BPS string solutions in non-Abelian Yang-Mills theories and confinement

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Starting from the bosonic part of $N=2$ super QCD with a "Seiberg-Witten" $N=2$ soft breaking mass term, we obtain string BPS conditions for arbitrary semisimple gauge groups. We show that the vacuum structure is compatible with a symmetry breaking scheme which allows the existence of Z_k-strings and which has $\mathrm{Spin}(10)\rightarrow SU(5)\times Z_2$ as a particular case. We obtain BPS Z_k-string solutions and show that they satisfy the same first order differential equations as the BPS string for the $U(1)$ case. We also show that the string tension is constant, which may cause a confining potential between monopoles increasing linearly with their distance.

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I. INTRODUCTION

String (and vortex) solutions [1] may have many important applications such as their possible relevance for quark confinement [2,3], for galaxy formation [4,5] and for superconductors [6]. These solutions may also be relevant for a field theory formalism for the "fundamental" string or the D-strings. Non-Bogomol'nyi-Prasad-Sommerfield (BPS) string solutions in non-Abelian theories were first analyzed in [7,8], for the particular case of $SO(10)$. There are various motivations for looking for BPS solutions. First, because they appear naturally in supersymmetric theories, often in connection with dualities. Secondly, because they satisfy first order differential equations which are easier to solve than the second order equations coming from the equations of motion. And finally because BPS (or almost-BPS) strings may be relevant for confinement [3] (for a recent review see [9]).

The BPS solutions for monopoles in Yang-Mills-Higgs theories are known for an arbitrary semisimple gauge group broken by a scalar in the adjoint representation [10]. However for strings (and vortices), the BPS solutions are only known for $U(1)$ Yang-Mills-Higgs theories broken to a Z exact symmetry group [11] (see a review [12]) and for some other particular examples ([13,14] and references therein). Our aim is thus to obtain the BPS string solutions in a Yang-Mills-Higgs theory with an arbitrary semisimple gauge group broken to a non-Abelian residual group.

In the paper of Seiberg and Witten [3], the authors consider an $SU(2)$ $N=2$ super Yang-Mills theory, and associated to the point in the moduli space where the monopole becomes massless they obtained an effective $U(1)$ $N=2$ super QED with an $N=2$ mass breaking term. In this effective theory, the $U(1)$ is broken to a Z group, the theory develops an Abelian string solution and as it happens Abelian confinement occurs. After this work, many interesting papers appeared [16] analyzing various related issues, considering either $U(1)$ or $U(1)^{N-1}$ effective theories broken to discrete groups. Since we are considering a non-Abelian generalization of Seiberg-Witten effective theory with a non-Abelian unbroken group, our BPS string solution may have some relevance for non-Abelian confinement. More precisely, in our theory the strings are associated to elements of a Z_k group, rather than the Z group, and the breaking of gauge symmetry by a scalar in the adjoint allows monopole solutions to arise belonging to representations of (the dual) non-Abelian unbroken symmetry [17], rather than $U(1)$ singlets.

Keeping in mind that our results can be specifically applied to the symmetry breaking scheme $\mathrm{Spin}(10)\xrightarrow{126}SU(5)\times Z_2$, our BPS string solution may also have some applications as a cosmic string.

We begin by obtaining, in Sec. II, the string BPS conditions considering the bosonic part of $N=2$ super QCD with one flavor and with an $N=2$ breaking mass term for the scalar in the vector multiplet, similar to the case considered by Seiberg-Witten [3]. Then, in Sec. III we show that the vacuum structure is compatible with a symmetry breaking scheme considered by Olive and Turok [15], which allows the existence of Z_k strings and which has $\mathrm{Spin}(10)\xrightarrow{126}SU(5)\times Z_2$ as a particular case. In Sec. IV, we consider a Z_k-string ansatz and obtain the first order differential equations which are exactly the same as the ones for the BPS string in the $U(1)$ theory. From this ansatz we obtain that the string tension is constant. This may ensure a confining potential between monopoles increasing linearly with their distance.

