

Implications of CP asymmetry limits for $B \rightarrow K\pi$ and $B \rightarrow \pi\pi$

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Recent experimental limits for the direct CP asymmetries in $B^0 \rightarrow K^+ \pi^-$, $B^+ \rightarrow K^+ \pi^0$, $B^+ \rightarrow K^0 \pi^+$, and $B^0 \rightarrow \pi^+ \pi^-$, and for the indirect CP asymmetry in $B^0 \rightarrow \pi^+ \pi^-$, are combined with information on CP -averaged branching ratios to shed light on weak and strong phases. At present such bounds favor $\gamma \geq 60^\circ$ at the 1σ level. The prospects for further improvements are discussed.

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I. INTRODUCTION

The decays of B mesons to the charmless final states $\pi\pi$ and $K\pi$ are a rich source of information on the fundamental parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, but the extraction of this information from data requires the separation of weak interaction effects from strong-interaction quantities such as magnitudes of operator matrix elements and strong phases. A number of model-independent analyses of these systems [1–4] have shown that when one combines data on CP asymmetries with branching ratios of CP -averaged final states, one can separate the strong interaction effects from fundamental CKM parameters, obtaining useful information on both sets of quantities.

In the present paper we apply several of these analyses [1–3] to the decays $B \rightarrow K\pi$ and $B \rightarrow \pi\pi$, using new upper limits quoted by the CLEO [5], BaBar [6,7], and Belle [8] Collaborations for several CP -violating asymmetries in these decays, as well as updated CP -averaged branching ratios for these states. Comparison of the CP -averaged rate for $B^0 \rightarrow K^+ \pi^-$ with that for $B^+ \rightarrow K^0 \pi^+$, given a small strong phase difference, excludes $31^\circ \leq \gamma \leq 60^\circ$ for the weak phase $\gamma \equiv \text{Arg}(-V_{ub}^* V_{ud} / V_{cb}^* V_{cd})$, while comparison of $B^+ \rightarrow K^+ \pi^0$ with $B^+ \rightarrow K^0 \pi^+$ sets a 1σ lower limit of $\gamma > 50^\circ$. Present 1σ bounds on the asymmetry parameter $S_{\pi\pi}$ in $B^0 \rightarrow \pi^+ \pi^-$ exclude roughly half the CKM parameter space allowed by other measurements.

We review the flavor decomposition of amplitudes in Sec. II and the relevant data in Sec. III. The decays $B^+ \rightarrow K^0 \pi^+$, expected to be dominated by the penguin amplitude and thus to have no CP -violating asymmetry, are discussed in Sec. IV. We then analyze rates and CP asymmetries for $B^0 \rightarrow K^+ \pi^-$, normalizing amplitudes in terms of the pure-penguin processes $B^+ \rightarrow K^0 \pi^+$, in Sec. V. The process $B^+ \rightarrow K^+ \pi^0$ and its comparison with $B^+ \rightarrow K^0 \pi^+$ are treated in Sec. VI, while Sec. VII deals with $B^0 \rightarrow \pi^+ \pi^-$. Section VIII concludes.

II. FLAVOR DECOMPOSITION OF AMPLITUDES

In order to put the observed rates and asymmetries in theoretical context, we review the $SU(3)$ flavor-decomposition of $B \rightarrow PP$ amplitudes, where $P = \pi, K$ [9].

Defining $t = T + P_{EW}^c$, $p = P - \frac{1}{3}P_{EW}^c - \frac{1}{3}P_{EW}^E$, $c = C + P_{EW}$, $a = A + P_{EW}^E$, and $e + pa = E + PA + \frac{1}{3}P_{EW}^A$, where T is a color-favored tree amplitude, P is a penguin amplitude, C is a color-suppressed tree amplitude, A is an annihilation amplitude, E is an exchange amplitude, PA is a penguin annihilation amplitude, and P_{EW} , P_{EW}^c , P_{EW}^E , and P_{EW}^A are respectively color-favored, color-suppressed, (γ, Z)-exchange, and (γ, Z)-direct-channel electroweak penguin amplitudes [10], we have

$$A(B^0 \rightarrow \pi^+ \pi^-) = -(t + p + e + pa),$$

$$A(B^+ \rightarrow \pi^+ \pi^0) = -(t + c)/\sqrt{2},$$

$$A(B^0 \rightarrow \pi^0 \pi^0) = (p - c + e + pa)/\sqrt{2},$$

$$A(B^0 \rightarrow K^0 \bar{K}^0) = p + pa,$$

(1)

$$A(B^+ \rightarrow \bar{K}^0 K^+) = p + a, \quad A(B^0 \rightarrow K^+ K^-) = -(e + pa),$$

$$A(B^0 \rightarrow K^+ \pi^-) = -(t' + p'), \quad A(B^+ \rightarrow K^0 \pi^+) = p' + a',$$

$$A(B^+ \rightarrow K^+ \pi^0) = -(p' + a' + t' + c')/\sqrt{2},$$

$$A(B^0 \rightarrow K^0 \pi^0) = (p' - c')/\sqrt{2}.$$

Here unprimed amplitudes denote $\Delta S = 0$ processes, while primed amplitudes involve $|\Delta S| = 1$. The $B^0 \rightarrow K^+ K^-$ decay is expected to be highly suppressed since it involves only amplitudes associated with interactions with the spectator quarks. Measurement of rates for this process can place upper limits on such spectator amplitudes (equivalently, on effects of rescattering [11]).

