Expected signals in relic neutrino detectors

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Here we estimate the magnitude of the signals expected for realistic cosmic neutrino backgrounds in detectors attempting to measure the mechanical forces exerted on macroscopic targets by the elastic scattering of relic neutrinos. We study effects proportional to the weak coupling constant G_F and to G_F^2 for Dirac and Majorana neutrinos, either relativistic or nonrelativistic, both gravitationally bound or not.

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I. INTRODUCTION

Along side the well measured background of cosmic photons, the big bang model predicts the existence of an elusive background of cosmic neutrinos. This neutrino background may be revealed in the near future by measurements of the microwave background radiation anisotropy and the large-scale structure of the Universe.

Here we reconsider the long-standing question of the detectability of the neutrino background in a laboratory experiment, a recurrent subject in the literature for the past thirty years [1–6]. We estimate how far macroscopic accelerations due to realistic neutrino backgrounds are from the smallest measurable acceleration at present, which is about 10^{-12} cm/sec^2 [7]. The new elements we consider in our estimates are the possibility of a very large lepton asymmetry in the cosmic neutrino background, and the study of Majorana as well as Dirac neutrinos.

With no lepton number asymmetry the number density n_{ν_i} of the neutrinos of each species in the background is equal to the number of antineutrinos of each species. The big bang model predicts this number density to be $n_\nu = (3/22) n_\gamma = 0.136 n_\gamma$, where n_γ is the density of the microwave background radiation photons. Since $n_\gamma = 412\ \text{cm}^{-3}$, therefore $n_{\nu_i} = 56\ \text{cm}^{-3}$. It is, however, possible for neutrinos to have very large lepton asymmetries. While charge neutrality requires the asymmetry in charged leptons to be the same as that in protons, for which $(n_B - n_{\bar{B}})/n_\gamma \simeq O(10^{-10})$, no such requirement limits the asymmetry in neutrinos. With a large asymmetry between neutrinos and antineutrinos only the excess of either ν_i or $\overline{\nu_i}$ remains after annihilation ceases with a density

$$n_{\nu_i} = n_\gamma 0.0252 [\xi_i \pi^2 + \xi_i^3] \tag{1}$$

where $\xi_i = |\mu_{\nu_i}|/T_{\nu_i}$ and μ_{ν_i} is the chemical potential of the given neutrino species. The ratio ξ_i is constant after neutrinos decouple.

Thus, for example, with $\xi_i = 5$ [8–10], one obtains a neutrino density of $n_\nu \simeq 4 n_\gamma = 1700\ \text{cm}^{-3}$. Equation (1) is valid for $\xi_i < 12$ [8] (for which the decoupling temperature T_{dec} is smaller than $2m_\mu$; see below). Bounds coming from big bang nucleosynthesis (BBN), the anisotropy of the microwave background radiation and data on the large scale structure of the Universe limit ξ_i to be much lower than 12. Kang and Steigman in 1992 [8] obtained

$$-0.06 \leq \xi_e \leq 1.1, \quad |\xi_{\mu,\tau}| \leq 6.9. \tag{2}$$

More recent studies yield bounds that depend on the presence of a cosmological constant (with energy density Ω_Λ in units of the critical density). If the cosmological constant is zero or small, $\Omega_\Lambda < 0.1$, the value $\xi_i \simeq 5$, namely $n_{\nu_i}/n_\gamma \simeq 4$ is favored [9,10]. For $\Omega_\Lambda \simeq 0.5$, one obtains $n_{\nu_i}/n_\gamma < 4$, and for $\Omega_\Lambda > 0.7$, the lepton asymmetry must be very small, so the neutrino density goes back to the standard value, $n_{\nu_i}/n_\gamma \simeq 0.14$ [9,11]. Notice that in models with large neutrino asymmetries, the standard BBN mechanism is altered, allowing for the existence of many more neutrinos than in the standard case. BBN and recent cosmic microwave background anisotropy data provide constraints on neutrino asymmetries which depend strongly on cosmological parameters [12]. In what follows we will take an upper bound $n_{\nu_i}/n_\gamma \leq 4$, i.e., $n_{\nu_i} \leq 1700\ \text{cm}^{-3}$, when considering the case of large lepton asymmetries.

The decoupling temperature of neutrinos $T_{\rm dec}$, which with no lepton asymmetry is a few MeV, increases with the lepton asymmetry. However, only for $\xi_i \geq 12$ would $T_{\rm dec}$ become larger than $2m_\mu$, i.e., neutrinos would decouple before $\mu^+\mu^-$ annihilation. In this case, not only would e^+e^- annihilations increase the number of photons relative to that of neutrinos due to entropy conservation, but $\mu^+\mu^-$ annihilations would as well, leading to a lower neutrino temperature relative to the photon temperature. For $\xi_i < 12$ the temperature of ν_i is now the usual one, $T_{\nu_i} = (4/11)^{1/3}\ T_\gamma = 0.71 T_\gamma$. With $T_\gamma = 2.728$ K, this means $T_{\nu_i} = 1.95\ \text{K} = 1.68 \times 10^{-4}$ eV. Therefore, if m_{ν_i} is smaller than 1.7×10^{-4} eV background neutrinos are relativistic at present while for larger masses they are non-relativistic. Moreover neutrinos may be Dirac or Majorana particles, a distinction that is important only for non-relativistic neutrinos.

Relativistic neutrinos are only in left-handed chirality states (and anti-neutrinos only in right-handed chirality states). These are the only states produced by weak interactions. For relativistic neutrinos chirality and helicity coincide (up to mixing terms of order $m_{\nu_i}/E_\nu \simeq m_{\nu_i}/T_\nu$). Bounds from structure formation in the Universe imply that stable neutrino masses are at most of the order of a few eV [13]. This means that neutrinos were relativistic at decoupling [$T_{\rm dec} \geqslant O$ (MeV)], even if they may be non-relativistic at present.

We will call left (right) chirality eigenstates ν_L (ν_R) and left (right) helicity eigenstates $\nu_l(\nu_r)$. At decoupling, neutrinos ν_L were only in left-handed helicity states and antineutrinos ν_R^c (or ν_R in the case of Majorana neutrinos) in right-handed ones. Helicity is an eigenstate of propagation and therefore it does not change while neutrinos propagate freely, even if they become non-relativistic. For Majorana neutrinos chirality acts as lepton number, so we are calling ''neutrinos'' those particles produced at $T > T_{dec}$ as ν_L and ''anti-neutrinos'' those produced as ν_R. Thus, neglecting intervening interactions, non-relativistic background neutrinos are in left-handed helicity eigenstates (which consist of equal admixtures of left- and right-handed chiralities) and antineutrinos are in right-handed helicity eigenstates (which also consist of equal admixtures of left- and right-handed chiralities). If the non-relativistic neutrinos are Dirac particles, only the left-handed chirality states (right for anti-neutrinos) interact, since the other chirality state is sterile, while if the neutrinos are Majorana, both chirality states interact (the right-handed ''neutrino'' state is the right-handed anti-neutrino).

Slow enough non-relativistic neutrinos eventually fall into gravitational potential wells, become bound and, after a characteristic orbital time, their helicities become well mixed up, since momenta are reversed and spins are not. Thus, gravitationally bound background neutrinos have well mixed helicities. Most background neutrinos however are not gravitationally bound at present because they are too light.