II. BPS STRING CONDITIONS IN NON-ABELIAN YANG-MILLS-HIGGS THEORIES

Let us consider the Lagrangian in 3+1 dimensions

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$$L=\mathrm{Tr}\left\{-\frac{1}{4}G^{\mu\nu}G_{\mu\nu}+\frac{1}{2}D_\mu S^\dagger D^\mu S\right\}+\frac{1}{2}D_\mu\phi^\dagger D^\mu\phi -V(S,\phi), \tag{1}$$

with an arbitrary semisimple gauge group, where S is a complex scalar field in the adjoint representation and ϕ is another complex scalar whose representation we shall specify below. As in the $U(1)$ theory (for a review see [12]), let us consider a static field configuration with cylindrical symmetry not depending on x^3 and the only nonvanishing component of $G_{\mu\nu}$ being $G_{12}\equiv -B$. The string tension is then

$$T=\int d^2x\left\{\frac{1}{2}\mathrm{Tr}[B^2+|D_\mu S|^2]+\frac{1}{2}|D_\mu\phi|^2+V(S,\phi)\right\}$$
$$\geqslant\int d^2x\left\{\frac{1}{2}\mathrm{Tr}[B^2+|D_1 S|^2+|D_2 S|^2]+\frac{1}{2}|D_1\phi|^2+\frac{1}{2}|D_2\phi|^2+V(S,\phi)\right\}. \tag{2}$$

Let us denote by ρ the distance from the string axis. In order for T to be finite, the field must tend to vacua configurations at $\rho\to\infty$, satisfying the conditions

$$D_\mu S=D_\mu\phi=O(1/\rho^2),\quad V(S,\phi)=O(1/\rho^3),\quad B=O(1/\rho^2). \tag{3}$$

Let $D_\pm=D_1\pm iD_2$. Using the identity

$$[D_\pm\phi]^\dagger[D_\pm\phi]-|D_1\phi|^2-|D_2\phi|^2=\pm[i\epsilon_{ij}\partial_i(\phi^\dagger D_j\phi)+e\phi^\dagger G_{12}\phi],$$

and the fact that

$$\int d^2x\,\epsilon_{ij}\partial_i(\phi^\dagger D_j\phi)=0$$

(which follows from the above boundary conditions) and similar results for the scalar S, it results that

$$T=\int d^2x\left\{\mathrm{Tr}\left[\frac{1}{2}B^2+\frac{1}{2}|D_\mp S|^2\mp\frac{e}{2}S^\dagger[B,S]\right]+\frac{1}{2}|D_\pm\phi|^2\pm\frac{e}{2}(\phi^\dagger B\phi)+V(S,\phi)\right\}$$
$$\geqslant\int d^2x\left\{\frac{1}{2}B_a^2\pm\frac{e}{2}(S_b^*if_{bca}S_c+\phi^\dagger T_a\phi)B_a+V(S,\phi)\right\}.$$

Note that we used the above identities with opposite signs for the fields ϕ and S, in order to make contact with the supersymmetric Lagrangian, as will become clear below. Let

$$Y_a=\frac{e}{2}(S_b^*if_{bca}S_c+\phi^\dagger T_a\phi)+X_a \tag{4}$$

where

$$X_a=-\frac{me}{2}\left(\frac{S_a+S_a^*}{2}\right).$$

Then T can be written as

$$T\geqslant\int d^2x\left\{\frac{1}{2}[B_a\pm Y_a]^2\mp X_aB_a-\frac{1}{2}Y_b^2+V(S,\phi)\right\}$$
$$\geqslant\int d^2x\left\{\mp X_aB_a-\frac{1}{2}Y_a^2+V(S,\phi)\right\}.$$

If $V(S,\phi)-Y_a^2/2\geqslant 0$, then

$$T\geqslant\int d^2x\{\mp X_aB_a\} \tag{5}$$

and the bound is saturated if and only if

$$D_0\phi=D_3\phi=D_0S=D_3S=0, \tag{6}$$

$$D_\pm\phi=0, \tag{7}$$

$$D_\mp S=0, \tag{8}$$

$$B_a\pm Y_a=0, \tag{9}$$

$$V(S,\phi)-\frac{1}{2}Y_a^2=0, \tag{10}$$

which are BPS equations for the string. The first conditions (6) imply that $W_0=0=W_3$.

