdot p')^2}{M'^2}-M^2}}$$

describes the polarization along the the recoil momentum $\vec{p'}$, and the P-wave $(L=1)$ orbital "polarization" $\epsilon_\alpha^\lambda(p)$ in moving $(p^2=m^2)$ is

$$\epsilon_\alpha^{\pm1}(p)=\mp\sqrt{\frac{3}{8\pi}}[f_\alpha^1(p)\pm if_\alpha^2(p)],$$

$$\epsilon_\alpha^0(p)=\sqrt{\frac{3}{4\pi}}f_\alpha^3(p),$$

with

$$f_\mu^i(p)=\frac{-m^2g_{i\mu}-m(Eg_{i\mu}-p_ig_{0\mu})+p_ip_\mu}{m(E+m)}$$
$$(i=1,2,3;\quad \mu=0,1,2,3).$$

In general, the matrix elements for the weak-binding systems with the approximation Eq. (38) may depend on only two "universal" functions which are generated directly by the overlapping integrations of the initial and final wave function: one is the integration without the relative momentum $q'_{p'\perp}$ being inserted between the two wave functions, i.e., ξ_1, and the other is with the relative momentum $q'_{p'\perp}$ being inserted linearly, i.e., ξ_2. Note that if the integration is with a higher power of the relative momentum $q'_{p'\perp}$ than is linearly being inserted, it will be the relativistic corrections to ξ_1 or ξ_2. For instance, those with $q'^2_{p'\perp}, q'^4_{p'\perp}, \ldots$ being inserted are the corrections to ξ_1 and those with $q'^3_{p'\perp}, q'^5_{p'\perp}, \ldots$ being inserted are the corrections to ξ_2. Therefore, there are only two such "universal" functions ξ_1,ξ_2 in the decays. In the case in which the decays are from an S wave to another S-wave state, due to the fact that the wave functions are normalized, ξ_1 must be much greater than ξ_2, even when the recoil momentum is great. Thus ξ_2 may be ignored safely, i.e., in that case it is enough to consider ξ_1 only. Furthermore, as found in Ref. [8], the "universal" function ξ (just ξ_1 here) may be directly related to the Isgur-Wise function appearing in heavy flavor effective theory (HFET) for heavy meson decays since spin-flavor symmetry [18], if taking the mass of c quark approaching to infinity. Whereas in the present case it is about the decays from B_c to a P-wave charmonium, it is different from those from B_c to an S-wave charmonium, hence an interesting aspect happens. Since the decays from an S-wave state to a P-wave state are considered as the common and familiar cases in the atomic and nuclear transitions, the function ξ_2 is dominant in the region of a small recoil momentum and is decreasing slowly as the momentum recoil is increasing, whereas the function ξ_1, as expected, is zero or tiny when the recoil momentum is zero or small, but turns out to be comparable to or even greater than ξ_2 in the region of a great momentum recoil. We should note that there is no conflict with our knowledge about the atomic and nuclear transitions, where the functions ξ_1 are always ignored due to the fact that the momentum recoil in all the transitions is very small, i.e., only the function ξ_2 "acts." But in the present case of the decay of B_c to a P-wave charmonium, the momentum recoil may even be relativistic, so we do not have any reason to ignore one of the two "universal" functions ξ_1 and ξ_2 to obtain correct results. Instead, we need to keep two of them precisely. In order to see this feature clearly, in Sec. V we evaluate the two functions ξ_1 and ξ precisely, and we put the curves of their values versus momentum transfer into a figure (Fig. 2).

Substituting the BS wave functions Eq. (24) and Eqs. (26) and (27) into the equation of current matrix elements and using Eq. (39), the precise formula for the form factors, i.e., the precise dependence of the form factors on ξ_1 and ξ_2, can be obtained and we put them in the Appendix. The curves of ξ_1 and ξ_2 obtained by numerical calculations are shown in a figure. With the functions ξ_1, ξ_2 and the form factors, the decay rates of the semileptonic decays and the spectrum of the charged lepton for the decays can be obtained by straightforward numerical calculations.

Note that in our calculations on the form factors, we have used the relations

$$\sum_{\lambda,\lambda'}\epsilon_\mu^\lambda(S,p')\epsilon_\nu^{\lambda'}(L,p')\langle1\lambda;1\lambda'|00\rangle$$
$$=\sqrt{\frac{1}{3}}\left(g_{\mu\nu}-\frac{p'_\mu p'_\nu}{M'^2}\right),$$

$$\sum_{\lambda,\lambda'} \epsilon^{\lambda}_{\mu}(S,p')\epsilon^{\lambda'}_{\nu}(L,p')\langle 1\lambda;1\lambda'|1\lambda''\rangle$$

$$=\sqrt{\frac{1}{2}}\frac{i}{M'}\epsilon_{\mu\nu\alpha\beta}p'^{\alpha}\epsilon^{\beta}_{\lambda''}(J,p'), \tag{40}$$

$$\sum_{\lambda,\lambda'} \epsilon^{\lambda}_{\mu}(S,p')\epsilon^{\lambda'}_{\nu}(L,p')\langle 1\lambda;1\lambda'|2\lambda''\rangle$$

$$\equiv \epsilon^{\lambda''}_{\mu\nu}(J,p'),$$

where $\langle 1S_z;1L_z|JJ_z\rangle$ as before are CG coefficients. The polarization of a vector with $(J=1;\ \lambda=1,0,-1)$, $\epsilon^{\lambda}_{\mu}(p')$, and that of a tensor with $(J=2;\ \lambda=-2,-1,0,1,2)$, $\epsilon^{\lambda}_{\mu\nu}(p')$, have the "projections"

