Large gauged Q balls

K. N. Anagnostopoulos,[1,*] M. Axenides,[2,†] E. G. Floratos,[2,3,‡] and N. Tetradis[1,3,§]
[1]*Department of Physics, University of Crete, GR-71003 Heraklion, Greece*
[2]*Institute of Nuclear Physics, N.C.R.P.S. Demokritos, GR-15310 Athens, Greece*
[3]*Nuclear and Particle Physics Sector, University of Athens, GR-15771 Athens, Greece*
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We study Q balls associated with local $U(1)$ symmetries. Such Q balls are expected to become unstable for large values of their charge because of the repulsion mediated by the gauge force. We consider the possibility that the repulsion is eliminated through the presence in the interior of the Q ball of fermions with charge opposite to that of the scalar condensate. Another possibility is that two scalar condensates of opposite charge form in the interior. We demonstrate that both these scenarios can lead to the existence of classically stable, large, gauged Q balls. We present numerical solutions, as well as an analytical treatment of the "thin-wall" limit.

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I. INTRODUCTION

Nontopological solitons named Q balls can appear in scalar field theories with $U(1)$ symmetries [1]. (For a review of the early literature, see Ref. [2].) These objects can be viewed as coherent states of the scalar field with a fixed total $U(1)$ charge. The case of global $U(1)$ symmetries has attracted much attention. The reason is the presence of such symmetries in the standard model, related to baryonic or leptonic charge. In supersymmetric extensions of the standard model, the scalar superpartners of baryons or leptons can form coherent states with a fixed baryon or lepton number, making the existence of Q balls possible. Their properties [3,4], cosmological origin [5], and experimental implications [6] have been the subject of several recent studies. Non-Abelian global symmetries can also lead to the existence of Q balls [7].

We are interested in the less popular case of Q balls resulting from local $U(1)$ symmetries [8,9]. Such Q balls become unstable for large values of their charge because of the repulsion mediated by the gauge force. However, small Q balls can still exist. A possibility that has not been considered before is that the repulsion is eliminated through the presence in the interior of the Q ball of fermions with charge opposite to that of the scalar condensate. The fermions must carry an additional conserved quantum number that prevents their annihilation against the condensate. This scenario can lead to the existence of large Q balls. The fermion gas may also be replaced by another scalar condensate, of opposite charge to the first, such that the interior of the Q ball remains neutral.

In the following we discuss, in detail, the above scenarios in the context of a toy model. We show that arbitrarily large Q balls can exist and examine the constraints imposed on the parameters by the requirement of classical stability.

*Email address: konstant@physics.uoc.gr
†Email address: axenides@mail.demokritos.gr
‡Email address: manolis@mail.demokritos.gr
§Email address: tetradis@physics.uoc.gr

II. SMALL GAUGED Q BALLS

For completeness we summarize briefly the basic properties of gauged Q balls in a toy model (see Ref. [9] for the details). We consider a complex scalar field $\phi(\vec{r},t) = f(\vec{r},t)\exp[-i\,\theta(\vec{r},t)]/\sqrt{2}$, coupled to an Abelian gauge field A^μ. The Lagrangian density is

$$\mathcal{L}=\frac{1}{2}\partial_\mu f\partial^\mu f+\frac{1}{2}f^2(\partial_\mu\theta-eA_\mu)^2-U(f)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{1}$$

The total $U(1)$ charge of a particular field configuration is

$$Q_\phi=\int d^3\vec{r} f^2(\dot{\theta}-eA_0). \tag{2}$$

Without loss of generality we assume $e,Q_\phi\geqslant 0$ in the following. The value $e=0$ leads to decoupling of the scalar from the gauge field.

We consider a spherically symmetric ansatz that neglects the spatial components of the gauge field $A_i=0$, $i=1,2,3$ and assumes $\theta=\omega t$ [1]. The component A_0 of the gauge field corresponds to the electrostatic potential that is responsible for the repulsive force destabilizing the Q ball. The equations of motion for the fields are

$$f''+\frac{2}{r}f'+fg^2-\frac{dU(f)}{df}=0 \tag{3}$$

$$g''+\frac{2}{r}g'-e^2f^2g=0, \tag{4}$$

with $r=|\vec{r}|$, $g(r)=\omega-eA_0(r)$, and primes denoting derivatives with respect to r. The total charge and energy are

$$Q_\phi=\int dV\,\rho_\phi=4\pi\int r^2dr\,f^2g \tag{5}$$

$$E=\int dV\,\epsilon$$
$$=4\pi\int r^2dr\left[\frac{1}{2}f'^2+\frac{1}{2e^2}g'^2+\frac{1}{2}f^2g^2+U(f)\right]. \quad (6)$$

