BPS branes in cosmology

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The possibility to study the M or string theory cosmology via 5D bulk and brane action is investigated. The role of the 4-form field in the theory of Bogomol'nyi-Prasad-Sommerfield (BPS) branes in 5D is clarified. We describe arguments suggesting that the effective 4D description of the universe in the ekpyrotic scenario should lead to contraction rather than expansion of the universe. To verify these arguments, we study the full 5D action prior to its integration over the fifth dimension. We show that if one adds the potential $V(Y)$ to the action of the bulk brane, then the metric ansatz used in the ekpyrotic scenario does not solve the dilaton and gravitational equations. To find a consistent cosmological solution one must use a more general metric ansatz and a complete 5D description of the brane interaction instead of simply adding an effective 4D bulk brane potential $V(Y)$.

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I. INTRODUCTION

During the last few years there were many attempts to construct a consistent brane cosmology, see e.g. Refs. [1–3], and references therein. One of the most interesting possibilities is to use supersymmetric Bogomol'nyi-Prasad-Sommerfield (BPS) branes in cosmology. Studies of this idea developed from the no-go theorems for nonsingular supersymmetric domain walls [4] to the construction of supergravity in singular spaces [5] where the bulk and brane actions are supersymmetric. Investigation of the BPS brane cosmology, i.e. the theory of interacting and moving near-BPS branes, has brought an additional level of complexity, both on the technical and on the conceptual level.

One of the most challenging recent attempts to construct a consistent cosmology based on a picture of colliding BPS branes is the ekpyrotic scenario [6]. In this paper we will discuss some of the general issues of the BPS brane cosmology by critically analyzing the ekpyrotic scenario.

It was claimed in Ref. [6] that the ekpyrotic scenario is based on the Hořava-Witten (HW) phenomenology, and it solves all major cosmological problems without using inflation [6]. However, in Ref. [7] it was argued that these claims were overly optimistic. First of all, the standard HW phenomenology (including most of its versions with nonstandard embedding) is based on the assumption that we live on the positive tension brane [8]. Meanwhile, the model proposed in Ref. [6] was based on an unconventional approach to HW theory, assuming that we live on the negative tension brane. This required a substantial reformulation of HW phenomenology [9,10]. In particular, one must revise the standard assumption [8] that the gauge coupling on the hidden brane is large. In Ref. [7] a different version of this scenario was proposed, assuming that we live on the positive tension brane, in accordance with Ref. [8]. It was called the pyrotechnic scenario.[1] We will discuss this in Sec. II of our paper and explain that the relevant issue is not the standard versus nonstandard embedding, but Hořava-Witten phenomenology [8] versus Benakli-Lalak-Pokorski-Thomas [11] phenomenology.

Other concerns include extreme fine tuning of the brane potential $V(Y)$ required in the ekpyrotic scenario. In particular, the potential must be extremely small (suppressed by a factor at least as small as 10^{-50}) near the hidden brane. This makes it very difficult to understand how this scenario could be made consistent with the brane stabilization [7]. To solve the homogeneity problem in this scenario, one would need the branes from the very beginning to be parallel to each other with an accuracy better than 10^{-60} on a scale 10^{30} times greater than the distance between the branes. In our opinion, these problems, as well as several other problems of the ekpyrotic scenario pointed out in Ref. [7], remain unresolved.

In this paper we would like to consider some other aspects

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[1]In the pyrotechnic scenario, unlike in the ekpyrotic scenario, we do not make any attempts to avoid inflation. In fact we have argued in Ref. [6] that avoiding inflation in this scenario may require additional fine tuning.

of the ekpyrotic scenario. First of all, it was claimed in Ref. [6] that the action and the cosmological solution describing three static branes in the ekpyrotic scenario have been obtained in Refs. [12,13]. However, the 5D bulk and brane action in Ref. [13] was found for the case of the two boundary branes only. We will show that it cannot be generalized to the case of boundary and bulk branes using the formalism of Refs. [12,13].

To add the bulk brane to this construction one must use the 4-form field, which was introduced in the context of 5D supergravity in singular spaces by Bergshoeff, Kallosh and Van Proeyen [5] and recently generalized in Ref. [14]. We will show that the corresponding part of the 5D action and the part of the solution given in Ref. [6] are not quite correct as far as coefficients are concerned; we will present the corrected action and static BPS solution describing boundary and bulk brane in Sec. III (some technical details are given also in the Appendix).

In Sec. IV we will discuss an effective 4D description of the 5D cosmology and give an argument that the 4D space in the ekpyrotic scenario can only collapse.[2] According to Ref. [6], this is indeed the case. The scale factor of the universe, which was obtained after integrating the 5D action over y (the 5th dimension) and solving equations in the effective 4D theory, decreases. The authors of Ref. [6] argued that one can go back from the effective 4D theory to the 5D theory and show that the scale factor of the visible brane grows. We believe, however, that in order to go back from the effective 4D description to the 5D theory one should perform an explicit investigation of the 5D geometry before the integrating over the 5th dimension.

This is not an easy task since the bulk brane potential $V(Y)$ driving expansion of the universe in the ekpyrotic scenario was added in Ref. [6] by hand to the 4D formulation of the theory, rather than to the 5D theory. One can only speculate how one should interpret this term from the point of view of the 5D theory.

In Ref. [6] this term was interpreted as a correction to the bulk brane action, which means that it represents an effective delta-functional addition to the total energy concentrated on the bulk brane. We will show in Sec. V that in this case the time-dependent ansatz for the metric and fields used in Ref. [6] does not provide a consistent solution to the 5D gravitational equations and to the equation for the dilaton field. In such a situation it is dangerous to take averages of the 5D action or of the 5D equations over y. If one does so with the 5D dilaton equation, one finds that the whole universe, including the visible brane, should exponentially collapse rather than expand.

In order to find a consistent solution of 5D equations one needs to make at least two important modifications discussed in Sec. VI. First of all, when one considers moving branes, or branes having an energy momentum tensor different from the 4D cosmological constant, one should use a general ansatz for the metric and for the fields which respects the (planar) symmetry in the problem [2] rather than the factorized ansatz used in Ref. [6]. The typical situation there when the junction conditions on the branes are taken into account is that the 5D metric has a nonfactorizable dependence on time and the fifth direction.