The quark subprocesses describing the above amplitudes for \bar{b} quark decay are summarized in Table I. We use the unitarity of the CKM matrix, $V_{tb}^* V_{tq} = -V_{cb}^* V_{cq} - V_{ub}^* V_{uq}$, ($q = d, s$), to eliminate elements involving the top quark in favor of those involving the charm and up quarks in penguin amplitudes, and then incorporate up quark contributions into redefined tree contributions. In this convention tree amplitudes involve CKM factors $V_{ub}^* V_{uq}$, while penguin and

TABLE I. Weak phases of amplitudes in the flavor decomposition.

Amplitude	Quark subprocess	CKM element	Weak phase
T, C	$\bar{b} \rightarrow \bar{u}u\bar{d}$	$V_{ub}^* V_{ud}$	γ
$P, P_{EW}, P_{EW}^c, P_{EW}^E$	$\bar{b} \rightarrow \bar{d}$	$V_{cb}^* V_{cd}$	0
E	$\bar{b}d \rightarrow \bar{u}u$	$V_{ub}^* V_{ud}$	γ
A	$\bar{b}u \rightarrow \bar{d}u$	$V_{ub}^* V_{ud}$	γ
PA, P_{EW}^A	$\bar{b}d \rightarrow \text{vacuum}$	$V_{cb}^* V_{cd}$	0
T', C'	$\bar{b} \rightarrow \bar{u}u\bar{s}$	$V_{ub}^* V_{us}$	γ
$P', P_{EW}', P_{EW}^c, P_{EW}^E$	$\bar{b} \rightarrow \bar{s}$	$V_{cb}^* V_{cs}$	π
E'	$\bar{b}s \rightarrow \bar{u}u$	$V_{ub}^* V_{us}$	γ
A'	$\bar{b}u \rightarrow \bar{s}u$	$V_{ub}^* V_{us}$	γ
PA', P_{EW}^A	$\bar{b}s \rightarrow \text{vacuum}$	$V_{cb}^* V_{cs}$	π

electroweak penguin amplitudes contain factors $V_{cb}^* V_{cq}$. The weak phases of amplitudes for B decays occur in the last column of Table I.

A useful flavor SU(3) relation between tree and electroweak penguin amplitudes holds when keeping only dominant $(V-A)(V-A)$ electroweak operators in the effective weak Hamiltonian. Neglecting very small (a few percent) electroweak penguin contributions from operators having a different chiral structure, tree and electroweak penguin operators carrying a given SU(3) representation are proportional to each other, and one finds, in the SU(3) limit [2],

$$t' + c' = (T' + C')(-\delta_{EW}e^{-i\gamma}), \quad (2)$$

where δ_{EW} is given in terms of ratios of Wilson coefficients and CKM factors:

$$\delta_{EW} = -\frac{3}{2} \frac{c_9 + c_{10}}{c_1 + c_2} \frac{|V_{cb}^* V_{cs}|}{|V_{ub}^* V_{us}|} = 0.65 \pm 0.15. \quad (3)$$

The central value is obtained for $|V_{ub}/V_{cb}| = 0.09$.

TABLE II. Branching ratios in units of 10^{-6} for B^0 or B^+ decays to pairs of light pseudoscalar mesons. Averages over decay modes and their CP conjugates are implied.

Mode	CLEO [12]	Belle [13]	BaBar [14,15]	Average [16]
$\pi^+ \pi^-$	$4.3^{+1.6}_{-1.4} \pm 0.5$	$5.6^{+2.3}_{-2.0} \pm 0.4$	$4.1 \pm 1.0 \pm 0.7$	4.4 ± 0.9
$\pi^+ \pi^0$	5.4 ± 2.6	$7.8^{+3.8+0.8}_{-3.2-1.2}$	$5.1^{+2.0}_{-1.8} \pm 0.8$	5.6 ± 1.5
$K^+ \pi^-$	$17.2^{+2.5}_{-2.4} \pm 1.2$	$19.3^{+3.4+1.5}_{-3.2-0.6}$	$16.7 \pm 1.6 \pm 1.3$	17.4 ± 1.5
$K^0 \pi^+$	$18.2^{+4.6}_{-4.0} \pm 1.6$	$13.7^{+5.7+1.9}_{-4.8-1.8}$	$18.2^{+3.3}_{-3.0} \pm 2.0$	17.3 ± 2.4
$K^+ \pi^0$	$11.6^{+3.0+1.4}_{-2.7-1.3}$	$16.3^{+3.5+1.6}_{-3.3-1.8}$	$10.8^{+2.1}_{-1.9} \pm 1.0$	12.2 ± 1.7
$K^0 \pi^0$	$14.6^{+5.9+2.4}_{-5.1-3.3}$	$16.0^{+7.2+2.5}_{-5.9-2.7}$	$8.2^{+3.1}_{-2.2} \pm 1.2$	10.4 ± 2.6
$\pi^0 \pi^0$	$< 5.6(90\% \text{ C.L.})$			
$K^0 \bar{K}^0$	$< 6.1(90\% \text{ C.L.})$		$< 7.3(90\% \text{ C.L.})$	
$K^+ K^-$	$< 1.9(90\% \text{ C.L.})$	$< 2.7(90\% \text{ C.L.})$	$< 2.5(90\% \text{ C.L.})$	
$\bar{K}^0 K^+$	$< 5.1(90\% \text{ C.L.})$	$< 5.0(90\% \text{ C.L.})$	$< 2.4(90\% \text{ C.L.})$	

III. RATE AND ASYMMETRY DATA AND AVERAGES

A. Rates

The CLEO [12], Belle [13], and BaBar [14,15] CP -averaged branching ratios for several $B \rightarrow PP$ modes are summarized in Table II, along with averages from Ref. [16]. We first note several general properties of these branching ratios.

