Model for fermion mass matrices and the origin of quark-lepton mass relations

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Several phenomenological features of fermion masses and mixings can be accounted for by a simple model for fermion mass matrices, which suggests an underlying $U(2)$ horizontal symmetry. In this context, a way in which approximate relations between quark and lepton mass matrices can be achieved without unified gauge theories is proposed.

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Understanding the pattern of fermion masses and mixings is a key subject in modern particle physics. Many approaches have been followed [1]. For example, discrete and continuous as well as Abelian and non-Abelian horizontal symmetries often have been used. Moreover, unified gauge theories, which are based on vertical symmetries, generally predict relations between quark and lepton masses [2]. Horizontal symmetries relate particles of different generations, while vertical symmetries relate particles of the same generation. Even the $SU(3)\times SU(2)\times U(1)$ symmetries of the standard model or the $SU(3)\times SU(2)\times SU(2)\times U(1)$ symmetries of the left-right model are vertical. In this paper we show that several phenomenological issues, among which are quark-lepton symmetry, the seesaw mechanism, and large neutrino mixing, lead to a simple model for quark and lepton mass matrices, suggesting an underlying broken $U(2)$ horizontal symmetry [3]. Speculations on the origin of the relations between quark and lepton mass matrices, not relying on unified models, and hence not leading to proton instability, are also presented.

It is well known that quark masses and mixings exhibit a hierarchical pattern

$$\frac{m_u}{m_c}\sim\frac{m_c}{m_t}\sim\lambda^4, \tag{1}$$

$$\frac{m_d}{m_s}\sim\frac{m_s}{m_b}\sim\lambda^2, \tag{2}$$

$$V_{us}\sim\lambda,\ \ V_{cb}\sim\lambda^2,\ \ V_{ub}\sim\lambda^4, \tag{3}$$

where $\lambda=0.22$ is the sine of the Cabibbo angle, and from Eq. (3) we see that quark mixings are small. Moreover, the hierarchy of charged lepton masses is similar to that of down quark masses,

$$\frac{m_e}{m_\mu}\sim\frac{m_\mu}{m_\tau}\sim\lambda^2. \tag{4}$$

For the masses of the third generation we have

$$m_t\gg m_b\sim m_\tau, \tag{5}$$

and the top quark mass is nearly equal to the vacuum expectation value (VEV) of the standard model Higgs boson.

Quark masses and mixings arise when quark mass matrices are diagonalized by means of biunitary transfor mations $V_{uL}^\dagger M_u V_{uR}=D_u$, $V_{dL}^\dagger M_d V_{dR}=D_d$. The Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [4] is given by the formula $V_{\mathrm{CKM}}=V_{uL}^\dagger V_{dL}$. In recent papers [5] a parallel structure for up and down quark mass matrices has been obtained:

$$M_u\simeq\begin{pmatrix}0 & \sqrt{m_u m_c} & 0\\ \sqrt{m_u m_c} & m_c & \sqrt{m_u m_t}\\ 0 & \sqrt{m_u m_t} & m_t\end{pmatrix}, \tag{6}$$

$$M_d\simeq\begin{pmatrix}0 & \sqrt{m_d m_s} & 0\\ \sqrt{m_d m_s} & m_s & \sqrt{m_d m_b}\\ 0 & \sqrt{m_d m_b} & m_b\end{pmatrix}. \tag{7}$$

By using Eqs. (1) and (2) we get these mass matrices in terms of powers of λ and an overall mass scale:

$$M_u\sim\begin{pmatrix}0 & \lambda^6 & 0\\ \lambda^6 & \lambda^4 & \lambda^4\\ 0 & \lambda^4 & 1\end{pmatrix}m_t, \tag{8}$$

$$M_d\sim\begin{pmatrix}0 & \lambda^3 & 0\\ \lambda^3 & \lambda^2 & \lambda^2\\ 0 & \lambda^2 & 1\end{pmatrix}m_b. \tag{9}$$

This structure for M_u and M_d strongly suggests the presence of a broken $U(2)$ horizontal symmetry [6], if we consider the zeros as approximate. In particular, for symmetric forms, the similarity $M_{22}\sim M_{23}$ favors the $U(2)$ symmetry, rather than the $U(1)$ symmetry, where we have $M_{22}\sim(M_{23})^2$ (see, for example, Ref. [7]). Mixings come out mainly from the down sector, where the hierarchy is less pronounced.