[2]Various versions of this argument were suggested to us independently by Banks, Dvali, and Maldacena [15].

Moreover, we believe that the brane potential $V(Y)$ should be interpreted as the ‘‘radion potential.’’ It represents the total energy of the configuration involving the bulk brane standing at the distance Y from the visible brane [16]. In such a situation it may be incorrect to represent $V(Y)$ as a delta-functional contribution to the energy density localized on the bulk brane. Instead of that, one should find out the distribution of the fields responsible for the emergence of the long range interaction described by the potential $V(Y)$, and substitute their y-dependent energy-momentum tensor [rather than $V(Y)$] into the 5D equations. A simplest example of a similar situation is given by the energy of an electric capacitor. The energy of interaction of the charged plates of a capacitor $V(R)$ is proportional to the distance R between the plates. However the electrostatic energy density $\sim E^2$ is concentrated not at the plates but in the bulk.

We conclude that in addition to many other problems of the ekpyrotic scenario discussed in Ref. [6], there exists another one. The ansatz for the metric and fields used in Ref. [6] does not provide a correct solution to the dilaton equation and the 5D Einstein equations.

II. BRANE TENSION AND HW PHENOMENOLOGY

According to the Hořava-Witten theory [8,13], the universe consists of two branes in 5D space, which appeared after 6 dimensions of the 11D space were compactified on Calabi-Yau (CY) space. The unification of weak, strong and electromagnetic interactions is achieved on the visible brane, with the gauge coupling $\alpha_{GUT}\sim 0.04$. The standard Hořava-Witten phenomenology [8], including its versions with nonstandard embedding [17], is based on the assumption that we live on the positive tension brane, and the volume of the CY space decreases towards the hidden brane, whereas the gauge coupling constant increases. This leads to the strong gauge coupling on the hidden brane, $\alpha_{\rm hid}=O(1)$. In the strong coupling regime one can obtain gaugino condensation, which plays an important role in the HW phenomenology.

The ratio of the gauge couplings is inversely proportional to the ratio of Calabi-Yau volumes at the positions of the branes [8],

$$\alpha_{\rm hid}=\alpha_{\rm GUT}\frac{V_{\rm vis}}{V_{\rm hid}}. \tag{1}$$

The visible brane is located at $y=0$ and the hidden one at $y=R$ so that $V_{\rm vis}=e^{\phi(0)}$ and $V_{\rm hid}=e^{\phi(R)}$. This formula is valid for the versions of HW theory with the standard embedding without M5-branes, as well as for the versions with the nonstandard embedding with M5-branes present in the bulk, see e.g. a discussion of the phenomenology of the theories with the nonstandard embedding by Lukas, Ovrut and Waldram [17].

In Refs. [6,9] it was assumed that the tension of the visible brane $-\alpha$ is negative. The volume of the CY space in Ref. [6] is proportional to $D^3(y)$, where $D(y)=C+\alpha y$. The function $D(y)$ grows at large y. As a result, the value of $D^3(y)$ near the hidden brane is 50 times greater than near the visible brane for the parameters used in Ref. [6]; it is 27 times greater for the parameters chosen in the replaced version of their paper. The authors of the ekpyrotic scenario did not calculate the value of $\alpha_{\rm hid}$ in their scenario [6,9], so we will estimate it now. If one has $\alpha_{GUT}\sim 0.04$ on the visible brane, one finds $\alpha_{\rm hid}=O(10^{-3})$. This is way too small to lead to the usual HW phenomenology, which requires $\alpha_{\rm hid}\sim 1$ and gaugino condensation [8]. We are not saying that a consistent phenomenology with $\alpha_{\rm hid}\leqslant\alpha_{GUT}$ is impossible, see e.g. Ref. [11], but this is a rather unconventional possibility.

Thus, we believe that the original version of the ekpyrotic scenario was at odds with the standard HW phenomenology as defined in Ref. [8]. So why was the tension of the visible brane chosen to be negative in Ref. [6], despite all complications associated with such a choice? In the original version of Ref. [6] we read: *As we will see in Section V B, it will be necessary for the visible brane to be in the small-volume region of space-time.* This statement, as well as the related conclusion that the spectrum of perturbations in the ekpyrotic scenario is blue, was emphasized in many places of the text. The reason of this statement, as explained in Sec. V B of Ref. [6], was rooted in the idea that the decrease of $D(Y)$ is required for generation of density perturbations in the ekpyrotic scenario.

In Ref. [7] a simple description of generation of density perturbations in the ekpyrotic scenario was presented[3] using the methods developed in the theory of tachyonic preheating [19]. It was shown in Ref. [7], in particular, that the requirement that the visible brane must be in the small-volume region of space-time is not necessary. Thus there is no reason to abandon the standard HW phenomenology and assume that we live on the negative tension brane. An improved version of the ekpyrotic scenario based on the assumption that we live on the positive tension brane was called the ''pyrotechnic universe'' [7].

[3]Recently it was claimed [18] that if one takes into account gravitational back reaction, no perturbations of metric are produced in the ekpyrotic scenario. If this is the case, there is no need to continue discussion of this scenario. However, it is not obvious to us that the absence of fluctuations of the effective 4D metric discussed in Ref. [18] implies the absence of the 5D metric perturbations and the absence of the time delay of the brane collision. Perturbations investigated in Refs. [6,7] appeared not because of the fluctuations of the curvature of the 4D space-time prior to the brane collision, but because of the ''radion'' perturbations δY (related to δg_{yy}). These perturbations, describing an inhomogeneous embedding of the bulk brane in the 5D space-time, cannot be reduced to perturbations of the 4D geometry. In any case, before studying perturbations, one should first carefully examine the behavior of the nonperturbed metric. This is what we are going to do in our paper.

III. M OR STRING THEORY AND 5D BPS DOMAIN WALLS

Compactification of M or string theory with extended objects down to 5D sometimes leads to the appearance of supersymmetric domain walls. It has been explained in Ref. [5] that the supersymmetric domain walls in 5D must be charged. The relevant 4-form A and 5-form field strength $F=dA$ play an important role in the supersymmetry of the bulk and brane construction of [5]. It is the consequence of the M or string theory supersymmetry where RR fields and M-theory form-field provide the balance of forces between BPS extended objects.

