Localization of bulk form fields on dilatonic domain walls

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We study the localization properties of bulk form potentials on dilatonic domain walls. We find that bulk form potentials of any ranks can be localized as form potentials of the same rank or one lower rank, for any values of the dilaton coupling parameter. For large enough values of the dilaton coupling parameter, bulk form potentials of any ranks can be localized as form potentials of both the same rank and one lower rank.

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The Randall-Sundrum (RS) scenario [1,2] provides an alternative compactification method, where our four-dimensional spacetime is realized as a three brane on which the bulk graviton can be localized even with noncompact extra spatial dimensions due to warped spacetime. Localization of various bulk fields on the brane has been studied. In particular, it was shown that, whereas the bulk scalar can be localized on the RS domain wall, bulk photon and form potentials cannot be localized [3,4]. Later, it was found out [5] that the bulk three-form potential, which is Hodge dual to a bulk scalar in five-dimensional bulk spacetime, can also be localized as a two-form potential in one lower dimension with a choice of a modified Kaluza-Klein (KK) zero mode ansatz. (cf. see also Ref. [6].) It was proposed in Ref. [7], whose work was extended to the supersymmetric case in Ref. [8], that a $U(1)$ field on the brane is originated rather from two bulk two-form potentials. Alternative methods for localizing the bulk $U(1)$ field on the brane through topological Higgs mechanism [9] and by adding a potential of the bulk $U(1)$ field to the brane action [10] were also proposed.

We showed [11–13] that dilatonic domain walls can localize bulk gravity, provided that the tension of the wall is positive. Bulk fields with various spins, including the bulk $U(1)$ field, were shown [12] to be localized on such dilatonic domain walls, in the sense that the KK zero modes of the bulk fields are normalizable. It is the purpose of this paper to study localization of bulk form potentials of various ranks, which were not considered in our previous work. Unlike the case of a nondilatonic RS domain wall, any bulk p-form potentials can be localized on the dilatonic domain wall both as p-form potentials and as $(p-1)$-form potentials in one lower dimension, provided the dilaton coupling parameter a is large enough. Furthermore, for any values of a, any bulk p-form potential can be localized on the dilatonic wall as a p-form potential or as a $(p-1)$-form potential in one lower dimension.

We begin by summarizing dilatonic domain wall solution and localization of bulk graviton, studied in Refs. [11–13]. The total action for the dilatonic domain wall solution is the sum of the D-dimensional action in the bulk of the domain wall:

$$S_{\text{bulk}}=\frac{1}{2\kappa_D^2}\int d^Dx\sqrt{-G}\left[\mathcal{R}-\frac{4}{D-2}\partial_M\phi\partial^M\phi-e^{-2a\phi}\Lambda\right], \tag{1}$$

and the $(D-1)$-dimensional action on the domain wall (DW) world volume:

$$S_{\text{DW}}=-\int d^{D-1}x\sqrt{-\gamma}\sigma_{DW}e^{-a\phi}, \tag{2}$$

where γ is the determinant of the induced metric $\gamma_{\mu\nu}=\partial_\mu X^M\partial_\nu X^N G_{MN}$ on the domain wall world volume, σ_{DW} is the energy density (or tension) of the domain wall, $M,N=0,1,\dots,D-1$ and $\mu,\nu=0,1,\dots,D-2$. In this paper, we consider the case in which the gravity can be localized on the DW. For such a case, the DW solution is $\mathbf{Z}_2$ symmetric and has naked singularities on both sides of the wall [11]:

$$G_{MN}dx^Mdx^N=\mathcal{W}[-dt^2+dx_1^2+\cdots+ds_{D-2}^2]+dy^2,$$

$$\phi=a^{-1}\ln(1-K|y|),\quad \mathcal{W}=(1-K|y|)^{8/(D-2)^2a^2},$$

$$K=\frac{(D-2)a^2}{2}\sqrt{\frac{\Lambda}{2\Delta}},\quad \Delta\equiv\frac{(D-2)a^2}{2}-2\frac{D-1}{D-2}, \tag{3}$$

