Constraining quark-hadron duality at large N_c

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Quark-meson duality for two-point functions of vector and axial-vector QCD currents is investigated in the large-N_c approximation. We find that the joint constraints of duality and chiral symmetry suggest degeneracy of excited vector and axial-vector mesons in the large-N_c limit. We compare model-independent constraints with expectations based on the Veneziano-Lovelace-Shapiro string model. Several models of duality are constructed, and phenomenological implications are discussed.

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I. INTRODUCTION

In the large-N_c limit, QCD correlators of quark bilinears can be expressed as sums of zero-width meson tree graphs [1]. These sums must be infinite in order to be consistent with perturbative QCD logarithms at large momentum transfer. The detailed matching of hadronic and partonic degrees of freedom, known as quark-hadron duality [2–4], has been explicitly verified in QCD in $1+1$ dimensions in the large-N_c limit [5]. It has long been thought that the exchange of an infinite number of vector mesons is in some sense dual to the perturbative QCD continuum [6]. Early work uncovered various intriguing similarities between the simplest models of quark-meson duality and hadronic string models. Given the widespread belief that large-N_c QCD is in some sense equivalent to a string theory, these similarities have received recent attention [7]. Following Ref. [7], in this paper we investigate duality in the large-N_c limit in the simplest correlators that have an operator product expansion (OPE): i.e., two-point functions of vector and axial-vector currents. We point out that there are nontrivial chiral symmetry constraints that must be satisfied in addition to those constraints implied by duality. We discuss the interesting dilemma raised by simultaneous satisfaction of all constraints. These constraints suggest that there is an infinite tower of *degenerate* vector and axial-vector mesons in the large-N_c limit. The phenomenological implications of this conjecture are considered in a simple model. As an example of a system with an infinite spectrum of mesons, we consider how chiral symmetry is satisfied in the Lovelace-Shapiro-Veneziano (LSV) string model [8,9] and we investigate the implications of that model for duality.

II. DUALITY CONSTRAINTS

In large-N_c QCD, mesons have the most general quantum numbers of the quark bilinear $\bar{q}\Gamma q$, where Γ is some arbitrary spin structure [1]. Hence all mesons have zero or unit isospin and transform as $(\mathbf{2},\mathbf{2})$, $(\mathbf{3},\mathbf{1})$, and /or $(\mathbf{1},\mathbf{3})$ with respect to SU(2)×SU(2) [to be precise, large-N_c QCD has a U(2)×U(2) chiral symmetry]. We will assume that the order parameter of chiral symmetry breaking in QCD with two massless flavors transforms as $(\mathbf{2},\mathbf{2})$. Assuming confinement, it then follows that chiral symmetry is spontaneously broken [10]. The conserved vector and axial-vector currents V_μ^a and A_μ^a form a six-dimensional multiplet; hence they transform as $(\mathbf{3},\mathbf{1})\oplus(\mathbf{1},\mathbf{3})$. Consider the time-ordered product of vector currents

$$\Pi_{VV}^{\mu\nu}(q)\,\delta_{ab}=2i\int d^4x\, e^{iqx}\langle 0|T[V_a^\mu(x)V_b^\nu(0)]|0\rangle. \quad (1)$$

Here Π_{VV} transforms as $(\mathbf{1},\mathbf{1})\oplus(\mathbf{3},\mathbf{3})\oplus\cdots$ with respect to SU(2)×SU(2). Lorentz invariance and current conservation allow the decomposition

$$\Pi_{VV}^{\mu\nu}(q)=(q^\mu q^\nu-g^{\mu\nu}q^2)\Pi_V(Q^2), \quad (2)$$

where $Q^2=-q^2$. Identical considerations for the AA correlator lead to $\Pi_A(Q^2)$. One can write a dispersive representation of the function $\Pi_{V,A}(Q^2)$ and saturate it with an infinite number of zero-width meson states. This dispersion relation requires one subtraction; however, we will assume an unsubtracted dispersion relation and track the divergent part. We find, in the large-N_c limit,

$$\Pi_V(Q^2)=2\sum_{n=0}^{\infty}\frac{F_V^2(n)}{Q^2+M_V^2(n)}, \quad (3a)$$

$$\Pi_A(Q^2)=2\frac{F_\pi^2}{Q^2}+2\sum_{n=0}^{\infty}\frac{F_A^2(n)}{Q^2+M_A^2(n)}, \quad (3b)$$

where $F_{V,A}(n)$ and $M_{V,A}(n)$ are the vector and axial-vector decay constants and masses, respectively. Because the functions $\Pi_{V,A}(Q^2)$ transform as $(\mathbf{1},\mathbf{1})$, they have perturbative components that are easily computed in QCD perturbation theory. The Euler-Maclaurin summation formula implies the duality matching condition

$$2\int_0^\infty dn\,\frac{F_{V,A}^2(n)}{Q^2+M_{V,A}^2(n)}+\mathcal{O}(Q^{-2}) \xrightarrow[Q^2\to\infty]{} -\frac{N_c}{12\pi^2}\log Q^2+\cdots+\sum_{m=1}^{\infty}\frac{\langle\mathcal{O}\rangle_{V,A}^{d=2m}}{Q^{2m}}, \quad (4)$$