We shall consider

$$V(S,\phi)=\frac{1}{2}(Y_a^2+F^\dagger F),\quad F\equiv e\left(S^\dagger-\frac{\mu}{e}\right)\phi. \tag{11}$$

Then the BPS condition (10) implies that $F=0$. When $m=0$, this potential coincides with the one for the bosonic part of $N=2$ super QCD with one flavor (A2) (see Appendix A) if the scalar $\phi_2=0$ of the hypermultiplet vanishes. The case $m\neq 0$ clearly breaks $N=2$ supersymmetry since it gives a bare mass to S_a. This is akin to the situation considered by Seiberg and Witten [3] where the authors obtained confinement by introducing a bare mass to the scalar in the vector supermultiplet. This X_a term is important in order to change the vacuum structure of the theory.

The equations of motion that follow from our Lagrangian are

$$(D_\mu G^{\mu\nu})_a-\frac{ie}{2}(\phi^\dagger T_aD^\nu\phi-D^\nu\phi^\dagger T_a\phi-S_b^*if_{abc}D^\nu S_c+D^\nu S_b^*if_{abc}S_c)=0,$$

$$D_\mu D^\mu\phi_k+eY_aT^a_{kl}\phi-e^2\left[\left(S-\frac{\mu}{e}\right)\left(S^\dagger-\frac{\mu}{e}\right)\phi\right]_k=0,$$

$$D_\mu D^\mu S_d + eY_a\left(if_{adc}S_c - \frac{m}{2}\delta_{ad}\right) - e^2\phi^\dagger\left(S - \frac{\mu}{e}\right)T_d\phi = 0.$$

Let us check if the BPS equations for the string are consistent with them. Acting with the covariant derivative D_i, $i=1,2$, on Eq. (9) and using the other BPS conditions it results that

$$D_\mu G_a^{\mu\nu} + \frac{ie}{2}\left[D^\nu\phi^\dagger T_d\phi - \phi^\dagger T_d D^\nu\phi - (D^\nu S)_b^* if_{dbc}S_c + S_b^* if_{dbc}D^\nu S_c - \frac{m}{2}(D^\nu S_d - D^\nu S_d^*)\right] = 0.$$

This relation is consistent with the first equation of motion if $m=0$. Similarly, from the BPS conditions we obtain

$$\begin{aligned}0 &= D_\mp D_\pm\phi - e\left(S - \frac{\mu}{e}\right)F\\ &= -D_\mu D^\mu\phi \mp eG_{12}\phi - e\left(S - \frac{\mu}{e}\right)F\\ &= -D_\mu D^\mu\phi - eY_aT_a\phi - e\left(S - \frac{\mu}{e}\right)F\end{aligned}$$

and

$$\begin{aligned}0 &= D_\pm D_\mp S_d - eF^\dagger T_d\phi\\ &= -D_\mu D^\mu S_d \pm e(G_{12}S)_d - eF^\dagger T_d\phi\\ &= -D_\mu D^\mu S_d - ieY_af_{adb}S_b - eF^\dagger T_d\phi.\end{aligned}$$

Once again, this last relation is consistent with the equations of motion only when m vanishes. However this condition must be understood in the limiting case $m\to 0$ as we shall discuss in the next section. Therefore it is only in this limit that we can have BPS strings satisfying Eqs. (6)–(11). One can check that 1/4 of the $N=2$ supersymmetry transformations (A7) vanish for field configurations satisfying the BPS conditions (6)–(11) in the limit $m\to 0$.

III. VACUA SOLUTIONS

The total energy for this theory is non-negative and it vanishes (vacuum) if and only if

$$D_\mu\phi = D_\mu S = G_{\mu\nu} = 0, \tag{12}$$

$$V=0 \Leftrightarrow Y_a = F = 0$$

in all spacetime. In order for the string to have finite tension T, the fields at $\rho\to\infty$ must tend to vacuum configurations. Moreover, a necessary condition for the existence of string solutions is that these vacua must break the gauge group G into G_ϕ such that

$$\Pi_1(G/G_\phi) = Z_k, \tag{13}$$

which allows the existence of Z_k strings.

