How does the cosmic microwave background plus big bang nucleosynthesis constrain new physics?

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Recent cosmic microwave background (CMB) results from BOOMERANG, MAXIMA, and DASI provide cosmological constraints on new physics that can be competitive with those derived from big bang nucleosynthesis (BBN). In particular, both CMB and BBN can be used to place limits on models involving neutrino degeneracy and additional relativistic degrees of freedom. However, for the case of the CMB, these constraints are, in general, sensitive to the assumed priors. We examine the CMB and BBN constraints on such models and study the sensitivity of "new physics" to the assumed priors. If we add a constraint on the age of the universe ($t_0 \gtrsim 11$ Gyr), then for models with a cosmological constant, the range of baryon densities and neutrino degeneracy parameters allowed by the CMB and BBN is fairly robust: $\eta_{10}=6.0\pm0.6$, $\Delta N_\nu \lesssim 6$, $\xi_e \lesssim 0.3$. In the absence of new physics, models without a cosmological constant are only marginally compatible with recent CMB observations (excluded at the 93% confidence level).

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I. INTRODUCTION

Until recently, big bang nucleosynthesis (BBN) provided the only precision estimates of the baryon density of the universe. Based on recent deuterium observations [1], BBN identifies a value for the baryon density which has been variously estimated (depending on the choice for the primordial deuterium abundance) as $\Omega_B h^2 = 0.015$–0.023 [2], and $\Omega_B h^2 = 0.017$–0.021 [3], or incorporating the most recent data $\Omega_B h^2 = 0.017$–0.024 [4], where Ω_B is the baryon density expressed as a fraction of the critical density, h is the Hubble parameter in units of 100 km/sec/Mpc, and the ranges quoted are intended to be at the 95% confidence level.

In the past year, observations of the cosmic microwave background (CMB) fluctuations have become a competitive means for estimating the baryon density. These data have been used both alone and in combination with other observations (such as type Ia supernovae and large-scale structure) to set limits on $\Omega_B h^2$. The preliminary CMB data from BOOMERANG [5] and MAXIMA [6] suggested a higher baryon density ($\Omega_B h^2 \sim 0.03$) than that predicted from BBN, due to the unexpectedly low second acoustic peak in these CMB observations (see, for example, Refs. [7,8]). This discrepancy has vanished in the wake of more recent data from BOOMERANG [9], MAXIMA [10] and DASI [11].

This original discrepancy between the BBN and CMB predictions for $\Omega_B h^2$ led to the suggestion that perhaps new physics must be invoked to reconcile the BBN and CMB predictions for $\Omega_B h^2$. The problems for BBN at the high baryon density suggested by Refs. [7,8] are that the BBN-predicted abundance of deuterium is too low while those of helium-4 and lithium-7 are too high when compared to the observationally inferred primordial abundances. If, however, the universal expansion rate were increased during the BBN epoch by, for example, the contribution to the total energy density of "new" neutrinos and/or other relativistic particles, the BBN-predicted abundance of deuterium would increase (less time for D destruction), while that of lithium would decrease (less time for production of ^{7}Be). This increase in the expansion rate results in a higher helium abundance, but the BBN-predicted helium abundance can be reduced by a non-zero chemical potential for the electron neutrinos. An excess of ν_e over $\overline{\nu_e}$ can drive the neutron-proton ratio down, leading to reduced production of helium-4. Thus, reconciling BBN with a high baryon density would require two kinds of "new physics:" the expansion rate should be faster than the standard value and ν_e should be "degenerate." Although these two effects may be unrelated, neutrino degeneracy can provide an economic mechanism for both, since the energy density contributed by degenerate neutrinos exceeds that from non-degenerate neutrinos, leading to an enhanced expansion rate during the epoch of BBN. As Kang and Steigman [12] and Olive *et al.* [13] have shown, the observed primordial abundances of the light nuclides can be reconciled with very large baryon densities provided sufficient neutrino degeneracy is permitted.