(1) Dominance of $B^0 \rightarrow \pi^+ \pi^-$ and $B^+ \rightarrow \pi^+ \pi^0$ by the color-favored tree amplitude would imply the relation

$$\frac{2\mathcal{B}(B^+ \rightarrow \pi^+ \pi^0)}{r_\tau \mathcal{B}(B^0 \rightarrow \pi^+ \pi^-)} = 1, \quad (4)$$

where $r_\tau \equiv \tau_{B^+}/\tau_{B^0} = 1.068 \pm 0.016$ is the ratio of B^+ and B^0 lifetimes [17]. The observed ratio corresponding to the left-hand side of (4) is 2.4 ± 0.8 , or 1.7σ above 1. The color-suppressed tree amplitude c with $\text{Re}(c/t) \approx 0.2$ [9,18] adds about 44% to the predicted $B^+ \rightarrow \pi^+ \pi^0$ branching ratio, converting the right-hand side of Eq. (4) to 1.44 and reducing the discrepancy to 1.2σ .

(2) Dominance of the $B \rightarrow K\pi$ decays by penguin amplitudes would imply

$$\begin{aligned} \mathcal{B}(B^0 \rightarrow K^+ \pi^-) &= \mathcal{B}(B^+ \rightarrow K^0 \pi^+)/r_\tau \\ &= 2\mathcal{B}(B^+ \rightarrow K^+ \pi^0)/r_\tau \\ &= 2\mathcal{B}(B^0 \rightarrow K^0 \pi^0), \end{aligned} \quad (5)$$

while these quantities are in the ratio

$$1.08 \pm 0.18 : 1 \text{ (def.)} : 1.41 \pm 0.28 : 1.29 \pm 0.37 \quad (6)$$

(normalizing to the pure-penguin amplitude for $B^+ \rightarrow K^0 \pi^+$). Thus the strongest evidence for amplitudes other than the penguin appears at the 1.46σ level in the ratio

$$R_c \equiv \frac{2\mathcal{B}(B^+ \rightarrow K^+ \pi^0)}{\mathcal{B}(B^+ \rightarrow K^0 \pi^+)} = 1.41 \pm 0.28. \quad (7)$$

(3) To first order in subleading amplitudes, one has the sum rule [19–21]

TABLE III. Asymmetries \mathcal{A}_{CP} for $B \rightarrow PP$ decays.

Mode	CLEO [5]	BaBar [6,7]	Belle [8]	Average
$S_{\pi\pi}$		$0.03^{+0.53}_{-0.56} \pm 0.11$		0.03 ± 0.56
$C_{\pi\pi}$		$-0.25^{+0.45}_{-0.47} \pm 0.14$		-0.25 ± 0.48
$K^+ \pi^-$	-0.04 ± 0.16	$-0.07 \pm 0.08 \pm 0.02$	$0.044^{+0.186+0.018}_{-0.167-0.021}$	-0.048 ± 0.068
$K^+ \pi^0$	-0.29 ± 0.23	$0.00 \pm 0.18 \pm 0.04$	$-0.059^{+0.222+0.055}_{-0.196-0.017}$	-0.096 ± 0.119
$K^0 \pi^+$	0.18 ± 0.24	$-0.21 \pm 0.18 \pm 0.03$	$0.098^{+0.430+0.020}_{-0.343-0.063}$	-0.047 ± 0.136

$$2\mathcal{B}(B^+ \rightarrow K^+ \pi^0)/r_\tau + 2\mathcal{B}(B^0 \rightarrow K^0 \pi^0) \\ = \mathcal{B}(B^+ \rightarrow K^0 \pi^+)/r_\tau + \mathcal{B}(B^0 \rightarrow K^+ \pi^-). \quad (8)$$

The left- and right-hand sides of this relation are $(43.6 \pm 6.1) \times 10^{-6}$ and $(33.6 \pm 2.7) \times 10^{-6}$, respectively. These relations are fairly general, so any violation of them would most likely signal systematic experimental errors.

B. Asymmetries

In Table III we summarize data on CP asymmetries in $B \rightarrow PP$, defined by

$$\mathcal{A}_{CP} \equiv \frac{\Gamma(\bar{B} \rightarrow \bar{f}) - \Gamma(B \rightarrow f)}{\Gamma(\bar{B} \rightarrow \bar{f}) + \Gamma(B \rightarrow f)}, \quad (9)$$

while coefficients of $\sin \Delta m_d t$ and $\cos \Delta m_d t$ measured in time-dependent CP asymmetries of $\pi^+ \pi^-$ states produced in asymmetric $e^+ e^-$ collisions at the $Y(4S)$ are [22]

$$S_{\pi\pi} \equiv \frac{2 \operatorname{Im}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2}, \quad C_{\pi\pi} \equiv \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2}, \quad (10)$$

where

$$\lambda_{\pi\pi} \equiv e^{-2i\beta} \frac{A(\bar{B}^0 \rightarrow \pi^+ \pi^-)}{A(B^0 \rightarrow \pi^+ \pi^-)}. \quad (11)$$

The smallness of these asymmetries will lead to useful constraints on CKM parameters, though reduction of statistical errors will be quite helpful. In some cases, however, the reduction of statistical errors on *ratios of branching ratios* described in the previous section will actually be of greater use.