Let us consider H_i, E_α to be generators of a Lie algebra in the Cartan-Weyl basis, with $H_i^\dagger = H_i$ and $E_\alpha^\dagger = E_{-\alpha}$, $\mathrm{Tr}(H_iH_j) = \delta_{ij}$, $\mathrm{Tr}(E_\alpha E_{-\beta}) = 2\delta_{\alpha\beta}/\alpha^2$ and satisfying the commutation relations

$$[H_i, E_\alpha] = \alpha^i E_\alpha,$$

$$[E_\alpha, E_{-\alpha}] = \alpha^{\rm v}\cdot H, \quad \alpha^{\rm v} \equiv \frac{2\alpha}{\alpha^2}.$$

Moreover $H_i|\lambda_a\rangle = \lambda_a^i|\lambda_a\rangle$. A symmetry breaking satisfying Eq. (13) can be realized in the following way [15]: let λ_ϕ be an arbitrary fundamental weight and let $S^{\rm vac}$ and $\phi^{\rm vac}$ be two scalars in the vacuum configuration. As is well known, a scalar in the adjoint representation of the form $S^{\rm vac}\propto\lambda_\phi\cdot H$ breaks the gauge group into

$$G\to G_S = U(1)\times K/Z_l,$$

where K is the subgroup of G associated with the algebra whose Dynkin diagram is given by removing the dot corresponding to α_ϕ from that of G, $U(1)$ is generated by $\lambda_\phi\cdot H$ and Z_l is a subgroup of $U(1)$ and K and is generated by

$$v_0 = \exp(2\pi iz\lambda_\phi^{\rm v}\cdot H), \quad z\equiv\frac{|Z(G)|}{|Z(K)|}, \tag{14}$$

where $|Z(G)|$ is the order of the center of G and $|Z(K)|$ is the order of the center of K. This symmetry breaking pattern allows the existence of monopoles. If the theory has another scalar $\phi^{\rm vac}\propto|k\lambda_\phi\rangle$, k being an integer, the gauge group G is further broken into [15]

$$G\to G_\phi = Z_{kl}\times K/Z_l \subset G_S$$

where Z_{kl} is generated by $v_0^{1/k}$ and K is as before. Then, $\Pi_1(G/G_\phi) = Z_k$. In order for $\phi^{\rm vac}\propto|k\lambda_\phi\rangle$ we may consider that ϕ belongs to the irrep $R_{k\lambda_\phi}$ with $k\lambda_\phi$ as highest weight.[1] The symmetry breaking scheme $\mathrm{Spin}(10)\xrightarrow{126} SU(5)\times Z_2$ considered by Kibble *et al.* [5] for the cosmic string corresponds to a particular case of this general result.

Let us show that the vacuum conditions (12) admit solutions of this form. Consider $\phi^{\rm vac} = a|k\lambda_\phi\rangle$ and $S^{\rm vac} = v\cdot H$ where $a, v\in R$ and $\langle k\lambda_\phi|k\lambda_\phi\rangle = 1$. The condition $Y_a=0$ is equivalent to

[1]If $k=2$, it can also be interesting to consider ϕ belonging to the symmetric part of the tensor product of two fundamental representations with highest weight λ_ϕ, $[R_{\lambda_\phi}\times R_{\lambda_\phi}]_S \equiv R_{2\lambda_\phi}^{\rm sym}\supset R_{2\lambda_\phi}$. [For $SU(N)$, $R_{2\lambda_{N-1}}^{\rm sym} = R_{2\lambda_{N-1}}$. For $SO(10)$, $R_{2\lambda_5}^{\rm sym} = 126\oplus 10$ and $R_{2\lambda_5} = 126$.] A physical motivation to consider $\phi\in R_{2\lambda_\phi}^{\rm sym}$ is because it allows a Yukawa coupling with two spinors in the fundamental representation R_{λ_ϕ}, and this term gives rise to the mass term for the spinors when ϕ has a nontrivial expectation value. In this case one could also consider ϕ as a difermion condensate as in the BCS theory.