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where the ellipsis corresponds to the logarithmic divergence that appears on both sides of the equation and the $\langle\mathcal{O}\rangle$'s are Wilson coefficients of mass dimension d. The coefficient of the logarithm is computed in perturbative QCD [11,7]. The duality matching condition implies

$$F^2_{V,A}(n)/M^2_{V,A}(n) \xrightarrow[n\to\infty]{} n^{-1}. \tag{5}$$

In addition to this asymptotic constraint, there are constraints on the n dependences of the couplings and masses: (i) the existence of an OPE implies that the sums over n in Eq. (3) must generate functions that, aside from perturbative logarithms, are analytic in $1/Q^2$; (ii) the coefficients of the OPE must have factorial behavior in n;[1] (iii) chiral symmetry must be preserved. We will address the issue of chiral symmetry in detail in the next section.

III. CHIRAL CONSTRAINTS

A. Matching to perturbation theory

In the $Q^2\to\infty$ limit, duality dictates that the infinite sums over vector and axial-vector meson states match to a perturbative expansion in α_s. This expansion is defined in the asymptotically free phase where chiral symmetry is unbroken. Therefore, in the matching region, each vector meson in the infinite sum must be paired with a *degenerate* axial-vector chiral partner; pair by pair they fill out *irreducible* $(\mathbf{1},\mathbf{3})\oplus(\mathbf{3},\mathbf{1})$ representations of the chiral group. This leads to the asymptotic constraints

$$F^2_V(n)/F^2_A(n) \xrightarrow[n\to\infty]{} 1, \tag{6a}$$

$$M^2_V(n)/M^2_A(n) \xrightarrow[n\to\infty]{} 1. \tag{6b}$$

We will see that these constraints are naturally incorporated in more general statements of chiral symmetry which will be derived below. Notice that, if $M^2_{V,A}(n)$ is linear in n, Eq. (6b) implies a "universal" slope parameter.

B. Matching to the OPE

The procedures of expanding in $1/Q^2$ and summing over n in $\Pi_{V,A}(Q^2)$ do not commute. This is due to the presence of logarithms which reorder the $1/Q^2$ expansion. Matching to the OPE must be achieved by summing over n and only then expanding in $1/Q^2$. However, this noncommutativity is not true of the correlator

$$\Pi_{LR}(Q^2)\equiv\frac{1}{2}[\Pi_V(Q^2)-\Pi_A(Q^2)] \xrightarrow[Q^2\to\infty]{} \sum_{m=1}^{\infty}\frac{\langle\mathcal{Q}\rangle^{d=2m}_{(3,3)}}{Q^{2m}}. \tag{7}$$

The subscript labeling the Wilson coefficients indicates that this correlator transforms as $(\mathbf{3},\mathbf{3})$ and therefore contains no perturbative logarithm. Hence performing the sum over n does not rearrange the $1/Q^2$ expansion, and one can expand in $1/Q^2$ before performing the infinite sum over n. We will see in the next section that this commutativity is protected by chiral symmetry. Since the first two OPE coefficients in Eq. (7) vanish in QCD in the chiral limit, one reads directly from Eq. (3) the spectral-function sum rules in the large-N_c limit [13,14]:

$$\sum_{n=0}^{\infty}F^2_V(n)-\sum_{n=0}^{\infty}F^2_A(n)=F^2_\pi, \tag{8a}$$

$$\sum_{n=0}^{\infty}F^2_V(n)M^2_V(n)-\sum_{n=0}^{\infty}F^2_A(n)M^2_A(n)=0. \tag{8b}$$

These sum rules must be satisfied by any model of large-N_c QCD consistent with chiral symmetry. Because of duality, the asymptotic constraints of Eq. (6) are enforced by these sum rules.

C. Constraints from the chiral algebra

It will prove useful to give a derivation of Eq. (8) that is independent of the OPE [15,16] as it will allow contact with hadronic string models. For this purpose, it is convenient to work in the infinite-momentum frame. This is a natural choice given our interest in large (Euclidean) momenta. Of course, the results that we derive are true in all frames. A useful property of the infinite-momentum frame is that the axial charges annihilate the vacuum, $Q^a_5|0\rangle=0$. If we boost the vector mesons along the three-axis to $p_\mu=(p_0,0,0,p_3)$, we can write, in the $p_3\to\infty$ limit,

$$\langle 0|A^a_\mu|\pi^b\rangle=\delta^{ab}F_\pi p_\mu, \tag{9a}$$

$$\langle 0|A^a_\mu|A^b\rangle^{(0)}=\delta^{ab}F_A M_A\epsilon^{(0)}_\mu=\delta^{ab}F_A p_\mu+\mathcal{O}(p_3^{-1}), \tag{9b}$$

$$\langle 0|V^a_\mu|V^b\rangle^{(0)}=\delta^{ab}F_V M_V\epsilon^{(0)}_\mu=\delta^{ab}F_V p_\mu+\mathcal{O}(p_3^{-1}), \tag{9c}$$