Now we have to consider lepton masses and mixings. In unified theories, such as the $SO(10)$ model, the charged lepton mass matrix M_e is related to M_d and the Dirac neutrino mass matrix M_ν is related to M_u,

$$M_e\sim\frac{m_\tau}{m_b}M_d, \tag{10}$$

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$$M_\nu \sim \frac{m_\tau}{m_b} M_u, \tag{11}$$

where the factor m_τ/m_b takes into account approximate running of quark masses with respect to lepton masses, and $m_b \simeq m_\tau$ at the unification scale in the supersymmetric case [8]. This is called quark-lepton symmetry. Moreover, the seesaw mechanism [9] holds, according to the formula

$$M_L \simeq M_\nu M_R^{-1} M_\nu, \tag{12}$$

where M_R is the Majorana mass matrix of heavy right-handed neutrinos and M_L is the effective Majorana mass matrix of light left-handed neutrinos. The seesaw mechanism can explain the smallness of the effective neutrino mass, since the mass of the right-handed neutrino is generated at the unification scale $M_U \sim 10^{16}$ GeV in the supersymmetric case and the Dirac mass at the weak scale $M_W \sim 10^2$ GeV. Lepton masses and mixings arise when M_L and M_e are diagonalized through the transformations $V_L^\dagger M_L V_L^* = D_L$, $V_{eL}^\dagger M_e V_{eR} = D_e$. The Maki-Nakagawa-Sakata (MNS) lepton mixing matrix [10] is given by $V_{\rm MNS} = V_L^\dagger V_{eL}$. In Eqs. (10) and (11) we may neglect the running of quark mixings, which is not relevant for our analysis. In fact, also the running of quark masses is unimportant. However, we keep the factor m_τ/m_b to remember the presence of the running effect, which is responsible for the mass difference between m_b and m_τ at the low scale. Quark-lepton symmetry is due to the $SU(4)\times SU(2)\times SU(2)$ subgroup of $SO(10)$ and in particular to the $SU(4)$ component, which includes the lepton number as fourth color [11]. In the $SU(5)$ model, only relation (10) is achieved.

It can be shown [12] that the large mixing angle solutions to the solar neutrino problem [13], which are favored by recent data fits [14], along with the large mixing of atmospheric neutrinos, give the remarkable result of a nearly democratic

$$M_L^{-1} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{1}{m_1}, \tag{13}$$

where m_1 is the mass of the lightest left-handed neutrino in the normal hierarchy case [15]. Then, by using Eqs. (8), (11), (12), and (13) we obtain

$$M_R \sim \left(\frac{m_\tau}{m_b}\right)^2 \begin{pmatrix} 0 & 0 & \lambda^6 \\ 0 & \lambda^8 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1 \end{pmatrix} \frac{m_t^2}{m_1}, \tag{14}$$

where again the zeros are approximate. The overall scale of M_R is consistent with or higher than the unification scale. Consistence is achieved for $m_1 \sim 10^{-4}$ eV.

Therefore, all fermion mass matrices can now be written in a form suggested by the breaking of the $U(2)$ horizontal symmetry [16–18], and filling the zero elements by higher powers of λ we get

$$M_u \sim \begin{pmatrix} (\lambda^6)^2 & \lambda^6 & \lambda^4\lambda^6 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^4\lambda^6 & \lambda^4 & 1 \end{pmatrix} m_t \sim M_\nu, \tag{15}$$

$$M_d \sim \begin{pmatrix} (\lambda^3)^2 & \lambda^3 & \lambda^2\lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^2\lambda^3 & \lambda^2 & 1 \end{pmatrix} m_b \sim M_e, \tag{16}$$

$$M_R \sim \begin{pmatrix} (\lambda^6)^2 & \lambda^4\lambda^6 & \lambda^6 \\ \lambda^4\lambda^6 & (\lambda^4)^2 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1 \end{pmatrix} M_3, \tag{17}$$

with M_3 the mass of the heaviest right-handed neutrino. Coefficients are greater than λ and less than $1/\lambda$. We stress that the heavy neutrino mass matrix is found, and not built, consistent with the broken $U(2)$ symmetry, as a consequence of the democratic form (13). The $U(2)$ breaking parameters are given by