Although the most recent CMB observations suggest that no new physics need be invoked to reconcile the CMB and BBN observations, these measurements also provide another tool, independent of BBN, to constrain such new physics. From the contribution of Ref. [14] and the combined CMB and BBN analyses of Refs. [15,16], it is already clear that the constraints on "new physics" are strongly dependent on the priors assumed in the analysis for the other, non-BBN related cosmological parameters. Here we explore this issue further. In particular, we consider the concordance between the CMB and BBN predictions for $\Omega_B h^2$ in models with neutrino degeneracy using four different representative sets of priors. In

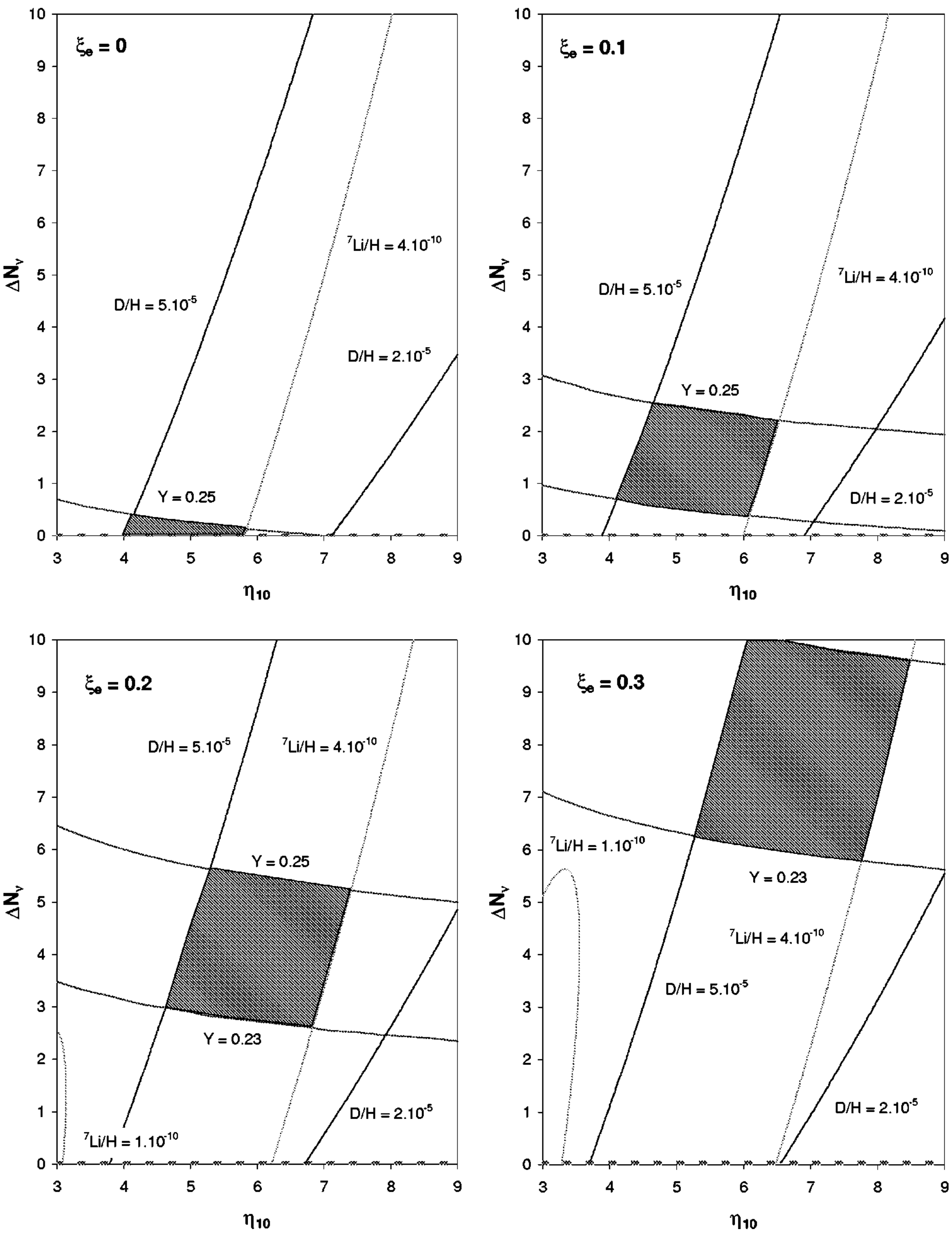

FIG. 1. Iso-abundance contours for deuterium (D/H), lithium (Li/H) and helium (mass fraction, Y) in the $\Delta N_\nu - \eta_{10}$ plane for four choices of ν_e degeneracy (ξ_e). The shaded areas highlight the range of parameters consistent with the adopted abundance ranges [see Eqs. (10)–(12)].

the next section we discuss our calculation and give results for our four models. Our conclusions are summarized in Secs. III and IV.

II. CALCULATIONS

Our first step is the calculation of element abundances in BBN for models with degenerate neutrinos. This is a well-understood calculation with a long history, and the reader is referred to Refs. [12,13,17] for the details.