The Q ball solution of the equations of motion involves an almost constant non-zero scalar field $f(r)=F$ in the interior of the Q ball, which moves quickly to zero (the vacuum value) at the surface. We are interested in the limit in which the radius R of the Q ball is much larger than the thickness of its surface and the total charge can be big. In this limit and for $eFR\ll 1$, the energy can be expressed as [9]

$$E=Q_\phi\left[\frac{2U(F)}{F^2}\right]^{1/2}\left[1+\frac{C^{2/3}}{5}\right]=Q_\phi\left[\frac{2U(F)}{F^2}\right]^{1/2}+\frac{3e^2Q_\phi^2}{20\pi R}, \quad (7)$$

with

$$R=\left[\frac{3Q_\phi}{4\pi F\sqrt{2U(F)}}\right]^{1/3}\left[1+\frac{C^{2/3}}{45}\right] \quad (8)$$

$$C=\frac{3e^3Q_\phi}{4\pi}\sqrt{\frac{F^4}{2U(F)}}. \quad (9)$$

The ratio E/Q_ϕ increases with Q_ϕ because of the presence of the electrostatic term in Eq. (7). This means that large Q balls are unstable and tend to evaporate scalar particles from their surface in order to increase their binding energy. For small Q_ϕ the above expressions are not applicable. Numerical solutions indicate that the ratio E/Q_ϕ becomes large again because of the contribution from the field derivative terms that we neglected. Therefore, there is a value $(Q_\phi)_{min}$ for which E/Q_ϕ is minimized. Classical stability requires that $(E/Q_\phi)_{min}<d^2U(0)/df^2$ (assuming that the absolute minimum of the potential is at $f=0$), so that the Q ball does not disintegrate into scalar particles of unit charge.

III. LARGE GAUGED Q BALLS WITH FERMIONS

It is clear from the above discussion that gauged Q balls with very large Q_ϕ become unstable because of electrostatic repulsion. One possibility that could remedy this problem is that fermions with charge opposite to that of the scalar background neutralize the electrostatic field and eliminate the repulsion. A model that realizes this scenario has a Lagrangian density

$$\mathcal{L}=\frac{1}{2}\partial_\mu f\partial^\mu f+\frac{1}{2}f^2(\partial_\mu\theta-eA_\mu)^2-U(f)$$
$$+i\overline{\psi}_\alpha\gamma^\mu(\partial_\mu+ie'A_\mu)\psi^\alpha-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \quad (10)$$

In the absence of Yukawa couplings, the scalar and fermionic fields carry independent conserved $U(1)$ charges. A linear combination of these charges is gauged while the orthogonal one remains global. We assume that there are N fermionic degrees of freedom, labeled by $\alpha=1\ldots N$, with gauge coupling e' and negligible mass. Realistic scenarios could involve condensates of electrically charged mesonic fields, with characteristic scales for their potentials $\mathcal{O}$(100 MeV–1 GeV), and Higgs or squark fields, with characteristic scales $\mathcal{O}$(100 GeV–1 TeV). In all these cases, neutralizing fermions, such as electrons, can be considered effectively massless.

The equation of motion (3) is not altered by the presence of fermions, but Eq. (4) becomes

$$g''+\frac{2}{r}g'-e^2f^2g-ee'\psi_\alpha^\dagger\psi^\alpha=0. \quad (11)$$

Instead of solving the Dirac equation, we approximate the fermions as a non-interacting Fermi gas with position dependent density. This is the Thomas-Fermi approximation [10]. The fermionic $U(1)$ charge and energy density are

$$\langle\psi_\alpha^\dagger\psi^\alpha\rangle=N\rho_\psi=N\frac{k_F^3}{3\pi^2},$$
$$\langle\psi_\alpha^\dagger(-i\vec{\alpha}\cdot\vec{\nabla})\psi^\alpha\rangle=N\epsilon_\psi=N\frac{k_F^4}{4\pi^2}, \quad (12)$$

in terms of the Fermi momentum k_F. The Dirac equation for a fermion near the Fermi surface results in the expression

$$\mu_\psi=k_F(r)+e'A_0(r)=k_F(r)+\frac{e'}{e}[\omega-g(r)]. \quad (13)$$

We see that μ_ψ can be interpreted as the chemical potential, i.e., the energy cost in order to add an extra fermion on the top of the Fermi sea. The fermions rearrange themselves so that μ_ψ is position independent. It is convenient to define the gauge-invariant chemical potential