$$\sum_{\lambda=\pm 1,0} \epsilon^{\lambda}_{\mu}(p')\epsilon^{\lambda}_{\nu}(p')=\left(\frac{p'_{\mu}p'_{\nu}}{M'^2}-g_{\mu\nu}\right)\equiv P'_{\mu\nu},$$

$$\sum_{\lambda=\pm 2,\pm 1,0} \epsilon^{\lambda}_{\mu\nu}(p')\epsilon^{\lambda}_{\alpha\beta}(p')=\frac{1}{2}(P'_{\mu\alpha}P'_{\nu\beta}+P'_{\mu\beta}P'_{\nu\alpha})$$

$$-\frac{1}{3}P'_{\mu\nu}P'_{\alpha\beta}. \tag{41}$$

IV. THE TWO-BODY NONLEPTONIC DECAYS

In this section, we outline how to calculate the two-body nonleptonic decays $B_c\to\chi_c(h_c)+h$ (here h denotes a meson) with the factorization assumption on the decay amplitudes, which is still widely adopted in estimating the nonleptonic decays for various mesons in literature. According to the assumption, the weak current matrix elements appear as a factor in the calculations precisely and they are related to the form factors just obtained in the previous section. For the noneptonic decay modes $B_c\to\chi_c(h_c)+h$ (caused by the decay $b\to c$), the following effective Lagrangian $L_{\rm eff}$ (QCD corrections are involved) is responsible:

$$L_{eff}=\frac{G_F}{\sqrt{2}}\{V_{cb}[c_1(\mu)Q_1^{cb}+c_2(\mu)Q_2^{cb}]+\text{H.c.}\}$$

$$+\text{penguin operators}. \tag{42}$$

G_F is the Fermi constant, V_{ij} are CKM matrix elements, and $c_i(\mu)$ are scale-dependent Wilson coefficients. The four-quark operators Q_1^{cb} and Q_2^{cb} (CKM favored only) are

$$Q_1^{cb}=[V^*_{ud}(\overline{d}u)_{V-A}+V^*_{us}(\overline{s}u)_{V-A}+V^*_{cd}(\overline{d}c)_{V-A}$$

$$+V^*_{cs}(\overline{s}c)_{V-A}](\overline{c}b)_{V-A}, \tag{43}$$

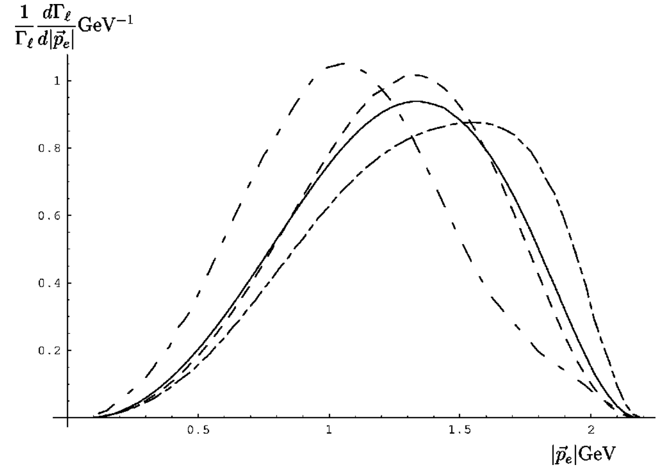

FIG. 3. The energy spectrum of the charged lepton for the decays $B_c\to\chi_c+e(\mu)+\nu$, where the solid line is the result of the $h_c[^1P_1]$ state, the dotted-blank-dashed line is of $\chi_c[^3P_0]$, the dashed line is of $\chi_c[^3P_1]$, and the dotted-dashed line is $\chi_c[^3P_2]$.

$$Q_2^{cb}=[V^*_{ud}(\overline{c}u)_{V-A}(\overline{d}b)_{V-A}+V^*_{us}(\overline{c}u)_{V-A}(\overline{s}b)_{V-A}$$

$$+V^*_{cd}(\overline{c}c)_{V-A}(\overline{d}b)+V^*_{cs}(\overline{c}c)_{V-A}(\overline{s}b)],$$

where $(\overline{q}_1q_2)_{V-A}$ denotes $\overline{q}_1\gamma_{\mu}(1-\gamma_5)q_2$.

Because at this moment we restrict ourselves to considering the decays to which the "penguin" operators in the effective Lagrangian contribute little in comparison with the two main ones c_1 and c_2, the contributions from penguin terms are neglected in the present calculations, although in the Ref. [11] it is pointed out that the "penguin" operator may contribute about 3–4% to the total decay width owing to its interference with the main ones. Moreover, at this stage we also restrict ourselves to considering only the decay modes where the weak annihilation contributions are small due to the precise reasons, e.g., the helicity suppression,

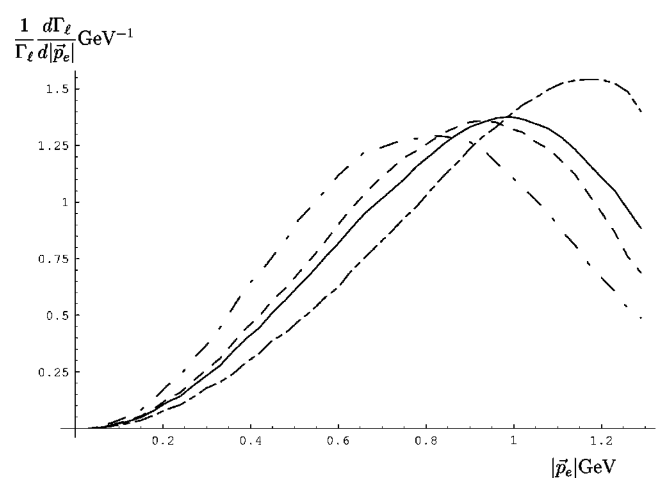

FIG. 4. The energy spectrum of the charged lepton for the decays $B_c\to\chi_c+\tau+\nu_{\tau}$, where the solid line is the result of $h_c[^1P_1]$ state, the dotted-blank-dashed line is of $\chi_c[^3P_0]$, the dashed line is of $\chi_c[^3P_1]$, and the dotted-dashed line is $\chi_c[^3P_2]$.