IV. $B^+ \rightarrow K^0 \pi^+$

The decay $B^+ \rightarrow K^0 \pi^+$ is expected to be dominated by the penguin amplitude, with a small contribution from the quark subprocess $\bar{b}u \rightarrow \bar{s}u$ proportional to the ratio $f_B/m_B \simeq 1/25$. An equivalent contribution is generated by rescattering, e.g., from such final states as $K^+ \pi^0$. Since the weak phase of the annihilation and penguin amplitudes are different, the annihilation amplitude can lead to a small CP asymmetry in the rate for $B^+ \rightarrow K^0 \pi^+$ vs its CP -conjugate decay. There is no evidence for such an asymmetry at present, but the experimental upper bounds are no stronger than for processes in which the penguin amplitude is expected to be

accompanied by tree amplitudes, such as $B^0 \rightarrow K^+ \pi^-$ and $B^+ \rightarrow K^+ \pi^0$. Much larger CP asymmetries could occur in those processes if strong phases were sufficiently large.

A useful way to estimate the effect of the annihilation amplitude in $B^+ \rightarrow K^0 \pi^+$ [23] is to use the U-spin [24,25] transformation $s \leftrightarrow d$ to relate it to $B^+ \rightarrow \bar{K}^0 K^+$. Under this substitution the penguin amplitude (proportional to $V_{cb}^* V_{cd}$) is reduced by a factor of $\lambda = |V_{cd}/V_{cs}|$, while the annihilation amplitude is *increased* by a factor $\lambda^{-1} = |V_{ud}/V_{us}|$, where $\lambda \simeq 0.22$. Thus, not only should the CP asymmetry in $B^+ \rightarrow \bar{K}^0 K^+$ be substantially larger than that in $B^+ \rightarrow K^0 \pi^+$, but if the annihilation amplitude is large enough it could lead to an enhancement of the rate for $B^+ \rightarrow \bar{K}^0 K^+$ over that expected if the penguin amplitude P were dominant, which corresponds to a branching ratio of about $\mathcal{B}(B^+ \rightarrow \bar{K}^0 K^+) \simeq |V_{cd}/V_{cs}|^2 \mathcal{B}(B^+ \rightarrow K^0 \pi^+) = 8 \times 10^{-7}$ [26,27].

Evidence for rescattering [11] would also be forthcoming from the process $B^0 \rightarrow K^+ K^-$, for which the contributions of the E and PA amplitudes are expected to lead to a branching ratio below 10^{-7} . Present experimental limits are an order of magnitude above this value.

V. $B^0 \rightarrow K^+ \pi^-$

Fleischer and Mannel [28] pointed out that a useful ratio giving information on the weak phase γ is

$$R \equiv \frac{r_\pi [\mathcal{B}(\bar{B}^0 \rightarrow K^- \pi^+) + \mathcal{B}(B^0 \rightarrow K^+ \pi^-)]}{\mathcal{B}(B^- \rightarrow \bar{K}^0 \pi^-) + \mathcal{B}(B^+ \rightarrow K^0 \pi^+)}. \quad (12)$$

Within the assumption of a dominant penguin amplitude and a subdominant tree amplitude, one finds

$$R = 1 - 2r \cos \gamma \cos \delta_0 + r^2, \quad (13)$$

where $r \equiv |T'/P'|$ is the ratio of tree to penguin amplitudes for strangeness-changing B decays to charmless final states, and $\delta_0 \equiv \delta_{T'} - \delta_{P'}$ is the strong final-state phase difference between tree and penguin amplitudes. Independently of r and δ_0 it can then be shown [28] that $R \geq \sin^2 \gamma$, so that a value of R below 1 could place useful bounds on γ .

The present experimental data summarized in Table II indicate $R = 1.08 \pm 0.18$, so that no useful bound arises from the Fleischer-Mannel procedure. However, it was shown in Ref. [1] that if one combined data on R with the CP pseudo-asymmetry

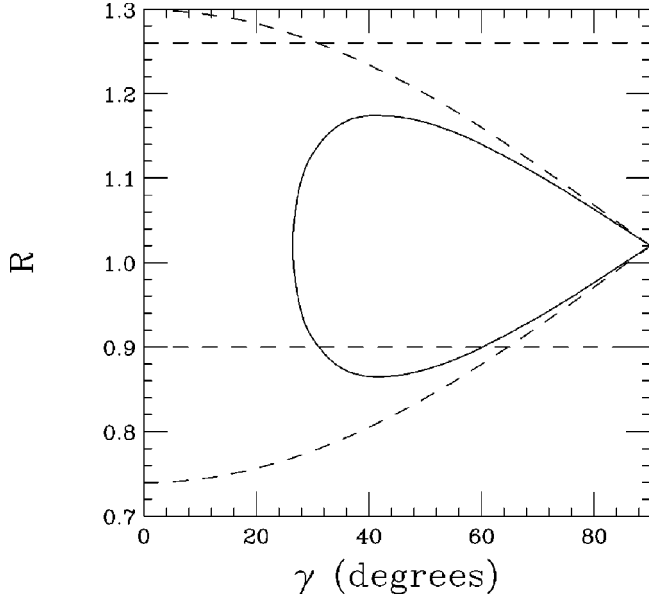


FIG. 1. Behavior of R for $r=0.14$ and $A_0=0$ (dashed curves) or $|A_0|=0.125$ (solid curve) as a function of the weak phase γ . Horizontal dashed lines denote $\pm 1\sigma$ experimental limits on R . The upper branches of the curves correspond to the case $\cos \gamma \cos \delta_0 < 0$, while the lower branches correspond to $\cos \gamma \cos \delta_0 > 0$.