the DW tension, which can be fixed by the boundary condition at $y=0$, has the fine-tuned positive value

$$\sigma_{\text{DW}}=\frac{1}{\kappa_D^2}\frac{8K}{(D-2)a^2}=\frac{4}{\kappa_D^2}\sqrt{\frac{\Lambda}{2\Delta}}, \tag{4}$$

and the effective $(D-1)$-dimensional gravitational constant has the nonzero value $(\Delta+4)\sqrt{\Lambda/2\Delta}\kappa_D^2/2$. Note, from the expression for K in Eq. (3) we see that such solution exists only for $\Lambda<0$ [$\Lambda>0$] when $\Delta<0$ [$\Delta>0$]. We have shown [11–13] that the normalizable KK zero mode for the bulk graviton exists and therefore the gravity can be localized on such a dilatonic domain wall, for any values of a.

We consider p-form potentials in the bulk of such a dilatonic domain wall. The action for a massless p-form potential $A_{M_1\dots M_p}$ with the field strength $F_{M_1\dots M_{p+1}}=(p+1)\partial_{[M_1}A_{M_2\dots M_{p+1}]}$ in the D-dimensional bulk spacetime is given by

$$S_p=-\frac{1}{2\cdot(p+1)!}\int d^Dx\sqrt{-G}G^{M_1N_1}\cdots \times G^{M_{p+1}N_{p+1}}F_{M_1\dots M_{p+1}}F_{N_1\dots N_{p+1}}. \tag{5}$$

If we choose the following KK zero mode ansatz for the p-form potential

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$$A_{\mu_1\dots\mu_p}(x^\mu,y)=a_{\mu_1\dots\mu_p}(x^\mu), \tag{6}$$

then the action (5) becomes[1]

$$S_p=-\frac{1}{2\cdot(p+1)!}\int_{-1/K}^{1/K}dy(1-K|y|)^{4(D-2p-3)/(D-2)^2a^2}\times\int d^{D-1}x\sqrt{-g}g^{\mu_1\nu_1}\cdots g^{\mu_{p+1}\nu_{p+1}}\times f_{\mu_1\dots\mu_{p+1}}f_{\nu_1\dots\nu_{p+1}}, \tag{7}$$

where $g_{\mu\nu}(x^\rho)$ is the KK zero mode for the bulk metric $G_{MN}(x^\mu,y)$ and $f_{\mu_1\dots\mu_{p+1}}\equiv(p+1)\partial_{[\mu_1}a_{\mu_2\dots\mu_{p+1}]}$ is the field strength of the $(D-1)$-dimensional p-form potential $a_{\mu_1\dots\mu_p}$. The bulk p-form potential $A_{M_1\dots M_p}$ can be localized on the dilatonic domain wall, if the y integral in Eq. (7) is finite, which is the case when

$$p<\frac{D-3}{2}+\frac{(D-2)^2a^2}{8}=\frac{(D-2)(\Delta+4)}{4}. \tag{8}$$

As observed in Ref. [4], a nondilatonic DW $(a=0)$ cannot localize p-form potentials with $p\geqslant(D-3)/2$. In particular, the RS DW [the $(D,a)=(5,0)$ case] can localize only a 0-form field or a scalar field. However, the dilatonic DWs $(a\neq0)$ can additionally localize the higher rank p-form potentials, as long as the dilaton coupling parameter a is large enough: The higher the rank p of the form potential, the larger the value of a required for localizing the form potential. All the bulk form potentials (up to rank $D-1$) can be localized on the wall when

$$a^2\geqslant4(D+3)/(D-2)^2\Leftrightarrow\Delta\geqslant8/(D-2). \tag{9}$$

So, for example, the five-dimensional dilatonic domain wall in string theories obtained by compactifying branes with one type of constituent brane on a Ricci flat manifold, for which $\Delta=4$, can localize bulk form potentials of all ranks.