TABLE I. The semileptonic decay widths (in the unit 10^{-15} GeV).

	$\Gamma(B_c\to {}^1P_1\ell\nu_\ell)$	$\Gamma(B_c\to {}^3P_0\ell\nu_\ell)$	$\Gamma(B_c\to {}^3P_1\ell\nu_\ell)$	$\Gamma(B_c\to {}^3P_2\ell\nu_\ell)$
$e(\mu)$	2.509	1.686	2.206	2.732
τ	0.356	0.249	0.346	0.422

etc.,[3] so we do not take the contributions into account here [19].

Precisely by means of the factorization assumption, the decay amplitudes for the nonleptonic decays can be formulated into three factors: the so-called leptonic decay constants, which are defined by the matrix elements $\langle 0|A_\mu|M(p)\rangle = if_M p_\mu$ [or $\langle 0|V_\mu|V(p,\epsilon)\rangle = f_V M_V \epsilon_\mu$]; the weak current matrix elements $\langle\chi_c|V_\mu(A_\mu)|B_c\rangle$, which are the semileptonic decays; and the relevant coefficients in the combinations: $a_1=c_1+\kappa c_2$ and $a_2=c_2+\kappa c_1$, where $\kappa=1/N_c$ and N_c is the number of color. The coefficients in the combination a_1,a_2 are due to the weak currents being "Fierz-reordered." In the numerical calculations later on, we will choose $a_1=c_1$ and $a_2=c_2$, i.e., we take $\kappa=0$ in the spirit of the large N_c limit, and the QCD correction coefficients c_1 and c_2 are computed at the energy scale of m_b. Therefore with the relations between the currents and form factors obtained as in the semileptonic decays, finally the factorized amplitudes for the nonleptonic decays can be formulated in terms of the form factors and the decay constants by the definitions $\langle 0|A_\mu|M(p)\rangle = if_M p_\mu$ and $\langle 0|V_\mu|V(p,\epsilon)\rangle = f_V M_V \epsilon_\mu$. Thus the decay widths for the two-body nonleptonic decays can be computed straightforwardly.

V. NUMERICAL RESULTS AND DISCUSSIONS

In the numerical calculations, according to the potential model, the parameters are taken as follows:

$\lambda=0.24\,\mathrm{GeV}^2$, $\alpha=0.06$ GeV, $\Lambda_{\mathrm{QCD}}=0.18$ GeV, $a=e=2.7183$, $V_0=-0.93$ GeV, $V_{bc}=0.04$ [13], $m_2=m_c=1.846$ GeV, $m_1=m_b=5.243$ GeV.

With the parameters, the masses of B_c and χ_c, h_c are

$$M=m_{B_c}=6.33\ \mathrm{GeV},\quad M'=m_{\chi_c(h_c)}=3.50\ \mathrm{GeV},$$

and the corresponding radial wave functions of the B_c meson and P-wave charmonium χ_c, h_c numerically, which are consistent with those in the literature. Note here that since in the present evaluations we only carry out the lowest-order ones without considering the splitting caused by L-S and S-S couplings, M', the masses of all the bound states ${}^3P_J$ ($J=0,1,2$) and ${}^1P_1$, are degenerated.

The behaviors of the functions $\xi_1(t_m-t)$ and $\xi_2(t_m-t)$, i.e., the two overlapping integrations of the wave functions of the initial and final states, are computed numerically and plotted in Fig. 2, where $t_m=(M-M')^2$ and $t=(P-P')^2$.

[3]We will consider the contribution from the penguin operator and weak annihilation carefully elsewhere.

The spectra of the charged lepton energy for the decays $B_c\to\chi_c+e(\mu)+\nu$ (the mass of the charged lepton can be ignored) are shown in Fig. 3, and those for the decays $B_c\to\chi_c+\tau+\nu$, where the mass of the charged lepton τ cannot be ignored, are shown in Fig. 4, where $|\vec{p}_l|$ is the momentum of lepton. The difference, shown in Fig. 3 and Fig. 4, is due to the fact that τ lepton has a sizable mass. For the semileptonic decays, we put the widths of the decays correspondingly in Table I.

As for the nonleptonic two-body decays $B_c\to\chi_c(h_c)+h$, we only evaluate some typical channels, whose widths are relatively larger, and put the results in Table II. In the numerical calculations, c_1 and c_2 are computed at the energy scale of m_b, and the coefficients $a_1=c_1$ and $a_2=c_2$, i.e., $\kappa=0$ is taken. The decay constant values $f_{\pi^+}=0.131$ GeV, $f_{\rho^+}=0.208$ GeV, $f_{a_1}=0.229$ GeV, $f_{K^+}=0.159$ GeV, $f_{K^{*+}}=0.214$ GeV, $f_{D_s}=0.213$ GeV, $f_{D_s^*}=0.242$ GeV, $f_{D^+}=0.209$ GeV, and $f_{D^{*+}}=0.237$ GeV are determined by fitting decays of B and D mesons.