$$Y_a T_a = \frac{e}{2}\left[(\phi^\dagger T_a \phi) T_a + [S^\dagger, S] - m\left(\frac{S+S^\dagger}{2}\right)\right] = 0. \quad (15)$$

Using that

$$\begin{aligned}(\phi_i^* T_{aij}\phi_j) T_a &= \mathrm{Tr}(\phi\phi^\dagger T_a) T_a = \phi\phi^\dagger \\ &= (\phi^\dagger H_i \phi) H_i \\ &\quad + \frac{\alpha^2}{2}(\phi^\dagger E_\alpha \phi) E_{-\alpha},\end{aligned}$$

from Eq. (15) follows that

$$v = \frac{ka^2}{m}\lambda_\phi .$$

On the other hand, from the condition $F=0$, it results that $v\cdot\lambda_\phi = \mu/ke$, which together with the previous relation leads to

$$a^2 = \frac{m\mu}{k^2 e \lambda_\phi^2}.$$

Then,

$$\begin{aligned}\phi^{\mathrm{vac}} &= a|k\lambda_\phi\rangle, \\ mS^{\mathrm{vac}} &= ka^2\lambda_\phi\cdot H, \qquad (16) \\ W_\mu^{\mathrm{vac}} &= 0,\end{aligned}$$

is a solution of the vacuum conditions (12) which satisfy $\Pi_1(G/G_\phi) = Z_k$.

Expanding the fields around this vacuum ($S = S^q + S^{\mathrm{vac}}$, $W_\mu = W_\mu^q + W_\mu^{\mathrm{vac}}$, etc.) and considering

$$W_\mu^q = W_\mu^\phi H_{\alpha_\phi} + \sum_{i\neq\phi} W_\mu^i H_{\alpha_i} + \sum_\alpha W_\mu^\alpha E_\alpha ,$$

where $H_\alpha \equiv \alpha^{\mathrm{v}}\cdot H$, from the kinetic terms of S and ϕ one finds the gauge particle mass terms

$$\sum_{\alpha>0} (\lambda_\phi\cdot\alpha^{\mathrm{v}})\left[\frac{(\alpha)^4 e^2 k^2 a^4}{4m^2} + \frac{e^2a^2k}{2}\right] W^{\alpha\mu} W_\mu^{-\alpha} + \frac{e^2a^2k^2}{2} W^{\phi\mu} W_\mu^\phi .$$

As we mentioned before, the BPS conditions are compatible with the equations of motion when m vanishes. However, if we do this, $a=0$ and there is no symmetry breaking, which is necessary in order for string solutions to exist. This result is very similar to what happens for the BPS monopole (see for instance [18]). In that case, one of the BPS conditions is $V(\phi) = \lambda(\phi^2 - a^2)^2/4 = 0$, which implies the vanishing of the coupling λ. (Note that for the string and the monopole, X_a and V are terms which break $N=2$ supersymmetry and which vanish for the BPS configurations.) However, that condition must be understood in the Prasad-Sommerfield limiting case $\lambda\to 0$ [19] in order to retain the boundary condition $|\phi|\to a$ as $r\to\infty$, and to have symmetry breaking. In our case, we have the same situation with a small difference: if one considers $m\to 0$, then $a\to 0$. We can avoid this problem by allowing $\mu\to\infty$ such that $m\mu$, or equivalently a, remains constant, implying that the field ϕ becomes infinitely heavy. The same happens for the gauge fields W_μ^α in which $\lambda_\phi\cdot\alpha^{\mathrm{v}} \neq 0$.

It is important to mention that if we take $m=0$, Eq. (16) is no longer a vacuum solution, but it is possible to consider other vacuum solutions such that $\Pi_1(G/G_\phi)\neq 0$. However, in this case, we were not able to construct a string ansatz satisfying the BPS conditions.

