Stability analysis of anisotropic inflationary cosmology

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The stability analysis of an anisotropic inflationary universe of the four-dimensional Neveu-Schwarz–Neveu-Schwarz string model with a nonvanishing cosmological constant is discussed in this paper. The accelerating expansion solution found earlier is shown to be stable against the perturbations with respect to the dilaton and axion fields once the dilaton field falls close to the local minimum of the symmetry-breaking potential. This indicates that the Bianchi type-I space tends to evolve to an isotropic flat Friedmann-Robertson-Walker space. This expanding solution is also shown to be stable against the perturbation with respect to anisotropic spatial directions.

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I. INTRODUCTION

Observations of the cosmic microwave background radiation, galaxies, and other astronomical objects reveal that our universe, on a very large scale, is remarkably uniform [1–3] and is currently under accelerated expansion [4–7]. Therefore, a physically acceptable cosmology should provide a self-contained mechanism to smear out the primordial anisotropy and achieve an accelerating expansion at the present time. Furthermore, it is firmly believed that Einstein's theory of general relativity breaks down at high enough energy, which, at least, occurred during the early epoch of our Universe. It is known that string theories are promising candidates for all particles and interactions, including gravity, in a unified formulation. Thus the astrophysical and cosmological implications of string theories have become a highly developing research theme (see Ref. [8] and references therein).

An anisotropic cosmology was considered recently in the framework of four-dimensional Neveu-Schwarz–Neveu-Schwarz (NS-NS) effective string theory in which the gravitational field is coupled with the dilaton and axion fields [9]. In the de Sitter geometry configuration, i.e., with a positive cosmological constant $\Lambda>0$, an inflationary solution can be found in an exact parametric form. At large time limit, this universe is made isotropic and its expansion is accelerating, which is consistent with our current astronomical observations. A stability analysis of any solution may provide more information of the theory under consideration [10–18]. Therefore it is interesting and is the purpose of the present paper to investigate the stability conditions of this anisotropic inflationary string cosmology. Our analysis indicates that the inflationary solution found in Ref. [9] remains *stable* when the potential $U(\phi)$ is close to the local minima of the potential.

This paper is organized as follows. In Sec. II we briefly review the NS-NS effective string theory and the exact solution found in Ref. [9]. The stability analysis with respect to the perturbations of the dilaton and axion fields will be performed in Sec. III. The perturbation with respect to the gravitational field will be studied in Sec. IV. In Sec. V, we will draw some conclusions.

II. ANISOTROPIC SOLUTION

The four-dimensional NS-NS effective action, which is common to both the heterotic and the type-II string theories [8], is given by

$$\mathcal{S}=\int d^4x\sqrt{-g}\left\{R-\kappa(\partial\phi)^2-\frac{1}{12}e^{-4\phi}H_{[3]}^2-U\right\}, \quad (1)$$

where ϕ is the so-called dilaton and $H_{\mu\nu\lambda}=\partial_{[\mu}B_{\nu\lambda]}$ is a totally antisymmetric tensor. Moreover, κ denotes the generalized dilaton coupling constant ($\kappa=2$ for typical superstring theories) and $U=U(\phi)$ is a dilaton potential. The field equations of the action (1) can be shown to be

$$R_{\mu\nu}-\kappa\partial_\mu\phi\partial_\nu\phi-\frac{1}{2}g_{\mu\nu}U-\frac{1}{4}e^{-4\phi}\times\left(H_{\mu\alpha\beta}H_\nu{}^{\alpha\beta}-\frac{1}{3}g_{\mu\nu}H^2\right)=0, \quad (2)$$

$$\nabla_\mu(e^{-4\phi}H^{\mu\nu\lambda})=0, \quad (3)$$

$$\nabla^2\phi+\frac{1}{6\kappa}e^{-4\phi}H^2-\frac{1}{2\kappa}\partial_\phi U=0. \quad (4)$$

In addition, the totally antisymmetric tensor H obeys the Bianchi identity $\partial_{[\mu}H_{\nu\lambda\rho]}=0$.