By comparing the present results in Table I with those for the decays of B_c to S-wave charmonium states J/ψ and η_c, e.g., $\Gamma(B_c\to J/\psi+l+\nu)\sim 25\times 10^{-15}$ GeV, which can be found in Refs. [8,9], one may realize that the semileptonic decays of B_c to the P-wave charmonium states in magnitude are about one-tenth of the decay $B_c\to J/\psi+l+\nu_l$. As for the two-body nonleptonic decays, due to the fact that the recoil momentum is fixed in each specific decay so that it varies in various decays the decays, $B_c\to\chi_c(h_c)+h$ can be greater than one-twentieth of the one corresponding to the S-wave state, $B_c\to J/\psi(\eta_c)+h$.

Based on the fact that the first observation of B_c by the CDF group is through the semileptonic decays $B_c\to J/\psi+l+\nu_l$ in run I of Tevatron, we can conclude that most of the decays concerned here are accessible at LHC and at Tevatron in run II. It is expected that the B_c meson events will be more than 20 times more frequent at Tevatron and LHC than those at run I, and the detectors at the two colliders will be much improved, especially the two-detector BTeV and LHCB, which were designed particularly for B physics. These may accumulate many more B_c events of better quality, thus not only may the concerned decays be accessible, but also rare decays, even CP-violation processes, may be too.

Since a specific detector always has a limited efficiency to record a photon, i.e., a photon may be missed in the detector with a precise possibility, the cascade decays of $B_c\to\chi_c+\cdots$ and $\chi_c\to J/\psi+\gamma$ may appear as an indication of the meson B_c through the decays $B_c\to J/\psi+\cdots$ when the photon in the second decay is missed. Furthermore, two of the P-wave charmonia have quite a large branching ratio (about

TABLE II. Two-body non-leptonic B_c^+ decay widths in unit 10^{-15} GeV.

Channel	Γ	$\Gamma(a_1=1.132)$	Channel	Γ	$\Gamma(a_1=1.132)$
${}^1P_1\pi^+$	$a_1^2 0.569$	0.729	${}^1P_1\rho$	$a_1^2 1.40$	1.79
${}^3P_0\pi^+$	$a_1^2 0.317$	0.407	${}^3P_0\rho$	$a_1^2 0.806$	1.03
${}^3P_1\pi^+$	$a_1^2 0.0815$	0.104	${}^3P_1\rho$	$a_1^2 0.331$	0.425
${}^3P_2\pi^+$	$a_1^2 0.277$	0.355	${}^3P_2\rho$	$a_1^2 0.579$	0.742
${}^1P_1A_1$	$a_1^2 1.71$	2.19	${}^1P_1K^+$	$a_1^2 4.26\times10^{-3}$	5.46×10^{-3}
${}^3P_0A_1$	$a_1^2 1.03$	1.33	${}^3P_0K^+$	$a_1^2 2.35\times10^{-3}$	3.02×10^{-3}
${}^3P_1A_1$	$a_1^2 0.671$	0.859	${}^3P_1K^+$	$a_1^2 0.583\times10^{-3}$	0.747×10^{-3}
${}^3P_2A_1$	$a_1^2 1.05$	1.34	${}^3P_2K^+$	$a_1^2 1.99\times10^{-3}$	2.56×10^{-3}
${}^1P_1K^*$	$a_1^2 7.63\times10^{-3}$	9.78×10^{-3}	${}^1P_1D_s$	$a_1^2 2.32$	2.98
${}^3P_0K^*$	$a_1^2 4.43\times10^{-3}$	5.68×10^{-3}	${}^3P_0D_s$	$a_1^2 1.18$	1.51
${}^3P_1K^*$	$a_1^2 2.05\times10^{-3}$	2.63×10^{-3}	${}^3P_1D_s$	$a_1^2 0.149$	0.191
${}^3P_2K^*$	$a_1^2 3.48\times10^{-3}$	4.47×10^{-3}	${}^3P_2D_s$	$a_1^2 0.507$	0.650
${}^1P_1D_s^*$	$a_1^2 1.99$	2.56	${}^1P_1D^+$	$a_1^2 0.0868$	0.111
${}^3P_0D_s^*$	$a_1^2 1.48$	1.89	${}^3P_0D^+$	$a_1^2 0.0443$	0.0568
${}^3P_1D_s^*$	$a_1^2 2.21$	2.83	${}^3P_1D^+$	$a_1^2 0.00610$	0.00782
${}^3P_2D_s^*$	$a_1^2 2.68$	3.44	${}^3P_2D^+$	$a_1^2 0.0209$	0.0267
${}^1P_1D^{*+}$	$a_1^2 0.0788$	0.101			
${}^3P_0D^{*+}$	$a_1^2 0.0567$	0.0726			
${}^3P_1D^{*+}$	$a_1^2 0.0767$	0.0983			
${}^3P_2D^{*+}$	$a_1^2 0.0972$	0.124			

a few tenths) for their radiative decays $\chi_c[{}^3P_1]\to J/\psi+\gamma$ ($Br=27.3\%$) and $\chi_c[{}^3P_2]\to J/\psi+\gamma$ ($Br=13.5\%$) [13], hence the cascade decays may contribute a substantial background for the observation of the B_c meson through $B_c\to J/\psi+\cdots$. In particular, we find here that the decays $B_c\to\chi_c[{}^3P_{1,2}]+l+\nu_l$ have quite a sizable branching ratio, the meson B_c observed by the CDF group, as pointed out in the Introduction, is through the semileptonic decays $B_c\to J/\psi+l+\nu_l$, thus the efficiency in detecting photons of the detector may affect the observation at a certain level. In conclusion, we think the results concerning the decay values obtained here are useful references in estimating the background quantitatively.