$$\tilde{\mu}=\mu-\frac{e'}{e}\omega=k_F(r)-\frac{e'}{e}g(r). \quad (14)$$

The total energy is now given by

$$E=4\pi\int r^2dr\left[\frac{1}{2}f'^2+\frac{1}{2e^2}g'^2+\frac{1}{2}f^2g^2+U(f)+N\epsilon_\psi\right.$$
$$\left.+(\vec{E}\cdot\vec{\nabla}A_0+Ne'\rho_\psi A_0+e\rho_\phi A_0)\right], \quad (15)$$

where ρ_ϕ is given by Eq. (5), ρ_ψ by the first of Eqs. (12), and $\vec{E}$ is the electric field. The equations of motion can be obtained by minimizing the energy under constant scalar and fermionic charge. This can be achieved through the use of Lagrange multipliers ω and μ_ψ. Minimization of $E-\omega\int\rho_\phi dV-\mu_\psi N\int\rho_\psi dV$ with respect to A_0, f, and k_F results in Eqs. (11), (3), and (13), respectively. Finally, the quantity in parentheses in the right-hand side of Eq. (6) vanishes through the application of Gauss' law, Eq. (11).

The above considerations provide a simple method for the determination of the properties of large Q balls. We are interested in the "thin-wall" limit, in which the effects of the surface of the Q ball may be neglected. (A more careful discussion of the validity of this approximation is given in the next section.) In this limit, the total energy is given by

$$E=\left(\frac{1}{2}f^2g^2+U(f)+N\frac{k_F^4}{4\pi^2}\right)V, \tag{16}$$

with V the volume of the Q ball. The charges of the scalar condensate and the fermions are

$$Q_\phi=f^2gV, \quad Q_\psi=N\frac{k_F^3}{3\pi^2}V. \tag{17}$$

In terms of the constant scalar and fermion charges Q_ϕ and Q_ψ, the total energy to be minimized is given by

$$E=\frac{1}{2}\frac{Q_\phi^2}{f^2V}+U(f)V+\frac{(3\pi^2)^{4/3}}{4\pi^2N^{1/3}}\frac{Q_\psi^{4/3}}{V^{1/3}}. \tag{18}$$

Minimization with respect to f and use of the first of Eqs. (17) gives

$$fg^2=\frac{dU(f)}{df}\equiv U'. \tag{19}$$

This relation could have been obtained by requiring that Eq. (3) be satisfied for constant fields. The existence of such a solution for Eq. (11) leads to

$$ef^2g=-e'N\frac{k_F^3}{3\pi^2}. \tag{20}$$

This implies

$$e'Q_\psi+eQ_\phi=0 \tag{21}$$

and guarantees the electric neutrality of the interior of the Q ball.

We can also obtain the equilibrium volume of our large fermion Q ball. It is given by

$$V=\frac{Q_\phi}{\sqrt{f^3U'}}. \tag{22}$$

Minimization of Eq. (16) with respect to V results in the relation

$$U(f)=\frac{1}{2}f^2g^2+\frac{1}{3}N\frac{k_F^4}{4\pi^2}=\frac{1}{2}f^2g^2+\frac{1}{3}\epsilon_\psi. \tag{23}$$

It can be put in a more convenient equivalent form, for which the scaling between Q_ϕ and V is explicit. Expressed solely in terms of f it takes the form

$$2U=fU'+\frac{(3\pi^2)^{4/3}}{6\pi^2}\left|\frac{e}{e'}\right|^{4/3}\frac{1}{N^{1/3}}f^2[U']^{2/3}. \tag{24}$$

Equations (19), (20), and (24) uniquely determine the values of f, g, k_F in the interior of a large Q ball. As a consequence, $\tilde{\mu}$ can be also specified through Eq. (14). It is now obvious that the total energy scales linearly with Q_ϕ for a given set of values for f and g

$$\frac{E}{Q_\phi}=\left[\frac{1}{2}\sqrt{\frac{U'}{f}}+\frac{U}{\sqrt{f^3U'}}+\frac{(3\pi^2)^{4/3}}{4\pi^2}\left|\frac{e}{e'}\right|^{4/3}\frac{1}{N^{1/3}}(f^3U')^{1/6}\right]. \tag{25}$$

For massless fermions, the stability condition $\min(E/Q_\phi)<\sqrt{U''(0)}$ guarantees that a large gauged Q ball cannot disintegrate into a collection of free particles. One could also consider the possibility that a ϕ particle is surrounded by a "cloud" of fermions (or the other way around), so that the resulting "atom" is approximately neutral. A collection of such states would probably be energetically favorable to a collection of free particles, due to the electrostatic attraction. However, for couplings $e,|e'|\lesssim 1$, the electrostatic binding energy is expected to be much smaller than the mass of the free scalars, similar to the situation in normal atoms. For this reason, the above relation gives a sufficiently accurate criterion for the classical stability of Q balls.