In four dimensions, any three-form can be mapped one-to-one onto its dual one-form. This mapping relates the three rank tensor H to a pseudo-vector A via $H={}^*A$ with $A=A_\mu dx^\mu$ the dual one-form. Moreover, one can show that the tensor H can be solved to give

$$H^{\mu\nu\lambda}=e^{4\phi}\,\epsilon^{\mu\nu\lambda\rho}\,\partial_\rho h \quad (5)$$

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following the field equation (3). Here the totally antisymmetric tensor ϵ denotes the Levi-Civita tensor and $h=h(t)$ is known as the Kalb-Ramond axion. Note that we use the convention that $\epsilon^{\mu\nu\lambda\rho}=-\delta^{\mu}_{[0}\delta^{\nu}_{1}\delta^{\lambda}_{2}\delta^{\rho}_{3]}/\sqrt{-g}$ in this paper. In addition, the above solution holds if and only if the first homology group for the space considered is trivial [19–21]. Consequently, the Bianchi identity shown earlier becomes

$$\partial_\mu(\sqrt{-g}\,e^{4\phi}\,\partial^\mu h)=0. \tag{6}$$

We will consider the case in which the potential of the dilaton field is a positive constant

$$U(\phi)=\Lambda\geqslant 0, \tag{7}$$

which represents a de Sitter space. An exact solution for an anisotropic (Bianchi type-I) cosmology has been found in Ref. [9]. Note that the line element of a Bianchi type-I space, an anisotropic generalization of the flat Friedmann-Robertson-Walker (FRW) geometry, can be written as

$$ds^2=-dt^2+a_1^2(t)dx^2+a_2^2(t)dy^2+a_3^2(t)dz^2, \tag{8}$$

with $a_i(t)$, $i=1,2,3$ the expansion factors on each spatial directions. With the identities (5) and (7), the field equations (2), (4), and (6) will take the following form:

$$3\dot\theta+\sum_{i=1}^{3}\theta_i^2+\kappa\dot\phi^2+\frac{1}{2}e^{4\phi}\dot h^2-\frac{1}{2}\Lambda=0, \tag{9}$$

$$\frac{1}{V}\partial_t(V\theta_i)-\frac{1}{2}\Lambda=0, \quad i=1,2,3, \tag{10}$$

$$\ddot h+3\theta\dot h+4\dot\phi\dot h=0, \tag{11}$$

$$\frac{1}{V}\partial_t(V\dot\phi)-\frac{1}{\kappa}e^{4\phi}\dot h^2=0. \tag{12}$$

Here we have introduced the *volume scale factor*, $V:=\Pi_{i=1}^3 a_i$, *directional Hubble factors*, $\theta_i:=\dot a_i/a_i$, $i=1,2,3$, and the *mean Hubble factor*, $\theta:=\Sigma_{i=1}^3\theta_i/3=\dot V/3V$ for convenience. In addition, we will also introduce two basic physical observational quantities in cosmology: the *mean anisotropy parameter*, $A:=\Sigma_{i=1}^3(\theta_i-\theta)^2/3\theta^2$, and the *deceleration parameter*, $q:=\partial_t\theta^{-1}-1$. Note that $A\equiv 0$ for an isotropic expansion. Moreover, the sign of the deceleration parameter indicates how the Universe expands. Indeed, a positive sign corresponds to "standard" decelerating models, whereas a negative sign indicates an accelerating expansion.

It was shown that the general exact solution of this model can be parametrized in the following form [9]:

$$a_i(\tau)=a_{i0}\sinh^{\alpha_i^+}\frac{\tau}{2}\cosh^{\alpha_i^-}\frac{\tau}{2}, \quad i=1,2,3 \tag{13}$$

$$e^{2\phi(\tau)}=\varphi_0^2\left(\tanh^{\omega}\frac{\tau}{2}+\tanh^{-\omega}\frac{\tau}{2}\right), \tag{14}$$

$$h(\tau)=h_0+\frac{\kappa\sqrt{\phi_0}}{C}\frac{\tanh^{2\omega}(\tau/2)-1}{\tanh^{2\omega}(\tau/2)+1}. \tag{15}$$