We would also like to point out that we find that the decays $B_c\to h_c+l+\nu_l$ and/or $B_c\to h_c+h$ have sizable branching ratios, so potentially they can create a new way to observe the charmonium state $h_c[{}^1P_1]$. In particular, the charmonium state $h_c[{}^1P_1]$ has not been well-established experimentally yet, thus it is worthwhile to try to observe the state h_c through the B_c decays at LHC and Tevatron, especially with the detectors LHCB and BTeV. Hopefully, it will be a complementary possibility experimentally to see the ${}^1P_1$ charmonium state, in addition to those through $\psi(2S)$ decay and proton-antiproton annihilation.

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APPENDIX

In this appendix, we present the form factors and the formulas for α, β_{++}, and γ, which are required in the calculations on the exclusive semileptonic decays of B_c to X. Here X denotes one of the states $h_c[{}^1P_1]$, $\chi_c[{}^3P_0]$, $\chi_c[{}^3P_1]$, and $\chi_c[{}^3P_2]$ as indicated precisely in each case below.

For convenience, we introduce the parameters below:

$$\omega_{10}\equiv\sqrt{\omega_{20}^2-m_2^2+m_1^2},$$

$$nep=\sqrt{\frac{(p\cdot p')^2}{M'^2}-M^2}.$$

1. B_c meson to charmonium $h_c[{}^1P_1]$

The matrix elements for the vector and axial currents are

$$\langle X(p',\epsilon)|V_\mu|B_c(p)\rangle\equiv r\epsilon_\mu^*+s_+(\epsilon^*\cdot p)(p+p')_\mu+s_-(\epsilon^*\cdot p)(p-p')_\mu,$$

$$\langle X(p',\epsilon)|A_\mu|B_c(p)\rangle\equiv iv\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}(p+p')^\rho(p-p')^\sigma,$$

where

$$r=\frac{(m_1'-m_2)(m_1+\omega_{10}-m_2-\omega_{20})\xi_2}{8m_1'\omega_{10}\omega_{20}}-\frac{(m_1'+m_2)(m_1+\omega_{10}+m_2+\omega_{20})\xi_2(p\cdot p')}{8M'Mm_1'\omega_{10}\omega_{20}}, \tag{A1}$$

$$\begin{aligned}s_+=&\frac{m_2[M(m_2+\omega_{20}-m_1-\omega_{10})-M'(m_1+\omega_{10}+m_2+\omega_{20})]\xi_2}{8M'M^2\omega_{10}{\omega_{20}}^2}\\&+\frac{m_2[M(m_2+\omega_{20}-m_1-\omega_{10})-M'(m_1+\omega_{10}+m_2+\omega_{20})]\xi_1}{8M'M\omega_{10}\omega_{20}nep}\\&+\frac{m_2[M(m_2\omega_{20}+\omega_{20}^2-m_1\omega_{20}-\omega_{10}^2)-M'(m_1\omega_{20}-\omega_{10}^2+m_2\omega_{20}+\omega_{20}^2)]\xi_2}{8M'M^2\omega_{10}^3\omega_{20}}\\&+\frac{(m_1'+m_2)(m_1+\omega_{10}+m_2+\omega_{20})\xi_2}{16M'Mm_1'\omega_{10}\omega_{20}}-\frac{(M'-M)\xi_1}{8M'M^2\omega_{10}},\end{aligned} \tag{A2}$$

$$\begin{aligned}s_-=&\frac{m_2[-M(m_2+\omega_{20}-m_1-\omega_{10})-M'(m_1+\omega_{10}+m_2+\omega_{20})]\xi_2}{8M'M^2\omega_{10}{\omega_{20}}^2}\\&+\frac{m_2[-M(m_2+\omega_{20}-m_1-\omega_{10})-M'(m_1+\omega_{10}+m_2+\omega_{20})]\xi_1}{8M'M\omega_{10}\omega_{20}nep}\\&+\frac{m_2[-M(m_2\omega_{20}+\omega_{20}^2-m_1\omega_{20}-\omega_{10}^2)-M'(m_1\omega_{20}-\omega_{10}^2+m_2\omega_{20}+\omega_{20}^2)]\xi_2}{8M'M^2\omega_{10}^3\omega_{20}}\\&-\frac{(m_1'+m_2)(m_1+\omega_{10}+m_2+\omega_{20})\xi_2}{16M'Mm_1'\omega_{10}\omega_{20}}+\frac{(M'-M)\xi_1}{8M'M^2\omega_{10}},\end{aligned} \tag{A3}$$

$$v=-\frac{(m_1'+m_2)(m_1+m_2+\omega_{10}+\omega_{20})\xi_2}{16M'Mm_1'\omega_{10}\omega_{20}}. \tag{A4}$$

The dependences of α, β_{++}, and γ on the above form factors are

$$\alpha=r^2+4M^2\vec{p}'^2v^2, \tag{A5}$$

$$\begin{aligned}\beta_{++}=&\frac{r^2}{4M'^2}-M^2yv^2+\frac{1}{2}\left[\frac{M^2}{M'^2}(1-y)-1\right]rs_+\\&+M^2\frac{\vec{p}'^2}{M'^2}s_+^2,\end{aligned} \tag{A6}$$

$$\begin{aligned}\beta_{+-}=&-\frac{r^2}{4M'^2}+(M^2-M'^2)v^2\\&+\frac{1}{4}\left[-\frac{M^2}{M'^2}(1-y)-3\right]rs_+\\&+\frac{1}{4}\left[\frac{M^2}{M'^2}(1-y)-1\right]rs_-+M^2\frac{\vec{p}'^2}{M'^2}s_+s_-,\end{aligned} \tag{A7}$$

$$\beta_{-+}=\beta_{+-}, \tag{A8}$$

$$\begin{aligned}\beta_{--}=&\frac{r^2}{4M'^2}+[M^2y-2(M^2+M'^2)]v^2\\&+\frac{1}{2}\left[-\frac{M^2}{M'^2}(1-y)-3\right]rs_-+M^2\frac{\vec{p}'^2}{M'^2}s_-^2,\end{aligned} \tag{A9}$$

$$\gamma=2rv. \tag{A10}$$

2. B_c meson to charmonium $\chi_c[^3P_0]$

Here we show the matrix element for the vector current vanishes in the present decay.