The present upper bound from structure formation in the Universe for neutrinos without a lepton asymmetry, for which $n_\nu \simeq 100\ \text{cm}^{-3}$, is of a few eV [13]. Neutrinos carrying a large lepton asymmetry must necessarily be lighter. The upper bound on the mass of degenerate neutrinos with a large asymmetry should be lower, since their number density is larger, at most of the order of 0.1 eV. The Tremaine-Gunn kinematical constraint [14] gives the scale at which neutrinos gravitationally cluster. Light neutrinos with masses $m_\nu < eV$ would be gravitationally bound only to the largest structures, large clusters of galaxies. We can see this using simple velocity arguments. Only cosmic neutrinos with velocities smaller than the escape velocity of a given structure can be bound to it. The escape velocity from a large galaxy like ours is about 600 km/s and from a large cluster of galaxies is about 2000 km/s. Considering that the average velocity modulus of non-relativistic neutrinos of mass m and temperature T_ν is (using a Maxwell-Bolztman distribution) $\langle|\vec{\beta}_\nu|\rangle = \sqrt{8kT_\nu/\pi m} = \sqrt{4.3\times 10^{-4}\ \text{eV/m}}$ (namely $\langle|\vec{v}_\nu|\rangle = 6200$ km/s for $m=1$ eV, and $\langle|\vec{v}_\nu|\rangle = 19\,600$ km/s for $m = 0.1$ eV) it is obvious that only about a third of 1 eV mass neutrinos and a very small fraction of lighter neutrinos could be gravitationally bound to large clusters at present. Fermi degenerate neutrinos may have even larger average velocities depending on their chemical potential $\xi = \mu/T_\nu$ (constant after neutrinos decouple), but the conclusions remain the same. For $\xi \gg 1$ $\langle|\vec{\beta}_\nu|\rangle = \sqrt{6\xi T_\nu/5m} \simeq \sqrt{\xi 1.68\ 10^{-4}\ \text{eV}/m}$ (namely $\langle|\vec{v}_\nu|\rangle = \sqrt{\xi}\ 12\,300$ km/s for $m = 0.1$ eV) and both expressions coincide for $\xi = 2.5$. In all cases the amount of neutrinos in the tail of the velocity distribution with velocities smaller that 600 km/s which would be gravitationally bound to galaxies is much smaller.

In the following we will give our results for both clustered (C-NR) and nonclustered (NC-NR) nonrelativistic background neutrinos as well as for relativistic (R) background neutrinos.

With zero lepton asymmetry the background consists of equal numbers of neutrinos and anti-neutrinos. Thus we have equal numbers of left and right either chirality or helicity states in all cases, relativistic or non-relativistic, Dirac or Majorana neutrinos. In the case of a large lepton asymmetry which favors, say, neutrinos so that there are only ν_L at the moment neutrinos decouple (left and right are to be exchanged in the following argument for an asymmetry favoring anti-neutrinos), the background consists of left chirality particles in the case of R or NC-NR neutrinos, and it consists of totally mixed helicity states for C-NR neutrinos

These properties of the different possible neutrino backgrounds are relevant to the effect linear in the weak coupling constant G_F, first studied by Stodolsky [1].

II. THE G_F EFFECT

This effect was proposed by Stodolsky in 1974 (before weak neutral currents were proven to exist, so he did not include them). It is the energy split of the two spin states of non-relativistic electrons in the cosmic neutrino bath. This energy split is proportional to the difference between the densities of neutrinos and antineutrinos in the neutrino background for Dirac neutrinos, and proportional to the net helicity of the background for Majorana neutrinos, as we will now see.

The Hamiltonian density of the ν-e interaction is

$$\mathcal{H}(x) = \frac{G_F}{\sqrt{2}} \bar{e}\gamma^\mu (g_V - g_A\gamma_5) e \bar{\nu}\gamma_\mu (1-\gamma_5)\nu. \tag{3}$$

For $\nu = \nu_e$, $g_A = 1/2$ and $g_V = 1/2 + 2\sin^2\theta_W$ while for $\nu = \nu_\mu$ or $\nu = \nu_\tau$, with which electrons only have neutral weak interactions, $g_A = -1/2$ and $g_V = -1/2 + 2\sin^2\theta_W$.

A. Dirac neutrinos

We compute first the energy shift of a single electron of momentum p and spin s in the neutrino background: $\Delta E_e = \langle p,s|H|p,s\rangle$. Let us call p and k the momenta of the e and ν respectively, and s (s') the incoming (outgoing) spins.

Working in momentum space and using the second quantization form for the neutrino fields, we obtain, for the case of Dirac neutrinos,

$$\langle H^D\rangle=\frac{G_F}{\sqrt{2}}\frac{m_e}{E_eV}\left(\sum_{s,s'}\overline{u_e}(p,s')\gamma^{\mu}(g_V-g_A\gamma_5)u_e(p,s)\right)\times\left\{\int d^3k\left(\frac{m_\nu}{E_\nu}\right)\sum_{s,s'}b^{\dagger}(k,s')b(k,s)[\overline{u_\nu}(k,s')\gamma_\mu\times(1-\gamma_5)u_\nu(k,s)]-\int d^3k\left(\frac{m_{\nu^c}}{E_{\nu^c}}\right)\sum_{s,s'}d^{\dagger}(k,s')d(k,s)\times[\overline{v_\nu}(k,s)\gamma_\mu(1-\gamma_5)v_\nu(k,s')]\right\} \quad (4)$$

where u and v are the usual spinors for particles and antiparticles and b, d ($b^\dagger$, $d^\dagger$) are the respective annihilation (creation) operators, for neutrinos and antineutrinos respectively, of momentum k. In the rest frame of the electron ($p=0$) the diagonal matrix element of the weak current is $g_A\sigma_e^i$. In this frame the neutrino current is $J_\nu^i=-\beta^i_{earth}(n_\nu-n_{\nu^c})$, since the non-zero average velocity $-\vec{\beta}_{earth}$ of the neutrinos is due to the motion of the earth relative to the neutrino bath (with velocity $\vec{\beta}_{earth}$). Dotting the two together yields $\Delta E=-(G_F/\sqrt{2})g_A(\vec{\sigma}_e\cdot\vec{\beta}_{earth})(n_\nu-n_{\nu^c})$.

We would like to present however a more careful, systematic, and first principle derivation. Using the decomposition $u_e^T(p,s)=\sqrt{(E_e+m_e)/2m_e}\;(\chi_s^T,[(\vec{\sigma}_e\cdot\vec{p})/(E_e+m_e)]\chi_s^T)$ for the electron spinors and the Gordon decomposition and similar relations containing γ_5 for the neutrino vertices, one obtains

$$\langle H^D\rangle=\frac{G_F}{\sqrt{2}}\left(\frac{E_e+m_e}{E_e}\right)\left\{g_V\left[1+\left(\frac{2\vec{\sigma}_e\cdot\vec{p}}{E_e+m_e}\right)^2\right]-2g_A\frac{2\vec{\sigma}_e\cdot\vec{p}}{E_e+m_e},\; 2\vec{\sigma}_e\left(2g_V\frac{2\vec{\sigma}_e\cdot\vec{p}}{E_e+m_e}-g_A\left[1+\left(\frac{2\vec{\sigma}_e\cdot\vec{p}}{E_e+m_e}\right)^2\right]\right)\right\}\times\frac{1}{V}\left\{\int d^3k\sum_{s,s'}b^{\dagger}(k,s')b(k,s)\frac{1}{E_\nu}[\overline{u_\nu}(k,s')(k_\mu-2W_\mu)u_\nu(k,s)]+\int d^3k\sum_{s,s'}d^{\dagger}(k,s')d(k,s)\frac{1}{E_{\nu^c}}[\overline{v_\nu}(k,s)(k_\mu-2W_\mu)v_\nu(k,s')]\right\}. \quad (5)$$