The degeneracy of any of the three neutrinos increases the total relativistic energy density, leading to an increase in the overall expansion rate. During "radiation dominated" epochs the expansion rate (Hubble parameter) is proportional to the square root of the total energy density in extremely relativistic (ER) particles so the speedup factor, S, is

$$S \equiv H'/H = (\rho'/\rho)^{1/2}. \tag{1}$$

In addition, the *electron* neutrino separately affects the rates of the weak reactions which interconvert protons and neutrons, and so it is convenient to parametrize the neutrino degeneracy in terms of ξ_e and ΔN_ν, where $\xi_e = \mu_e / T_\nu$ is the ratio of the electron neutrino chemical potential μ_e to the neutrino temperature T_ν, and ΔN_ν ($\equiv N_\nu - 3$) is the additional energy density contributed by all the degenerate neutrinos *as well as any other energy density not accounted for in the standard model of particle physics* (e.g., additional relativistic particles) expressed in terms of the equivalent number of extra, non-degenerate, two-component neutrinos:

$$\rho' - \rho \equiv \Delta\rho_{\mathrm{ER}} \equiv \Delta N_\nu \rho_\nu (\xi = 0). \tag{2}$$

The contribution to ΔN_ν from one species of degenerate neutrinos is [12],

$$\Delta N_\nu = 15/7[(\xi/\pi)^4 + 2(\xi/\pi)^2]. \tag{3}$$

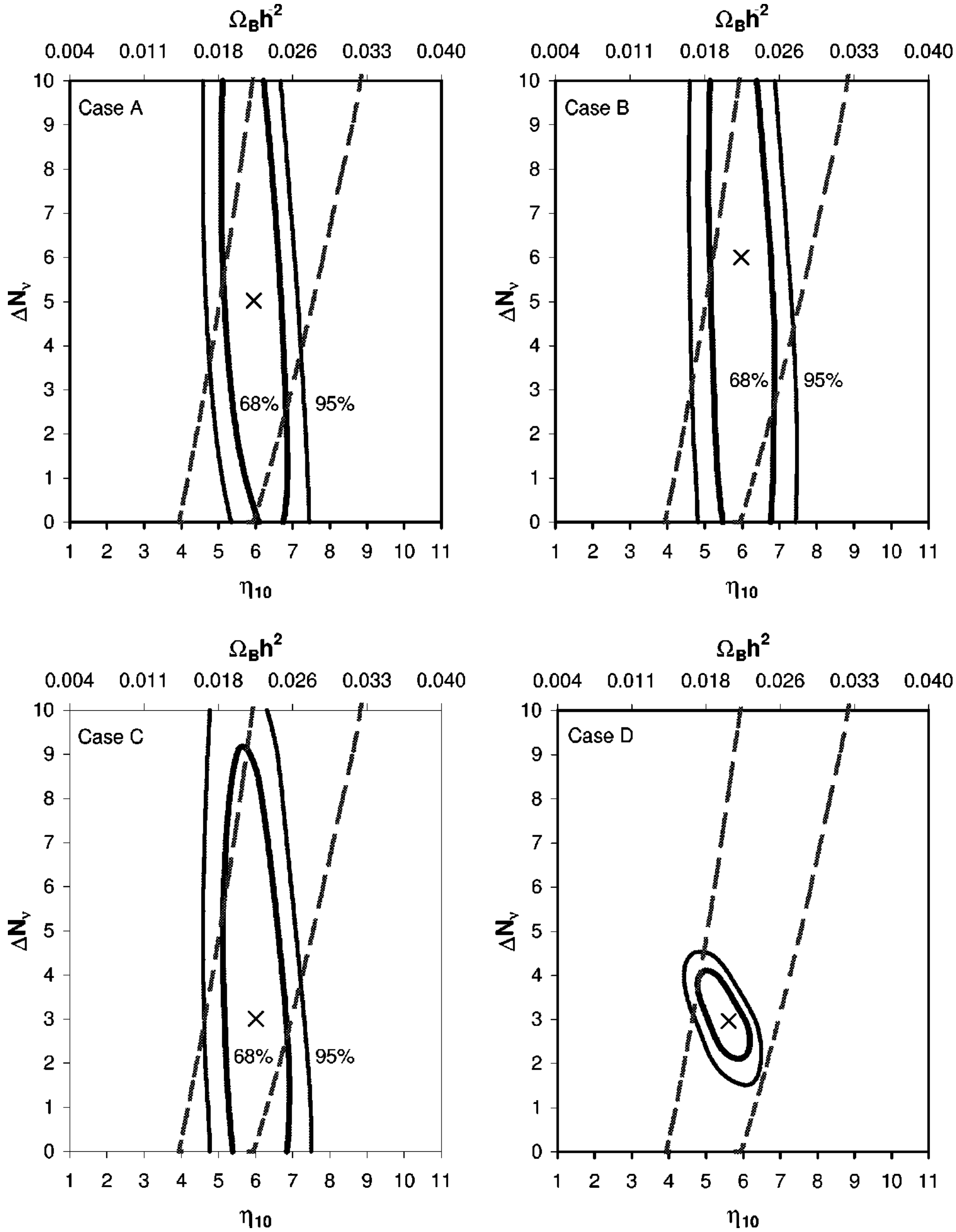

FIG. 2. 68% and 95% contours (solid lines) for the BOOMERANG, MAXIMA, and DASI CMB anisotropies in the $\Delta N_\nu - \eta_{10}$ plane (the upper horizontal axes show $\Omega_B h^2$) for cases A–D. The crosses indicate the best fit values. The consistency band for BBN is shown by the dashed lines.