$$\begin{aligned}
 A_0 &\equiv \frac{\Gamma(\bar{B}^0 \rightarrow K^- \pi^+) - \Gamma(B^0 \rightarrow K^+ \pi^-)}{\Gamma(B^- \rightarrow \bar{K}^0 \pi^-) + \Gamma(B^+ \rightarrow K^0 \pi^+)} \\
 &= \mathcal{A}_{CP}(B^0 \rightarrow K^+ \pi^-) R \\
 &= -2r \sin \gamma \sin \delta_0,
 \end{aligned} \tag{14}$$

one could eliminate the strong phase difference between tree and penguin amplitudes and obtain useful information on the weak phase γ . The result is

$$R = 1 + r^2 \pm \sqrt{4r^2 \cos^2 \gamma - A_0^2 \cot^2 \gamma}. \tag{15}$$

Plots of R as a function of γ for various values of r and A_0 were given in Ref. [1]. Note that this function is invariant under the replacement $\gamma \rightarrow \pi - \gamma$, so it only need be plotted for $0 \leq \gamma \leq 90^\circ$. However, the expression (13) indicates that the upper branches of the curves correspond to $\cos \gamma \cos \delta_0 < 0$, while the lower branches correspond to $\cos \gamma \cos \delta_0 > 0$.

Using the experimental asymmetries summarized in Table III, one finds $A_0 = -0.052 \pm 0.073$. In Ref. [1] we estimated $r = 0.16 \pm 0.06$. Using the most recent experimental data for $B^+ \rightarrow K^0 \pi^+$ to estimate $|P'|$ and factorization in $B \rightarrow \pi l \nu$ [29] and flavor SU(3) [9] to estimate $|T'|$, an updated result is $r = 0.184 \pm 0.044$.

The most conservative bounds on γ are obtained using the smallest value of r and the largest value of $|A_0|$. A plot of R for $r=0.14$ (the 1σ lower bound) and both $A_0=0$ and $|A_0|=0.125$ (the 1σ upper bound) is shown in Fig. 1. With present experimental errors, no useful bound on γ emerges from the consideration of R unless additional assumptions are made. Reduction of errors on R by roughly a factor of two could have a considerable impact even given present

errors on A_0 and r . Since the curves for $A_0=0$ and $|A_0|=0.125$ are fairly close to one another for a considerable range of γ , improvement of bounds on A_0 is less likely to sharpen the bounds on γ unless that angle differs considerably from 90° .

Theoretical estimates [18] of small final-state phases imply $\cos \delta_0 > 0$, so that with $\gamma \leq 90^\circ$ one should have destructive tree-penguin interference in $B^0 \rightarrow K^+ \pi^-$ and thus should be on the lower branch of the curves in Fig. 1. The 1σ lower bound on R then would exclude $31^\circ \leq \gamma \leq 60^\circ$.

The expressions for R and A_0 are invariant under the interchange of γ and δ_0 , so that Fig. 1 can also be used in principle for bounds on δ_0 . At present, no useful bounds emerge. However, writing $\sin \delta_0 = -A_0 / (2r \sin \gamma)$ and using the 1σ range $-0.125 \leq A_0 \leq 0.021$ and the lower bounds $r \geq 0.14$ from the above discussion and $\gamma \geq 32^\circ$ from a fit to CKM parameters [30], one finds $-8^\circ \leq \delta_0 \leq 57^\circ$ up to a discrete ambiguity which also permits a solution $\delta_0 \rightarrow \pi - \delta_0$.

VI. $B^+ \rightarrow K^+ \pi^0$

The ratio

$$R_c \equiv \frac{2[\mathcal{B}(B^- \rightarrow K^- \pi^0) + \mathcal{B}(B^+ \rightarrow K^+ \pi^0)]}{\mathcal{B}(B^- \rightarrow \bar{K}^0 \pi^-) + \mathcal{B}(B^+ \rightarrow K^0 \pi^+)} \tag{16}$$

also contains useful information on the weak phase γ . Initially it was proposed to use this ratio in an amplitude triangle construction [31] in which the amplitude $t' + c' = -A(B^+ \rightarrow K^0 \pi^+) - \sqrt{2}A(B^+ \rightarrow K^+ \pi^0)$ was evaluated using flavor SU(3) from the corresponding amplitude $t + c = -\sqrt{2}A(B^+ \rightarrow \pi^+ \pi^0)$. However, this procedure neglected important electroweak penguin (EWP) contributions [32]. It was then shown that these could be taken into account [2] through the SU(3) relation (2). Neglecting a' contributions in decay amplitudes, and writing

$$-\sqrt{2}A(B^+ \rightarrow K^+ \pi^0) = p' + (T' + C')(1 - \delta_{EW} e^{-i\gamma}), \tag{17}$$

one finds

$$\begin{aligned}
 R_c &= 1 - 2r_c \cos \delta_c (\cos \gamma - \delta_{EW}) \\
 &\quad + r_c^2 (1 - 2\delta_{EW} \cos \gamma + \delta_{EW}^2),
 \end{aligned} \tag{18}$$

where $r_c \equiv |T' + C'| / |p'|$, $\delta_c \equiv \delta_{T'+C'} - \delta_{p'}$, and δ_{EW} is given in Eq. (3). Note that the latter parameter involves a sizable uncertainty from $|V_{ub}/V_{cb}|$. In order to demonstrate possible constraints on weak and strong phases, we will explore the effect of $\pm 1\sigma$ deviations from the central value of $\delta_{EW} = 0.65 \pm 0.15$.