Therefore, one has

$$\theta(\tau)=\sqrt{\frac{\Lambda}{6}}\coth\tau, \tag{16}$$

$$V(\tau)=V_0\sinh\tau, \tag{17}$$

where $\tau:=\sqrt{3\Lambda/2}(t-t_0)$, $\alpha_i^{\pm}:=1/3\pm\sqrt{2/3\Lambda}K_i/V_0$, $\omega:=\sqrt{8\phi_0/3\Lambda}/V_0$ and $\varphi_0^2:=\sqrt{C^2/8\kappa\phi_0}$. Here t_0, a_{i0}, K_i, $C\geqslant 0$, $\phi_0>0$, and h_0 are free parameters representing the constants of integration. In addition, one writes $V_0=\Pi_{i=1}^3 a_{i0}/2$ for convenience. One can also show that there is an additional constraint $\Sigma_{i=1}^3 K_i=0$ following the field equations. These integration constants also obey the consistency condition,

$$K^2:=\sum_{i=1}^{3}K_i^2=\Lambda V_0^2-\kappa\phi_0, \tag{18}$$

which follows from Eq. (9). Thereby the mean anisotropy and the deceleration parameter are given by

$$A(\tau)=\frac{2K^2}{\Lambda V_0^2}\,\mathrm{sech}^2\tau, \tag{19}$$

$$q(\tau)=3\,\mathrm{sech}^2\tau-1. \tag{20}$$

This exact solution indicates that the evolution of the Bianchi type-I universe starts from a singular state, but with finite values of the mean anisotropy and deceleration parameters. In the large time limit the mean anisotropy tends to zero, $A\to 0$, and the Universe approaches an isotropic inflationary de Sitter phase with a negative deceleration parameter, $q<0$, providing an accelerating expanding Universe consistent with the present observations. Furthermore, in the large time limit the dilaton and axion fields become constants, $\lim_{t\to\infty}h(t)=h_0=\text{constant}$, $\lim_{t\to\infty}e^{2\phi(t)}=2\varphi_0^2=\text{constant}$. Note that the dynamics and evolution of the Universe is determined only by the presence of a cosmological constant (or a dilaton field potential). There is no direct coupling between the metric and the dilaton and axion fields other than the constraint on the integration constants.

Note that the mixmaster model [22–26] discussed earlier deals with a model with perfect fluid. Earlier work focuses on the behavior of the solution near the singularity. The static field equation studied in Ref. [9] is similar to the mixmaster model with axion and dilaton behaving like the perfect fluid. The solution shown in this section is, however, the first time such an exact solution is found for the NS-NS string effective theory that exhibits the desired property driving an initially anisotropic Bianchi type-I space to an isotropic space. We will focus on the stability property of this solution in the following section.

III. PERTURBATION AND STABILITY

Perturbations of the fields of a gravitational system against the background evolutionary solution should be checked to ensure the stability of the exact or approximated background solution. In principle, the stability analysis should be performed against the perturbations of all possible fields in all possible manners subject to the field equations and boundary conditions of the system. In the following section, we will divide the perturbations into two disjoint classes: (a) the perturbations of the scale factors, or equivalently the metric field, and the axion field; and (b) the perturbations of the dilaton and axion fields.

Note further that the axion field can be solved as a combination of the dilaton field and the scale factors according to Eq. (11). The result is

$$\dot{h}=C\,e^{-4\phi}\,V^{-1}, \tag{21}$$

with a constant of integration $C\geqslant 0$. Here $C=0$ means that the axion field does not couple to any other field. Therefore, the effect of the perturbation of the axion field can be replaced by Eq. (21). In fact, Eq. (26) indicates that if one perturbs the axion field, one has to perturb either the dilaton field or a_i altogether unless the constant C vanishes. Note also that Eq. (21) indicates that $\dot{h}\sim a^{-3}$, which is much smaller than a in the large time limit where $a_i \rightarrow a$ for all directions. This is an indication that the effect of the axion field perturbation can be ignored in the metric perturbation we will discuss later.

Once we replace the effect of the axion field in the field equations (9), (10), and (12), one can further simplify the perturbations into two disjoint class: (a) the perturbations of the scale factors and (b) the perturbations of the dilaton fields. We will argue that the most complete stability conditions we are looking for can be obtained from class (a) and class (b) perturbations; even the backreaction of the scalar field perturbation on the metric field perturbations is known to be important [27]. We will show that this backreaction does not bring in any further restriction on the stability conditions.

The reason is rather straightforward. One can write the linearized perturbation equation as

$$D^i_{a_j}\delta a_j + D^i_{\phi}\delta\phi = 0 \tag{22}$$

for the system we are interested. Here the axion perturbations have been replaced according to Eq. (21). Moreover, perturbations are defined as $a_i=a_i^0+\delta a_i$ and $\phi=\phi_0+\delta\phi$ with the index 0 denoting the background field solution. Note also that the operators $D^i_{a_j}$ and D^i_{ϕ} denote the differential operator one obtained from the linearized perturbation equation with all fields evaluated at the background solutions. The exact form of these differential operators will be shown later in the following arguments.