Ekpyrotic scenario is based on 11D theory with Hořava-Witten domain walls and some M5-branes between them. A compactification of this system may lead to 5D theory with charged 3-branes. A complete M-theory derivation of the 5D theory with boundary HW domain walls and bulk branes between them was not actually worked out. Only the part of it with 2 HW walls was reduced to 5D in Ref. [13]. The 4-form field describing HW domain walls and compactified M5-branes was introduced recently in Ref. [6] and it was argued in Ref. [10] that the 4-form formulation of the action in Ref. [6] is equivalent to the action presented in Ref. [13]. It was claimed that this is easily seen by eliminating the 4-form using its equation of motion.

Since the purpose of these notes is to find the correct equations in 5D which are related to M or string theory and BPS construction, we have examined this claim and found that it is not quite correct. We use in this analysis the supersymmetric 5D theory with the 4-form field introduced in Ref. [5].

In Ref. [13] there are only 2 HW walls, no branes in between. We will show that in the case when in addition to the 2 boundary branes there are bulk branes present, the covariant action with the bulk dilaton potential (replacing the 5-form contribution) of the type [13] does not exist. Only the formulation with the 4-form field can describe the two boundary branes and a brane in between. We refer the reader to the Appendix where all technical details are explained.

In case of only 2 HW walls the action in Ref. [6] is not equivalent to the action in Ref. [13] unless the factor in front of the $\mathcal{F}^2$ kinetic term is changed.

The expression for the 4-form for the static solution in Ref. [6] is not correct. The numerical factor has to be modified and the sign has to be changed to make it a BPS solution.

We start with the corrected version of the action in Ref. [6] (note the factor 3/2 in front of $\mathcal{F}^2$):

$$S=\frac{M_5^3}{2}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left(R-\frac{1}{2}(\partial\phi)^2-\frac{3}{2}\frac{e^{2\phi}\mathcal{F}_5^2}{5!}\right) \tag{2}$$

$$-3\sum_{i=1}^{3}\alpha_i M_5^3\int_{\mathcal{M}_4^{(i)}} d^4\xi_{(i)}\left(\sqrt{-h_{(i)}}e^{-\phi}-\frac{\epsilon^{\mu\nu\kappa\lambda}}{4!}\mathcal{A}_{\gamma\delta\epsilon\zeta}\partial_\mu X^\gamma_{(i)}\partial_\nu X^\delta_{(i)}\partial_\kappa X^\epsilon_{(i)}\partial_\lambda X^\zeta_{(i)}\right), \tag{3}$$

where $\alpha_1 = -\alpha,\ \alpha_2 = \alpha - \beta,\ \alpha_3 = \beta$. The corrected form of the static solution is[4]

$$ds^2 = D(y)(-N^2dt^2 + A^2d\vec{x}^2) + B^2D^4(y)dy^2, \tag{4}$$

$$e^{\phi} = BD^3(y), \tag{5}$$

$$D(y) = \alpha y + C \qquad \text{for} \quad y < Y, \tag{6}$$

$$= (\alpha - \beta)y + C + \beta Y \quad \text{for} \quad y > Y, \tag{7}$$

where A,B,C,N are constants, $C>0$,

$$\mathcal{A}_{0123} = +A^3NB^{-1}D^{-1}(y),$$
$$\mathcal{F}_{0123y} = -A^3NB^{-1}D^{-2}(y)D'(y), \tag{8}$$

and

$$[D(y)]'' = 2[\alpha\delta(y) - (\alpha-\beta)\delta(y-R)] - \beta\delta(y-Y) - \beta\delta(y+Y). \tag{9}$$

The factor $-A^3NB^{-1}$ in $\mathcal{F}$ in Eq. (8) was absent in Ref. [6]. The sign of $\mathcal{A}_{0123}$ and $\mathcal{F}_{0123y}$ (which differs from the one in Ref. [6]), is easily checked by observing that the force on a static probe brane parallel to the source branes must vanish.[5]

The static BPS solution is valid for branes that are not moving. It may serve as a starting point for finding time-dependent cosmological solutions.

IV. 4D VIEW ON EKPYROTIC UNIVERSE

Before investigating time-dependent cosmological solutions in 5D, let us see what could be expected from the point of view of the effective 4D theory. Indeed, if one considers the situation when the distance between the branes is very small and their motion is slow, one could expect that in the first approximation it should be possible to describe low energy theory from the point of view of 4D Einstein gravity (or Brans-Dicke theory) coupled to matter. Deviations from this description could occur if there were some processes with the energies comparable to the inverse distance between the branes. However, in the ekpyrotic scenario all energy scales (reheating temperature, Hubble constant, etc.) are several orders of magnitude smaller than the inverse distance between the branes $1/R$.

Therefore one may expect that the ekpyrotic scenario can be described entirely in terms of the 4D theory. But in such a case the universe, which was static in the beginning of the process, can only collapse. We present here, in a slightly modified form, the basic argument of Ref. [15]; see also Ref. [7].

Let us write down the Einstein equations for a homogeneous flat universe. The first equation is

$$H^2 = \frac{8\pi G}{3}\rho. \tag{10}$$

The second equation, which follows from the first one and the energy conservation, can be represented in the following form:

$$\dot{H} = -4\pi G(\rho + p). \tag{11}$$

Here ρ and p are the density and pressure in the effective 4D theory, and $H = \dot{a}/a$, where a is the scale factor in the 4D space on the visible brane.

In the beginning, the branes do not expand, $H=0$, and the effective energy density and pressure vanish for the static brane configuration considered in the previous section.

This situation changes when one adds by hand the potential energy $V(Y)$ associated with the position of the bulk brane. According to Ref. [6], $V(Y)<0$. This, however, would be inconsistent with Eq. (10) unless one assumes that the bulk brane has nonzero velocity from the very beginning, so that the total energy density is non-negative. We do not want to speculate on how this configuration could emerge; see Ref. [7] for a discussion of the problem of initial conditions in the ekpyrotic scenario.

What is more important, Eq. (10) implies that $\dot{H} \leqslant 0$ because $\rho + p \geqslant 0$ in accordance with the null energy condition. Thus, if the universe begins in a static state, $H=0$, then it can only collapse, since $H = \dot{a}/a \leqslant 0$. Therefore one may argue that the ekpyrotic scenario cannot describe an expanding universe [15].