We emphasize that our results are independent of whether ΔN_ν (or, equivalently, the corresponding value of S) arises from neutrino degeneracy, from "new" (ER) particles, or from some other source. Note that a non-zero value of ξ_e implies a non-zero contribution to ΔN_ν from the electron neutrinos alone; we have included this contribution in our calculations. However, for the range of ξ_e which proves to be of interest for BBN consistency ($\xi_e \lesssim 0.5$), the degenerate electron neutrinos contribute only a small fraction of an additional neutrino species to the energy density ($\Delta N_\nu \lesssim 0.1$).

The question we address is: for a given value of the baryon-photon ratio η ($\eta_{10} \equiv 10^{10}\eta = 274\Omega_B h^2$), are there values for ξ_e and for ΔN_ν which result in agreement between the BBN predictions and the known limits on the primordial element abundances? Through the hard work of many observers, aided by better detectors and bigger telescopes, the statistical uncertainties in the observationally inferred primordial abundances have been reduced significantly in recent years. In contrast, the systematic errors are still quite large (cf. [2]). For this reason we adopt generous ranges for the primordial abundances of ^{4}He, D, and ^{7}Li. Furthermore, even for fixed values of η, ξ_e, and ΔN_ν, there are uncertainties in the BBN-predicted abundances due to uncertainties in the nuclear and/or weak reaction rates. We have chosen the ranges for the primordial abundances large enough to encompass these uncertainties as well. For the primordial helium-4 mass fraction, we take the limits to be

$$0.23 \leqslant Y_P \leqslant 0.25. \tag{4}$$

For deuterium and lithium-7, expressed as number ratios to hydrogen, we take the limits

$$2\times10^{-5} \leqslant \mathrm{D/H} \leqslant 5\times10^{-5} \tag{5}$$

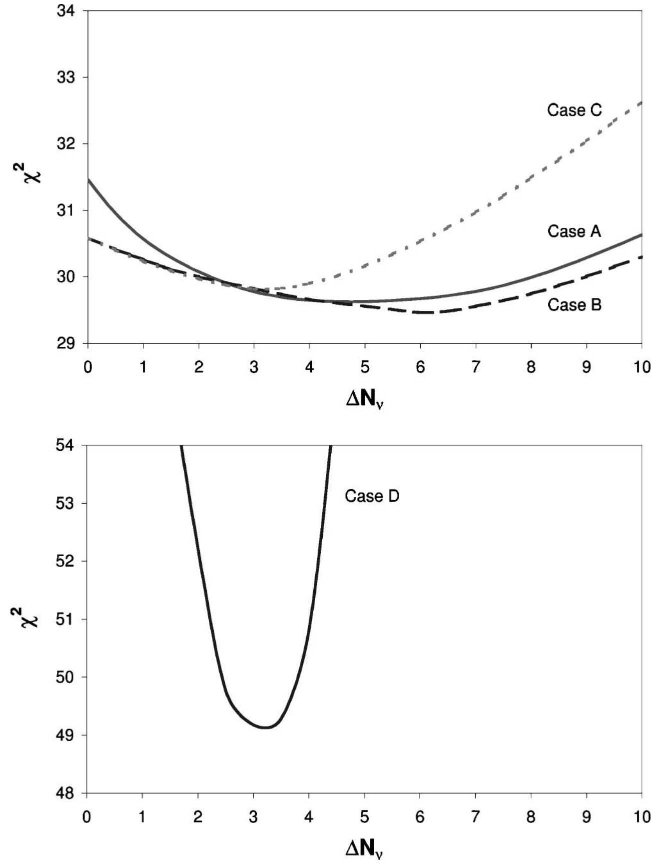

FIG. 3. χ^2 distributions for ΔN_ν for the four sets of priors corresponding to cases A–D.