The CP -violating asymmetry in $B^+ \rightarrow K^+ \pi^0$ decays then provides a constraint on the relative strong phase δ_c . We define a pseudo-asymmetry

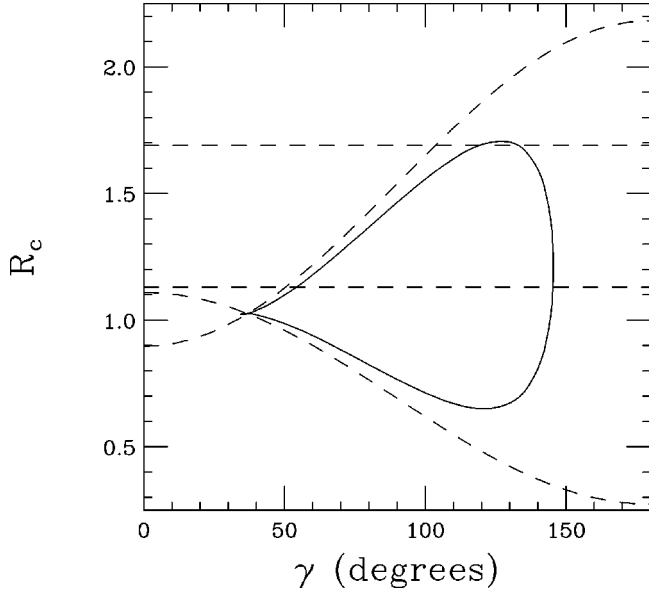


FIG. 2. Behavior of R_c for $r_c=0.265$ (1σ upper limit) and $A_c=0$ (dashed curves) or $|A_c|=0.30$ (solid curve) as a function of the weak phase γ . Horizontal dashed lines denote $\pm 1\sigma$ experimental limits on R_c . Upper branches of curves correspond to $\cos \delta_c(\cos \gamma - \delta_{EW}) < 0$, while lower branches correspond to $\cos \delta_c(\cos \gamma - \delta_{EW}) > 0$. Here we have taken $\delta_{EW}=0.80$ [the 1σ upper limit in Eq. (3)], which leads to the most conservative bound on γ .

$$\begin{aligned} A_c &\equiv \frac{2[\mathcal{B}(B^- \rightarrow K^- \pi^0) - \mathcal{B}(B^+ \rightarrow K^+ \pi^0)]}{\mathcal{B}(B^- \rightarrow \bar{K}^0 \pi^-) + \mathcal{B}(B^+ \rightarrow K^0 \pi^+)} \\ &= R_c \mathcal{A}_{CP}(B^+ \rightarrow K^+ \pi^0) \\ &= -2r_c \sin \delta_c \sin \gamma, \end{aligned} \quad (19)$$

and, using the experimental averages in Tables II and III, we find $A_c = -0.13 \pm 0.17$.

Eliminating δ_c , we can plot R_c as a function of γ for various values of δ_{EW} , r_c and A_c , to see if any constraints on γ emerge when taking a 1σ lower limit on R_c , $R_c \geq 1.13$. The ratio r_c , obtained from [31]

$$r_c = \sqrt{2} \frac{V_{us} f_K}{V_{ud} f_\pi} \left[\frac{\mathcal{B}(B^- \rightarrow \pi^- \pi^0) + \mathcal{B}(B^+ \rightarrow \pi^+ \pi^0)}{\mathcal{B}(B^- \rightarrow \bar{K}^0 \pi^-) + \mathcal{B}(B^+ \rightarrow K^0 \pi^+)} \right]^{1/2}, \quad (20)$$

was estimated in Ref. [2] to be $r_c \equiv \epsilon_{3/2} = 0.24 \pm 0.06$. We can update this estimate using the new branching ratios quoted in Table II, finding $r_c = 0.230 \pm 0.035$. The resulting plot is shown in Fig. 2 for the $+1\sigma$ values of r_c and δ_{EW} (which lead to the weakest lower bound on γ), both for $A_c=0$ and for the 1σ upper limit $A_c=0.30$. The weakest 1σ bound on γ in this case, as opposed to the case of $B^0 \rightarrow K^+ \pi^-$, occurs when $A_c=0$, and is $\gamma \geq 50^\circ$. As a result of the electroweak penguin term, the value of R_c is not symmetric under the replacement $\gamma \rightarrow \pi - \gamma$, in contrast to the case of R for $B^0 \rightarrow K^+ \pi^-$. In Table IV we show the minimum values of γ

TABLE IV. Minimum values of γ (in degrees) for $R_c \geq 1.13$, given central and $\pm 1\sigma$ values of r_c and δ_{EW} . First figure denotes value with $A_c=0$ while second figure denotes value with $|A_c|=0.30$.

r_c :	0.195	0.230	0.265
δ_{EW} :			
0.50	75/82	71/74	68/70
0.65	66/74	62/67	60/62
0.80	57/68	53/59	50/54

obtained on the basis of the 1σ inequality $R_c \geq 1.13$ for $r_c = 0.230 \pm 0.035$ and $\delta_{EW} = 0.65 \pm 0.15$, both for $A_c=0$ and for $A_c=0.3$.

As in the case of $B^0 \rightarrow K^+ \pi^-$, there is little difference on the bounds one obtains for zero CP asymmetry and for the maximum allowed value. The greatest leverage on bounds would be provided by reducing the experimental error on R_c , with some additional help associated with reduction of the errors on r_c and δ_{EW} . The limits of Table IV correspond to the branches of the curves that would be chosen if $\cos \delta > 0$, as expected in some theoretical treatments [18].