Here the comma in the first key bracket separates the zero and spatial components of the electron amplitude and $W_\mu=i\sigma_{\mu\nu}k^\nu\gamma_5=-\frac{1}{2}\epsilon_{\mu\nu\sigma\rho}\sigma^{\sigma\rho}k^\nu$ is the Pauli-Lubansky pseudo-vector (one uses here $\gamma_5\sigma_{\mu\nu}=(i/2)\epsilon_{\mu\nu\sigma\rho}\sigma^{\sigma\rho}$). In terms of the four dimensional "spin operator"

$$\vec{\Sigma}=\begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix} \quad (6)$$

the Pauli-Lubansky pseudovector is given by

$$W_\mu=\left(\frac{\vec{\Sigma}}{2}\cdot\vec{k},\;-\frac{E_\nu}{2}\vec{\Sigma}+\frac{i}{2}\gamma_5(\vec{k}\times\vec{\Sigma})\right), \quad (7)$$

so that for particles at rest ($\vec{k}=0$) it is proportional to the spin operator $W_\mu=(0,-m_\nu\vec{\Sigma}/2)$ and for relativistic particles it is proportional to the helicity $h=\vec{\Sigma}\cdot\hat{k}$, i.e. $W_\mu|k\rangle=(h/2)k_\mu|k\rangle$. Although $\vec{W}$ is the spin operator in the rest frame and $W^\mu W_\mu=-m^2s(s+1)$ is a Casimir operator of the Poincaré algebra, in general $\vec{W}$ is not the spin operator: it is easy to see that the W_i do not have the SU(2) commutation relations required for angular momentum components, i.e. $[W_i,W_j]\neq i\epsilon_{ijk}W_k$. However, one can write $\vec{W}$ in terms of the true spin operator [15]

$$\vec{S}=\frac{1}{m_\nu}\left(\vec{W}-\frac{W_0\vec{k}}{E_\nu+m_\nu}\right). \quad (8)$$

Considering non-relativistic electrons with velocity $\vec{\beta}_e=\vec{p}/E_e$ [thus $(E_e+m_e)/E_e=2$], using $\vec{\beta}_\nu=\vec{k}/E_\nu$, and writing $\vec{W}$ in terms of $\vec{S}$, we obtain, to first order in $\vec{\beta}_e$,

$$\langle H^D\rangle=\frac{G_F}{\sqrt{2}}\{(g_V-g_A 2\vec{s}_e\cdot\vec{\beta}_e),\ \ 2\vec{s}_e(-g_A+g_V 2\vec{s}_e\cdot\vec{\beta}_e)\}\frac{1}{V}\Bigg\{\int d^3k\sum_{s,s'} b^\dagger(k,s')b(k,s)$$
$$\times\left[\overline{u}_\nu(k,s')\left(1-\vec{\Sigma}\cdot\vec{\beta}_\nu,\ \ -\vec{\beta}_\nu+\frac{2m_\nu}{E_\nu}\vec{S}+\frac{E_\nu\vec{\beta}_\nu}{E_\nu+m_\nu}\vec{\Sigma}\cdot\vec{\beta}_\nu\right)u_\nu(k,s)\right]+\int d^3k\sum_{s,s'} d^\dagger(k,s')d(k,s)$$
$$\times\left[\overline{v}_\nu(k,s)\left(1-\vec{\Sigma}\cdot\vec{\beta}_\nu,\ \ -\vec{\beta}_\nu+\frac{2m_\nu}{E_\nu}\vec{S}+\frac{E_\nu\vec{\beta}_\nu}{E_\nu+m_\nu}\vec{\Sigma}\cdot\vec{\beta}_\nu\right)v_\nu(k,s')\right]\Bigg\}. \tag{9}$$

In the neutrino "rest-frame" the term of the form $\vec{s}_e\cdot\vec{S}$ in Eq. (9) vanishes for the following simple reason. We have calculated our interaction between a "sea" of neutrinos and a single electron. Thus $\vec{s}_e\cdot\vec{S}$ gives the projection of the individual neutrino spin along the single electron's spin axis. But we expect the neutrinos to be distributed isotropically, and thus there is equal probability for the neutrino spin projected along this axis to be positive or negative. Hence when the sum over spins and the integration over the isotropic neutrino distribution are performed, these terms becomes zero. However, in the electron rest-frame the neutrino distribution is no longer isotropic and this term does not vanish. Using the basis of neutrinos of definite helicity, we can take the spin to lie completelty parallel or anti-parallel to the direction of motion, i.e., $\vec{S}=(h/2)\hat{\beta}$ where h is the helicity. Thus the term in Eq. (9) containing the neutrino spin will be proportional to $\langle(m_\nu/E_\nu)\hat{\beta}\rangle$.

Instead of utilizing the spin basis, we now work in the helicity basis so that all spin dependent terms involve the helicity operator $\vec{\Sigma}\cdot\hat{\beta}$. We can use now the completeness relations for the spinors u and v,

$$\overline{u}_\nu(k,h')u_\nu(k,h)=\delta_{hh'},\quad \overline{v}_\nu(k,h')v_\nu(k,h)=-\delta_{hh'}. \tag{10}$$

Furthermore, in the helicity basis we have

$$\overline{u}_\nu(k,h')\vec{\Sigma}_\nu\cdot\vec{\beta}_\nu u_\nu(k,h)=h|\vec{\beta}_\nu|\delta_{hh'}, \tag{11}$$

$$\overline{v}_\nu(k,h')\vec{\Sigma}_\nu\cdot\vec{\beta}_\nu v_\nu(k,h)=-h|\vec{\beta}_\nu|\delta_{hh'}, \tag{12}$$

where $h=2\vec{s}\cdot\hat{\beta}$ is the helicity. Thus we obtain

$$\langle H^D\rangle=\frac{G_F}{\sqrt{2}}\frac{1}{V}\{(g_V-g_A 2\vec{s}_e\cdot\vec{\beta}_e),\ \ 2\vec{s}_e(-g_A+g_V 2\vec{s}_e\cdot\vec{\beta}_e)\}$$
$$\times\int d^3k\Bigg\{\sum_h b^\dagger(k,h)b(k,h)\left(1-h,-\vec{\beta}_\nu+\frac{E_\nu h}{E_\nu+m_\nu}|\vec{\beta}_\nu|\vec{\beta}_\nu+\frac{m_\nu}{E_\nu}h\hat{\beta}_\nu\right)$$
$$-\sum_h d^\dagger(k,h)d(k,h)\left(1-h,-\vec{\beta}_\nu+\frac{E_\nu h}{E_\nu+m_\nu}|\vec{\beta}_\nu|\vec{\beta}_\nu+\frac{m_\nu}{E_\nu}h\hat{\beta}_\nu\right)\Bigg\}. \tag{13}$$

The number density of neutrinos and antineutrinos are respectively

$$n_\nu=\frac{1}{V}\int d^3k\sum_h b^\dagger(k,h)b(k,h), \tag{14}$$

$$n_{\nu^c}=\frac{1}{V}\int d^3k\sum_h d^\dagger(k,h)d(k,h), \tag{15}$$

and obviously

$$\frac{1}{V}\int d^3k\sum_h b^\dagger(k,h)b(k,h)\vec{\beta}_\nu=\langle\vec{\beta}_\nu\rangle n_\nu, \tag{16}$$

and similarly for antineutrinos, where $\langle\ \rangle$ denotes average values.