The matrix element for the axial current is

$$\langle X(p')|A_\mu|B_c(p)\rangle\equiv u_+(p+p')_\mu+u_-(p-p')_\mu,$$

where

$$u_{+}=\frac{[M(\omega_{10}+m_1+\omega_{20}+m_2)\xi_1+M'(\omega_{20}+m_2-\omega_{10}-m_1)]m_2\xi_1}{8\sqrt{3}M'\omega_{10}\omega_{20}nep}$$
$$-\frac{M'(\omega_{10}+m_1+\omega_{20}+m_2)\xi_1+M(\omega_{20}+m_2-\omega_{10}-m_1)\xi_1}{8\sqrt{3}M'\omega_{10}nep}$$
$$+\frac{3\xi_2[M'(m_1'+m_2)(\omega_{10}+m_1+\omega_{20}+m_2)+M(m_1'-m_2)(\omega_{20}+m_2-\omega_{10}-m_1)]}{16\sqrt{3}M'Mm_1'\omega_{10}\omega_{20}}$$
$$+\frac{\xi_2m_2[M'(-\omega_{10}-m_1+\omega_{20}+m_2)+M(\omega_{20}+m_2+\omega_{10}+m_1)]}{8\sqrt{3}M'M\omega_{10}\omega_{20}^2}$$
$$+\frac{\xi_2[M'm_2(m_2\omega_{20}+\omega_{20}^2-m_1\omega_{20}-\omega_{10}^2)+M\omega_{20}(m_2\omega_{20}+\omega_{20}^2+m_1\omega_{20}-\omega_{10}^2)]}{8\sqrt{3}M'M\omega_{10}^3\omega_{20}}$$
$$+\frac{\xi_2[-M'(m_2\omega_{20}+\omega_{20}^2+m_1\omega_{20}-\omega_{10}^2)+M(-m_2\omega_{20}-\omega_{20}^2+m_1\omega_{20}+\omega_{10}^2)]}{8\sqrt{3}M'M\omega_{10}^3}+\frac{\xi_2(M'-M)(m_2+\omega_{20})}{8\sqrt{3}M'Mm_2\omega_{10}}, \quad \text{(A11)}$$

$$u_{-}=\frac{[-M(\omega_{10}+m_1+\omega_{20}+m_2)\xi_1+M'(\omega_{20}+m_2-\omega_{10}-m_1)]m_2\xi_1}{8\sqrt{3}M'\omega_{10}\omega_{20}nep}$$
$$-\frac{M'(\omega_{10}+m_1+\omega_{20}+m_2)\xi_1-M(\omega_{20}+m_2-\omega_{10}-m_1)\xi_1}{8\sqrt{3}M'\omega_{10}nep}$$
$$+\frac{3\xi_2[M'(m_1'+m_2)(\omega_{10}+m_1+\omega_{20}+m_2)-M(m_1'-m_2)(\omega_{20}+m_2-\omega_{10}-m_1)]}{16\sqrt{3}M'Mm_1'\omega_{10}\omega_{20}}$$
$$+\frac{\xi_2m_2[M'(-\omega_{10}-m_1+\omega_{20}+m_2)-M(\omega_{20}+m_2+\omega_{10}+m_1)]}{8\sqrt{3}M'M\omega_{10}\omega_{20}^2}$$
$$+\frac{\xi_2[M'm_2(m_2\omega_{20}+\omega_{20}^2-m_1\omega_{20}-\omega_{10}^2)-M\omega_{20}(m_2\omega_{20}+\omega_{20}^2+m_1\omega_{20}-\omega_{10}^2)]}{8\sqrt{3}M'M\omega_{10}^3\omega_{20}}$$
$$+\frac{\xi_2[-M'(m_2\omega_{20}+\omega_{20}^2+m_1\omega_{20}-\omega_{10}^2)-M(-m_2\omega_{20}-\omega_{20}^2+m_1\omega_{20}+\omega_{10}^2)]}{8\sqrt{3}M'M\omega_{10}^3}+\frac{\xi_2(M'+M)(m_2+\omega_{20})}{8\sqrt{3}M'Mm_2\omega_{10}}. \quad \text{(A12)}$$

The dependences of α, β_{++}, and γ on the above form factors are

$$\alpha=0, \quad \text{(A13)}$$

$$\beta_{++}=u_+^2, \quad \beta_{+-}=u_+u_-,$$

$$\beta_{-+}=u_-u_+, \quad \beta_{--}=u_-^2, \quad \text{(A14)}$$

$$\gamma=0. \quad \text{(A15)}$$

3. B_c meson to charmonium $\chi_c[^3P_1]$

The matrix elements for the vector and axial currents are

$$\langle X(p',\epsilon)|V_\mu|B_c(p)\rangle\equiv l\epsilon_\mu^*+c_+(\epsilon^*\cdot p)(p+p')_\mu+c_-(\epsilon^*\cdot p)(p-p')_\mu,$$

$$\langle X(p',\epsilon)|A_\mu|B_c(p)\rangle\equiv iq\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}(p+p')^\rho(p-p')^\sigma.$$