$$\epsilon_u \simeq \epsilon_d^2 \simeq \lambda^4, \tag{18}$$

$$\epsilon_u' \simeq \epsilon_d'^2 \simeq \lambda^6, \tag{19}$$

so that mass matrices can be written as

$$M_u \sim \begin{pmatrix} \epsilon_u'^2 & \epsilon_u' & \epsilon_u\epsilon_u' \\ \epsilon_u' & \epsilon_u & \epsilon_u \\ \epsilon_u\epsilon_u' & \epsilon_u & 1 \end{pmatrix} m_t \sim M_\nu, \tag{20}$$

$$M_d \sim \begin{pmatrix} \epsilon_d'^2 & \epsilon_d' & \epsilon_d\epsilon_d' \\ \epsilon_d' & \epsilon_d & \epsilon_d \\ \epsilon_d\epsilon_d' & \epsilon_d & 1 \end{pmatrix} m_b \sim M_e, \tag{21}$$

$$M_R \sim \begin{pmatrix} \epsilon_u'^2 & \epsilon_u\epsilon_u' & \epsilon_u' \\ \epsilon_u\epsilon_u' & \epsilon_u^2 & \epsilon_u \\ \epsilon_u' & \epsilon_u & 1 \end{pmatrix} M_3. \tag{22}$$

The parameters ϵ are involved in the breaking of $U(2)$ to $U(1)$, and the parameters ϵ' in the further breaking of this $U(1)$. According to the $U(2)$ horizontal symmetry, the three generations ψ_i transform as a doublet ψ_a plus a singlet ψ_3. This is attractive for supersymmetry, because it leads to the degeneracy between first and second generation, which is needed to suppress the flavor-changing neutral currents in the squark sector [3]. Thus we discuss the origin of the matrices in Eqs. (15), (16), and (17) within the supersymmetric standard model with right-handed neutrinos. We do not assume the existence of a unified model. In fact, in unified theories with $U(2)$ as horizontal symmetry, the mechanisms for the enhancement of the up quark mass hierarchy with respect to the down quark mass hierarchy are quite involved [6,18]. Moreover, it is not clear if a simple Higgs structure is able to account for fermion masses and mixings.

In the supersymmetric model, Dirac masses are generated by two distinct Higgs doublets, H_d and H_u, with VEVs v_d and v_u, through the Yukawa terms

$$Y_u Q H_u u + Y_d Q H_d d, \tag{23}$$

$$Y_\nu L H_u \nu + Y_e L H_d e, \tag{24}$$

where Q and L are quark and lepton weak doublets, and u, d, ν, e are weak singlets. We see that M_u and M_ν are generated by the same Higgs doublet H_u, that is $M_{u,\nu} = Y_{u,\nu} v_u$. In a similar way M_d and M_e are generated by the Higgs doublet H_d, that is $M_{d,e} = Y_{d,e} v_d$. This fact could already suggest the assumption of an approximate relation between quark and lepton sectors. The hierarchy (5) is obtained for $v_u \gg v_d$, keeping all Yukawa couplings for the third generation of order 1, so that $m_t \simeq v_u$ and $m_b \simeq v_d$. We find that $Y_{u,\nu}$ and $Y_{d,e}$ are produced through two independent Yukawa potentials, with breaking parameters ϵ_u, ϵ'_u and ϵ_d, ϵ'_d, respectively. The potential for a Dirac mass matrix is in the form [16]

$$L \simeq v\left(\psi_3 \psi_3 + \psi_3 \frac{\varphi^a}{M} \psi_a + \psi_a \frac{S^{ab}}{M} \psi_b + \psi_a \frac{A^{ab}}{M} \psi_b \right), \tag{25}$$

where φ^a, S^{ab}, A^{ab} are doublet, triplet, and singlet flavon fields, with VEVs given by

$$\frac{\varphi}{M} \simeq \begin{pmatrix} \epsilon\epsilon' \\ \epsilon \end{pmatrix}, \quad \frac{S}{M} \simeq \begin{pmatrix} \epsilon'^2 & \epsilon\epsilon' \\ \epsilon\epsilon' & \epsilon \end{pmatrix}, \quad \frac{A}{M} \simeq \begin{pmatrix} 0 & \epsilon' \\ -\epsilon' & 0 \end{pmatrix}, \tag{26}$$

and the mass M is the cutoff scale of the effective theory, corresponding to the existence of very massive states. The flavons are weak singlets.