Recall that we denote left- and right-handed helicity states with lower case indices, ν_l and ν_r. In general the neutrino and the anti-neutrino are admixtures of states of left and right helicity. Thus we write

$$n_\nu=n_{\nu_l}+n_{\nu_r},\quad n_{\nu^c}=n_{\nu_l^c}+n_{\nu_r^c}. \tag{17}$$

With this notation the terms in $\langle H^D\rangle$ linear in $\vec{s}_e$ are $H^D_{s_e}$,

$$H^D_{s_e}=(G_F/\sqrt{2})\bigg\{-g_A\ 2\vec{s}_e\cdot\vec{\beta}_e[(n_\nu-n_{\nu^c})+\langle|\vec{\beta}_\nu|\rangle\times(n_{\nu_l}+n_{\nu^c_l}-n_{\nu_r}-n_{\nu^c_r})]+g_A 2\vec{s}_e\cdot\bigg[\langle\vec{\beta}_\nu\rangle(n_\nu-n_{\nu^c})+\left(\left\langle\frac{E_\nu}{E_\nu+m_\nu}\vec{\beta}_\nu|\vec{\beta}_\nu|\right\rangle+\left\langle\frac{m_\nu}{E_\nu}\hat{\beta}_\nu\right\rangle\right)\times(n_{\nu_l}+n_{\nu^c_l}-n_{\nu_r}-n_{\nu^c_r})\bigg]\bigg\}. \quad (18)$$

In the rest frame of the ν-bath due to isotropy the average of all vectors proportional to neutrino velocities $\langle\vec{\beta}_\nu\rangle=0$, $\langle\hat{\beta}_\nu\rangle=0$, $\langle(E_\nu/E_\nu+m_\nu)\vec{\beta}_\nu|\vec{\beta}_\nu|\rangle=0$ and $\langle(m_\nu/E_\nu)\ \hat{\beta}_\nu\rangle=0$, thus only the first term in Eq. (18) remains. In the rest frame of the electron, the frame in which experiments are performed, $\vec{\beta}_e=0$ and only the second term in Eq. (18) remains. In this frame the non-zero value of $\langle\vec{\beta}_\nu\rangle$ is due to the motion of the earth relative to the neutrino bath, $\langle\vec{\beta}_\nu\rangle=-\vec{\beta}_{earth}$, which is approximately 10^{-3}. One can compute the averages in Eq. (18) explicitly in the relativistic (R) and non-relativistic (NR) limits, using a Gaussian distribution for the neutrinos in the latter case. Up to first order in $\vec{\beta}_{earth}$ we have

$$\left\langle\frac{E_\nu}{E_\nu+m_\nu}\vec{\beta}_\nu|\vec{\beta}_\nu|\right\rangle_R=-\langle|\vec{\beta}_\nu|\rangle\vec{\beta}_{earth}=-\vec{\beta}_{earth}, \quad (19)$$

$$\left\langle\frac{m_\nu}{E_\nu}\hat{\beta}_\nu\right\rangle_R=-\frac{\vec{\beta}_{earth}}{\gamma}, \quad (20)$$

$$\left\langle\frac{E_\nu}{E_\nu+m_\nu}\vec{\beta}_\nu|\vec{\beta}_\nu|\right\rangle_{NR}=-\frac{2}{3}\langle|\vec{\beta}_\nu|\rangle\vec{\beta}_{earth}+O(\vec{\beta}^3_{earth}), \quad (21)$$

$$\left\langle\frac{m_\nu}{E_\nu}\hat{\beta}_\nu\right\rangle_{NR}=-\frac{16}{3\pi}\frac{\vec{\beta}_{earth}}{\langle|\vec{\beta}_\nu|\rangle}+O(\vec{\beta}^3_{earth}), \quad (22)$$

where $\langle|\vec{\beta}_\nu|\rangle$ is always the average of the velocity modulus in the neutrino rest frame. We see in Eqs. (19) and (21) that $\langle(E_\nu/E_\nu+m_\nu)\vec{\beta}_\nu|\vec{\beta}_\nu|\rangle$ is either of the same order of magnitude or smaller than $\vec{\beta}_{earth}$. The remaining average, $\langle(m_\nu/E_\nu)\ \hat{\beta}_\nu\rangle$ is negligible for relativistic neutrinos with $\gamma\gg 1$, but is larger than $\vec{\beta}_{earth}$ for non-relativistic neutrinos, for which $\langle|\vec{\beta}_\nu|\rangle<1$ [see Eq. (22)]. The factor $(n_{\nu_l}+n_{\nu^c_l}-n_{\nu_r}-n_{\nu^c_r})$, which multiplies both these averages in Eq. (18), becomes zero for C-NR neutrinos (since the helicities are well mixed) but could be large for NC-NR neutrinos in the case of a large lepton asymmetry. In this case $-\langle(m_\nu/E_\nu)\hat{\beta}_\nu\rangle=1.7\sqrt{m/\xi 1.7\times 10^{-4}\text{ eV}}\ \vec{\beta}_{earth}\leqslant(14/\sqrt{\xi})\vec{\beta}_{earth}$ (considering that the mass of neutrinos with a very large lepton asymmetry is at most of the order of 0.1 eV) and this term is dominant.

From Eqs. (18) to (22) we have that in the rest frame of the electron, the frame in which experiments are performed, to first order in $\vec{\beta}_{earth}$,

$$H^D_{s_e}=-\frac{G_F}{\sqrt{2}}g_A 2\vec{s}_e\cdot\vec{\beta}_{earth}\times[(n_\nu-n_{\nu^c})+(x\langle|\vec{\beta}_\nu|\rangle^{-1}+y\langle|\vec{\beta}_\nu|\rangle)\times(n_{\nu_l}+n_{\nu^c_l}-n_{\nu_r}-n_{\nu^c_r})] \quad (23)$$

where $y=1$ and $x=0$ for relativistic neutrinos and $y=2/3$ and $x=16/3\pi=1.7$ for non-relativistic neutrinos [these numbers are the prefactors in Eqs. (19), (21), and (22) above]. While in the rest frame of the neutrino bath, where $\langle\vec{\beta}_\nu\rangle=0$ and $\vec{\beta}_e=\vec{\beta}_{earth}$ we find

$$H^D_{s_e}=-\frac{G_F}{\sqrt{2}}g_A 2\vec{s}_e\cdot\vec{\beta}_{earth}\times[(n_\nu-n_{\nu^c})+\langle|\vec{\beta}_\nu|\rangle(n_{\nu_l}+n_{\nu^c_l}-n_{\nu_r}-n_{\nu^c_r})]. \quad (24)$$

For relativistic neutrinos, for which chirality and helicity coincide, we find in both reference frames that $H^D_{s_e}=H^D_R$ is

$$H^D_R=-\sqrt{2}G_F g_A 2\vec{s}_e\cdot\vec{\beta}_{earth}(n_{\nu_L}-n_{\nu^c_R})=-\sqrt{2}G_F g_A 2\vec{s}_e\cdot\vec{\beta}_{earth}(n_\nu-n_{\nu^c}), \quad (25)$$

and for non-relativistic gravitationally bound (C-NR) Dirac neutrinos (with well mixed helicities) we find to order $\vec{\beta}_{earth}$, again in both reference frames, that $H^D_{s_e}=H^D_{C\text{-}NR}$ is

$$H^D_{C\text{-}NR}=-\frac{G_F}{\sqrt{2}}g_A 2\vec{s}_e\cdot\vec{\beta}_{earth}(n_\nu-n_{\nu^c})=\frac{1}{2}H^D_R, \quad (26)$$

which is smaller than the effect in Eq. (25) for relativistic neutrinos by a factor of two.

For most of the non-relativistic relic neutrinos, those non-clustered, in the presence lepton asymmetry, we find that in the rest frame of the electron the dominant contribution to $H^D_{s_e}=H^D_{NC\text{-}NR}$ is

$$H^D_{NC\text{-}NR}\simeq-\frac{G_F}{\sqrt{2}}g_A 2\vec{s}_e\cdot\vec{\beta}_{earth}1.7\langle|\vec{\beta}_\nu|\rangle^{-1}\times(n_{\nu_l}+n_{\nu^c_l}-n_{\nu_r}-n_{\nu^c_r}), \quad (27)$$

with $\langle|\vec{\beta}_\nu|\rangle^{-1}=\sqrt{m/\xi 1.7\times 10^{-4}\text{ eV}}\ \vec{\beta}_{earth}\leqslant(8.2/\sqrt{\xi})\vec{\beta}_{earth}$.