One can place a one-sided 1σ limit on the strong phase δ_c using the present range $-0.30 \leq A_c \leq 0.04$. With

$$\sin \delta_c = -A_c / (2r_c \sin \gamma), \quad (21)$$

$\gamma \geq 32^\circ$, and $r_c \geq 0.195$ one has $-0.19 \leq \sin \delta_c \leq 1.44$, so $\delta_c \geq -11^\circ$. The upper limit on $|A_c|$ [equivalently, on $|\mathcal{A}_{CP}(B^+ \rightarrow K^+ \pi^0)|$] must be reduced to about 2/3 of its present value if a two-sided constraint on δ_c is to be obtained.

VII. $B^0 \rightarrow \pi^+ \pi^-$

The implications of the BaBar [7] limits on $S_{\pi\pi}$ and $C_{\pi\pi}$ quoted in Table III have been partially explored in Ref. [29]. Here we review these limits, discuss their implications for CKM parameters, and discuss prospects for their improvement.

As mentioned in Sec. III, the present experimental ratio (4) of $B^+ \rightarrow \pi^+ \pi^0$ and $B^0 \rightarrow \pi^+ \pi^-$ branching ratios is somewhat larger than that expected from tree-dominance alone, even accounting for a color-suppressed contribution to the former process. For this reason, as well as for the purpose of estimating the ‘‘penguin pollution’’ correction to the time-dependent CP asymmetry in $B^0 \rightarrow \pi^+ \pi^-$, it is useful to estimate the ratio $|P/T|$ of penguin to tree amplitudes in $\Delta S = 0$ B decays. Using this estimate it is then possible to place limits on the weak phase α even given the crude limits on $S_{\pi\pi}$ and $C_{\pi\pi}$ noted in Table III.

Many previous attempts have been made to estimate $|P/T|$ in a model-independent way, including an isospin analysis requiring the measurement of $B^+ \rightarrow \pi^+ \pi^0$, $B^0 \rightarrow \pi^0 \pi^0$, and corresponding charge-conjugate decays [33], methods which use only part of the above information [34–36], and numerous applications of flavor $SU(3)$ [9,37,38]. There have been hints, based on earlier data, that the penguin

amplitude was interfering destructively with the tree in $B^0 \rightarrow \pi^+ \pi^-$ [39].

The method of Ref. [29] is capable in principle of giving a good value of $|T|$ based on naive factorization and measurement of the spectrum of $B \rightarrow \pi l \nu$ near $q^2=0$, where q^2 is the squared effective mass of the $l \nu$ system. Present experimental measurements and some theoretical estimates of form factor shapes based on lattice gauge theory lead to an estimate $|T|=2.7 \pm 0.6$, where all amplitudes are quoted as square roots of B^0 branching ratios multiplied by 10^3 . This is the same value obtained [40] from $B^+ \rightarrow \pi^+ \pi^0$ with additional assumptions about the color-suppressed amplitude.

The penguin amplitude can be estimated from $B^+ \rightarrow K^0 \pi^+$. The average of the branching ratios for that process in Table II is

$$\mathcal{B}(B^+ \rightarrow K^0 \pi^+) = (17.2 \pm 2.4) \times 10^{-6}, \quad (22)$$

leading to $|P'|^2 = (17.2 \pm 2.4)/r_\tau$, $|P'| = 4.02 \pm 0.28$.

We now estimate the strangeness-preserving $\bar{b} \rightarrow \bar{d}$ amplitude $|P|$ which is proportional to the CKM factor $V_{cd}V_{cb}^*$ in our convention. We find

$$|P/P'| = \left| \frac{V_{cd}}{V_{cs}} \right| = 0.22, \quad |P| \approx 0.91 \pm 0.06. \quad (23)$$

Assuming factorization of penguin amplitudes [18], this estimate is corrected by an SU(3) breaking factor of f_π/f_K and becomes $|P| \approx 0.74 \pm 0.05$.

With the present method of estimating errors on $|P|$ and $|T|$, we then find $|P/T| = 0.34 \pm 0.08$ without introducing SU(3) breaking in P/P' , or $|P/T| = 0.276 \pm 0.064$ when SU(3) breaking in P/P' is introduced through f_π/f_K . The latter number is to be compared with a value of 0.285 ± 0.076 obtained by [18] on the basis of a theoretical calculation which includes small annihilation corrections. A value of 0.26 ± 0.09 was obtained [29] when defining P and P' as the amplitudes containing V_{td} and V_{ts} , respectively, without introducing SU(3) breaking in the ratio of these amplitudes.

The decay amplitudes to $\pi^+ \pi^-$ for B^0 and \bar{B}^0 are

$$\begin{aligned} A(B^0 \rightarrow \pi^+ \pi^-) &= -(|T|e^{i\delta_T}e^{i\gamma} + |P|e^{i\delta_P}), \\ A(\bar{B}^0 \rightarrow \pi^+ \pi^-) &= -(|T|e^{i\delta_T}e^{-i\gamma} + |P|e^{i\delta_P}), \end{aligned} \quad (24)$$

where δ_T and δ_P are strong phases of the tree and penguin amplitudes, and $\delta \equiv \delta_P - \delta_T$. The CP -averaged branching ratio in Table II then implies

$$|T|^2 + |P|^2 + 2|TP|\cos\gamma\cos\delta = 4.4 \pm 0.9, \quad (25)$$

which suggests but does not prove, given our errors on $|T|$ and $|P|$, that the tree and penguin amplitudes are interfering destructively with one another in $B^0 \rightarrow \pi^+ \pi^-$. For $\cos\delta > 0$ as favored theoretically [18], this would require $\cos\gamma < 0$, which is not favored by CKM fits [16].