Let us compare these expectations with the results obtained in Ref. [6]. To describe the motion of the bulk brane in the ekpyrotic scenario the authors started with the factorized ansatz based on Eqs. (4)–(7) where it was assumed that some of the parameters of the static solution become functions of time but not of y $[A,N,Y \to A(t),N(t),Y(t)]$, whereas some other parameters remain constant $(\dot{B} = \dot{C} = 0)$. They substituted this modified ansatz into the action (3), and *integrated over y*. In this way they obtained the 4D "moduli space" action with the Lagrangian density $\mathcal{L} = \mathcal{L}^{4d}_{bulk} + \mathcal{L}_{\beta}$, where

$$\mathcal{L}^{4d}_{bulk} = -2\frac{3A^3BM_5^3}{N}\int_0^R dy D^3(y,Y) \times\left[\left(\frac{\dot{A}}{A}\right)^2 + 3\frac{\dot{A}}{A}\frac{\dot{D}}{D} + \frac{1}{2}\left(\frac{\dot{D}}{D}\right)^2\right], \tag{12}$$

and

$$\mathcal{L}_{\beta} = \frac{3\beta M_5^3A^3B}{N}\left[\frac{1}{2}D^2(Y)\dot{Y}^2\right]. \tag{13}$$

[4]Here y is a point of S^1/Z_2, i.e. $-R<y\leqslant R$, with 0 and R as fixed points, R identified with $-R$. This explains the factor of 2 for the fixed-point brane sources at $0,R$ (accounting for their images), and the presence of the brane in the bulk at $y=Y$ and its image at $y=-Y$.

[5]The world-volume term cancels against the Wess-Zumino term in the static probe brane action. We are assuming the standard convention $\epsilon^{0123} = +1$.

We gave here a form of the Lagrangian in which it is clear that it is integrated over the full range of the 5th coordinate.

In Ref. [6] the notation $I_k(Y) \equiv \int_{-R}^{R} dy D^k(y,Y) = 2\int_0^R dy D^k(y,Y)$ $(k=1,2,\ldots)$ and the explicit form of D was used so that $\dot{D}=0$ at $|y|<Y$ and $\dot{D}=\beta\dot{Y}$ at $|y|\geqslant Y$. After obtaining the effective 4D action by integration over y, the authors added by hand an important term, which plays a crucial role in their scenario:

$$\Delta\mathcal{L}_\beta = -3\beta M_5^3 A^3 BN\, V(Y). \tag{14}$$

The effective potential $V(Y)$ may appear, e.g., as a result of the nonperturbative effects associated with open M2-brane instantons [20]. It was assumed in Ref. [6] that $V(Y) \sim e^{-\alpha m Y}$ at large Y, and that it vanishes at $Y=0$. Here m is some positive numerical constant specified (together with the tension α) in Ref. [6].

The next step was to replace A and N by $a = A(BI_3M_5)^{1/2}$ and $n = N(BI_3M_5)^{1/2}$. This gives the effective 4D theory in the following form:

$$\mathcal{L} = \frac{3a^3M_5^2}{n}\left\{ -\left(\frac{\dot{a}}{a}\right)^2 + J(Y)\beta^2\dot{Y}^2 + \frac{\beta}{I_3}\left[\frac{1}{2}D(Y)^2\dot{Y}^2 - n^2\frac{V(Y)}{BI_3M_5}\right]\right\}, \tag{15}$$

where $J(Y) \equiv (9I_{2b}^2/4I_3^2 - I_{1b}/2I_3)$ and $D(Y) = \alpha Y + C$. Then the authors of Ref. [6] neglected the terms proportional to β^2 (we will return to this point in the next section) and studied the 4D theory with the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{3a^3M_5^2}{n}\left\{ -\left(\frac{\dot{a}}{a}\right)^2 + \frac{\beta}{I_3}\left[\frac{1}{2}D(Y)^2\dot{Y}^2 - n^2\frac{V(Y)}{BI_3M_5}\right]\right\}. \tag{16}$$

Since this Lagrangian leads to the equations which look exactly like the usual Friedmann equations for the scale factor a, the authors obtained the result which we expected on the basis of our general arguments: $H = \dot{a}/a < 0$, i.e. the 4D universe *contracts*.

At this stage it is very important to realize that we are not talking here about contraction of the bulk brane, or the visible brane, or the hidden brane. We are investigating an effective 4D geometry where the distinction between these branes completely disappeared. It was ''washed away'' by the integration over y. In a certain sense, one may imagine that integration over y ''glues'' the three branes together. Thus we are talking about the contraction of the whole 4D space rather than of one of the branes. All the difference between the branes in 4D must be encoded in the dynamics of the moduli fields, but not in the different rate of expansion or contraction of different branes.

Note that the contraction of the universe was crucial for ekpyrotic scenario in order to gain a residual kinetic energy of the moving brane and transfer it to the radiation upon the collision. This residual energy was suppressed by the small coefficient $O(\beta/\alpha)$ as compared to the maximal kinetic energy of the brane.

So where did the expansion of the universe come from in Ref. [6]? After solving the effective 4D equations, the authors decided to ''unglue'' the branes and go back to 5D. They studied the difference between the expansion of the universe as seen by different observers living on different branes and concluded that whereas the universe described by the overall scale factor a collapses, the visible brane, described by the scale factor a_-, may expand.

It is this point that is in an apparent contradiction with our 4D expectations suggesting that the scale factor of the universe in the ekpyrotic scenario cannot expand. However, the statement that a_- expands was based on the assumption that the function A in the metric (4) depends only on t but not on y, and that the effective 4D description with the metric ansatz of Ref. [6] provides a correct solution for the scale factor of the universe in the full 5D theory. As we will see in the next section, our analysis of the 5D solutions does not confirm this assumption.