The heavy neutrino mass is generated by a bare mass term $M_R \nu\nu$ and is related to the $U(2)$ breaking involved in the up sector, as expected since the ν fields appear. It is noteworthy that we find this result from the independent input of large neutrino mixing. Contrary to Dirac masses, which are generated at the weak scale, the Majorana mass of the right-handed neutrino is not constrained and can be very large, allowing the seesaw mechanism. The potential for the Majorana mass matrix of heavy neutrinos looks like

$$L_R \simeq M_3 \left(\nu_3 \nu_3 + \nu_3 \frac{\phi^a}{M} \nu_a + \nu_a \frac{\phi^a \phi^b}{M^2} \nu_b \right), \tag{27}$$

with the VEV of ϕ^a given by

$$\frac{\phi}{M} \simeq \begin{pmatrix} \epsilon' \\ \epsilon \end{pmatrix}, \tag{28}$$

and without the triplet (and of course the antisymmetric singlet) contribution. We can also assume, in analogy to Eqs. (23) and (24), that M_R is generated through a Yukawa term $Y_R \nu H \nu$ with a singlet Higgs boson H having a very high VEV $v_R \simeq M_3$, in such a way as to couple different Higgs fields to different flavon fields.

The present model ascribes the hierarchy (5), and its analogue in the lepton sector, to the coupling with two distinct Higgs doublets, which is necessary in the supersymmetric case [19], and the hierarchies (1) and (2), and their analogues in the lepton sector, to the coupling with two corresponding different sets of flavon fields, responsible for the $U(2)$ breaking. As a consequence, an approximate relation between quark and lepton mass matrices can be realized without the $SO(10)$ grand unification or the $SU(4) \times SU(2) \times SU(2)$ partial unification, avoiding a possible conflict with proton decay [20]. It is worth noting that the model is in some sense economic with respect to unified theories, since many gauge and Higgs bosons need not be introduced, while flavon fields must in any case appear, in order to account for the hierarchies. However, we do not exclude a possible realization within the $SO(10)$ model.

A shortcoming of the present framework, common to the class of horizontal $U(2)$ models, is that the different flavor structure between the up and down sectors is set by hand. Moreover, the flavon sector for the heavy Majorana neutrino has a structure of VEVs different from that of Dirac particles. We have been led to the proposed scenario by following a kind of bottom-up approach. The hope is that a more fundamental theory could explain the features mentioned above.

In summary, from quark-lepton symmetry, the seesaw mechanism, and the large lepton mixing, we have inferred a simple model for fermion mass matrices in the supersymmetric standard model with right-handed neutrinos and horizontal $U(2)$ symmetry. Such a model suggests an alternative to the usual origin of quark-lepton mass relations. The basic features of masses and mixings are produced by VEVs of scalar fields. In particular, Higgs fields determine the overall scales of mass matrices, and flavon fields their internal hierarchies. The ratio $v_u / v_d = \tan\beta$ is predicted to be very large.

Finally, we discuss the implications for baryogenesis via leptogenesis [21]. This mechanism for the generation of a baryon asymmetry Y_B in the universe is based on the decay of the heavy right-handed neutrinos, producing a lepton asymmetry which is partially converted into a baryon asymmetry by sphaleron processes. See Ref. [22] for a collection of the relevant formulas. Using the approximate expression for the supersymmetric case and Eqs. (15) and (17),

$$Y_B \sim \frac{M_1}{M_P} \frac{[(M_\nu^\dagger M_\nu)_{12}]^2}{[(M_\nu^\dagger M_\nu)_{11}]^2} \frac{M_1}{M_2}, \tag{29}$$

where $M_P \sim 10^{19}$ GeV is the Planck mass, we obtain

$$Y_B \sim \lambda^{12} \frac{M_3}{M_P}. \tag{30}$$

The required value $Y_B \sim \lambda^{15}$ is achieved for $M_3 \sim 10^{17}$ GeV, by one order higher than the unification scale, and corresponding to $m_1 \sim 10^{-5}$ eV. This result has to be taken with care, due to the approximation and the dependence on Yukawa coefficients.

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