It is obvious that the effect is non-zero only in the presence of a lepton asymmetry, where $n_\nu\neq n_{\nu^c}$. The effect is maximum if the relic bath consists only of neutrinos (or only of antineutrinos) so that $n_\nu\neq 0$ and $n_{\nu^c}=0$ (or vice versa), which is possible with a large lepton asymmetry ($\xi > 2$). In this case, Eq. (25) becomes

$$H_R^D = -\sqrt{2} G_F g_A 2\vec{s}_e \cdot \vec{\beta}_{earth} n_\nu, \tag{28}$$

and Eq. (27) becomes

$$H_{NC\text{-}NR}^D \simeq 0.85 \sqrt{\frac{m_\nu}{\xi 1.7\times 10^{-4}\ \text{eV}}} H_R^D \leqslant \frac{7}{\sqrt{\xi}} H_R^D, \tag{29}$$

since $m_\nu \leqslant 0.1$ eV, in the presence of a large lepton asymmetry.

B. Majorana neutrinos

In comparison to Dirac neutrinos, Majorana neutrinos satisfy an additional constraint, $\nu = \nu^c$. We write the Majorana field as $\Psi_\nu^M = (\Psi_\nu^D + (\Psi_\nu^D)^c)/\sqrt{2}$. Therefore, using the ordinary decomposition for Dirac neutrinos Ψ_ν^D one arrives at

$$\Psi_\nu^M(x,t) = \int \frac{d^3k}{(2\pi)^{3/2}} \left(\frac{m_\nu}{E_\nu}\right)^{1/2} \sum_s \left[\left(\frac{b(k,s)+d(k,s)}{\sqrt{2}}\right) \times u(k,s) e^{-ik\cdot x} + \left(\frac{b(k,s)+d(k,s)}{\sqrt{2}}\right)^\dagger v(k,s) e^{ik\cdot x}\right]. \tag{30}$$

Defining a new operator $\tilde{b}(k,s) = (b(k,s)+d(k,s))/\sqrt{2}$ the general wave expansion for a Majorana field is given in terms of only one creation and one annihilation operator, $\tilde{b}$ and $\tilde{b}^\dagger$. Note that the factor $1/\sqrt{2}$ is included in the definition of $\tilde{b}$ and $\tilde{b}^\dagger$, instead of keeping it as an overall factor. The overall normalization of a Majorana field is trickier than that of a Dirac field. Our Dirac fields are normalized such that $\int d^3x \Psi_\nu^{D\dagger} \Psi_\nu^D = N_\nu - N_{\bar{\nu}}$. However, in the case of Majorana fields, $\int d^3x \Psi_\nu^{M\dagger} \Psi_\nu^M = 0$, and obviously such a condition cannot be used to determine an overall normalization. However, one can check the anti-commutation relations of the creation and annihilation operators $\tilde{b}$ and $\tilde{b}^\dagger$. One finds that $\{\tilde{b}(k,s), \tilde{b}^\dagger(k,s')\} = \delta_{s,s'} \delta^3(\vec{k} - \vec{k}')$, which is the correct relation and proves that the definitions of $\tilde{b}$ and $\tilde{b}^\dagger$ are correct and that no further normalization of Ψ_ν^M is needed.

Since the general wave expansion of the Majorana field is written in terms of only one creation and one annihilation operators, one can recover the result for Majorana neutrinos directly from the calculation for Dirac neutrinos by taking $b = d$ in Eq. (9). Effectively the vertex $\gamma_\mu(1-\gamma_5)$ in Eq. (3) becomes a pure axial $\gamma_\mu\gamma_5$ vertex. This amounts to the absence of the k_μ terms in Eq. (5) and thus the terms proportional to $(n_\nu - n_{\nu^c})$ in Eq. (18) are absent. Therefore, the effect depends on having a net helicity in the Majorana-neutrino bath. The terms linear in $\vec{s}_e$ in H are now

$$H_{s_e}^M = \sqrt{2} G_F g_A \left\{ -2\vec{s}_e \cdot \vec{\beta}_e \langle|\vec{\beta}_\nu|\rangle + 2\vec{s}_e \cdot \left(\left\langle \frac{E_\nu}{E_\nu + m_\nu} \vec{\beta}_\nu |\vec{\beta}_\nu| \right\rangle + \left\langle \frac{m_\nu}{E_\nu} \hat{\beta}_\nu \right\rangle \right) \right\} (n_{\nu_l} - n_{\nu_r}). \tag{31}$$

In the rest frame of the electron, the frame relevant for experiments, this is

$$H_{s_e}^M = -\sqrt{2} G_F g_A 2\vec{s}_e \cdot \vec{\beta}_{earth} [x\langle|\vec{\beta}_\nu|\rangle^{-1} + y\langle|\vec{\beta}_\nu|\rangle] \times (n_{\nu_l} - n_{\nu_r}) \tag{32}$$

where, as before, $x = 0$ and $y = 1$ for relativistic and $x = 16/3\pi$ and $y = 2/3$ for nonrelativistic neutrinos. For relativistic Majorana neutrinos, for which helicity coincides with chirality $\nu_l = \nu_L$ and $\nu_r = \nu_R^c$, this term coincides with the result for relativistic Dirac neutrinos in Eq. (25), as it should be since Dirac and Majorana neutrinos cannot be distinguished when relativistic. In the absence of interactions the helicity is conserved, even when as the temperature of the universe decreases neutrinos become non-relativistic. In the case of a large lepton asymmetry favoring, say, neutrinos, so that only $\nu_L = \nu_l$ are present at the moment of decoupling (when neutrinos are relativistic) a net left helicity remains in the bath of Majorana neutrinos, while not gravitationally bound. In this case, without gravitational binding, Majorana neutrinos would have a net left helicity $n_{\nu_l} = n_{\nu_L} \neq 0$, $n_{\nu_r} = n_{\nu_R^c} = 0$. As in the case of Dirac neutrinos the term proportional to $\langle|\vec{\beta}_\nu|\rangle^{-1}$ dominates for non relativistic neutrinos. This is important only for unclustered (NC-NR) neutrinos. As already mentioned, gravitationally bound non-relativistic (C-NR) neutrinos lose their net helicity. After a characteristic orbital time, helicities become maximally mixed thus l and r helicities become equally abundant, so $(n_{\nu_l} - n_{\nu_r}) = 0$, and the Stodolsky effect vanishes.

Thus, to summarize, in general

$$H_R^M = H_R^D \tag{33}$$

$$H_{C\text{-}NR}^M = 0 \tag{34}$$

and for neutrinos with a large lepton asymmetry, so that say $n_{\nu_l} \simeq n_\nu$,

$$H_{NC\text{-}NR}^M \simeq 1.7 \sqrt{\frac{m_\nu}{\xi 1.7\times 10^{-4}\ \text{eV}}} H_R^D \leqslant \frac{14}{\sqrt{\xi}} H_R^D. \tag{35}$$

In this last equation H_R^D is that given in Eq. (28).