V. 5D THEORY IN THE BULK

An assumption of the ekpyrotic cosmology in Ref. [6] was that the time dependent solution can be obtained from the static solution using the ''moduli space'' approximation, i.e. by replacing the constant moduli parameters of the static solution by time-dependent functions. More precisely, the ansatz used in Ref. [6] was to introduce only time *but not y* dependence into A,N,Y which were constants in the static BPS solution, so that they become $A(t)$, $N(t)$, $Y(t)$. All dependence on time in $D(y)$ then enters only via $Y(t)$, i.e. for the cosmological solution one takes $D = D[y,Y(t)]$. The two extra parameters B,C which appeared in the metric, in the dilaton and in the 4-form field were assumed to be time-independent.

As already discussed above, in Ref. [6] this ansatz was plugged into the 5D action, and then integration over y gave an action for time-dependent functions only. That action was taken as a starting point for a cosmological analysis. However, it is not clear *a priori* why this procedure is actually *consistent*, i.e. why the solutions of the resulting effective equations represent at the same time the solutions of the original 5-*dimensional* gravitational equations.

Indeed, it is well-known (see e.g. Ref. [21]) that the moduli space approximation, i.e. replacing moduli by time-dependent function may not always be a consistent. Unless the dependence on the internal coordinate (y in the present case) is ''homogeneous'' so that it effectively ''scales out'' of the time-dependent higher-dimensional equations, one cannot simply replace these equations by their integrated (averaged over y) version—the moduli space approximation ansatz will not be consistent with the full set of the gravitational equations.

The question now arises whether it is possible to set up a 5D theory of ekpyrotic cosmology before integrating over y.

Since in Ref. [6] the interaction between the branes in 5D bulk was not specified but only the potential $V(Y)$ which lives on the bulk brane at $y=Y$ was introduced, we will also take this ansatz as part of the definition of the 5D theory. As we shall see, this will lead to a major problem with ekpyrotic cosmology, a contradiction with 5D solutions of equations of motion.

It could be expected that the 5D theory (3) does not describe the physics of the brane collision and does not address the moduli stabilization problem [6]. The problem we are discussing now is completely different. Here we will analyze equations in the bulk during the roll of the bulk brane until it reaches the minimum of its potential $V(Y)$. If the 5D equations do not work even at this stage, then we do not have a consistent scenario not only during the brane collision but even before it.

To make the discussion as clear as possible, we shall start with the simplest equation—the one for the dilaton ϕ. The dilaton equation of motion in the bulk (away from the branes where the source terms vanish) is given by

$$\frac{2}{M_5^3}\frac{\delta S_{bulk}}{\delta\phi(t,y)}=\partial_\gamma(\sqrt{-g}\,g^{\gamma\delta}\partial_\delta\phi)-3\sqrt{-g}\,\frac{e^{2\phi}\mathcal{F}^2}{5!}=0. \tag{17}$$

It is satisfied for the above static solution (4)–(9). Plugging in the time-dependent ansatz of Ref. [6], we find that the new term in the dilaton equation, which was absent in static case, is simply the time-derivative one ($\dot{D}\equiv\partial_t D$):

$$\partial_t(\sqrt{-g}\,g^{tt}\partial_t\phi)=-3\partial_t(A^3D^2BN^{-1}\dot{D}). \tag{18}$$

The other terms in the dilaton equation, $\partial_y(\sqrt{-g}\,g^{yy}\partial_y\phi)-3(e^{2\phi}\mathcal{F}^2/5!)$, cancel not only for the static ansatz but for the time dependent ansatz as well, for the corrected action and solution given in Eqs. (3)–(8). Following Ref. [6] in assuming that B and C are constant, we may study the dilaton equation which then reduces to the condition that Eq. (18) should vanish for all values of y away from the branes. For $y<Y$ the term (18) vanishes since for $D=C+\alpha y$ we find that $\dot{D}=0$. However, for y behind the moving brane, i.e. $y>Y$, we find that $\dot{D}=\beta\dot{Y}\neq 0$, and therefore the correction to the dilaton equation due to the time evolution does not vanish automatically. In this case Eq. (17) looks as follows:

$$-\frac{2}{3M_5^3}\frac{\delta S_{bulk}}{\delta\phi}=\partial_t(A^3D^2BN^{-1}\beta\dot{Y})=0,\quad y>Y(t). \tag{19}$$

If we assume as in Ref. [6] that B is constant and $N(t)=A(t)$ (which corresponds to the choice of t as a conformal time), we get the condition

$$\partial_t(A^2D^2\dot{Y})=0,\quad y>Y(t). \tag{20}$$

The solution of this equation is

$$A^2(t)D^2[y,Y(t)]\dot{Y}(t)=f(y),\quad y>Y(t), \tag{21}$$

where $f(y)$ is an arbitrary function of y that does not depend on t.

Let us first look at this equation in the spirit of Ref. [6], replacing $D^2[y,Y(t)]$ in the first approximation by its average value. When Y changes from R to 0 in the ekpyrotic scenario, $D(y,Y)$ for any $y>Y$ changes just few times. Indeed, consider the maximum and minimum values of $D(y,Y)$. At $y=0$ in ekpyrotic cosmology it takes the minimum value $D=C$. At $y=R$ it takes the maximum value $D(y,Y)=C+(\alpha-\beta)R+\beta Y$. Thus $C\leqslant D(y,Y)\leqslant C+\alpha R$. In the examples of Ref. [6] $D_{min}=10^2$ and $D_{max}=3\times10^2$ or $D_{min}=10^3$ and $D_{max}=2\times10^3$. Therefore, the function D changes 3 and 2 times in these two examples, respectively, so one can in the first approximation replace $D(y,Y)$ by its average value.

On the other hand, in the ekpyrotic cosmology one had $D\dot{Y}=-\sqrt{-2V(Y)}=-\sqrt{2v}e^{-\alpha mY/2}$ [6]. According to Refs. [6,7], when Y changes from R to 0, the function $e^{\alpha mY(t)/2}$ changes from some value greater than e^{60} to 1. This means that during this time the scale factor $A(t)\sim f(y)/D^2\dot{Y}(t)$ contracts at least e^{30} times.

To compare this with the conclusions of Ref. [6] note that they define the scale factor of the visible brane as

$$a_-(t)=A(t)\sqrt{C}, \tag{22}$$

where C is the constant in D. They conclude, using the 4D approximation, that $a_-(t)$ grows in time and the visible universe expands. But as we have seen, this contradicts strongly the solution of the 5D dilaton equation in the bulk, which shows that $a_-(t)$ exponentially contracts.