The problem with the KK zero mode ansatz of the form (6) is that the ansatz of such form for the pair $A_{\mu_1\dots\mu_p}$ and $A_{\mu_1\dots\mu_{D-p-2}}$ are not compatible with the following Hodge-duality formula in the bulk spacetime:

$$\sqrt{-G}G^{M_1N_1}\cdots G^{M_{D-p-1}N_{D-p-1}}\tilde{F}_{N_1\dots N_{D-p-1}}=\frac{1}{(p+1)!}\epsilon^{M_1\dots M_{D-p-1}N_1\dots N_{p+1}}F_{N_1\dots N_{p+1}}. \tag{10}$$

Another problem is that a higher rank form potential hodge dual to a lower rank form potential which can be localized on the wall may not be localizable, as can be seen from the criterion (8). To resolve such contradictions, it was proposed in Ref. [5] to choose the KK zero mode ansatz for a p-form potential with $p>(D-1)/2$ as

$$A_{\mu_1\dots\mu_{p-1}y}(x^\mu,y)=\mathcal{W}^{-(D-2p-1)/2}(y)a_{\mu_1\dots\mu_{p-1}}(x^\mu), \tag{11}$$

with all the other components vanishing. Then, ansatz (6) and (11) are compatible with with Eq. (10), and the bulk Hodge-duality formula (10) reduces to the following Hodge-duality formula in $D-1$ dimensions:

$$\sqrt{-g}g^{\mu_1\nu_1}\cdots g^{\mu_{D-p-1}\nu_{D-p-1}}\tilde{f}_{\nu_1\dots\nu_{D-p-1}}=\frac{1}{p!}\epsilon^{\mu_1\dots\mu_{D-p-1}\nu_1\dots\nu_p}f_{\nu_1\dots\nu_p}. \tag{12}$$

Substituting the ansatz (11) into the bulk action (5), we obtain

$$S_p=-\frac{1}{2\cdot p!}\int_{-1/K}^{1/K}dy(1-K|y|)^{-4(D-2p-1)/(D-2)^2a^2}\times\int d^{D-1}x\sqrt{-g}g^{\mu_1\nu_1}\cdots g^{\mu_p\nu_p}f_{\mu_1\dots\mu_p}f_{\nu_1\dots\nu_p}. \tag{13}$$

From this we see that the criterion for a bulk p-form potential with the KK zero mode of the form (11) to be localized on the wall is

$$p>(D-1)/2-(D-2)^2a^2/8=-(D-2)\Delta/4. \tag{14}$$

So, for any values of a, all the bulk p-form potentials with $p>(D-1)/2$ can be localized on the wall as a $(p-1)$-form potential. In the bulk of dilatonic domain wall $(a\neq0)$, bulk form potentials with the lower ranks can be additionally localized on the wall in such a manner: The larger the value of a, the less stringent the lower bound on p for such localization. All the bulk p-form potentials with $p\geqslant1$ can be localized on the dilatonic wall as $(p-1)$-form potentials, if

$$a^2\geqslant4(D-1)/(D-2)^2\Leftrightarrow\Delta\geqslant0. \tag{15}$$

So, for example, all the DWs obtained from (intersecting) branes (with equal charges) in string theories through the compactification on Ricci flat manifolds, for which $\Delta=4/N$ with $N\in\mathbf{Z}_+$, can localize any rank bulk p-form potentials as $(p-1)$-form potentials in $D-1$ dimensions.