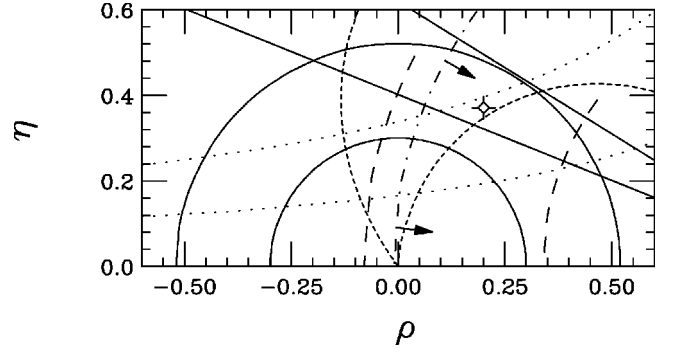


FIG. 3. Constraints on parameters of the CKM matrix. Solid circles denote limits on $|V_{ub}/V_{cb}| = 0.090 \pm 0.025$ from charmless b decays. Dashed arcs denote limits from B^0 - \bar{B}^0 mixing. Dot-dashed arc denotes limit from B_s - \bar{B}_s mixing. Dotted hyperbolae are associated with limits on CP -violating K^0 - \bar{K}^0 mixing (the parameter ϵ). Limits of $\pm 1\sigma$ from CP asymmetries in $B^0 \rightarrow J/\psi K_S$ leading to $\sin(2\beta) = 0.79 \pm 0.10$ are shown by the solid rays. The small dashed lines represent the constraint due to $S_{\pi\pi}$, with $0.21 \leq |P/T| \leq 0.34$. The plotted point lies in the middle of the allowed region.

The BaBar Collaboration [7] has recently reported the first results for the CP -violating asymmetries (10) in $B^0 \rightarrow \pi^+ \pi^-$ decays. Our expressions for the decay amplitudes imply

$$\lambda_{\pi\pi} = e^{2i\alpha} \left(\frac{1 + |P/T|e^{i\delta}e^{i\gamma}}{1 + |P/T|e^{i\delta}e^{-i\gamma}} \right). \quad (26)$$

In the absence of the penguin amplitude we would have $S_{\pi\pi} = \sin(2\alpha)$. If $|P/T| \neq 0$ but δ is small [18], we have $S_{\pi\pi} \approx \sin(2\alpha_{\text{eff}})$, where $\alpha_{\text{eff}} = \alpha + \Delta\alpha$, with

$$\Delta\alpha = \tan^{-1} \left[\frac{|P/T|\sin\gamma}{1 + |P/T|\cos\gamma} \right]. \quad (27)$$

Using

$$\tan\alpha = \frac{\eta}{\eta^2 - \rho(1-\rho)}, \quad \tan\Delta\alpha = \frac{\eta|P/T|}{\sqrt{\rho^2 + \eta^2 + \rho|P/T|}}, \quad (28)$$

we plot in Fig. 3 the $\pm 1\sigma$ contours of $-0.53 \leq S_{\pi\pi} \leq 0.59$, along with other CKM constraints taken from Ref. [16]. The 1σ $S_{\pi\pi}$ bounds exclude about half of the (ρ, η) parameter space allowed by all other constraints. Similar constraints under slightly different technical assumptions were obtained in Ref. [29].

The quantity $C_{\pi\pi}$ is also consistent at present with zero. Its observed range is not yet tightly enough constrained to provide much information, but reduction in errors will eventually be useful mainly in constraining the strong phase difference δ . For one such example, see Ref. [29].

VIII. CONCLUSIONS

While a CP -violating indirect asymmetry (associated with B^0 - \bar{B}^0 mixing) has been observed in the decays B^0

$\rightarrow J/\psi K_S$, no direct asymmetries have yet been observed in $B \rightarrow K\pi$ decays, and no asymmetries of any sort have been seen in $B \rightarrow \pi^+ \pi^-$. Nonetheless, the present upper limits on $K\pi$ and $\pi\pi$ asymmetries, crude as they are, already are beginning to provide useful information on CKM phases. As one example, the deviation of the ratio $2\mathcal{B}(B^+ \rightarrow K^+ \pi^0)/\mathcal{B}(B^+ \rightarrow K^0 \pi^+)$ from 1 is able at the 1σ level to provide a lower bound $\gamma \geq 50^\circ$ independently of the CP asymmetry in $B^+ \rightarrow K^+ \pi^0$. The proximity of the ratio $(\tau_+/\tau_0)\mathcal{B}(B^0 \rightarrow K^+ \pi^-)/\mathcal{B}(B^+ \rightarrow K^0 \pi^+)$ to unity, when combined with the expectation that the final-state strong phase is small in the $K^+ \pi^-$ system, allows one to exclude a range $31^\circ \leq \gamma \leq 60^\circ$ at the 1σ level. Finally, the $\pm 1\sigma$ bounds on $S_{\pi\pi}$ allow one to exclude (at the 1σ level) roughly half of the parameter space in the (ρ, η) plane allowed by other observables. The 1σ bound $\gamma \geq 60^\circ$ is the strongest constraint of these.

Uncertainties in theoretical parameters, including the ratios of tree to penguin amplitudes in $B \rightarrow K\pi$ and $B \rightarrow \pi\pi$, should be reduced in the future with a larger amount of data.

More severe constraints are expected for small rescattering and color-suppressed electroweak amplitudes. With the increased data samples expected to be available from BaBar and Belle, one can look forward to greatly improved limits on CKM parameters from analyses such as ours even if no CP asymmetries are observed in $B \rightarrow K\pi$ and $B \rightarrow \pi\pi$ decays.

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