C. Accelerations

The Stodolsky effect consists of a difference in energy between the two helicity states of the electron interacting with the neutrino bath in the rest frame of the Earth in which experiments are performed (which we take to coincide with the rest frame of the electrons). This energy difference ΔE is obtained in each case by replacing $\vec{s}_e \cdot \vec{\beta}_{earth}$ by $|\vec{\beta}_{earth}|$ in the respective expressions for the H^D and H^M given in the previous sections. Therefore, the Stodolsky effect vanishes except in the presence of a lepton asymmetry which in the case of Majorana neutrinos persists in the neutrino bath as an asymmetry in helicity as long as neutrinos are not gravitationally bound. In the case of relativistic, light neutrinos of density n_ν, with a very large lepton asymmetry favoring, say, neutrinos ν_L, so that $n_{\nu_L}=n_{\nu_l}=n_\nu$, from Eqs. (25) and (33) one has

$$(\Delta E)^D_R=(\Delta E)^M_R=2\sqrt{2}G_F g_A|\vec{\beta}_{earth}|n_\nu. \tag{36}$$

In the case of non-relativistic, unclustered neutrinos, we see from Eqs. (29) and (35) that

$$\begin{aligned}(\Delta E)^M_{NC\text{-}NR}&=2(\Delta E)^D_{NC\text{-}NR}\\ &\simeq 1.7\sqrt{\frac{m_\nu}{\xi 1.7\times 10^{-4}\ \text{eV}}}(\Delta E)^D_R\\ &\leqslant \frac{14}{\sqrt{\xi}}(\Delta E)^D_R.\end{aligned} \tag{37}$$

In the case of gravitationally bound (C-NR) neutrinos (so that there is no net helicity in the bath) of density n_ν, if there is a very large lepton asymmetry favoring neutrinos ν_L, for Dirac neutrinos $n_{\nu_R^c}=0$, and for Majorana neutrinos $n_{\nu_l}=n_{\nu_r}$ and the effect vanishes. Thus,

$$(\Delta E)^D_{C\text{-}NR}\simeq\sqrt{2}G_F g_A|\vec{\beta}_{earth}|n_\nu, \tag{38}$$

$$(\Delta E)^M_{C\text{-}NR}\simeq 0. \tag{39}$$

Equivalent results would be obtained with an asymmetry favoring antineutrinos.

From now on, we will use the energy difference ΔE in Eq. (36) to compute the maximal possible strength of the Stodolski effect, recalling that the effect could be at most one order of magnitude larger for non-clustered non-relativistic (NC-NR) neutrinos (most of the relic neutrinos if they are non-relativistic).

The difference in energy ΔE between the two helicity states of an electron in the direction of the bulk velocity of the neutrino background $\langle\vec{\beta}_\nu\rangle=-\vec{\beta}_{earth}$ implies a torque of magnitude $\Delta E/\pi$ applied on the spin of the electron. Since the spin is "frozen" in a magnetized macroscopic piece of material with N polarized electrons, the total torque applied to the piece has a magnitude $\tau=N\Delta E/\pi$. Given a linear dimension R and mass M of the macroscopic object, its moment of inertia is parametrized as $I=MR^2/\gamma$, where γ is a geometrical factor. In the typical case of one polarized electron per atom in a material of atomic number A, the number N above is $N=(M/gr)N_{\rm AV}/A$ (using cgs units), where $N_{\rm AV}$ is Avogadro's number. Thus, the effect we are considering would produce an angular acceleration of order $\alpha=\tau/I$ and a linear accelerations of order $a_{G_F}=R\alpha$ in the magnet, given by

$$a_{G_F}=\frac{N_{\rm AV}}{A}\frac{\Delta E}{\pi}\frac{\gamma}{R(gr)} \tag{40}$$

where the G_F subindex indicates the mechanism we have described.

Using the expression in Eq. (36) for ΔE with $n_\nu=f\ 100\ \text{cm}^{-3}$ we then find

$$a_{G_F}=10^{-27}f\cdot\left(\frac{\gamma}{10}\right)\left(\frac{100}{A}\right)\left(\frac{\text{cm}}{R}\right)\left(\frac{\beta_{earth}}{10^{-3}}\right)\frac{\text{cm}}{\text{sec}^2} \tag{41}$$

where f accounts for a possible local enhancement of the standard background neutrino number density. This acceleration could be at most one order of magnitude larger for non-relativistic (NC-NR). These accelerations are rather weak.

As mentioned above the present upper bound from structure formation in the Universe for neutrinos without a lepton asymmetry, for which $n_\nu\simeq 100\ \text{cm}^{-3}$, is of a few eV. The upper bound on the mass of degenerate neutrinos with a large asymmetry should be lower, since their number density is larger. Light neutrinos with masses $m_\nu<eV$ would be either unbound or gravitationally bound to very large structures and thus local enhancement to the neutrino density due to gravitational clustering would be very small.

Throughout we have used $\beta_{earth}\approx 10^{-3}$, which is the velocity of the Earth with respect to the Galaxy; however, if neutrinos are gravitationally clustered on much larger scales, one must consider the relative motion of the Earth with respect to the rest frame of these larger objects. At such scales the neutrino bath rest frame can be taken to coincide with the cosmic microwave background (CMB) rest frame. The Sun's motion with respect to the CMB is believed to be responsible for the largest anisotropy in the Cosmic Background Explorer (COBE) Differential Microwave Radiometer (DMR) maps, the 3 mK dipole, and thus has been determined with great accuracy. From the COBE data, $v_{sun}=369.0\pm 2.5$ km/sec which corresponds to $\beta_{earth}=1.231\times 10^{-3}$ [16]. Therefore, even if background neutrinos are bound to such large objects as super-clusters, we are justified in using still $\beta_{earth}\simeq 10^{-3}$.

Thus with very little gravitationally clustering locally, a neutrino density enhancement could only be due to the asymmetry itself, in which case (as explained in the Introduction) $f<20$ ($n_{\nu_i}\leqslant 1700\ \text{cm}^{-3}$).

However, it is amusing to note that if there is a local cloud of ν_e of density $\simeq 10^{17}\ \text{cm}^{-3}$ as required for a rather audacious explanation of the tritium end point anomaly [17] then $a_{G_F}\simeq 10^{-12}\ \text{cm/sec}^2$, and the effect would be observable.

D. Comparisons with past results

The first calculation of this order G_F effect was done by Stodolsky in 1974 [1]. In our notation, he found the energy difference between the two spin states of an electron moving with velocity $\vec{\beta}_e$ in the rest frame of the neutrino bath to be

$$\Delta E = 2\sqrt{2} G_F \frac{\beta_e}{\sqrt{1-\beta_e^2}} (n_\nu - n_{\nu^c}). \tag{42}$$

Stodolsky performed his calculation before the discovery of neutral currents and thus did not include them (hence he took g_A to be 1). He considered only relativistic neutrinos. Stodolsky's result coincides with ours in Eq. (36) with $\beta_e = |\vec{\beta}_{earth}|$ (and $\sqrt{1-\beta_e^2} \approx 1$).

Langacker, Leveille, and Sheiman also explored this effect [4]. Although the majority of their paper deals with exposing the flaws in several relic neutrino detection schemes, they conclude that the only legitimate effect of order G_F is that proposed by Stodolsky. They studied only Dirac neutrinos and found

$$\Delta E = 2\sqrt{2} G_F \beta_e N_a^{tot} \sum_i (n_{\nu_i} - n_{\nu_i^c}) g_A^{ai} K(p, m_i), \tag{43}$$

where N_a^{tot} is the total number of electrons in the sample, and K is a function of E_ν and m_ν which reduces to 1 if $m_\nu = 0$ or 1/2 if $k_\nu \ll m_\nu$. Their calculation is performed in the rest frame of the "neutrino sea." Their result agrees exactly with Eqs. (36) and (38) in which we give the ΔE for Dirac neutrinos in the rest frame of the electron.

Ferreras and Wasserman [5] recently tackled the subject of the detectability of relic neutrinos, and also concluded that there are no order G_F effects without lepton asymmetry. However, they point out that density fluctuations in the ν (ν^c) background could give rise to non-steady forces of order G_F. Such forces would give rise to displacements in massive objects proportional to $t^{3/2}$ rather than t^2 as in constant acceleration. They caution that although such accelerations may be more readily detectable, they could also act as an additional source of noise. Here we have not considered neutrino density fluctuations.

III. THE G_F^2 EFFECT

Nucleons are continuously bombarded by the relic neutrinos and a momentum $\Delta\vec{p} \simeq \vec{p}_\nu$ is imparted in each collision. The momentum of the neutrinos p_ν is $p_\nu = E_\nu/c \approx 4T_\nu/c$ for relativistic neutrinos (R) and also for non-clustered, non-relativistic neutrinos (NC-NR) due to momentum red-shift, and is $m_\nu v_\nu \simeq m_\nu v_{virial}$ for non-relativistic clustered neutrinos (C-NR).