When the KK zero mode is chosen to be of the form (6) for the lower rank form potentials and of the form (11) for the higher rank form potentials, the only cases in which the nondilatonic domain walls $(a=0)$ cannot localize the bulk form potentials are $p=(D-3)/2$ and $p=(D-1)/2$, as can be seen from Eqs. (8) and (14). For the RS domain wall $(D=5)$, these correspond to the bulk $U(1)$ field and two-form field. On the other hand, dilatonic domain walls can localize bulk p-form potentials with $p=(D-3)/2$ and $p=(D-1)/2$ when KK zero modes are chosen to be of the forms (6) and (11), respectively, for any values of a. So, bulk p-form potentials of any ranks can be localized on the dilatonic domain wall (with any a) as p-form potentials or $(p-1)$-form potentials in $D-1$ dimensions. Furthermore, from Eqs. (9) and (15) we see that bulk p-form potentials of any ranks can be localized on the dilatonic domain walls as

[1] Note, spacetime is undefined beyond the singularities at $y=\pm1/K$, so the integration interval is $-1/K\leqslant y\leqslant1/K$.

both p-form potentials and $(p-1)$-form potentials in $D-1$ dimensions, if the condition (9) is satisfied.

We now explicitly study the KK modes of massless bulk p-form potentials by considering the following equations of motion obtained from the bulk action (5):

$$\frac{1}{\sqrt{-G}}\partial_{M_1}[\sqrt{-G}G^{M_1N_1}\cdots G^{M_{p+1}N_{p+1}}F_{N_1\ldots N_{p+1}}]=0. \tag{16}$$

We will find above all that the KK zero modes of a bulk form potentials are indeed given by Eqs. (6) and (11).

First, we consider the dimensional reduction of a bulk p-form potential to a form potential of the same rank p. By using the gauge degrees of freedom, we can take the gauge conditions $A_{M_1\ldots M_{p-1}y}=0$. We consider the following KK mode ansatz for the bulk p-form potential:

$$A_{\mu_1\ldots\mu_p}(x^\mu,y)=a^{(m)}_{\mu_1\ldots\mu_p}(x^\mu)u_m(y), \tag{17}$$

where $a^{(m)}_{\mu_1\ldots\mu_p}$ is assumed to satisfy the following field equations for a massive p-form potential in $(D-1)$-dimensional flat spacetime:

$$\partial^{\mu_1}f^{(m)}_{\mu_1\ldots\mu_{p+1}}+m^2a^{(m)}_{\mu_2\ldots\mu_{p+1}}=0, \tag{18}$$

along with the gauge conditions $\partial^{\mu_1}a^{(m)}_{\mu_1\ldots\mu_p}=0$, where $f^{(m)}_{\mu_1\ldots\mu_{p+1}}\equiv(p+1)\partial_{[\mu_1}a^{(m)}_{\mu_2\ldots\mu_{p+1}]}$ is the field strength of $a^{(m)}_{\mu_1\ldots\mu_p}$. Then, the equations of motion (16) for the bulk p-form potential with the bulk metric (3) substituted reduce to the following form of the Sturm-Liouville equation satisfied by $u_m(y)$:

$$\partial_y[\mathcal{W}^{(D-2p-1)/2}\partial_y u_m]=m^2\mathcal{W}^{(D-2p-3)/2}u_m. \tag{19}$$

The operator $\mathcal{L}=\partial_y(\mathcal{W}^{(D-2p-1)/2}\partial_y)$ is self-adjoint, provided the boundary condition $[(\mathcal{W}^{(D-2p-1)/2}u_n')u_m-(\mathcal{W}^{(D-2p-1)/2}u_m')u_n]|^{1/K}_{-1/K}=0$ is satisfied. For such a case, the eigenvalue m^2 is real and the eigenfunctions u_m with different eigenvalues are orthogonal to each other with respect to the weighting function $w(y)=\mathcal{W}^{(D-2p-3)/2}$, i.e., $\int^{1/K}_{-1/K}dy\,u_m(y)u_n(y)w(y)=0$ for $m^2\neq n^2$. By using a new y-dependent function $\tilde{u}_m=\mathcal{W}^{(D-2p-1)/4}u_m$, we can bring the Sturm-Liouville equation (19) into the following form of the Schrödinger equation with zero-energy eigenvalue:

$$-d^2\tilde{u}_m/dy^2+V(y)\tilde{u}_m=0, \tag{20}$$

where the potential is given by

$$V(y)=\frac{D-2p-1}{4}\mathcal{W}^{-1}\mathcal{W}''+\frac{(D-2p-1)(D-2p-5)}{16}\times\mathcal{W}^{-2}(\mathcal{W}')^2+m^2\mathcal{W}^{-1}. \tag{21}$$