IV. BPS STRING SOLUTIONS

The string must tend at $\rho\to\infty$ to vacuum solutions in any angular direction φ. Let us denote $\phi(\varphi) = \phi(\varphi, \rho\to\infty)$, $S(\varphi) = S(\varphi,\rho\to\infty)$. Then, the vacuum conditions (12) imply that this asymptotic field configuration must be related by gauge transformations from a vacuum configuration, which we shall consider Eq. (16), i.e.,

$$\begin{aligned}W_i(\varphi) &= \frac{-1}{ie}(\partial_i g(\varphi)) g(\varphi)^{-1}, \qquad i=1,2, \\ \phi(\varphi) &= g(\varphi)\phi^{\mathrm{vac}}, \\ S(\varphi) &= g(\varphi) S^{\mathrm{vac}} g(\varphi)^{-1},\end{aligned}$$

for some $g(\varphi)\in G$. Then, in order for the field configurations to be single-valued, $g(2\pi)g(0)\in G_\phi$. Without lost of generality we shall consider $g(0)=1$. We shall also consider that G is simply connected (which can always be done by going to the universal covering group). Then, a necessary condition for the existence of strings is that $g(2\pi)$ belongs to a nonconnected component of G_ϕ [7]. Let $g(\varphi) = \exp i\varphi M$. Then, at $\rho\to\infty$,

$$\begin{aligned}\phi(\varphi) &= a e^{i\varphi M}|k\lambda_\phi\rangle, \\ mS(\varphi) &= ka^2 e^{iM\varphi}\lambda_\phi\cdot H e^{-iM\varphi}, \qquad (17) \\ W_i(\varphi) &= \frac{\epsilon_{ij}x^j}{e\rho^2} M, \qquad i,j=1,2.\end{aligned}$$

A possible choice for M is

$$M = \frac{n}{k}\frac{\lambda_\phi\cdot H}{\lambda_\phi^2},$$

with n being a nonvanishing integer defined modulo k. From Eq. (17), it is direct to see that for this choice $g(2\pi)\in G_\phi$. Indeed, since [20]

$$\lambda_\phi^2 = \frac{1}{2}\alpha_\phi^2 \frac{|Z(K)|}{|Z(G)|},$$

we see from Eq. (14) that $g(2\pi) = v_0^{n/k}$.

Let us consider the ansatz

$$\phi(\varphi,\rho)=f(\rho)e^{i\varphi M}a|k\lambda_{\phi}\rangle,$$

$$mS(\varphi,\rho)=h(\rho)ka^2e^{i\varphi M}\lambda_{\phi}\cdot He^{-i\varphi M}, \tag{18}$$

$$W_i(\varphi,\rho)=g(\rho)M\frac{\epsilon_{ij}x^j}{e\rho^2}\rightarrow B(\varphi,\rho)=\frac{M}{e\rho}g'(\rho),$$

$$W_0(\varphi,\rho)=W_3(\varphi,\rho)=0, \tag{19}$$

with the boundary conditions

$$f(\infty)=g(\infty)=h(\infty)=1,$$

in order to recover the configuration (17) at $\rho\rightarrow\infty$ and

$$f(0)=g(0)=0$$

in order to eliminate singularities at $\rho=0$.

Putting this ansatz in the BPS conditions (7)–(10), from the first order differential equations it results that

$$h(\rho)=\text{const}=1,$$

$$f'(\rho)=\pm\frac{n}{\rho}[1-g(\rho)]f(\rho),$$

$$g'(\rho)=\mp\frac{e^2a^2\rho k^2\lambda_{\phi}^2}{2n}[|f(\rho)|^2-1],$$

which are exactly the same differential equations which appear in the $U(1)$ case. These equations do not have analytic solutions, however their existence has been proven and some of their properties have been analyzed (see for instance [21]).

It is important to emphasize that the BPS conditions only hold when $m\rightarrow 0$ and for $m\neq 0$ the string becomes non-BPS as has been already pointed out in [22] for the $G=U(1)$ case.

Using the ansatz (18), it is straightforward to obtain the BPS bound for the string tension (5)

$$T=\pi a^2|n|,$$

which once more coincides with the $U(1)$ result. Since the tension is constant, it may cause a confining potential between monopoles increasing linearly with their distance, which is an interesting behavior since it may produce quark confinement in a possible dual theory.