If the Earth was at rest with respect to the relic neutrino "rest frame," i.e. the frame in which the neutrinos are isotropically distributed, then the average momentum transfer $\langle \Delta p \rangle$ would vanish. However the motion of the earth with velocity $c\beta_{earth} = v_{earth}$ induces a "dipole" distortion of $O(\beta_{earth})$ in the velocity distribution of the relic neutrinos. In the laboratory frame (the Earth's rest frame) this makes

$$\langle \Delta p \rangle_R \simeq \beta_{earth}(E_\nu/c),$$
$$\langle \Delta p \rangle_{NC\text{-}NR} \simeq \beta_{earth}(4T_\nu/c) = \langle \Delta p \rangle_R,$$
$$\langle \Delta p \rangle_{C\text{-}NR} \simeq \beta_{earth} c m_\nu. \tag{44}$$

The fluxes of infalling neutrinos are $\Phi_R = n_\nu c$ and $\Phi_{NR} = n_\nu v_\nu$ for relativistic and non-relativistic neutrinos respectively. To find the resulting accelerations we compute the force exerted on one gram of detector material containing $\mathcal{N} = N_{AV}/A$ nuclei (A,Z). The force is given by the momentum imparted per second: the latter is the microscopic Δp of Eq. (44) times $\mathcal{N}\Phi_\nu \sigma_{\nu-A}$, the number of collisions per second inside one gram:

$$\frac{F}{m} = a = \Phi_\nu \frac{N_{AV}}{A} \sigma_{\nu-A} \langle \Delta p \rangle. \tag{45}$$

The neutrino-nucleus cross sections $\sigma_{\nu-A}$ are extremely small even if we include a nuclear coherence enhancement factor $(A-Z)^2 \simeq A^2$ relative to the neutrino-nucleon $\sigma_{\nu-N}$ cross sections, which are of the order

$$\sigma_{\nu-N} \simeq \begin{cases} G_F^2 m_\nu^2/\pi \simeq 10^{-56} (m_{\nu\{\mathrm{eV}\}})^2 \ \mathrm{cm}^2 & \text{for (NR)} \\ G_F^2 E_\nu^2/\pi \simeq 5\times 10^{-63} (T_{\nu\{1.9^\circ\ \mathrm{K}\}})^2 \ \mathrm{cm}^2 & \text{for (R),} \end{cases} \tag{46}$$

where { } indicates the units of the mentioned quantities.

Collecting all terms in Eq. (45) above we find

$$a = \frac{N_{AV}}{A} n_\nu \frac{G_F^2}{\pi} A^2 \begin{cases} (4T_\nu)^3 \beta_{earth} & \text{for (R),} \\ m_\nu^2 4T_\nu \beta_{earth} & \text{for (NC-NR),} \\ m_\nu^3 v_\nu^2 & \text{for (C-NR).} \end{cases} \tag{47}$$

We will take the Earth's velocity v_{earth} and the clustered neutrino's velocity to be all $\simeq c\beta_{virial}$ with $\beta_{virial} \simeq 10^{-3}$ corresponding to typical galactic virial velocities. We use $n_\nu = f 100\ \mathrm{cm}^{-3}$. Hence we find

$$a_R = 3\times 10^{-54} f \frac{A}{100} (T_{\nu\{1.9^\circ\ \mathrm{K}\}})^3 \frac{\mathrm{cm}}{\mathrm{sec}^2}, \tag{48}$$

$$a_{NC\text{-}NR} = 0.6\times 10^{-47} f \frac{A}{100} (m_{\nu\{\mathrm{eV}\}})^2 T_{\nu\{1.9^\circ\ \mathrm{K}\}} \frac{\mathrm{cm}}{\mathrm{sec}^2}. \tag{49}$$

Here the density enhancement factor f may only be due to a large lepton asymmetry and $f < 20$ (see Sec. I). Finally for clustered non-relativistic neutrinos we have

$$a_{C\text{-}NR} = 10^{-46} f \frac{A}{100} (m_{\nu\{\mathrm{eV}\}})^3 \frac{\mathrm{cm}}{\mathrm{sec}^2}. \tag{50}$$

Here f contains also a clustering enhancement factor. Values of f up to 10^7 have been mentioned in the literature (see for example [6]). If neutrinos were sufficiently massive to cluster in our Galaxy and make up the local dark matter halo ($m_\nu \geq 20$ eV), only then would we have $f = \rho_{local}/m_\nu = 0.4\times 10^7/m_{\nu\{\mathrm{eV}\}}$. This possibility is already rejected by structure formation arguments. With sub-eV neutrino masses the enhacement f due to clustering could only be of order 1.

All of the above accelerations are extremely small and beyond the reach of any known experimental measurement technology. However, it has been noted (by Zeldovich and Khlopov [18] as well as Smith and Lewin [2]) that coherent scattering of neutrinos from domains of the size of the de Broglie neutrino wavelength $\lambda_\nu = 2\pi\hbar/p_\nu$ dramatically increases the scattering cross section. The extra factor due to coherence is the number of nuclei in this domain (since the nuclear coherence factor is already included),

$$N_c = \frac{N_{AV}}{A}\rho_{\{\mathrm{gr\,cm}^{-3}\}}(\lambda_{\nu\,\{\mathrm{cm}\}})^3. \tag{51}$$

For relativistic and for unclustered non-relativistic relic neutrinos with $\lambda_\nu = 2\pi\hbar/4T_\nu \simeq 0.2$ cm this enhancement factor is

$$N_c = \frac{5\times 10^{21}}{A\ \ T_{\nu\{1.9^\circ\ \mathrm{K}\}}{}^3}\rho_{\{\mathrm{gr\,cm}^{-3}\}}. \tag{52}$$

For clustered non-relativistic neutrinos with $p_\nu = m v_\nu \simeq 10^{-3} m_{\nu\{\mathrm{eV}\}}$ eV the wavelength is $\lambda_\nu \approx (0.12/m_{\nu\,\{\mathrm{eV}\}})$ cm so that the coherence enhancement factor is

$$N_c = \frac{10^{21}}{A\ \ m_{\nu\,\{\mathrm{eV}\}}{}^3}\rho_{\{\mathrm{gr\,cm}^{-3}\}}. \tag{53}$$

Thus the largest acceleration values with the N_C enhancement factors are

$$a_R = 2\times 10^{-34} f \rho_{\{\mathrm{gr\,cm}^{-3}\}}\ \frac{\mathrm{cm}}{\mathrm{sec}^2}, \tag{54}$$

$$a_{NC\text{-}NR} = 3\times 10^{-28} f (m_{\nu\,\{\mathrm{eV}\}})^2 (T_{\nu\{1.9^\circ\ \mathrm{K}\}})^{-2}\rho_{\{\mathrm{gr\,cm}^{-3}\}}\ \frac{\mathrm{cm}}{\mathrm{sec}^2}, \tag{55}$$

and finally, for clustered, nonrelativistic neutrinos

$$a_{C\text{-}NR} \simeq 10^{-27} f \rho_{\{\mathrm{gr\,cm}^{-3}\}}\ \frac{\mathrm{cm}}{\mathrm{sec}^2}. \tag{56}$$

The above discussion applies only to Dirac non-relativistic neutrinos as only these have coherent vectorial Z^o couplings in the static limit

$$A^{(o)}_{Z^o}\cdot \bar{\psi}\gamma_o\psi \simeq A^{(o)}_{Z^o}\psi^+\psi. \tag{57}$$

In the Majorana case we still have coherent $\psi^+\gamma_5\psi A^{(o)}_{Z^o}$ couplings. The latter are, however, suppressed by β_ν, the ratio of "small" and "large" components of the spinor. Recall that for non-relativistic spinors the lower components are "smaller" than the upper components by a factor of β. Without the γ_5 these terms are dwarfed by the leading order terms; however with the $\gamma_\mu\gamma_5$ vertex in the static limit these terms are the leading order term). Thus the analog of Eqs. (55) and (56) for non-relativistic Majorana neutrinos is suppressed by an extra factor of $\beta_\nu^2 \simeq 10^{-6}$.