where

$$l=\frac{(m_1+\omega_{10}+m_2+\omega_{20})\xi_1[(p\cdot p')^2-M^2M'^2]m_2}{4\sqrt{2}MM'^2nep\,\omega_{10}\omega_{20}}$$
$$-\frac{(m_1+\omega_{10}+m_2+\omega_{20})\xi_2(p\cdot p')}{2\sqrt{2}MM'\omega_{10}\omega_{20}}$$
$$-\frac{(m_1+\omega_{10}-m_2-\omega_{20})\xi_2(p\cdot p')}{4\sqrt{2}Mm_1'm_2\omega_{10}}$$
$$+\frac{(m_1+\omega_{10}-m_2-\omega_{20})M'\xi_2}{4\sqrt{2}m_1'\omega_{10}\omega_{20}}$$
$$-\frac{[M'^2M^2-(p\cdot p')^2]\xi_2}{4\sqrt{2}M'^2M^2\omega_{10}}$$
$$+\frac{\xi_2 m_2[(p\cdot p')^2-M^2M'^2]}{4\sqrt{2}M^2M'^2\omega_{10}\omega_{20}}\left[\frac{(\omega_{10}+m_1+\omega_{20}+m_2)}{\omega_{20}}\right.$$
$$\left.+\frac{(m_1\omega_{20}+m_2\omega_{20}+\omega_{20}^2-\omega_{10}^2)}{\omega_{10}^2}\right], \tag{A16}$$

$$c_+=\frac{(m_1+\omega_{10}+m_2+\omega_{20})\xi_1(M'^2-p\cdot p')m_2}{8\sqrt{2}MM'^2nep\,\omega_{10}\omega_{20}}$$
$$+\frac{(M'^2-p\cdot p')\xi_2}{8\sqrt{2}M'^2M^2\omega_{10}}+\frac{(m_1+\omega_{10}+m_2+\omega_{20})\xi_2}{4\sqrt{2}MM'\omega_{10}\omega_{20}}$$
$$+\frac{(m_1+\omega_{10}-m_2-\omega_{20})\xi_2}{8\sqrt{2}Mm_1'm_2\omega_{10}}+\frac{\xi_2 m_2(M'^2-p\cdot p')}{8\sqrt{2}M^2M'^2\omega_{10}\omega_{20}}$$
$$\times\left[\frac{(\omega_{10}+m_1+\omega_{20}+m_2)}{\omega_{20}}\right.$$
$$\left.+\frac{(m_1\omega_{20}+m_2\omega_{20}+\omega_{20}^2-\omega_{10}^2)}{\omega_{10}^2}\right],$$

$$c_-=\frac{(m_1+\omega_{10}+m_2+\omega_{20})\xi_1(M'^2+p\cdot p')m_2}{8\sqrt{2}MM'^2nep\,\omega_{10}\omega_{20}}$$
$$+\frac{(M'^2+p\cdot p')\xi_2}{8\sqrt{2}M'^2M^2\omega_{10}}-\frac{(m_1+\omega_{10}+m_2+\omega_{20})\xi_2}{4\sqrt{2}MM'\omega_{10}\omega_{20}}$$
$$-\frac{(m_1+\omega_{10}-m_2-\omega_{20})\xi_2}{8\sqrt{2}Mm_1'm_2\omega_{10}}+\frac{\xi_2 m_2(M'^2+p\cdot p')}{8\sqrt{2}M^2M'^2\omega_{10}\omega_{20}}$$
$$\times\left[\frac{(\omega_{10}+m_1+\omega_{20}+m_2)}{\omega_{20}}\right.$$
$$\left.+\frac{(m_1\omega_{20}+m_2\omega_{20}+\omega_{20}^2-\omega_{10}^2)}{\omega_{10}^2}\right],$$

$$q=\frac{(\omega_{10}+m_1+\omega_{20}+m_2)\xi_1}{8\sqrt{2}M'\omega_{10}nep}$$
$$-\frac{m_2(-\omega_{10}-m_1+\omega_{20}+m_2)\xi_1}{8\sqrt{2}M'\omega_{10}\omega_{20}nep}$$
$$+\frac{(\omega_{20}^2+m_1\omega_{20}+m_2\omega_{20}-\omega_{10}^2)\xi_2}{8\sqrt{2}M'M\omega_{10}^3}$$
$$+\frac{(m_2+\omega_{20})\xi_2}{8\sqrt{2}M'Mm_2\omega_{10}}$$
$$-\frac{\xi_2 m_2}{8\sqrt{2}MM'\omega_{10}\omega_{20}}\left[\frac{(-\omega_{10}-m_1+\omega_{20}+m_2)}{\omega_{20}}\right.$$
$$\left.+\frac{(-m_1\omega_{20}+m_2\omega_{20}+\omega_{20}^2-\omega_{10}^2)}{\omega_{10}^2}\right].$$

The dependences of α, β_{++}, and γ on the above form factors are

$$\alpha=l^2+4M^2\vec{p}'^2q^2, \tag{A17}$$

$$\beta_{++}=\frac{l^2}{4M'^2}-M^2yq^2+\frac{1}{2}\left[\frac{M^2}{M'^2}(1-y)-1\right]lc_+$$
$$+M^2\frac{\vec{p}'^2}{M'^2}c_+^2, \tag{A18}$$

$$\beta_{+-}=-\frac{l^2}{4M'^2}+(M^2-M'^2)q^2$$
$$+\frac{1}{4}\left[-\frac{M^2}{M'^2}(1-y)-3\right]lc_+$$
$$+\frac{1}{4}\left[\frac{M^2}{M'^2}(1-y)-1\right]lc_-+M^2\frac{\vec{p}'^2}{M'^2}c_+c_-, \tag{A19}$$

$$\beta_{-+}=\beta_{+-}, \tag{A20}$$

$$\beta_{--}=\frac{l^2}{4M'^2}+[M^2y-2(M^2+M'^2)]q^2$$
$$+\frac{1}{2}\left[-\frac{M^2}{M'^2}(1-y)-3\right]lc_-+M^2\frac{\vec{p}'^2}{M'^2}c_-^2, \tag{A21}$$