However, the statement about expansion of the universe [6], as well as our statement about its exponential contraction, was based on averaging over the 5th dimension, assuming that the solutions of the 5D equations satisfying the metric ansatz of Ref. [6] do actually exist. Now let us look at our exact result, Eq. (21), more carefully. Since $D[y,Y(t)]=(\alpha-\beta)y+C+\beta Y(t)$ is a function of both y and t, one can easily check that this equation is formally inconsistent, i.e. it does not have any solutions at all. This simply means that the ansatz for the metric and the fields used in Ref. [6] does not solve the 5D equations.

One can come to a similar conclusion using the gravitational equations in the bulk. First, note that general covariance leads to a relation between 5D Einstein equations, dilaton and 4-form equations. One can verify that the 4-form equation of motion in the bulk $\delta S_{bulk}/\delta A_{\alpha\beta\gamma\delta}=0$ is satisfied by the time dependent ansatz of Ref. [6]. Thus we get an identity:

$$\nabla^\gamma\frac{\delta S_{bulk}}{\delta g_{\gamma\delta}}-\partial^\delta\phi\frac{\delta S_{bulk}}{\delta\phi}=0. \tag{23}$$

For a solution of bulk equations of motion each term in this identity should vanish. However, if the dilaton equation is not satisfied, this identity shows that a particular combination of the gravitational field equations cannot be satisfied by the ansatz of Ref. [6] (note that the derivatives $\partial^\delta\phi$ in the time and 5th direction do not vanish).

To see this more explicitly let us insert the ansatz of Ref. [6] directly into the action, with $\dot{B}=\dot{C}=0$. We shall change the variables so that

$$\tilde{A}^2[t,y,Y(t)]=A^2(t)D^3[y,Y(t)],$$

$$\tilde{N}^2[t,y,Y(t)]=N^2(t)D^3[y,Y(t)]. \quad (24)$$

Then, up to total derivatives,

$$\mathcal{L}_{\rm bulk}(t,y)=\frac{M_3^2}{2}\sqrt{g}\left(R-\frac{1}{2}(\partial\phi)^2-\frac{3}{2}\frac{e^{2\phi}\mathcal{F}^2}{5!}\right)$$
$$=-\frac{3M_3^2\tilde{A}^3B}{\tilde{N}}\left[\left(\frac{\dot{\tilde{A}}}{\tilde{A}}\right)^2-\frac{7}{4}\left(\frac{\dot{D}}{D}\right)^2\right]. \quad (25)$$

Taking into account that $\dot{\phi}=3\dot{D}/D$ we may also rewrite the bulk action as

$$\mathcal{L}_{\rm bulk}(t,y)=-\frac{M_3^2\tilde{A}^3B}{\tilde{N}}\left[3\left(\frac{\dot{\tilde{A}}}{\tilde{A}}\right)^2-\frac{7}{12}\dot{\phi}^2\right]. \quad (26)$$

This is a *local action* that should be varied to derive the *local equations*. If we perform the variation of this action over ϕ, we will get the same dilaton equation as in Eq. (20) (note that $\tilde{A}$ and $\tilde{N}$ are independent of ϕ), namely, $\partial_t[(\tilde{A}^3B/\tilde{N})\dot{\phi}]=0$ for $y>Y$.

The reason why this equation was not satisfied in Ref. [6] is that they took the action of the form (26), integrated it over y, added the brane action and neglected the term $J(Y)\beta^2\dot{Y}^2(t)\sim\int dy[(M_3^2\tilde{A}^3B/\tilde{N})\dot{\phi}^2]$, as we have explained in the previous section [see Eqs. (15), (16)]. One may argue about whether this term is small or not as compared to the brane contribution in the integrated action. However, in 5 dimensions, i.e. before the integration over y, this term is given by $(\tilde{A}^3B\dot{\phi}^2/\tilde{N})(t,y)$. For $y>Y$ there is no other contribution to the dilaton equation. Therefore the variation of this term must vanish as Eq. (18) states.

VI. TOWARDS A CONSISTENT BRANE COSMOLOGY

As we have seen, different ways of taking averages in the situation where the 5D solutions do not exist can lead to dramatically different conclusions regarding expansion versus contraction of the universe. Thus, if one really wants to investigate cosmological consequences of the ekpyrotic scenario, one should find exact solutions of the corresponding 5D equations. This is especially important in the situation where the results of the averaging over the 5th dimension lead to the conclusions that are in an apparent contradiction with the Einstein equations in 4D; see Sec. IV.

The fact that the ansatz for the metric and the fields used in Ref. [6] does not solve the 5D equations is not very surprising. Indeed, it was shown in Ref. [2] that even in the simplest versions of brane cosmology describing one or two branes one should use a more general ansatz for the metric in order to satisfy the Israel junction conditions on the branes. A generic metric which respects the planar symmetry of the problem has the form

$$ds^2=-n^2(t,y)dt^2+a^2(t,y)d\vec{x}^2+b^2(t,y)dy^2. \quad (27)$$

Note that here the functions a, b, and n depend both on t and y, and there still is a residual freedom of transformation of the coordinates (t,y). Similarly, one may need to consider a more general ansatz for the fields as in Ref. [2].

Before one begins looking for exact solutions using a more general metric ansatz, one should reexamine other assumptions of the theory. Indeed, in Ref. [6] the potential $V(Y)$ was added by hand to the bulk brane action, whereas the bulk supergravity action remained unchanged. However, it is not obvious to us whether this is the proper way to introduce the interbrane interactions in 5D.

As an illustrative example, consider two charged plates of a capacitor in ordinary electrodynamics, positioned at $y=0$ and $y=R$. If they have charges q and $-q$, and the electric field between the plates is E, then the potential energy of the interaction between the plates can be represented as the "brane potential" $V(R)=-qE\,R$. However, it would be incorrect to think that this energy is localized on the plates. Rather it is concentrated in the electric field between the plates. It is possible to use the potential $V(R)$ to study the motion of the branes. For example, if each brane has mass M, one can write $m\ddot{R}=-V'(R)$, just as one does for the bulk brane acceleration in the ekpyrotic scenario. But if one studies gravitational backreaction of the electric field, it would be completely incorrect to replace the contribution of the electric field to the energy-momentum tensor in the bulk by the delta-functional term proportional to $V(R)$.