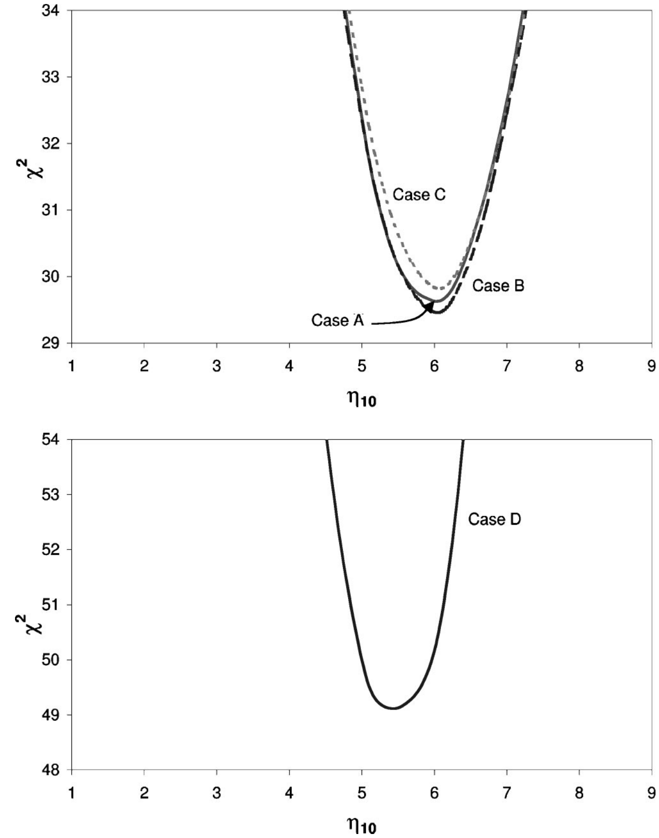

FIG. 4. χ^2 distributions for η_{10} for the four sets of priors corresponding to cases A–D.

and

$$1\times10^{-10}\leq{}^7\mathrm{Li/H}\leq4\times10^{-10}. \tag{6}$$

Our allowed parameter range is thus a three-dimensional volume in the space of η, ξ_e, and ΔN_ν. However, since we wish to compare our BBN constraints with the predictions of the CMB, which are sensitive to η and ΔN_ν, but independent of ξ_e, we project our allowed BBN region onto the $\eta-\Delta N_\nu$ plane. Our BBN results are shown in the four panels of Fig. 1 where, for four choices of ξ_e we show the iso-abundance contours for Y_P, D/H and Li/H in the $\eta-\Delta N_\nu$ plane. The shaded areas highlight the acceptable regions in our parameter space. As ξ_e increases, the allowed region moves to higher values of η and ΔN_ν, tracing out a BBN-consistent band in the $\eta-\Delta N_\nu$ plane. This band is shown by the dashed lines in Figs. 2 and 5. The trends are easy to understand (see Refs. [12,13]). As the baryon density increases the universal expansion rate (as measured by ΔN_ν) must increase to keep the deuterium and lithium unchanged, while the ν_e degeneracy (ξ_e) must increase to maintain the helium abundance at its SBBN value.

We then use CMBFAST [18] to calculate the CMB fluctuation spectrum as a function of η and ΔN_ν and compare with the BOOMERANG [9], MAXIMA [10] and DASI [11] observations. However, the CMB anisotropy spectrum is sensitive to a large number of other parameters which have no effect on BBN, including the fraction of the critical density in non-relativistic matter Ω_M (where Ω_M includes both baryonic and non-baryonic matter), the fraction of the critical density contributed by the cosmological constant Ω_Λ (or an equivalent vacuum energy density), the total Ω ($\equiv\Omega_M+\Omega_\Lambda$; $\Omega=1$ corresponds to a "flat" universe), the Hubble parameter h, and the slope of the primordial power spectrum n ("tilt"). Since we are interested in the way in which restricting these parameters affects the agreement between the CMB and BBN, we consider four representative sets of prior assumptions:

Case A: $\Omega=1,\ 0.4\leq h\leq1.0,\ \Omega_B\leq\Omega_M\leq1,\ n=1.$
Case B: $\Omega=1,\ 0.4\leq h\leq1.0,\ \Omega B\leq\Omega M\leq1,\ 0.7\leq n\leq1.3.$
Case C: $\Omega=1,\ 0.5\leq h\leq0.9,\ \Omega_B\leq\Omega_M\leq0.4,\ 0.7\leq n\leq1.3.$
Case D: $\Omega\leq1,\ 0.5\leq h\leq0.9,\ \Omega_B\leq\Omega_M=\Omega,\ 0.7\leq n\leq1.3.$

In models A–C, the inflation-inspired assumption that the universe is flat is adopted and a cosmological constant is assumed to give $\Omega=1$; in contrast, in model D the value of Ω_Λ is set to zero and the universe is allowed to be open or flat [14]. Tensor modes are ignored in all of these cases. Case A differs from case B only in the restriction of tilt to $n=1$. Case C differs from case B in the adoption of a slightly smaller range for the Hubble parameter and, of more significance, a more restricted range for the non-relativistic matter density, both of which are consistent with complementary observational data. Case C is closest to what is often referred

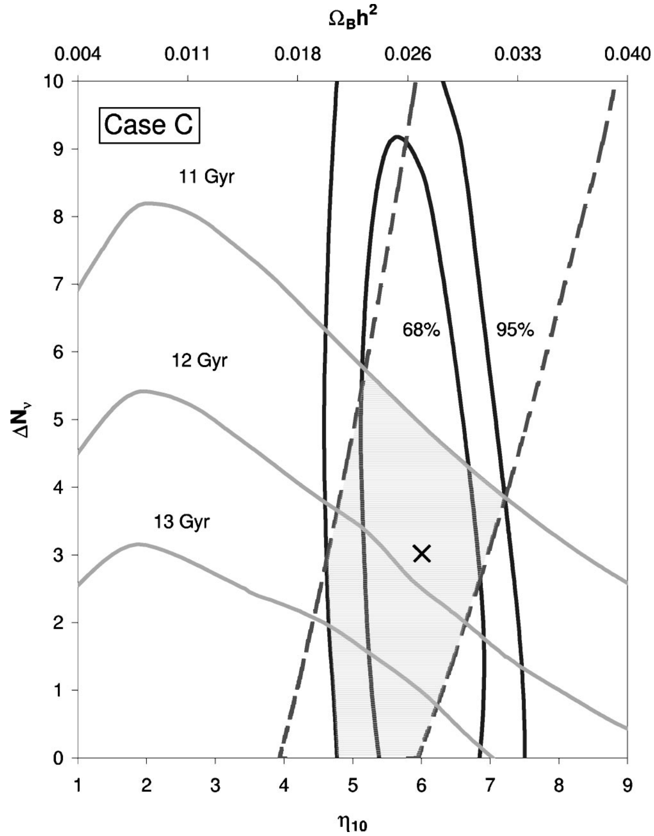

FIG. 5. The BBN (dashed) and CMB (solid) contours in the $\Delta N_\nu - \eta_{10}$ plane for the priors corresponding to Case C (see Fig. 2). The corresponding best fit isochrones are shown for 11, 12, and 13 Gyr. The shaded region delineates the parameters consistent with BBN, CMB (at 95%), and $t_0 > 11$ Gyr.

to as the ''concordance model'' (flat, cold dark matter model with a cosmological constant: ΛCDM) while Case D is closest to the ''standard cold dark matter (SCDM) model'' which is inconsistent with the supernova type Ia (SN Ia) data [19].

For each of these sets of priors, we determine the best fit CMB model for a given pair of values of ΔN_ν and η and assign a confidence limit based on the $\Delta\chi^2$ value calculated with RADPACK [20]. In the four panels of Fig. 2 we display the (two-parameter) 68% and 95% contours in the $\eta - \Delta N_\nu$ plane for the four choices of priors discussed above. The different shapes of the confidence interval contours highlight the sensitivity of the ''new physics'' (ΔN_ν) to the choices of priors for the other cosmological parameters.