Solar neutrinos and WIMPS

Solar neutrinos provide a directional fairly well known source of relativistic neutrinos, and it is interesting to estimate their contribution to the accelerations we have calculated. The acceleration due to solar neutrinos is

$$a = \Phi_{\mathrm{solar}\ \ \nu}\ \frac{N_{AV}}{A}\rho\sigma_{(\nu-A)}\frac{E_\nu}{c}. \tag{58}$$

For a gallium detector, for example, we take $\Phi_\nu \simeq 10^{11}$ cm^{-2}sec^{-1}, $E_\nu \simeq 0.3$ MeV, and $\sigma_{\nu\text{-gallium}} \simeq 10^{-44}$ cm^2. Thus for solar pp neutrinos in gallium we find

$$a \simeq 10^{-27}\ \ \mathrm{cm/sec}^2. \tag{59}$$

This exceeds most previously calculated accelerations, but it is still too small to be detectable at present.

Our main focus here is on neutrino induced forces. However, it is believed that much of the cosmological and most of the halo dark matter is made of massive weakly interacting particles (WIMPs).

Let us first estimate the effect of WIMPs if they had the cross section of Dirac neutrinos with masses $m_X \simeq O(100$ GeV). Then the nuclear cross section with $A \simeq 100$ targets would be large

$$\sigma_{x-A} \simeq \frac{G_F^2}{\pi}(m_X)^2 A^2 \simeq 10^{-30} m_{X\,\{\mathrm{GeV}\}} A^2\ \ \mathrm{cm}^2 \tag{60}$$

and the recoil energies, $m_X\beta_X^2/2 \simeq O$ (30 keV) (using $m_X \simeq 100$ GeV), detectable. Indeed such WIMPs have been excluded by direct searches in underground detectors. Realistic WIMP candidates at the "threshold of detectability" have smaller cross sections by a factor $10^{-\Delta}$. Using the analogue of Eq. (45),

$$a_{\mathrm{WIMP}} = \Phi_{\mathrm{WIMP}}\frac{N_{AV}}{A}\sigma_{(X-A)} m_X v_X$$
$$= n_X m_X \frac{N_{AV}}{A}\sigma_{(X-A)} v_X^2, \tag{61}$$

and a WIMP density $n_X m_X \simeq \rho_{\mathrm{dark(local)}} \simeq 10^{-24}$ g/cm^3, we find for $A=100$ and $\sigma_{X-A} \simeq 10^{-30-\Delta}$ cm^2

$$a_{\mathrm{WIMP}} \simeq 10^{-(17+\Delta)}\left(\frac{\beta_X}{10^{-3}}\right)^2 \frac{A}{100}\ \frac{\mathrm{cm}}{\mathrm{sec}^2}. \tag{62}$$

Clearly a_{WIMP} dominates over a very large range of cross sections [with $\Delta < 10$, using Eq. (39) or Eq. (54)] the corresponding accelerations due to the scattering of light, locally unclustered neutrinos.

IV. CONCLUSIONS

We have calculated the magnitude of the signals expected for realistic cosmic neutrino backgrounds in detectors attempting to measure the mechanical forces exerted on macroscopic targets by the elastic scattering of relic neutrinos.

We examined effects proportional to G_F and G_F^2 for both Dirac and Majorana neutrinos either relativistic or non-relativistic. We also estimated the contributions to macroscopic accelerations due to solar neutrinos and WIMPs in the galactic halo.

The effect linearly proportional to the weak coupling constant G_F vanishes in the case of no lepton asymmetry. With a lepton asymmetry, macroscopic accelerations for relativistic Dirac and Majorana neutrinos and clustered non-relativistic Dirac neutrinos were found in Eq. (41) to be of the order of $f\times 10^{-27}$ cm/sec^2, where f is a density enhancement factor which can be at most about 17 (with a large lepton asymmetry), and at most one order of magnitude larger (i.e. $f\times 10^{-26}$ cm/sec^2) for non-clustered Dirac or Majorana neutrinos (which are most of the relic neutrinos if they are non-relativistic at present). These accelerations are at most thirteen orders of magnitude smaller that the smallest measureable acceleration of 10^{-12} cm/sec^2. The acceleration of non-relativistic, gravitationally bound Majorana neutrinos vanishes.

For the effect proportional to G_F^2 accelerations of relativistic Dirac and Majorana neutrinos, taking advantage of coherent scattering effects, were found in Eq. (54) to be of the order of $f\times 10^{-34}$ cm/sec^2. The accelerations of non-relativistic Dirac neutrinos were calculated to be of the order of $f\times 10^{-28}(m_\nu/\mathrm{eV})^2$ cm/sec^2 in Eq. (55) for non-clustered neutrinos and $f\times 10^{-27}$ cm/sec^2 in Eq. (56) for clustered neutrinos, while the accelerations of non-relativistic Majorana neutrinos are down by a factor $\beta_\nu^2 \simeq 10^{-6}$. All accelerations are well beyond the smallest measureable acceleration, 10^{-12} cm/sec^2, mentioned above.

Additional calculations for the accelerations due to solar neutrinos and WIMPs in the galactic halo raise concerns that signals in a detector due to relic neutrinos may well be washed out by solar neutrino or WIMP events unless directionality can be used to reject them.

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[1] L. Stodolsky, Phys. Rev. Lett. **34**, 110 (1974).
[2] P. Smith and J. Lewin, Phys. Lett. **127B**, 185 (1983).
[3] V. Shvartsman, V. Braginski, S. Gershtein, Y. Zel'dovich, and M. Khlopov, JETP Lett. **36**, 277 (1982).
[4] P. Langacker, J. Leveille, and J. Sheiman, Phys. Rev. D **27**, 1228 (1983).
[5] I. Ferreras and I. Wasserman, Phys. Rev. D **52**, 5459 (1995).
[6] C. Hagmann, in *Proceedings of the COSMO-98 International Workshop on Particle Physics and the Early Universe*, edited by D. Caldwell (AIP, New York, 1999), pp. 460–463, astro-ph/9902102.
[7] E. Adelberger *et al.*, Annu. Rev. Nucl. Part. Sci. **41**, 269 (1991).
[8] H. Kang and S. Steigman, Nucl. Phys. **B372**, 494 (1992).
[9] J. Adams and S. Sarkar, talk presented at the ''ICTP Workshop on the Physics of Relic Neutrinos,'' Trieste, Italy, 1998.
[10] J. Lesgourgues and S. Pastor, Phys. Rev. D **60**, 103521 (1999).
[11] W. H. Kinney and A. Riotto, Phys. Rev. Lett. **83**, 3366 (1999).
[12] J. P. Kneller, R. J. Scherrer, G. Steigman, and T. P. Walker, Phys. Rev. D (to be published), astro-ph/0101386.
[13] See, for example, J. R. Primack and M. A. Gross, astro-ph/0007165.
[14] S. Tremaine and J. Gunn, Phys. Rev. Lett. **42**, 407 (1979).
[15] *Group Theoretical Concepts and Methods in Elementary Particle Physics*, edited by F. Gürsey (Gordon and Breach, New York, 1964).
[16] C. H. Lineweaver *et al.*, Astrophys. J. **470**, 38 (1996).
[17] R. G. H. Robertson, T. J. Bowles, G. J. Stephenson, Jr., D. L. Wark, J. F. Wilkerson, and D. A. Knapp, Phys. Rev. Lett. **67**, 957 (1991).
[18] Y. Zeldovich and M. Khlopov, Sov. Phys. Usp. **24**, 755 (1981).