Similarly, if the potential $V(Y)$ appears due to the membrane instantons stretched between the branes, one should check whether the energy-momentum tensor in the bulk, as well as the dilaton and the 4-form field, changes due to these nonperturbative effects. Otherwise the appearance of the potential depending on the interbrane separation would look as an example of the action at a distance.

In a certain sense, the potential $V(Y)$ is analogous to the effective potential $V(r)$ of the radion field introduced by Goldberger and Wise [16]. It is a very useful concept if the only goal is to describe the forces acting on the bulk brane, ignoring the change of the metric produced by these forces. However, if one wants to study the corresponding changes in space-time geometry (and this was the main goal of Ref. [6]), one should perform a full investigation of the interbrane interactions in 5D [22] and check whether one can add the fields responsible for the radion potential without additional fine tuning and strong modification of the 4D geometry [23].

Thus one has a lot of things to do. First of all, one needs to find a theory with the potential $V(Y)$ which behaves as $-e^{-\alpha mY}$ at large Y (the functional form is important). This potential should be smaller in absolute value than e^{-120} near the hidden brane, and should not have any positive contributions there with this accuracy. This fine tuning is necessary to produce desirable density perturbations and avoid inflation. Also, this potential should not receive any contributions proportional to $e^{-\alpha m(R-Y)}$ due to the interaction with the hidden brane [7]. One must make sure that this potential vanishes at $y=0$, to avoid the cosmological constant problem. Then one must take into account that the visible brane and the hidden brane should be stabilized by some strong forces so that the effective potential of the corresponding moduli field could have mass on the TeV scale or even greater. One should also check that the strong forces leading to the brane stabilization do not interfere with the extremely weak interaction responsible for the potential $e^{-\alpha m Y}$. One cannot ignore the unresolved problem of brane stabilization (which was the position taken in Ref. [6]) and speculate about the interbrane potentials suppressed by a factor of e^{-120}.

When/if the theory with the desirable $V(Y)$ is found, one should investigate its 5D nature and make sure that the effects producing $V(Y)$ do not induce large curvature on the branes. Then one should write down equations in 5D taking into account the energy-momentum tensor in the bulk together with the junction conditions, and solve them.

And finally, one should find out what happens at the moment of the brane collision: whether the visible brane collapses, expands, stays at the same place or oscillates, etc. These issues have not been addressed in Ref. [6], and they cannot be fully analyzed until the brane stabilization mechanism is understood.

Thus, if one wants to propose a consistent alternative to inflationary cosmology, one would need first to give a consistent formulation of the alternative theory, and then find a correct solution of the corresponding equations. As we have seen, this is a rather nontrivial task.

In this paper we studied a very limited part of this problem. We tried to check whether the basic assumptions of the ekpyrotic scenario (the ansatz for the metric and for the fields, and the modification of the bulk brane action proposed in Ref. [6]) can lead to a consistent 5D description of an expanding visible brane. We have found that this is not the case.

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APPENDIX: THE 4-FORM STORY

There are two points about the 4-form dependence in Ref. [6] which must be changed to get the correct setup for charged BPS 5D domain walls. One has to correct the coefficient in the action and the coefficient in the solution for the 4-form. These corrections in the form sector are important in order to test the time-dependent ansatz of Ref. [6].

1. Action

The action in Eq. (10) of Ref. [6] is not the one to which they refer as given in Refs. [12,13]. The one in Ref. [13] does not have a 4-form. We questioned the origin of their action in Ref. [7] and they replied in Ref. [10]: *The 4-form formulation of the action is equivalent to the action presented in [12,13]. This is easily seen by eliminating the 4-form using its equation of motion*.

In Ref. [13] there were indeed the relevant world-volume terms in the action (only 2 branes at the fixed points are present there, so there $\beta=0$), but there were no Wess-Zumino terms with the 4-form field:

$$S=-\frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left[R+\frac{1}{2V^2}\partial_\alpha V\,\partial^\alpha V+\frac{1}{3V^2}\alpha^2\right]-\sum_{i=1}^{2}\frac{\sqrt{2}}{\kappa_5^2}\alpha_i\int_{\mathcal{M}_4^{(i)}} d^4\xi_{(i)}\sqrt{-g}\,V^{-1}. \tag{A1}$$

We take the constants in the bulk and boundary actions as $\alpha=-\alpha^{(1)}=\alpha^{(2)}$. Then we make the redefinitions $V\rightarrow e^{\phi}$, $\alpha\rightarrow(3/\sqrt{2})\alpha$, $\kappa_5^{-2}\rightarrow M_5^3$, $R\rightarrow -R$ so that our notations agree with Ref. [6]. That leads to

$$S=\frac{M_5^3}{2}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left[R-\frac{1}{2}\partial_\alpha\phi\,\partial^\alpha\phi-\frac{3}{2}e^{-2\phi}\alpha^2\right]-3\sum_{i=1}^{2}M_5^3\alpha_i\int_{\mathcal{M}_4^{(i)}} d^4\xi_{(i)}\sqrt{-g}\,e^{-\phi}. \tag{A2}$$

The kinetic terms match the ones in Eq. (10) in Ref. [6]. Now we may perform the procedure suggested in Ref. [5]. We promote the constant α to a function $G(x)$ and add a Lagrange multiplier of the form $\varepsilon^{\alpha\beta\gamma\delta\epsilon}(\partial_\epsilon G)\mathcal{A}_{\alpha\beta\gamma\delta}$ and assign the supersymmetry transformation to the 4-form so that its variation will compensate the variation of terms in the rest of the action with derivatives of G. If there are no sources, from the equation for the 4-form we learn that $G(x)$ is a constant, as before in usual gauged supergravity in $d=5$ without a 4-form where there is a constant gauge coupling. In the presence of sources we will find that G is piecewise constant. If there are charged sources as in case of Ref. [13] we may also add the WZ term.