In the limit $e\to 0$, Eqs. (19)–(23) give $k_F=0$ and the well-known conditions for the existence of global Q balls are reproduced: $\omega^2=E^2/Q^2=2U(f)/f^2=U'/f$ [1].

IV. NUMERICAL SOLUTIONS

In this section we present numerical solutions of the equations of motion (11), (3), and (13). The two differential equations require four boundary conditions. We impose $f'(r=0)=g'(r=0)=0$, so that there are no singularities at the center of the Q balls. We also impose $f(r=\infty)=0$ and $g'(r=\infty)=0$, so that the solutions outside the Q balls correspond to the normal vacuum. We use a potential of the form $U(f)=f^2/2-f^4/4+\lambda^2f^6/6$, in order to make comparisons with the results of Ref. [9]. For the same reason we choose $\lambda^2=0.2$ and $e=0.1$. We assume that there are $N=10$ fermionic species of unit charge $e'=-0.2$. A large number of species results in a small fermionic kinetic energy that helps to keep the Q balls classically stable. Moreover, values $N=\mathcal{O}(10)$ are typical of realistic theories, such as the MSSM. All dimensionful quantities are considered to be renormalized with respect to the mass term in the potential (set equal to 1).

Q ball solutions of various sizes are obtained by fixing the value of μ and varying ω. The chemical potential μ is assumed to be negative. The reason is apparent through Eq. (13). If we would like to interpret A_0 as the electrostatic potential, we must choose a gauge such that $A_0(r)\propto r^{-1}$ for large r. By taking μ negative, we expect that k_F will become 0 at a finite radial distance. We assume that there are no fermions at larger distances, so that Eq. (13) is inapplicable. Instead we impose $k_F=0$ in Eq. (11), which results in the

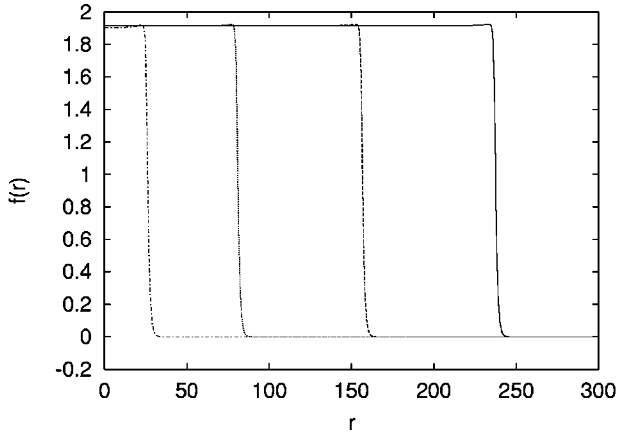

FIG. 1. The magnitude of the scalar field *f* as a function of the radial distance *r* for *Q* balls of increasing size.

expected behavior for $A_0(r)$. Positive values of μ would result in a non-zero fermionic density at arbitrary distances from the center of the *Q* ball.

In Figs. 1–3 we present a series of *Q* ball solutions of increasing size. In Fig. 1 we plot the scalar field *f* as a function of the radial distance from the center of the *Q* ball. We observe that $f(r)$ behaves as a step function to a very good approximation, even for fairly small *Q* balls. In Fig. 2 we depict the magnitude of the electric field for the same solutions. The smallest *Q* ball has the strongest electric field. The field vanishes at the center for symmetry reasons, but quickly grows with *r*. For large enough *r* it falls $\propto r^{-2}$. For larger *Q* balls the electric field is zero in the interior, because of the cancellation of the charge of the scalar field by that of the fermionic gas. The electric field is non-zero near the surface, while it falls again $\propto r^{-2}$ for large *r*.