III. DISCUSSION

The effect on the post-BBN universe of a non-zero ΔN_ν is to enhance the relativistic energy density, delaying the epoch of equal matter and radiation densities. This can be offset by increasing Ω_M, effectively restoring the original, $\Delta N_\nu = 0$, ratio of matter and radiation densities. This effect produces the large difference between cases A and B and case C. This may be seen in Fig. 3 where the sensitivity of the constraints on ΔN_ν to the priors adopted in the CMB fits is explored by comparing the χ^2 distributions for our four cases A–D. In cases A and B, very large values for ΔN_ν are allowed, corresponding to large values of Ω_M. Thus, cases A and B do not provide very effective upper limits on ΔN_ν when only the CMB data is taken into account (Fig. 3). For case C, in contrast, large values of Ω_M are not permitted. As seen in Fig. 3 this results in a stronger upper bound on ΔN_ν: at the 68% confidence level, $\Delta N_\nu < 6.7$. Case D yields a very different set of constraints. In this model, values of $\Omega_M < 1$ are compensated with curvature, rather than with a cosmological constant. But the position of the first acoustic peak strongly constrains the curvature to be nearly zero, forcing Ω to be nearly unity. Hence, in these models $\Omega_M \approx 1$, with almost no freedom to vary, and a change in ΔN_ν cannot be canceled by changing Ω_M. Thus, the allowed range for ΔN_ν is very small.

Despite the differences, there are some striking similarities in the parameter ranges identified in Fig. 2. With the exception of case D, the preferred ranges of baryon densities are very similar (see Fig. 4). At 68% confidence $5.4 \lesssim \eta_{10} \lesssim 6.6$ (at 95% confidence, $4.8 \lesssim \eta_{10} \lesssim 7.2$), for cases A–C; for case D, η_{10} is shifted downwards by ≈ 0.6. For all cases, a baryon density $\Omega_B h^2 \approx 0.02$ is a robust prediction of the CMB observations.

In contrast, constraints on the magnitude of the ''new physics'' (ΔN_ν) do depend sensitively on the choice of priors. As noted earlier, for the ΛCDM models (cases A–C), case C produces a stronger upper bound on ΔN_ν, than do cases A or B. Figure 3 also illustrates a point which is only marginally apparent from Fig. 2: case A prefers a non-zero value of ΔN_ν slightly more than do cases B and C (albeit not at a statistically significant level). Since case A fixes $n=1$, this suggests that a nonzero ΔN_ν can mimic, to some extent the effect of ''tilt.'' This point is further emphasized when the BBN data are included in Fig. 2: for $\Delta N_\nu = 0$, the overlap between the allowed values for η for CMB and BBN is smaller for case A (ruled out at the 68% confidence level), than for cases B and C. However, given the marginal level of exclusion (68%), this cannot be used to argue for ''new physics.'' In contrast, as already noted, case D is anomalous; in the absence of new physics it disagrees with the CMB data at the 93% confidence level.

It is clear from Fig. 3 that, with the exception of the statistically disfavored case D, the CMB provides only very weak constraints on ΔN_ν. The notable contrast between cases B and C, with very similar priors, demonstrates the significant sensitivity of ΔN_ν to the choice of priors. Because of this sensitivity, it is difficult to compare our results directly with those of Hannestad [21], Lesgourgues and Liddle [22], and of Hansen *et al.* [23]. We are in agreement with Hannestad [21] in that although $\Delta N_\nu \approx 3-6$ appears to be favored by the CMB data, the standard model value of $\Delta N_\nu = 0$ is entirely compatible with the present data.

IV. CONCLUSIONS

In Fig. 5 we choose the priors corresponding to Case C (ΛCDM) to illustrate the confrontation between the BBN constraints and those from the CMB. As already alluded to above, the points in the η–ΔN_ν plane (Fig. 2) are projections from a multi-dimensional parameter space and the relevant values of those additional parameters may not always be consistent with other, independent observational data. As an illustration, in Fig. 5 we also show three isochrones, for 11, 12, and 13 Gyr.

The trend in the isochrones is easy to understand: as ΔN_ν increases, so too do the corresponding values of the matter density (Ω_M) and the Hubble parameter (H_0) which minimize χ^2. In addition, since $\Omega_M+\Omega_\Lambda=1$, Ω_Λ decreases. All of these lead to younger ages for larger values of ΔN_ν. Note that if a constraint is imposed that the universe today is at least 11 Gyr old [24], then the BBN and CMB overlap is considerably restricted (to the shaded region in Fig. 5). Even with this constraint it is clear that there is room for modest "new physics" ($\Delta N_\nu \lesssim 6$; $\xi_e \lesssim 0.3$), for which there is a limited range of baryon density ($0.018 \lesssim \Omega_B h^2 \lesssim 0.026$) which is concordant with both the BBN and CMB constraints. If instead we were to impose a stricter, but still reasonable, constraint on the age, say that the universe be older than 13 Gyr, the acceptable range of baryon density and "new physics" would be considerably narrowed.