One is looking for stability conditions that the field parameters must obey in order to keep the evolutionary solution stable. One can show that class (a) and class (b) solutions are good enough to cover all domain of stability conditions. Let us denote the domain of solutions to class (a), (b), and (a+b) stability conditions as $S(a), S(b)$, and $S(a+b)$ respectively. Specifically, the definition of these domains are defined by $S(a)\equiv\{\delta a_i | D^i_{a_j}\delta a_j=0\}$, $S(b)\equiv\{\delta\phi | D^i_{\phi}\delta\phi=0\}$, and $S(a+b)\equiv\{(\delta a_i,\delta\phi) | D^i_{a_j}\delta a_j + D^i_{\phi}\delta\phi=0\}$.

Therefore, one only needs to show that $S(a)\cap S(b)\subset S(a+b)$. This is because that "$D^i_{a_j}\delta a_j=0$ and $D^i_{\phi}\delta\phi=0$" imply that "$D^i_{a_j}\delta a_j + D^i_{\phi}\delta\phi=0$." On the other hand, "$D^i_{a_j}\delta a_j + D^i_{\phi}\delta\phi h=0$" does not imply that "$D^i_{a_j}\delta a_j=0$ or $D^i_{\phi}\delta\phi=0$." Hence class (a) and class (b) solutions cover all the required stability conditions we are looking for. Hence we only need to consider these two separate cases for simplicity.

In addition, one knows that any small time-dependent perturbation against the metric field is known to be equivalent to a gauge choice [28]. This can be clarified as follows. Indeed, one can show that any small coordinate change of the form $x'^{\mu}=x^{\mu}-\epsilon^{\mu}$ will induce a gauge transformation on the metric field according to $g'_{\mu\nu}=g_{\mu\nu}+D_{\mu}\epsilon_{\nu}+D_{\nu}\epsilon_{\mu}$. Therefore, a small metric perturbation against a background metric is amount to a gauge transformation of the form $a'_i=a_i+\epsilon^t\dot{a}_i$ for the Bianchi type-I metric with $\epsilon_{\mu}=(\text{constant},\epsilon_i(t))$. This is then equivalent to small metric perturbations. If a background solution is stable against small perturbation with respect to small field perturbations, one in fact did nothing but a field redefinition.

If the background solution is, however, unstable against small perturbations, e.g., the small perturbation will grow exponentially as we will show momentarily, the resulting large perturbations cannot be classified as small gauge transformation any more. Therefore, the stability analysis performed in the literature [10–18] for various models against the unstable background solution served as a very simple method to check if the system supports a stable metric field background. This is the reason why we still perform a perturbation on the metric field for stability analysis; even a small perturbation is equivalent to a gauge redefinition.

Note that one should also consider a more general perturbation with space perturbation included. The formulation is, however, much more complicated than the one we will show in this paper. We will focus on the time-dependent case for simplicity in this paper. The space-dependent perturbation analysis is still under investigation. The time-dependent analysis alone will, however, bring us much useful information for the stability conditions about the model we are interested. For example, we will show in the following subsection that the solution found in Ref. [9] remains stable as long as the scalar field falls close to any local minimum of the potential $U(\phi)$. Note again that the solution found in Ref. [9] is an exact solution only when $U=$ constant.

A. Dilaton field perturbation

In this subsection, we will consider the perturbation $\delta\phi$ with respect to the exact background solution ϕ_B given in the preceding section.

We will consider a general symmetry-breaking potential $U(\phi)$ which has at least one local minimum. The solution obtained in Sec. II remains a good approximation as long as the dilation field is close to the local minimum ($\phi=v$) of the potential U such that $U_0 := U(v)=\Lambda$. Therefore, a perturbation on ϕ field can also be considered as a test to see whether such solution tends to stabilize the FRW space or not. We will show momentarily that the result indicates that FRW space tends to be a stable final state of any Bianchi type-I space once the dilaton field approaches the local minimum of the symmetry-breaking potential.