V. CONCLUSIONS

In this paper we showed the existence of BPS Z_k-string solutions for arbitrary semisimple gauge groups broken to non-Abelian groups. In order to obtain these solutions we considered the bosonic part of $N=2$ SQCD with one flavor and a $N=2$ breaking mass term. We showed that BPS conditions are compatible with the equations of motion only if $m\rightarrow 0$. We must also take $\mu\rightarrow\infty$, with $m\mu$ fixed, in order to allow gauge symmetry breaking, where m is the S bare mass and μ is the ϕ bare mass. We found vacua solutions compatible with the existence of string solutions and we were able to construct these string solutions satisfying the BPS conditions. Since our theory is a non-Abelian generalization of Seiberg-Witten effective theory, we hope that our BPS string solution may have some relevance for non-Abelian confinement. In particular, since in our theory the breaking of gauge symmetry by S allows for monopole solutions belonging to representations of (the dual) non-Abelian unbroken symmetry and the string solutions are associated to elements of a Z_k group, we expect that monopole bound states with properties more similar to the ones of quark bound states in QCD may appear in our theory. An indication of the existence of these monopole bound states comes from the fact that in our theory the BPS string tensions are constant which may give rise to a potential between monopoles increasing linearly with their distance. It would be interesting if one could find monopole bound solutions (in the classical theory) similar to the breathers in sine-Gordon theory.

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APPENDIX A: $N=2$ SQCD POTENTIAL

Using Sohnius' conventions [23] and considering $S=M+iN$, the bosonic part for the potential of $N=2$ super Yang-Mills theory with one hypermultiplet can be written as

$$V(S,\phi_m)=\frac{e^2}{8}\Bigg\{(S_b^* if_{bca}S_c)^2+(\phi_m^\dagger\sigma_{mn}^pT_a\phi_n-v_p\delta_{a0})^2+\frac{4\mu^2}{e^2}\phi_m^\dagger\phi_m-\frac{4\mu}{e}\phi_m^\dagger(S+S^\dagger)\phi_m+2\phi_m^\dagger\{S^\dagger,S\}\phi_m\Bigg\}, \tag{A1}$$

where σ^p are the Pauli matrices and the terms $v_p\delta_{a0}$ are the Fayet-Iliopoulos that may exist associated to a possible $U(1)$ factor[2] with a generator we shall denote T_0. This expression can be rewritten as

$$V(S,\phi_m)=\frac{1}{2}((d_a^1)^2+(d_a^2)^2+(D_a)^2+F_m^\dagger F_m), \tag{A2}$$

[2]The coupling constant for a possible $U(1)$ factor is not necessarily the same as the non-Abelian part, but for notational simplicity we shall consider the same constant e.

where

$$D_a = \frac{e}{2}(S_b^* i f_{bca} S_c) + d_a^3, \tag{A3}$$

$$d_a^p = \frac{e}{2}(\phi_m^\dagger \sigma_{mn}^p T_a \phi_n - v_p \delta_{a0}), \quad p=1,2,3, \tag{A4}$$

$$F_1 = e\left(S^\dagger - \frac{\mu}{e}\right)\phi_1, \tag{A5}$$

$$F_2 = e\left(S - \frac{\mu}{e}\right)\phi_2. \tag{A6}$$

From this expression it is easy to see that we recover Eq. (11), for $m \to 0$, when one puts $\phi_2 = 0$.

Let us denote by ψ and $\lambda^m, m=1,2$, the pseudo-Majorana spinors belonging to the vector supermultiplet and to the hypermultiplet respectively. Their $N=2$ supersymmetry transformations are given by [23]

$$\delta\lambda^m = \frac{i}{2} G_{\mu\nu}\gamma^{\mu\nu}\xi^m - \gamma^\mu D_\mu (M+\gamma_5 N)\xi^m - ie[M,N]\gamma_5\xi^m + i\xi^n \sigma_{nm}^p d^p, \tag{A7}$$

$$\delta\psi = -[i\gamma^\mu D_\mu + e(M+\gamma_5 N) - \mu]\xi^m \phi_m,$$

where $\gamma^{\mu\nu} \equiv i[\gamma^\mu, \gamma^\nu]/2$ and where the ξ^m are supersymmetry parameters.

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