In Fig. 3 we plot the scalar and fermionic charge densities as a function of the radial distance. For the smallest *Q* ball the fermions are not capable to neutralize the interior. There is a mismatch between the scalar and fermionic densities. Moreover, there is a large concentration of scalar charge near the surface. This is a result of the electrostatic repulsion that forces the positive unit charges to maximize the distance

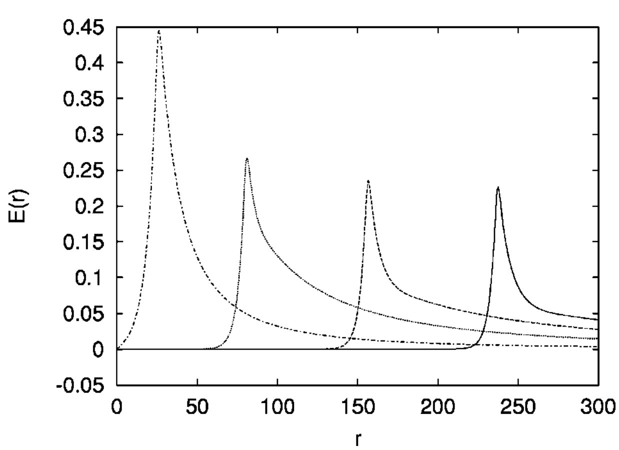

FIG. 2. The electric field *E* as a function of the radial distance *r* for *Q* balls of increasing size.

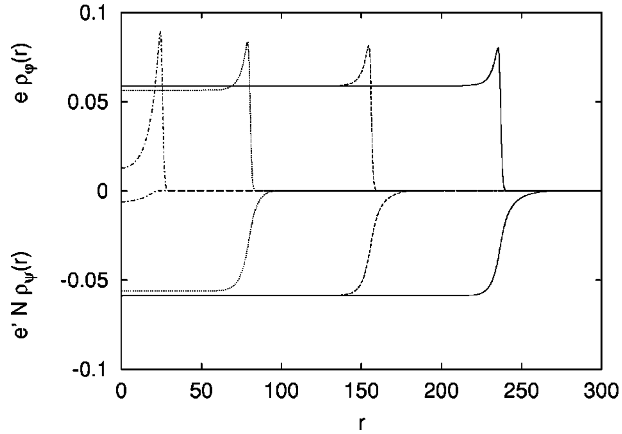

FIG. 3. The charge densities of the scalar condensate and the fermionic gas as a function of the radial distance *r* for *Q* balls of increasing size.

between themselves. As the size of the *Q* ball increases, the charge densities in the interior become opposite to each other. For large *Q* balls their magnitude is independent of the radius. This is the ''thin-wall'' limit we discussed in the previous section. The values of *f*, *g*, and k_F in the interior (and, therefore, the charge densities) should be uniquely determined by Eqs. (19)–(23). We have checked that $f(r=0)$, $g(r=0)$, and $k_F(r=0)$ for the large *Q*-ball solutions depicted in Figs. 1–3 satisfy Eqs. (19)–(23) with an accuracy of better than 1%.

A particular question merits some discussion at this point. From Figs. 1–3 one could infer naively that the profile of the surface is constant for large *Q* balls and merely displaced at different radii *R*. This would mean that the electric field is the same near the surface for all large *Q* balls. Such a field can only be produced if the net surface charge density is constant and the net surface charge scales $Q_s \propto R^2$. One implication would be that the electrostatic contribution to the total energy of the system $\sim Q_s^2/R \propto R^3$ would scale proportionally to the volume. As a result, our assumption that the surface effects are negligible in the ''thin-wall'' limit would be invalid. However, the numerical solutions do not confirm this picture. The shape of the numerical solution varies slightly at the surface even for large *Q* balls. The electric field at the surface becomes smaller for increasing radius, while the fermionic density is modified appropriately. Numerically we have not identified any residual surface effect. Moreover, we expect that a more rigorous treatment of the fermionic cloud that surrounds the *Q* ball would support this conclusion. Our simple approximation of the fermions as a non-interacting gas is adequate for the interior but very crude near the surface. We expect that, in a more careful treatment of the surface, a surrounding fermionic cloud will neutralize completely the *Q* ball (similarly to the neutralization of atoms). In this picture, the surface effects would be even less important in the ''thin-wall'' limit.