where the superscripts in parentheses label the helicity λ. It will prove worthwhile to consider matrix elements of the axial charges as well. The matrix element for a transition from a meson β to a meson α and a pion in the infinite-momentum frame is given by

$$\mathcal{M}(\beta(\lambda')\to\alpha(\lambda)+\pi_a)=(F_\pi)^{-1}(M^2_\alpha-M^2_\beta)^{(\lambda')}\times\langle\beta|Q^5_a|\alpha\rangle^{(\lambda)}\delta_{\lambda'\lambda}, \tag{10}$$

where the Kronecker delta ensures helicity conservation and Q^5_a is a conserved axial charge. We define

$$\langle\pi_b|Q^5_a|S\rangle=-i\,\delta_{ab}G_{S\pi}/F_\pi, \tag{11a}$$

$$\langle\pi_b|Q^5_a|V_c\rangle=-i\,\epsilon_{abc}G_{V\pi}/F_\pi, \tag{11b}$$

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[1] Reference [12] points out that for $F^2_{V,A}(n)=F^2_{V,A}$ the sums over n in Eq. (3) are Euler ψ functions which satisfy (i) and (ii). The occurrence of gamma functions is reminiscent of hadronic string models.

$$\langle A_b|Q_a^5|V_c\rangle = -i\epsilon_{abc}G_{VA}/F_\pi . \qquad (11c)$$

Here S, V, and A represent meson states with I^G (J^{PC}) given by 0^+ (even^{++}), 1^+ (odd^{--}), and 1^- (odd^{++}), respectively. We suppress the helicity labels on the states as we are interested only in zero-helicity transitions.

Consider the following matrix elements of the chiral algebra:

$$\langle 0|[Q_5^a, V_\mu^b]|A^e\rangle = i\epsilon^{abc}\langle 0|A_\mu^c|A^e\rangle, \qquad (12a)$$

$$\langle 0|[Q_5^a, A_\mu^b]|V^e\rangle = i\epsilon^{abc}\langle 0|V_\mu^c|V^e\rangle, \qquad (12b)$$

$$\langle 0|[Q_5^a, V_\mu^b]|\pi^e\rangle = i\epsilon^{abc}\langle 0|A_\mu^c|\pi^e\rangle, \qquad (12c)$$

$$\langle \pi_e|[Q_a^5, Q_b^5]|\pi_d\rangle = i\epsilon_{abc}\langle \pi_e|T_c|\pi_d\rangle. \qquad (12d)$$

By inserting a complete set of states in the commutators and using Eq. (9), Eq. (11), and $\langle \pi_a|T_b|\pi_c\rangle = i\epsilon_{abc}$, it is easy to derive a cornucopia of sum rules [16]. Consider, as an example, Eq. (12a); there is a sum rule for each axial-vector state, labeled by n'. Using $Q_5^a|0\rangle = 0$ and inserting a complete set of states yields

$$-\sum_{n=0}^{\infty} \langle 0|V_\mu^b|V^f;n\rangle \delta_{J,1}\langle V^f;n|Q_5^a|A^e;n'\rangle = i\epsilon^{abc}\langle 0|A_\mu^c|A^e;n'\rangle, \qquad (13)$$

where the Kronecker delta constrains the sum to spin-1 V states. Using Eqs. (9) and (11), it is easy to derive

$$\sum_{n=0}^{\infty} F_V(n)G_{VA}^{J=1}(n,n') = F_\pi F_A(n'), \qquad (14)$$

where the superscript indicates that the sum is over spin-1 V states. The sum rules from Eq. (12) that are of relevance to this paper are

$$\sum_{n=0}^{\infty} F_V^2(n) - \sum_{n=0}^{\infty} F_A^2(n) = F_\pi^2, \qquad (15a)$$

$$\sum_{n=0}^{\infty} F_V(n)G_{V\pi}^{J=1}(n) = F_\pi^2, \qquad (15b)$$

$$\sum_{n=0}^{\infty} G_{S\pi}^2(n) + \sum_{n=0}^{\infty} G_{V\pi}^2(n) = F_\pi^2. \qquad (15c)$$

The first sum rule is the first spectral-function sum rule. We now see that, in the large-N_c limit, this sum rule is true independent of the OPE; it is a simple consequence of chiral symmetry, which is encoded in the commutators of Eq. (12). The second and third sum rules constrain the pion vector form factor and π-π scattering, respectively [16].

There are additional sum rules which involve the meson masses, and which can be derived without the OPE [16]. These sum rules require the assumption that the order parameter of chiral symmetry breaking transforms purely as (**2**,**2**). Those of relevance here are

$$\sum_{n=0}^{\infty} F_V^2(n)M_V^2(n) - \sum_{n=0}^{\infty} F_A^2(n)M_A^2(n) = 0, \qquad (16a)$$

$$\sum_{n=0}^{\infty} G_{S\pi}^2(n)M_S^2(n) - \sum_{n=0}^{\infty} G_{V\pi}^2(n)M_V^2(n) = 0. \qquad (16b)$$

The first sum rule is the second spectral-function sum rule. The second sum rule constrains π - π scattering [16].