Due to the fluctuation $\delta\phi$, the deviation of the dilaton potential, up to the first order, is

$$U(\phi_B+\delta\phi)\rightarrow U(\phi_B)+\partial_\phi U(\phi_B)\ \delta\phi=\Lambda+\partial_\phi U(\phi_B)\ \delta\phi. \tag{23}$$

By keeping the metric fields unperturbed, one can show that the perturbation equations are

$$\dot{\phi}_B\ \delta\dot{\phi}-\frac{C^2}{\kappa V_B^2}e^{-4\phi_B}\ \delta\phi=0, \tag{24}$$

$$\partial_\phi U(\phi_B)\ \delta\phi=0, \tag{25}$$

$$\frac{1}{V_B}\partial_t(V_B\ \delta\dot{\phi})+4\dot{\phi}_B\ \delta\dot{\phi}+\frac{1}{2\kappa}\partial_\phi^2 U(\phi_B)\ \delta\phi=0. \tag{26}$$

Here we use Eq. (21) in the above equations. In addition, once the dilaton field falls close enough to one of the local minimum of the symmetry-breaking potential, the dynamics of the dilaton perturbation is inevitably much smaller than the scale factor perturbations. This confirms our claims earlier that class (a) perturbation and class (b) perturbation are not of the same order of magnitude.

Note also that the Eq. (25) indicates that one of the necessary stable conditions for the background solution is that ϕ_B should be a local minimum of the dilaton potential $U(\phi)$. In fact, close to local minimum condition is the only situation we are interested in this paper.

Combining the equations of $\delta\dot{\phi}$, one finds that the dilaton perturbation $\delta\phi$ is determined by

$$\partial_t(V_B e^{4\phi_B}\ \delta\dot{\phi})+\frac{1}{2\kappa}V_B e^{4\phi_B}\partial_\phi^2 U(\phi_B)\ \delta\phi=0. \tag{27}$$

Taking the large time limit, $t\rightarrow\infty$, one can show that the background variables V_B and ϕ_B approach

$$V_B\rightarrow\frac{V_0}{2}e^{\alpha t}, \qquad \phi_B\rightarrow v, \tag{28}$$

where $\alpha := \sqrt{3\Lambda/2}$ and $v := \ln(2\varphi_0^2)/2$. In addition, Eq. (27) reduces to

$$\delta\ddot{\phi}+\alpha\ \delta\dot{\phi}+\beta\ \delta\phi=0, \tag{29}$$

with $\beta=\partial_\phi^2 U(\phi)|_{\phi=v}/2\kappa$. For the case $\partial_\phi^2 U(\phi)|_{\phi=v}=0$, i.e., $\beta=0$, the perturbations of dilaton and axion fields behave as $\delta\phi\propto\exp(-\alpha t)$ and $\delta h\propto\exp(-2\alpha t)$ indicating both fields approach zero exponentially in the large time limit. On the other hand, the dilaton perturbation can be shown to be $\delta\phi\propto\exp(\gamma t)$ with $\gamma=(-\alpha\pm\sqrt{\alpha^2-4\beta})/2$ for the case $\beta\neq 0$. This implies that $|\delta\phi|$ vanishes rapidly near the local minimum v when $\partial_\phi^2 U(\phi)|_{\phi=v}>0$. $\delta h\propto\exp[(\gamma-\alpha)t]$ also converges to zero exponentially.

In short, the necessary condition for the stability of the background solutions is that the background dilaton field must be close to the local minimum v of the dilaton potential $U(\phi)$. Consequently, the perturbed fields will approach zero exponentially provided that $\partial_\phi^2 U(\phi)|_{\phi=v}$ is non-negative. This partially reflects the fact that the background solution remains a good approximated solution to the system as long as the dilaton potential $U(\phi)\sim$ constant near $\phi=v$.

Note that we know the evolutionary solution of the system only when the dilaton potential is a constant. Nonetheless, the argument shown in this subsection indicates that the system tends to bring the Universe to the FRW space once the dilaton field falls close to local minimum of the symmetry-breaking potential as $t\rightarrow\infty$. This result provides convincing evidence for the formation of the flat FRW space evolved from a Bianchi type-I anisotropic space-time.