In the case of the KK zero mode $(m=0)$, by substituting the expression for the warp factor (3) into Eq. (21), we obtain the following explicit expression for the potential:

$$V(y)=\frac{2(D-2p-1)K^2}{(D-2)^4a^4}\frac{2(D-2p-1)-(D-2)^2a^2}{(1-K|y|)^2}-\frac{4(D-2p-1)K}{(D-2)^2a^2}\delta(y). \tag{22}$$

From this we see that the solution to the Schrödinger equation (20) satisfies the boundary condition $\tilde{u}_0'(0^+)-\tilde{u}_0'(0^-)=-4(D-2p-1)K\tilde{u}_0(0)/(D-2)^2a^2$. The solution to Eq. (20) satisfying this boundary condition is $\tilde{u}_0(y)\sim(1-K|y|)^{2(D-2p-1)/(D-2)^2a^2}$. So, the KK zero mode is constant: $u_0(y)=\mathcal{W}^{-(D-2p-1)/4}\tilde{u}_0(y)=\text{constant}$. This zero mode $u_0(y)$ is normalizable if its norm

$$\int^{1/K}_{-1/K}dy\,u_0^2(y)w(y)\sim(1-K|y|)^{[4(D-2p-3)+(D-2)^2a^2]/(D-2)^2a^2}|^{1/K}_{-1/K} \tag{23}$$

is finite, which is the case when $4(D-2p-3)+(D-2)^2a^2>0$. This normalization condition coincides with the condition (8) obtained by considering the effective action (7). For such a case, the normalized zero mode is given by

$$u_0(y)=\sqrt{\frac{4(D-2p-3)+(D-2)^2a^2}{2(D-2)^2a^2}K}.$$

We make a couple of comments on the KK zero mode. First, had we just considered the Sturm-Liouville equation (19) with $m=0$, the most general form of the KK zero mode would have been given by

$$u_0(y)=c_1+c_2\mathcal{W}^{(D-2)^2a^2/8-(D-2p-1)/2}, \tag{24}$$

where c_1 and c_2 are integration constants. This solution expressed in terms of $\tilde{u}_0=\mathcal{W}^{(D-2p-1)/4}u_0$ is also a general solution to the Schrödinger equation (20) in the region $y\neq0$. The boundary condition at $y=0$ due to the δ-function term in the potential (22) requires that $c_2=0$. However, when just the Sturm-Liouville equation (19) is considered, such a restriction due to the boundary condition at $y=0$ does not exist. If we use the general zero mode (24), then the bulk action (5) takes the form

$$\begin{aligned}S_p=&-\frac{1}{2\cdot p!}\int^{1/K}_{-1/K}dy(c_1\varpi^{2(D-2p-3)/(D-2)^2a^2}\\&+c_2\varpi^{1-2(D-2p+1)/(D-2)^2a^2})^2\\&\times\int d^{D-1}x\sqrt{-g}f^{(0)}_{\mu_1\ldots\mu_{p+1}}f^{(0)\mu_1\ldots\mu_{p+1}}\\&+\tilde{c}_2\int^{1/K}_{-1/K}dy\int d^{D-1}x\sqrt{-g}a^{(0)}_{\mu_1\ldots\mu_p}a^{(0)\mu_1\ldots\mu_p},\end{aligned} \tag{25}$$