IV. TROUBLE WITH MASS SPLITTINGS

In this section, we consider how duality and chiral symmetry constrain the meson decay constants and masses when $M_{V,A}^2(n)$ is a linear function of n. The duality constraint Eq. (5) allows the general parametrization

$$F_V^2(n) = F_V^2 + \tilde{\chi}_V(n) + \chi_V(n), \qquad (17a)$$

$$F_A^2(n) = F_A^2 + \tilde{\chi}_A(n) + \chi_A(n), \qquad (17b)$$

where $\tilde{\chi}_{V,A}(n)$ and $\chi_{V,A}(n)$ are functions that vanish as $n \to \infty$. We will regulate the sums over states with a cutoff N on the number of vector and axial-vector mesons. The general decomposition in Eq. (17) is such that $\Sigma_{n=0}^{N}\tilde{\chi}_{V,A}(n)$ are divergent with no cutoff-independent finite parts, while $\Sigma_{n=0}^{N}\chi_{V,A}(n)$ are convergent. The first spectral-function sum rule Eq. (8a) implies that

$$F_V = F_A \equiv F, \qquad (18a)$$

$$\tilde{\chi}_V(n) = \tilde{\chi}_A(n) \equiv \tilde{\chi}(n), \qquad (18b)$$

$$\chi_V(n) - \chi_A(n) \equiv \chi(n), \qquad (18c)$$

$$\sum_{n=0}^{\infty} \chi(n) = F_\pi^2, \qquad (18d)$$

while the second spectral-function sum rule Eq. (8b) requires

$$M_{V,A}^2(n) = M_{V,A}^2 + \Lambda^2 n, \qquad (19a)$$

$$\sum_{n=0}^{N} n\chi(n) = \frac{(M_A^2 - M_V^2)}{\Lambda^2}\left(F^2(N+1) + \sum_{n=0}^{N} \tilde{\chi}(n) + \sum_{n=0}^{\infty} \chi_V(n) \right) - F_\pi^2 \frac{M_A^2}{\Lambda^2}, \qquad (19b)$$

where $M_{V,A}^2$ and Λ^2 are free parameters; Eq. (19b) should be satisfied in the limit $N \to \infty$. Note that Eqs. (18a) and (19a) ensure compliance with Eqs. (6a) and (6b), respectively.

This parametrization illustrates the difficulty in satisfying the chiral constraints and the duality constraints simultaneously. There would appear to be no solution $\chi(n)$ that satisfies Eqs. (18) and (19) with $M_V \neq M_A$. For instance, by naive power counting, Eq. (18d) requires that $\chi(n)$ vanish faster than n^{-1} for large n. But, with this asymptotic behavior, the sum in Eq. (19b) cannot generate the linear divergence necessary to balance the equation. Therefore, given the assumption that $M^2_{V,A}(n)$ is linear in n, we find no solution to the duality and chiral constraints in the large-N_c limit with $M_V \neq M_A$. However, we stress that we are not able to prove that there is no solution; it is conceivable that a solution without degeneracy exists, for instance, with $\chi(n)$ a complicated function of n with alternating sign.

If the vector and axial-vector mesons are degenerate, $M_V = M_A \equiv M$, and Eq. (19b) becomes

$$\sum_{n=0}^{\infty} n\chi(n) = -F_\pi^2 \frac{M^2}{\Lambda^2}. \tag{20}$$

By naive power counting, Eqs. (20) and (18d) can be satisfied simultaneously if $\chi(n)$ vanishes faster than n^{-2}. We will return to the degenerate case below.

Group theoretically the situation is as follows. As pointed out above, if the vector and axial-vector mesons are degenerate, pair by pair they fill out *irreducible* $(\mathbf{1},\mathbf{3})\oplus(\mathbf{3},\mathbf{1})$ representations of the chiral group, which is rather trivial. In the absence of degeneracy, the vector and axial-vector mesons generally fill out infinite-dimensional *reducible* sums of $(\mathbf{1},\mathbf{3})$, $(\mathbf{3},\mathbf{1})$, and $(\mathbf{2},\mathbf{2})$ representations. The degenerate case is certainly puzzling from the point of view of symmetry. Since chiral symmetry is spontaneously broken there is no obvious symmetry to protect the degeneracy between vectors and axial vectors. One possibility is that there is an enhanced global symmetry in the large-N_c limit. It has recently been conjectured that QCD exhibits an enhanced global symmetry as the number N_f of QCD flavors is increased to some critical value [17]. This global symmetry leads to degeneracy of vectors and axial vectors.