In Fig. 4 we plot the energy to charge ratio as a function of the charge Q_ϕ of the scalar condensate. The fermionic charge Q_ψ may differ substantially from Q_ϕ for small *Q* balls. As we have assumed that the fermions are massless

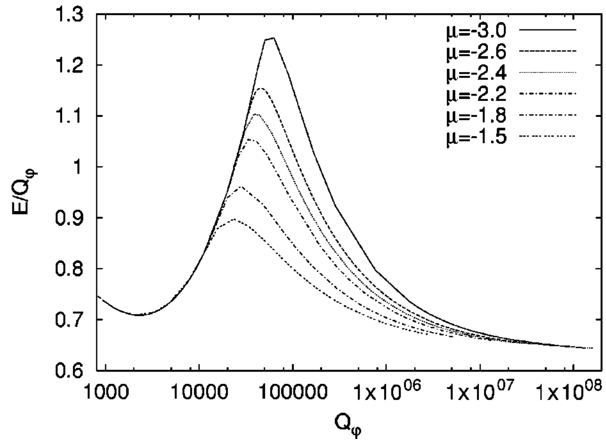

FIG. 4. The energy to charge ratio E/Q_ϕ as a function of the scalar charge Q_ϕ of the Q ball.

and normalized everything with respect to the scalar mass term, the classical stability requirement is $E/Q_\phi < 1$. In Fig. 4 we observe a series of curves that correspond to different (negative) values of μ. The fermionic content of a small Q ball is controlled through μ and, for the same Q_ϕ, the various curves have different ratios Q_ψ/Q_ϕ. In Fig. 5 we plot Q_ψ/Q_ϕ as a function of Q_ϕ for the same range values of μ as in Fig. 4. We observe that Q_ψ/Q_ϕ tends to increase with decreasing $|\mu|$. As the ratio E/Q_ϕ decreases for decreasing $|\mu|$, we conclude that, for fixed Q_ϕ, the Q balls become more stable by absorbing fermions and increasing their fermionic content. A limit to the process of fermion accretion is set by the requirement of a positive chemical potential, so that the fermions are bound to the Q ball.

For $Q_\phi \lesssim 10^4$, the fermionic content becomes negligible and we obtain the gauged Q balls of Ref. [9]. For $Q_\phi \lesssim 10^3$ the ratio E/Q_ϕ increases because of the contribution of the derivative terms to the energy [9]. For $Q_\phi \gtrsim 10^7$, $Q_\psi/Q_\phi = 1$ and the energy to charge ratio has a very weak dependence on μ and Q_ϕ. In this region the "thin-wall" approximation is valid. The gauge invariant quantity $\tilde{\mu}$ of Eq. (14) is almost constant. Asymptotically for $Q_\phi \to \infty$, the properties of the Q balls are completely determined by the values of f, g, and k_F in the interior as given by the solution of Eqs. (19)–(23).

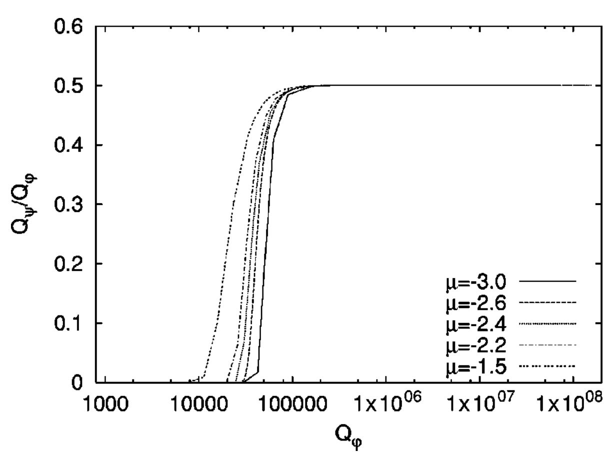

FIG. 5. The ratio of scalar to fermionic charge Q_ψ/Q_ϕ as a function of the scalar charge Q_ϕ of the Q ball.

TABLE I. The energy to charge ratio E/Q_ϕ in the "thin-wall" limit, for various values of the parameters N and e'.

	$N=1$	$N=2$	$N=4$	$N=6$	$N=10$	$N=15$	$N=20$
$e'=-0.1$	1.954	1.654	1.403	1.277	1.135	1.036	0.972
$e'=-0.2$	1.021	0.878	0.759	0.700	0.635	0.589	0.560
$e'=-0.3$	0.724	0.633	0.559	0.522	0.481	0.453	0.435

For our choice of parameters, the biggest Q balls are the most stable. Moreover, as we discussed above, the stability is enhanced by the absorption of fermions. These results suggest an efficient accretion mechanism for large Q balls with important astrophysical implications [11]. For different parameters it is possible that small Q balls with $Q_\phi \sim 10^3$ and without fermions become the most stable states. However, it is apparent from Fig. 4 that a barrier would still separate the large from the small Q balls. The decay of large Q balls into smaller fragments and free fermions would involve tunnelling and probably would proceed at a very slow rate.