BBN alone does not provide any significant constraint on the magnitude of the "new physics" arising from neutrino degeneracy; larger values of ξ_e and ΔN_ν simply correspond to larger values of η (see [12], [13]). In this paper we have shown that CMB observations can constrain ΔN_ν (and, correspondingly, ξ_e) but this constraint is sensitive to the priors chosen when fitting the CMB data. However, we have noted that if an additional cosmological constraint (on the age of the universe) is imposed, this ambiguity can be eliminated and a restricted range of parameters is identified: $\Omega_B h^2 \approx 0.018$–$0.026$, $\Delta N_\nu \lesssim 6$, and $\xi_e \lesssim 0.3$. If the extra relativistic energy density (ΔN_ν) is contributed by degenerate ν_μ, and/or ν_τ, then (see Eq. 3) $\xi_\mu \lesssim 3.1$ (for $\xi_\tau=0$ or, vice-versa) or, $\xi_\mu=\xi_\tau \lesssim 2.3$.

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[1] S. Burles and D. Tytler, Astrophys. J. **499**, 699 (1998); **507**, 732 (1998).
[2] K.A. Olive, G. Steigman, and T.P. Walker, Phys. Rep. **333**, 389 (2000).
[3] S. Burles, K.M. Nollett, and M.S. Turner, Phys. Rev. D **63**, 063512 (2001).
[4] J.M. O'Meara, D. Tytler, D. Kirkman, N. Suzuki, J.X. Prochaska, and A.M. Wolfe, Astrophys. J. **552**, 718 (2001).
[5] P. de Bernardis *et al.*, Nature (London) **404**, 995 (2000).
[6] S. Hanany *et al.*, Astrophys. J. Lett. **545**, L5 (2000).
[7] A.E. Lange *et al.*, Phys. Rev. D **63**, 042001 (2001).
[8] M. White, D. Scott, and E. Pierpaoli, Astrophys. J. **545**, 1 (2000).
[9] C.B. Netterfield *et al.*, astro-ph/0104460.
[10] A.T. Lee *et al.*, astro-ph/0104459.
[11] N.W. Halverson *et al.*, astro-ph/0104489.
[12] H.-S. Kang and G. Steigman, Nucl. Phys. **B372**, 494 (1992).
[13] K.A. Olive, D.N. Schramm, D. Thomas, and T.P. Walker, Phys. Lett. B **265**, 239 (1991).
[14] J. Lesgourgues and M. Peloso, Phys. Rev. D **62**, 081301(R) (2000).
[15] S. Esposito, G. Mangano, A. Melchiorri, G. Miele, and O. Pisanti, Phys. Rev. D **63**, 043004 (2001).
[16] S. Hannestad, Phys. Rev. Lett. **85**, 4203 (2000).
[17] R.V. Wagoner, W.A. Fowler, and F. Hoyle, Astrophys. J. **148**, 3 (1967); A. Yahil and G. Beaudet, *ibid.* **206**, 26 (1976); Y. David and H. Reeves, Philos. Trans. R. Soc. London **A296**, 415 (1980); R.J. Scherrer, Mon. Not. R. Astron. Soc. **205**, 683 (1983); N. Terasawa and K. Sato, Astrophys. J. **294**, 9 (1985); K. Kohri, M. Kawasaki, and K. Sato, *ibid.* **490**, 72 (1997).
[18] U. Seljak and M. Zaldarriaga, Astrophys. J. **469**, 437 (1996).
[19] S. Perlmutter *et al.*, Astrophys. J. **483**, 565 (1997); A.G. Riess *et al.*, Astron. J. **114**, 722 (1997); **116**, 1009 (1998); B.P. Schmidt *et al.*, Astrophys. J. **507**, 46 (1998); S. Perlmutter *et al.*, *ibid.* **517**, 565 (1999); A.V. Filippenko and A.G. Riess, astro-ph/0008057.
[20] J.R. Bond, A.H. Jaffe, and L. Knox, Astrophys. J. **533**, 19 (2000).
[21] S. Hannestad, Phys. Rev. D **64**, 083002 (2001).
[22] J. Lesgourgues and A.R. Liddle, astro-ph/0105361.
[23] S.H. Hansen, G. Mangano, A. Melchiorro, G. Miele, and O. Pisanti, astro-ph/0105385.
[24] B. Chaboyer (private communication); B. Chaboyer and L. Krauss (in preparation).