V. THE LOVELACE-SHAPIRO-VENEZIANO STRING MODEL

Ideally, one would like to find a smooth ansatz for $F^2_{V,A}(n)$ that generates both chiral and perturbative physics. For vector and axial-vector squared masses linear in n and degenerate, this involves finding the function $\chi(n)$ that satisfies Eqs. (18d) and (20). Hadronic string models are an interesting place to look for clues. Generally, these models are interesting for large-N_c QCD because there are an infinite number of mesons exchanged.[2] Consider the following representation of the π-π scattering amplitude:

$$A(s,t,u) = -\tfrac{1}{2}\lambda\{\Phi(\alpha_s,\alpha_t)+\Phi(\alpha_s,\alpha_u)-\Phi(\alpha_t,\alpha_u)\}, \tag{21}$$

where

$$\Phi(a,b) \equiv \frac{\Gamma(1-a)\Gamma(1-b)}{\Gamma(1-a-b)} = (1-a-b)B(1-a,1-b) \tag{22}$$

and the linear Regge trajectory is

$$\alpha_s = \alpha_0 + \alpha s. \tag{23}$$

The parameter λ, the intercept α_0, and the slope α determine the scattering. Chiral symmetry requires that the amplitude have an Adler zero at the point $s=t=u=0$. This determines $\alpha_0 = \frac{1}{2}$. Scattering is then consistent with the low-energy theorems of chiral symmetry if one takes $\pi\lambda\alpha = F_\pi^{-2}$. Normalizing the Regge slope to the lightest exchanged state gives $(2\alpha)^{-1} = M_\rho^2$. Using

$$\mathrm{Im}\,\Phi(\alpha_s,\alpha_t) = -\pi\sum_{n=1}^{\infty}\frac{\Gamma(\alpha_t+n)}{\Gamma(n)\Gamma(\alpha_t)}\delta(\alpha_s-n), \tag{24}$$

it is straightforward to extract the generalized couplings and masses as a function of n. We find

$$G^2_{V\pi}(n) = G^2_{S\pi}(n) = \frac{1}{2}\chi_{\mathrm{LSV}}(n), \quad n=0,1,\ldots, \tag{25}$$

where

$$\chi_{\mathrm{LSV}}(n) \equiv \frac{F_\pi^2}{\pi}\frac{\Gamma(\frac{1}{2}+n)}{\Gamma(\frac{1}{2})\Gamma(1+n)(n+\frac{1}{2})} \tag{26}$$

and

$$M_V^2(n) = M_S^2(n) = M_\rho^2(1+2n), \quad n=0,1,\ldots. \tag{27}$$

The sum rule Eq. (15c) is then

$$\sum_{n=0}^{\infty} G^2_{S\pi}(n) + \sum_{n=0}^{\infty} G^2_{V\pi}(n) = \sum_{n=0}^{\infty}\chi_{\mathrm{LSV}}(n) = F_\pi^2, \tag{28}$$

which is indeed satisfied. The states that participate in the string amplitude are therefore in an infinite-dimensional representation of the chiral group. This representation includes states of all spins. Notice that the mass sum rule Eq. (16b) is trivially satisfied by Eqs. (25) and (27), a consequence of the fact that the amplitude with $I=2$ in the t channel vanishes, by construction, in the LSV model [18,19].

VI. STRINGY IMPLICATIONS FOR DUALITY

A. The LSV model

The LSV model is notable in that it satisfies the chiral constraints with an infinite number of mesons and is therefore consistent with large-N_c QCD. Given the symmetric appearance of the chiral sum rules in Eq. (15) one might consider $\chi_{\mathrm{LSV}}(n)$ as an ansatz for duality in Eq. (17) when the vector and axial-vector mesons are degenerate.[3] However, for n large, $\chi_{\mathrm{LSV}}(n) \to n^{-3/2}$, and therefore the sum in Eq. (20) does not converge. There is a further related problem with this ansatz. The sum over n is easy to do in the correlators of Eq. (3). For large Q the resulting functions contain fractional powers of $1/Q^2$ and therefore do not match to the

[2] Reviews of this model are given in Refs. [18,19].

[3] A generalization of the LSV model to pion scattering on an arbitrary hadronic target suggests $M_A^2(n) - M_V^2(n) = (2\alpha)^{-1}$ [20].

OPE. This is no surprise since $\chi_{\mathrm{LSV}}(n)$ generates Regge asymptotic behavior in π-π scattering.

The chiral sum rule Eq. (15b) relates $F_V(n)$ to π-π scattering and thus links duality and the LSV model. Consider the ansatz

$$F_V(n)G_{V\pi}^{J=1}(n)=\chi_{\mathrm{LSV}}(n), \quad n=0,1,\ldots, \tag{29}$$

which satisfies Eq. (15b). Using the duality matching condition Eq. (5), this implies $[G_{V\pi}^{J=1}(n)]^2\to n^{-3}$ for n large. We can immediately put this to the test in the LSV model; partial-wave projection yields

$$[G_{V\pi}^{J=1}(n)]^2=\frac{3F_\pi^2}{\pi}\left(n+\frac{1}{2}\right)^{-4}\int_{-n}^{1/2}dx\,\frac{\Gamma(x+n+1)}{\Gamma(n+1)\Gamma(x)} \times\left(2x-\frac{1}{2}+n\right). \tag{30}$$

We have not succeeded in evaluating this integral to a simple expression. Asymptotically, one finds [21]

$$[G_{V\pi}^{J=1}(n)]^2 \xrightarrow[n\to\infty]{} n^{-5/2}(\log n)^{-1} \tag{31}$$

which is not (quite) consistent with Eq. (29).