The dependence of the ratio E/Q_ϕ on N and e' in the "thin-wall" limit is given in Table I. The various values have been obtained through the numerical solution of the algebraic system of Eqs. (19)–(23) for $e=0.1$. We observe that a small number N of fermionic species with $|e'|=e$ results in a high energy to charge ratio and, therefore, unstable Q balls. This behavior is caused by the big contribution from the fermionic kinetic energy. Large values of N permit the distribution of the compensating charge among various species, thus reducing the fermionic energy $\propto N^{-1/3}$.

V. LARGE GAUGED Q BALLS WITH TWO SCALAR CONDENSATES

Another possibility is that two scalar condensates with opposite charges form in the interior of a gauged Q ball. An appropriate Lagrangian density is

$$\mathcal{L} = \frac{1}{2}\partial_\mu f \partial^\mu f + \frac{1}{2} f^2(\partial_\mu \theta_1 - eA_\mu)^2 + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi + \frac{1}{2}\chi^2(\partial_\mu \theta_2 - e'A_\mu)^2 - U(f,\chi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{26}$$

The two scalar fields carry independent conserved $U(1)$ charges, a linear combination of which is gauged. We assume a time dependence for the two condensates of the form $\theta_1 = \omega_1 t$ and $\theta_2 = \omega_2 t$. The resulting equations of motion are

$$f'' + \frac{2}{r} f' + f(\omega_1 - eA_0)^2 - \frac{\partial U(f,\chi)}{\partial f} = 0 \tag{27}$$

$$\chi'' + \frac{2}{r}\chi' + \chi(\omega_2 - e'A_0)^2 - \frac{\partial U(f,\chi)}{\partial \chi} = 0 \tag{28}$$

$$A_0''+\frac{2}{r}A_0'+ef^2(\omega_1-eA_0)+e'\chi^2(\omega_2-e'A_0)=0. \tag{29}$$

The energy of the system is given by the expression

$$E=4\pi\int r^2dr\left\{\frac{1}{2}A_0'^2+\frac{1}{2}f'^2+\frac{1}{2}f^2(\omega_1-eA_0)^2+\frac{1}{2}\chi'^2\right.$$
$$+\frac{1}{2}\chi^2(\omega_2-e'A_0)^2+U(f,\chi)+[(\vec{E}\cdot\vec{\nabla}A_0$$
$$\left.+ef^2(\omega_1-eA_0)A_0+e'\chi^2(\omega_2-e'A_0)A_0]\right\}. \tag{30}$$

Similarly to the discussion in Sec. III, the equations of motion (27)–(29) can be obtained by minimizing the total energy under constant total charges of the two scalar condensates. The expression in the second line of Eq. (30) vanishes through application of Gauss' law, Eq. (29).

The numerical solution of the three second-order differential equations (27)–(29) is more difficult than in the case of a scalar condensate with compensating fermions. In that case we had to integrate two second-order differential equations and an algebraic one. Moreover, we expect a qualitative behavior very similar to the one studied in Secs. III and IV. For a large Q ball to remain classically stable, the net charge in its interior must be zero. A possible mismatch at the surface could result in non-zero electrostatic energy. However, in the "thin-wall" limit, this contribution is expected to become negligible. For this reason, we limit our discussion to the analytical treatment of the "thin-wall" limit, which is more useful for practical applications.

In this limit, the total energy is given by

$$E=\left(\frac{1}{2}f^2g_1^2+\frac{1}{2}\chi^2g_2^2+U(f,\chi)\right)V, \tag{31}$$

with V the volume of the Q ball and $g_1=\omega_1-eA_0$ and $g_2=\omega_2-e'A_0$. The charges of the two scalar condensates are

$$Q_\phi=f^2g_1V, \quad Q_\chi=\chi^2g_2V. \tag{32}$$

They are taken to satisfy an electric charge neutrality condition

$$eQ_\phi+e'Q_\chi=0. \tag{33}$$

Keeping each of them fixed means that we must minimize the quantity

$$E=\frac{1}{2}\frac{Q_\phi^2}{f^2V}+\frac{1}{2}\frac{Q_\chi^2}{\chi^2V}+U(f,\chi)V. \tag{34}$$