Thus we add a Lagrange multiplier term and a WZ term to the action of Ref. [13] given above in the form (A2). Its normalization is arbitrary, we thus put a constant c in front:

$$S_A=\frac{c}{4!}\int_{\mathcal{M}_5} d^5x\left[-\varepsilon^{\alpha\beta\gamma\delta\epsilon}(\partial_\epsilon G)\mathcal{A}_{\alpha\beta\gamma\delta}+\sum_{i=1}^{2}2\alpha_i\varepsilon^{\mu\nu\rho\sigma}\mathcal{A}_{\mu\nu\rho\sigma}\delta(x^5-x_i^5)\right]. \tag{A3}$$

The relative normalization between the two terms is arranged such that α jumps by $2\alpha_i$ at brane i. If we differentiate over the 4-form we are back to Eq. (A2). To have agreement with Ref. [6] we have to choose $c=\frac{3}{2}M_5^3$. We may however do something else, namely, add and subtract a term quadratic in $\mathcal{F}$. We find

$$S_G=\int_{\mathcal{M}_5} d^5x\left[-\frac{3M_5^3}{4}\sqrt{-g}e^{-2\phi}\left(G-\frac{2e^{2\phi}}{3M_5^3\sqrt{-g}}\frac{c}{5!}\times\varepsilon^{\alpha\beta\gamma\delta\epsilon}\mathcal{F}_{\alpha\beta\gamma\delta\epsilon}\right)^2-\sqrt{-g}\frac{e^{2\phi}}{5!3M_5^3}c^2\mathcal{F}^2\right]. \quad (A4)$$

Now we vary over the field G and from its equation we find that

$$G=\frac{2e^{2\phi}}{3M_5^3\sqrt{-g}}\frac{c}{5!}\varepsilon^{\alpha\beta\gamma\delta\epsilon}\mathcal{F}_{\alpha\beta\gamma\delta\epsilon}. \quad (A5)$$

We thus find the action given in Ref. [6] but where *the factor in front of the $\mathcal{F}^2$ term has to be changed by* 3/2. The correct action (i.e. the one which is equivalent to an action without 4-form in Ref. [13]) is thus

$$S=\frac{M_5^3}{2}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left(R-\frac{1}{2}(\partial\phi)^2-\frac{3}{2}\frac{e^{2\phi}\mathcal{F}^2}{5!}\right)-3\sum_{i=1}^{3}\alpha_i M_5^3\int_{\mathcal{M}_4^{(i)}} d^4\xi_{(i)}\left(\sqrt{-h_{(i)}}e^{-\phi}-\frac{\epsilon^{\mu\nu\kappa\lambda}}{4!}\mathcal{A}_{\gamma\delta\epsilon\zeta}\partial_\mu X^\gamma_{(i)}\partial_\nu X^\delta_{(i)}\partial_\kappa X^\epsilon_{(i)}\partial_\lambda X^\zeta_{(i)}\right). \quad (A6)$$

This action for the two branes at the fixed points (i.e. without the third brane) taken in the static gauge with $X^\mu=\xi^\mu$ and $X^4=Y=$const does agree with the action in Ref. [13].

2. Bulk brane and 4-form

As was promised, we will show here that the formulation of the 5D supersymmetric theory with a bulk brane between the orbifold planes is impossible without the use of the 4-form or, equivalently, without the gauge coupling field G dual to the field strength $F=dA$ [5].

In the case of two orbifold planes we may derive the action (A1) by solving the equation of motion for the 4-form. This leads to

$$G'=2\alpha[\delta(y)-\delta(y-R)]. \quad (A7)$$

The solution for the gauge coupling field is $G(y)=\alpha\epsilon(y)$. It simply means that the gauge coupling is positive for positive y and negative for negative y. Therefore, $[G(y)]^2=\alpha^2$ for all y and we find the bulk potential proportional to $e^{-2\phi}\alpha^2$ as it was given in the original form of the action (A2) where there was no 4-form, neither in the bulk, nor on the branes.

Now we may try to perform the same procedure of getting rid of the 4-form in case when the bulk brane is present. We find the gauge-coupling field dual to the 5-form field strength

$$G'=2\alpha[\delta(y)-\delta(y-R)]-\beta\delta(y-Y)-\beta\delta(y+Y). \quad (A8)$$

In this case the solution for G which is dual to the 5-form takes values

$$G^2=\alpha^2 \quad \text{at} \quad 0<|y|<Y,$$
$$G^2=(\alpha-\beta)^2 \quad \text{at} \quad Y<|y|<R. \quad (A9)$$

If we would try to plug this solution back into the action (into the term $e^{-2\phi}G^2$), we would find that the *bulk potential is a piecewise function, and, therefore, there is no local general covariant theory*. If, however, we keep the 4-form, there exists a local general covariant 5D bulk action. In the presence of sources, this will lead to piecewise values of the 5-form for the solutions, but the local general covariant action is available.

3. Solution

Let us suppose first that the action given in Ref. [6] were correct and let us try to check the solution for the 4-form. The corresponding equation is

$$\partial_y(\sqrt{-g}e^{2\phi}\mathcal{F}^{0123y})+3\left[-\alpha\delta(y)+(\alpha-\beta)\delta(y-R)+\frac{\beta}{2}\delta(y-Y)+\frac{\beta}{2}\delta(y+Y)\right]=0. \quad (A10)$$

For the ansatz in Ref. [6] we have $\sqrt{-g}=A^3BN[D(y)]^4$ while $\sqrt{-g_{(4)}}=A^3N[D(y)]^2$ and $e^{2\phi}=B[D(y)]^3$. We find then that the solution is

$$\mathcal{F}_{0123y}=-\frac{3}{2}A^3NB^{-1}D^{-2}(y)D'(y), \quad (A11)$$

where

$$[D(y)]''=2[\alpha\delta(y)-(\alpha-\beta)\delta(y-R)]-\beta\delta(y-Y)-\beta\delta(y+Y). \quad (A12)$$

This differs from the solution given in Ref. [6] by a factor of $\frac{3}{2}A^3NB^{-1}$ and by a sign. If we now start with the corrected form of the action in Eq. (3) (with extra kinetic term factor $\frac{3}{2}$), we find the corrected expression for the 4-form,

$$\mathcal{F}_{0123y}=-A^3NB^{-1}D^{-2}(y)D'(y). \quad (A13)$$

This solution differs from the one in Ref. [6] by the sign and by the factor A^3NB^{-1}.

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