B. A generalization of the LSV model

The success of the LSV model in incorporating the chiral symmetry constraints suggests that it might be profitable to search for simple generalizations of $\chi_{\mathrm{LSV}}(n)$ that are consistent with duality as well. Consider, for instance [22],

$$\chi(n,N_M,\alpha_0)\equiv F_\pi^2\frac{\Gamma(N_M+\alpha_0)(-1)^n}{\Gamma(\alpha_0)\Gamma(N_M-n)\Gamma(1+n)(n+\alpha_0)}, \quad N_M>0. \tag{32}$$

For integral values of N_M, $\chi(n,N_M,\alpha_0)$ vanishes for $n>N_M$, while for nonintegral values $\chi(n,N_M,\alpha_0)$ is nonvanishing for all n. Using this function one can define a one-parameter coupling which interpolates between a finite and an infinite number of mesons.[4] Note that $\chi(n,\frac{1}{2},\frac{1}{2})=\chi_{\mathrm{LSV}}(n)$. We now have the asymptotic behavior

$$\chi(n,N_M,\alpha_0) \xrightarrow[n\to\infty]{} n^{-(N_M+1)}. \tag{33}$$

Therefore $\chi(n,N_M,\alpha_0)$ with $N_M>1$ serves as an ansatz for duality when the vector and axial-vector mesons are degenerate. In effect, we find

$$\sum_{n=0}^{\infty}\chi(n,N_M,\alpha_0)=F_\pi^2, \tag{34a}$$

$$\sum_{n=0}^{\infty}n\chi(n,N_M,\alpha_0)=-\alpha_0F_\pi^2, \quad N_M>1, \tag{34b}$$

which is in agreement with Eqs. (18d) and (20) when $\alpha_0=M^2/\Lambda^2$. With $\alpha_0=\frac{1}{2}$ one finds the spectrum Eq. (27) of the LSV model. This is not really surprising since the sum rules of Eq. (34) are a statement of chiral symmetry, and the Regge intercept in Eq. (27) was fixed using chiral symmetry.

VII. MODELS OF DUALITY

A. A string-inspired model

In this section we build a model of duality that is consistent with chiral symmetry and has no discontinuity in n. The vector and axial-vector mesons are degenerate so it has little to do with the real world. In our model we choose $\tilde{\chi}(n)=0$ in Eq. (17).[5] An ansatz consistent with duality and chiral symmetry is

$$M_{V,A}^2(n)=M^2+\Lambda^2 n \tag{35a}$$

$$F_V^2(n)=F^2+\eta\chi(n,N_M,M^2/\Lambda^2), \tag{35b}$$

$$F_A^2(n)=F^2+(\eta-1)\chi(n,N_M,M^2/\Lambda^2), \tag{35c}$$

where η is a free parameter and $N_M>1$. Inserting this ansatz into Eq. (3) and doing the sums over n yields

$$\Pi_{V,A}(Q^2)=-\frac{2F^2}{\Lambda^2}\psi\left(\frac{M^2+Q^2}{\Lambda^2}\right)+\cdots+\frac{2\eta F_\pi^2}{Q^2}\left[1-\epsilon_{V,A}\times\frac{\Gamma(M^2/\Lambda^2+N_M)\Gamma((M^2+Q^2)/\Lambda^2)}{\Gamma(M^2/\Lambda^2)\Gamma(N_M+(M^2+Q^2)/\Lambda^2)}\right], \tag{36a}$$

$$\Pi_{LR}(Q^2)=-\frac{F_\pi^2}{Q^2}\frac{\Gamma(M^2/\Lambda^2+N_M)\Gamma((M^2+Q^2)/\Lambda^2)}{\Gamma(M^2/\Lambda^2)\Gamma(N_M+(M^2+Q^2)/\Lambda^2)}, \tag{36b}$$

where the ellipsis represents a logarithmic divergence; $\epsilon_V=1$ and $\epsilon_A=(\eta-1)/\eta$. At large Q^2 we then have

[4]Reference [22] considers $F_V(n)G_{V\pi}^{J=1}(n)=\chi(n,N_M,1/2)$ as an ansatz for the pion vector form factor. For integer values, N_M counts the number of vector mesons that contribute to the form factor. Evidently, a fit to data gives $N_M\sim1.3$, which implies an infinite number of vector mesons.

[5]If, for instance, $\tilde{\chi}(n)\to n^{-1}$ for n large, its effect on duality is to generate logarithmic corrections to the OPE coefficients.

$$\Pi_{V,A}(Q^2)=-\frac{2F^2}{\Lambda^2}\log Q^2+\cdots$$
$$+\left[2\eta F_\pi^2-\frac{2F^2}{\Lambda^2}\left(M^2-\frac{1}{2}\Lambda^2\right)\right]\frac{1}{Q^2}$$
$$+\frac{F^2}{\Lambda^2}\left(M^4-M^2\Lambda^2+\frac{1}{6}\Lambda^4\right)\frac{1}{Q^4}$$
$$-\frac{2F^2}{3\Lambda^2}\left(M^2-\frac{1}{2}\Lambda^2\right)(M^2-\Lambda^2)M^2\frac{1}{Q^6}$$
$$-2\eta\epsilon_{V,A}F_\pi^2\frac{\Gamma(M^2/\Lambda^2+N_M)}{\Gamma(M^2/\Lambda^2)}\frac{\Lambda^{2N_M}}{Q^{2N_M+2}}$$
$$+\mathcal{O}(Q^{-2N_M-4},Q^{-8}),\tag{37a}$$

$$\Pi_{LR}(Q^2)=F_\pi^2\frac{\Gamma(M^2/\Lambda^2+N_M)}{\Gamma(M^2/\Lambda^2)}\frac{\Lambda^{2N_M}}{Q^{2N_M+2}}$$
$$+\mathcal{O}(Q^{-2N_M-4}).\tag{37b}$$