Minimization with respect to V results in the relation

$$U(f,\chi)=\frac{1}{2}f^2g_1^2+\frac{1}{2}\chi^2g_2^2. \tag{35}$$

Three more constraints can be obtained by requiring that Eqs. (27)–(29) be satisfied for constant fields. They are

$$fg_1^2=\frac{\partial U(f,\chi)}{\partial f}, \tag{36}$$

$$\chi g_2^2=\frac{\partial U(f,\chi)}{\partial \chi}, \tag{37}$$

$$ef^2g_1+e'\chi^2g_2=0. \tag{38}$$

The first two could have been obtained through the minimization of the total energy of Eq. (34) with respect to f and χ. The last equation guarantees the electric neutrality of the interior of the Q ball. The four equations (35)–(38) uniquely determine the gauge-invariant quantities f, χ, g_1, and g_2 in the interior of a large Q ball. As a consequence, E, Q_ϕ, and Q_χ are also specified through Eqs. (31) and (32). Similarly to the fermionic case, the fixed charges Q_ϕ and Q_χ scale linearly with the volume for fixed values of their interior field variables f, χ, g_1, and g_2. One reason for this is the charge neutrality condition in Eq. (33) which they satisfy. Hence the total energy of the double condensate configuration scales linearly with respect to the scalar charge $Q_\phi=|e'/e|Q_\chi$. The classical stability condition becomes $\min(E)<m_\phi Q_\phi+m_\chi Q_\chi$, with $m_\phi^2=\partial^2U(0,0)/\partial\phi^2$, $m_\chi^2=\partial^2U(0,0)/\partial\chi^2$ the masses of the two scalars at the vacuum at $\phi=\chi=0$.

VI. CONCLUSIONS

The main emphasis in the studies of Q balls has been on theories with global $U(1)$ symmetries. Theories with local $U(1)$ symmetries can support Q balls as well. However, in the absence of a neutralizing mechanism, the electrostatic repulsion destabilizes the Q balls with significant charge. In the main part of this paper we pointed out that gauged Q balls can be stabilized through the neutralization of the scalar condensate by fermions of opposite charge. The total energy is increased because of the kinetic energy of the fermions. However, the resulting configuration can be stable even for an arbitrarily large charge of the scalar condensate.

From a cosmological perspective, the neutralization of gauged Q balls is expected. For example, one could envision the existence of electric Q balls that could be produced during phase transitions [5] when the Universe passes through an electric-charge breaking vacuum [12]. It seems likely that several fermionic species could be trapped within the Q ball during its formation. The ones with a charge of similar sign to the scalar condensate will be expelled, so that the resulting object will remain approximately neutral.

The existence of large electric fields can lead to spontaneous pair creation. The presence of a strong electrostatic field at the surface of the Q ball can separate a virtual fermion-antifermion pair and bring the particles on mass shell [13]. The fermion will be attracted towards the surface, while the antifermion will be expelled. In the vacuum, the critical field strength is $E_{crit}=m_\psi^2/|e'|$. Our assumption that the fermion mass is much smaller than the typical scale of the potential of the scalar field implies that this mechanism is very efficient. In the interior of a Q ball, the pair creation stops only when the fermionic energy levels are populated up to a Fermi momentum comparable to the scale of the scalar field potential. It seems, therefore, likely that large gauged Q balls can be neutralized through this mechanism, instead of disintegrating.

We mention that evaporation from the surface is possible if the scalar field has decay channels into light species [14]. In this case, simultaneous evaporation of the decay products and fermions maintains the approximate neutrality of the Q ball.

The tendency of gauged Q balls to trap fermions in their interior could have interesting experimental consequences. Even though we concentrated on massless fermions, heavy exotic species may have found their way to the interior of gauged Q balls. Thus the discovery of a Q ball may lead to the additional discovery of the exotic species trapped in its interior. The fact that the energy per charge of a Q ball is reduced when its fermionic content is increased (up to neutralization) indicates an efficient accretion mechanism with important astrophysical implications [11].

We also discussed the possibility of neutralization of a gauged Q ball through the presence of two scalar condensates of opposite charge in its interior. In this case the formation of Q balls seems less likely than in the case of one scalar condensate with compensating fermions. For neutral Q balls to be produced, two condensates with the appropriate properties (values of $f,\ \chi,\ \omega_1,\ \omega_2$) must be assumed to be generated dynamically after a phase transition [5]. This should be more difficult than the trapping of fermions from the thermal bath in the region with a non-zero charged condensate.

Finally, we point out that the neutralization mechanism is expected to work for general potentials of the scalar field. In particular, we expect it to be applicable to the case of potentials with flat directions, such as the ones appearing in supersymmetric extensions of the standard model. In this case the global Q balls do not approach the ''thin-wall'' limit, even though they can become very big, with energy that scales $E \propto Q^{3/4}$ [15]. The gauged Q balls with similar potentials cannot reach large sizes, unless a neutralization mechanism (through trapping of fermions for example) eliminates the electrostatic repulsion.

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