B. Metric perturbation

We will study the stability of the background solution with respect to perturbations of the metric in this subsection. Perturbations will be considered for all three expansion factors a_i via

$$a_i\rightarrow a_{Bi}+\delta a_i=a_{Bi}(1+\delta b_i), \tag{30}$$

hereafter. We will focus on the variables δb_i instead of δa_i from now on for convenience. Therefore, the perturbations of the following quantities can be shown to be

$$\theta_i\rightarrow\theta_{Bi}+\delta\dot{b}_i, \quad \theta\rightarrow\theta_B+\frac{1}{3}\sum_i\ \delta\dot{b}_i,$$

$$\sum_i\ \theta_i^2\rightarrow\sum_i\ \theta_{Bi}^2+2\sum_i\ \theta_{Bi}\ \delta\dot{b}_i, \quad V\rightarrow V_B+V_B\sum_i\ \delta b_i. \tag{31}$$

One can show that the metric perturbations δb_i, to the linear order in δb_i, obey the following equations:

$$\sum_i\ \delta\ddot{b}_i+2\sum_i\ \theta_{Bi}\delta\dot{b}_i-\frac{C^2}{V_B^2}e^{-4\phi_B}\sum_i\ \delta b_i=0, \tag{32}$$

$$\delta\ddot{b}_i+\frac{\dot{V}_B}{V_B}\delta\dot{b}_i+\sum_j\ \delta\dot{b}_j\ \theta_{Bi}=0, \tag{33}$$

$$\sum_i\ \delta\dot{b}_i\ \dot{\phi}_B=0. \tag{34}$$

Equation (34) indicates that $\Sigma_i\delta\dot{b}_i=0$. Therefore, Eq. (32) gives further that

$$\sum_i \delta b_i = 0. \tag{35}$$

Substituting it back to the other equations, one can show that

$$\delta\ddot{b}_i + \frac{\dot{V}_B}{V_B}\delta\dot{b}_i = 0. \tag{36}$$

Note that the background variables V_B and θ_{Bi} approach

$$V_B \to \frac{V_0}{2}e^{\alpha t}, \qquad \theta_{Bi} \to \sqrt{\Lambda/6}, \tag{37}$$

in the large time limit as $t \to \infty$. Here $\alpha \coloneqq \sqrt{3\Lambda/2}$. Note also that we keep only leading order in the above equations. Therefore, the solution for δb_i can be found to be

$$\delta b_i = c_i e^{-\alpha t}, \tag{38}$$

together with the following constraint on integration constants c_i:

$$\sum_i c_i = 0. \tag{39}$$

Moreover, the asymptotic form, in the large time limit, of background scale factors is $a_{Bi} \to a_{i0}\exp[\alpha t/3]/4^{1/3}$. Therefore, the ''actual'' fluctuations for each expansion factor, $\delta a_i = a_{Bi}\delta b_i$ is

$$\delta a_i \to \frac{a_{i0}c_i}{4^{1/3}} e^{-2\alpha t/3}. \tag{40}$$

Hence δa_i approaches zero exponentially since α is definite positive. Consequently, the background solution is *stable* against the perturbation of the graviton field.

Note that the perturbations of the three different expansion factors in Bianchi type-I cosmology are not independent due to the constraint (39). The constraint (39) indicates that scale factor perturbations along arbitrary directions are confined by the field equations. This also signifies the symmetry of the coordinate transformation among the scale factors. In fact, there should be only two independent coordinate perturbations reflecting the symmetry of the coordinate choice.

IV. CONCLUSION

In summary, we investigate the stability condition for an anisotropic inflationary string solution that evolves to the FRW space. This solution is consistent with the significant requirements constrained by present astronomical observations—homogeneity, isotropy, and accelerating expansion. We analyze in details the perturbed equations with respect to the dilaton fields and separately with respect to the expanding scale factors of the Bianchi type-I space. Our result indicates that the cosmological model considered in this paper is stable for both cases under certain constraints.

In the former case, the necessary condition for the stability of the background solutions is that the background dilaton field must be close to the local minimum v of the dilaton potential $U(\phi)$. Consequently, the perturbed fields will approach zero exponentially provided that $\partial_\phi^2 U(\phi)|_{\phi=v}$ is nonnegative. This partially reflects the fact that the background solution remains a good approximation to the system only when the dilaton potential $U(\phi)\sim$ constant near $\phi = v$.

Note that we do not know the exact evolutionary solution of the system when the dilaton potential is not close to a constant. Nonetheless, we show that the system tends to bring the Universe to the FRW space once the dilaton field falls close to local minimum of the symmetry-breaking potential as $t \to \infty$. This result provides a convincing evidence for the formation of the flat FRW space evolved from a Bianchi type-I anisotropic space-time that will work for models with a reasonable symmetry-breaking potential.

For the latter case, the perturbed expanding scale factors are also shown to approach zero exponentially in the large time limit. We also show that the perturbations of the three expanding scale factors are constrained by the field equations.

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