Here we see that N_M must be an integer in order to match to the OPE. Hence N_M counts the number of vector and axial-vector mesons that contribute to the $\Pi_{LR}(Q^2)$ correlator. In principle, one would expect N_M to be infinite. Taking N_M (arbitrarily) large and matching to the OPE gives

$$F^2=\frac{N_c}{24\pi^2}\Lambda^2,\tag{38a}$$

$$\langle\mathcal{O}\rangle_{V,A}^{d=2}=0=2\eta F_\pi^2-\frac{N_c}{12\pi^2}\left(M^2-\frac{1}{2}\Lambda^2\right),\tag{38b}$$

$$\langle\mathcal{O}\rangle_{V,A}^{d=4}=\frac{\alpha_s}{12\pi}\langle G_{\mu\nu}G_{\mu\nu}\rangle$$
$$=\frac{N_c}{24\pi^2}\left(M^4-M^2\Lambda^2+\frac{1}{6}\Lambda^4\right),\tag{38c}$$

$$\langle\mathcal{O}\rangle_{V}^{d=6}=-\frac{28}{9}\pi\alpha_s\langle\overline{q}q\rangle^2$$
$$=-\frac{N_c}{36\pi^2}\left(M^2-\frac{1}{2}\Lambda^2\right)(M^2-\Lambda^2)M^2,\tag{38d}$$

$$\langle\mathcal{O}\rangle_{A}^{d=6}=\frac{44}{9}\pi\alpha_s\langle\overline{q}q\rangle^2$$
$$=-\frac{N_c}{36\pi^2}\left(M^2-\frac{1}{2}\Lambda^2\right)(M^2-\Lambda^2)M^2,\tag{38e}$$

$$\vdots$$

$$\langle\mathcal{O}\rangle_{V}^{d=2N_M+2}=-2\eta F_\pi^2\frac{\Gamma(M^2/\Lambda^2+N_M)}{\Gamma(M^2/\Lambda^2)}\Lambda^{2N_M}+\cdots,\tag{38f}$$

$$\langle\mathcal{O}\rangle_{A}^{d=2N_M+2}=-2(\eta-1)F_\pi^2\frac{\Gamma(M^2/\Lambda^2+N_M)}{\Gamma(M^2/\Lambda^2)}\Lambda^{2N_M}+\cdots.\tag{38g}$$

Since there is no local QCD operator with $d=2$, we can fix η using Eq. (38b). For large N_M, there is no solution with $\langle\overline{q}q\rangle\neq 0$. This is not inconsistent with degenerate vector and axial-vector mesons. While this model is clearly unrealistic, it provides an existence proof of a smooth chirally invariant ansatz for duality with an infinite number of mesons.

B. A minimal realistic model

One way to satisfy all constraints is to make an artificial separation between the low-energy physics relevant to chiral symmetry and the high-energy physics relevant to duality [23,7]. This requires introducing a discontinuity in n. A simple ansatz [23] is

$$F_{V,A}^2(n)=\begin{cases}F_{\rho,a_1}^2, & n=0,\\ F_{V,A}^2, & n>0,\end{cases}\tag{39a}$$

$$M_{V,A}^2(n)=\begin{cases}M_{\rho,a_1}^2, & n=0,\\ M_{V,A}^2+\Lambda_{V,A}^2(n-1), & n>0.\end{cases}\tag{39b}$$

Here we have extracted the lowest-lying vector and axial-vector mesons ρ and a_1, respectively. This is the minimal nontrivial model consistent with chiral symmetry. The duality and chiral constraints then imply

$$F_V^2=F_A^2=\frac{N_c}{24\pi^2}\Lambda^2.\tag{40a}$$

$$M_V=M_A\equiv M,\tag{40b}$$

$$\Lambda_V=\Lambda_A\equiv\Lambda,\tag{40c}$$

where we have matched to the coefficient of the perturbative logarithm, and

$$F_\rho^2-F_{a_1}^2=F_\pi^2,\tag{41a}$$

$$F_\rho^2M_\rho^2-F_{a_1}^2M_{a_1}^2=0.\tag{41b}$$

Notice that the vector and axial-vector mesons in the infinite tower are degenerate. With respect to the $\Pi_{LR}(Q^2)$ correlator, this simple ansatz has been investigated in many places [13,24,25]. Here π, ρ, and a_1, together with an isoscalar S, fill out a reducible (ten-dimensional) $(\mathbf{1},\mathbf{3})\oplus(\mathbf{3},\mathbf{1})\oplus(\mathbf{2},\mathbf{2})$ representation, while all other vector and axial-vector mesons are in irreducible $(\mathbf{1},\mathbf{3})\oplus(\mathbf{3},\mathbf{1})$ representations. It is interesting that the chiral symmetry constraints effectively decouple the hadronic parameters M and Λ from low-energy chiral physics.[6] Inserting the ansatz Eq.