$$\gamma=2lq. \tag{A22}$$

4. B_c meson to charmonium $\chi_c[^3P_2]$

The matrix elements of the vector and axial currents are

$$\langle X(p',\epsilon)|V_\mu|B_c(p)\rangle \equiv ih_{+-}\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu\alpha}p_\alpha(p+p')^\rho \times(p-p')^\sigma,$$

$$\langle X(p',\epsilon)|A_\mu|B_c(p)\rangle \equiv k\epsilon^*_{\mu\nu}p^\nu + b_+(\epsilon^*_{\rho\sigma}p^\rho p^\sigma)(p+p')_\mu + b_-(\epsilon^*_{\rho\sigma}p^\rho p^\sigma)(p-p')_\mu .$$

where

$$k=-\frac{(m_1+\omega_{10}+m_2+\omega_{20})\xi_1}{4\omega_{10}nep} + \frac{m_2(-m_1-\omega_{10}+m_2+\omega_{20})\xi_1}{4\omega_{10}\omega_{20}nep} + \frac{(\omega_{10}^2-m_1\omega_{20}-m_2\omega_{20}-\omega_{20}^2)\xi_2}{4M\omega_{10}^3} - \frac{(m_2+\omega_{20})\xi_2}{4Mm_2\omega_{10}} + \frac{\xi_2 m_2}{4M\omega_{10}\omega_{20}}\left[\frac{(-\omega_{10}-m_1+\omega_{20}+m_2)}{\omega_{20}} + \frac{(-m_1\omega_{20}+m_2\omega_{20}+\omega_{20}^2-\omega_{10}^2)}{\omega_{10}^2}\right],$$

$$b_+=\frac{m_2(m_1+\omega_{10}+m_2+\omega_{20})\xi_1}{8M'M\omega_{10}\omega_{20}nep} + \frac{\xi_2}{8M'M^2\omega_{10}} + \frac{\xi_2 m_2}{8M^2M'\omega_{10}\omega_{20}}\left[\frac{(\omega_{10}+m_1+\omega_{20}+m_2)}{\omega_{20}} + \frac{(m_1\omega_{20}+m_2\omega_{20}+\omega_{20}^2-\omega_{10}^2)}{\omega_{10}^2}\right],$$

$$b_-=-\frac{m_2(m_1+\omega_{10}+m_2+\omega_{20})\xi_1}{8M'M\omega_{10}\omega_{20}nep} - \frac{\xi_2}{8M'M^2\omega_{10}} - \frac{\xi_2 m_2}{8M^2M'\omega_{10}\omega_{20}}\left[\frac{(\omega_{10}+m_1+\omega_{20}+m_2)}{\omega_{20}} + \frac{(m_1\omega_{20}+m_2\omega_{20}+\omega_{20}^2-\omega_{10}^2)}{\omega_{10}^2}\right],$$

$$h_{+-}=-\frac{m_2(m_1+\omega_{10}+m_2+\omega_{20})\xi_1}{8M'M\omega_{10}\omega_{20}nep} - \frac{\xi_2}{8M'M^2\omega_{10}} - \frac{\xi_2 m_2}{8M^2M'\omega_{10}\omega_{20}}\left[\frac{(\omega_{10}+m_1+\omega_{20}+m_2)}{\omega_{20}} + \frac{(m_1\omega_{20}+m_2\omega_{20}+\omega_{20}^2-\omega_{10}^2)}{\omega_{10}^2}\right].$$

The dependences of α, β_{++}, and γ on the above form factors are

$$\alpha=\frac{M^2\vec{p}'^2}{2M'^2}(k^2+4M^2\vec{p}'^2h^2), \tag{A23}$$

$$\beta_{++}=-\frac{yM^4\vec{p}'^2}{2M'^2}h^2+\frac{M^2k^2}{24M'^2}\left(y+\frac{4\vec{p}'^2}{M'^2}\right)+\frac{2b_+^2}{3}\frac{M^4\vec{p}'^4}{M'^4} + \frac{M^2\vec{p}'^2kb_+}{3M'^2}\left[\frac{M^2}{M'^2}(1-y)-1\right], \tag{A24}$$

$$\beta_{+-}=\frac{M^2\vec{p}'^2}{2M'^2}h^2(M^2-M'^2)+\frac{k^2}{24}\left(1-\frac{M^2}{M'^2}-\frac{4M^2\vec{p}'^2}{M'^4}\right) + \frac{2b_+b_-}{3}\frac{M^4\vec{p}'^4}{M'^4}+\frac{M^2\vec{p}'^2kb_+}{6M'^2}\left[-\frac{M^2}{M'^2}(1-y)-3\right] - \frac{M^2\vec{p}'^2kb_-}{6M'^2}\left[-\frac{M^2}{M'^2}(1-y)+1\right], \tag{A25}$$

$$\beta_{-+}=\beta_{+-} \tag{A26}$$

$$\beta_{--}=-\frac{M^2\vec{p}'^2}{2M'^2}h^2[2(M^2+M'^2)-M^2y]+\frac{2b_-^2}{3}\frac{M^4\vec{p}'^4}{M'^4} + \frac{k^2}{24}\left(2+\frac{M^2}{M'^2}(2-y)+\frac{4M^2\vec{p}'^2}{M'^4}\right) - \frac{M^2\vec{p}'^2kb_-}{3M'^2}\left[\frac{M^2}{M'^2}(1-y)+3\right], \tag{A27}$$

$$\gamma=\frac{M^2\vec{p}'^2kh}{M'^2}. \tag{A28}$$

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