where $\varpi \equiv 1 - K|y|$ and the constant $\tilde{c}_2$ is proportional to c_2. So, with $c_2 \neq 0$, we have a finite mass term[2] for the p-form potential in the effective action, which is contradictory to the fact that u_0 is the KK zero mode. Furthermore, the finiteness of the kinetic term in Eq. (25) requires a larger value of a than the value satisfying Eq. (9). Second, we have seen that the KK zero modes for the bulk p-form potentials are independent of the extra spatial coordinate y. Although such zero modes are normalizable, one might argue that the zero modes are not localized on the wall because they are spread evenly in the bulk (rather than localized sharply near the wall). On the other hand, the KK zero mode of the bulk scalar field, which is widely regarded as being localized on the wall, is also independent of y. Furthermore, the KK zero mode of the bulk graviton is also independent of y, if the bulk graviton $h_{\mu\nu}$ is defined as $\mathcal{W}(\eta_{\mu\nu}+h_{\mu\nu})dx^\mu dx^\nu + dy^2$. Perhaps, we might wish to choose to consider the product $u_0^2(y)w(y)$ for determining the distribution of the bulk field across the y direction rather than the zero mode u_0 itself, since it is this product that appears as the integrand of the y integral in the kinetic term of the effective action and may be interpreted as being related to the form potential charge density along the y direction. Then, we find that the bulk p-form potential is localized around $y=0$, spread evenly in the bulk and localized around $|y|=1/K$, when $p<(D-3)/2$, $p=(D-3)/2$ and $p>(D-3)/2$, respectively. So, for $D=5$, the distribution of the KK zero mode is localized around the wall only for the bulk scalar field case. For the KK zero mode ansatz (11), the bulk p-form potential is localized around $y=0$, spread evenly in the bulk and localized around $|y|=1/K$, when $p>(D-1)/2$, $p=(D-1)/2$, and $p<(D-1)/2$, respectively, as can be seen from the integrand of the y integration in Eq. (13). So, for $D=5$, the distribution of the KK zero mode is localized around the wall only for the bulk three form potential case. (Also, with such criterion, the KK zero mode for the bulk graviton is localized around both dilatonic and nondilatonic domain walls.)

[2]For the nondilatonic domain wall case, the integration interval for the y integration is infinite, so the mass term would diverge.

Second, we consider the dimensional reduction of a bulk p-form potential to a $(p-1)$-form potential. We choose $A_{\mu_1\ldots\mu_{p-1}y}(x^\mu,y)=a_{\mu_1\ldots\mu_{p-1}}(x^\mu)v(y)$ to be the KK ansatz with all the other components taken to vanish. Substituting this ansatz into Eq. (16), we obtain the following equation satisfied by $v(y)$: $\partial_y[\mathcal{W}^{(D-2p-1)/2}v]=0$, from which we obtain $v \sim \mathcal{W}^{-(D-2p-1)/2}$. So, the KK ansatz (26) coincides with the ansatz (11) proposed in Ref. [5], thereby providing a justification for such a choice.

It is pointed out in Ref. [5] that if we would additionally require the consistency with the bulk Einstein's equations then only the $p=0$ case with the ansatz (6) and the $p=D-2$ case with the ansatz (11) are allowed. Actually, considering the Einstein's equations means looking for the solution for gravitating (nondilatonic) charged $(p-1)$ brane or $(p-2)$ brane (coupled to the bulk p-form potential) within the domain wall, depending on the choice of the KK zero mode ansatz. We have seen in our previous works [14,15] that such solutions do not always exist due to the constraint following from the equations of motion. In this paper and other related papers, where just the KK modes of bulk fields are studied, the gravitational backreaction of the bulk fields are ignored (thereby, the bulk metric remaining as a static background in which the bulk fields propagate), assuming that bulk fields make very little contribution to the bulk energy density, and therefore the Einstein's equations should not be considered. If we would insist on consistency with the Einstein's equations, then we should rather consider the solution for (nondilatonic) charged brane within the wall, which, according to Ref. [14], does not exist except for the $p=0,D-2$ cases mentioned in the above, as the bulk metric, instead of the domain wall solution (3). In the case in which the kinetic term for the bulk form potential has the dilaton factor $e^{2a_p\phi}$, the solution for the charged brane exists for any p, provided a and a_p satisfy the constraint resulting from the equations of motion [14,15].

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