[6]The authors of Ref. [7] consider an ansatz given by Eq. (39) with $F_{a_1}^2=0$, match to the OPE, and experience no such decoupling. However, they do not impose the sum rules of Eq. (8); according to Eqs. (40) and (41), consistency of their ansatz with chiral symmetry requires $F_V^2=F_A^2$, $M_V^2(n)=M_A^2(n)$, *and* $M_\rho^2=0$. In this case, π and ρ are in an irreducible $(\mathbf{1},\mathbf{3})\oplus(\mathbf{3},\mathbf{1})$ representation.

(39) in Eq. (3), doing the sums over n, and matching to the OPE gives

$$\langle\mathcal{O}\rangle_{V,A}^{d=2}=0=2F_\rho^2-\frac{N_c}{12\pi^2}\left(M^2-\frac{1}{2}\Lambda^2\right), \tag{42a}$$

$$\langle\mathcal{O}\rangle_{V,A}^{d=4}=\frac{\alpha_s}{12\pi}\langle G_{\mu\nu}G_{\mu\nu}\rangle=-2F_\rho^2M_\rho^2+\frac{N_c}{24\pi^2}\times\left(M^4-M^2\Lambda^2+\frac{1}{6}\Lambda^4\right), \tag{42b}$$

$$\langle\mathcal{O}\rangle_{V}^{d=6}=-\frac{28}{9}\pi\alpha_s\langle\bar{q}q\rangle^2=2F_\rho^2M_\rho^4-\frac{N_c}{36\pi^2}\left(M^2-\frac{1}{2}\Lambda^2\right)\times(M^2-\Lambda^2)M^2, \tag{42c}$$

$$\langle\mathcal{O}\rangle_{A}^{d=6}=\frac{44}{9}\pi\alpha_s\langle\bar{q}q\rangle^2=2F_{a_1}^2M_{a_1}^4-\frac{N_c}{36\pi^2}\left(M^2-\frac{1}{2}\Lambda^2\right)\times(M^2-\Lambda^2)M^2 \tag{42d}$$

for the first few Wilson coefficients. One can develop a phenomenology for the QCD condensates with this (or other) simple parametrizations of duality. This is hampered by large uncertainties in the values of the condensates. The relations of Eq. (41) can be parametrized by a single mixing angle ϕ, via $F_\pi=F_\rho\sin\phi$, $F_{a_1}=F_\pi\cot\phi$, and $M_\rho=M_{a_1}\cos\phi$. The known vector excited states are $\rho'(1450)$, $\rho''(1700)$, and $\rho'''(2150)$ [26]. Fitting to $\rho'(1450)$, we have $M=1450$ MeV. Using Eq. (42) with F_π, M_ρ, and M as input we then find $\Lambda=1189$ MeV, which predicts $M_{\rho''}=1875$ MeV and $M_{\rho'''}=2220$ MeV. These values differ from the experimental values by amounts consistent with $\mathcal{O}(1/N_c)$ corrections. We also predict $\phi=44.4°$, compared to the value $\phi=37.4°$ resulting from fitting F_ρ directly to $\rho^0\to e^+e^-$ [26]. One then predicts $F_{a_1}=95$ MeV and $M_{a_1}=1078$ MeV, compared with the experimental values $F_{a_1}=122\pm23$ MeV and $M_{a_1}=1230\pm40$ MeV. The predicted condensates are $\alpha_s\langle G_{\mu\nu}G_{\mu\nu}\rangle=0.06$ GeV^4 and $\pi\alpha_s\langle\bar{q}q\rangle^2=1.5\times10^{-3}$ GeV^6, respectively. These values are somewhat large; recent determinations give $\alpha_s\langle G_{\mu\nu}G_{\mu\nu}\rangle=0.048\pm0.03$ GeV^4 [27] and $\pi\alpha_s\langle\bar{q}q\rangle^2=(9\pm2)\times10^{-4}$ GeV^6 [28].

This model predicts excited axial-vector states with masses $M_{a1'}=1450$ MeV, $M_{a1''}=1875$ MeV, and $M_{a1'''}=2220$ MeV. The Particle Data Group lists one excited axial-vector state $a_1'(1640)$ [26]. The splitting between this state and $\rho'(1450)$ is consistent with an $\mathcal{O}(1/N_c)$ correction. It will be very interesting to have new data on the spectrum of excited vector and axial-vector mesons. It is expected that the masses and widths of the low-lying excited vectors and axial vectors will be determined in the Hall D program at Jefferson Laboratory in the near future [29].

VIII. CONCLUSION

Two-point functions of conserved vector and axial-vector QCD currents offer an interesting system to investigate quark-hadron duality. In the large-N_c limit, the duality matching conditions are tractable and, in contrast with QCD in $1+1$ dimensions, there are chiral symmetry constraints, which take a particularly simple form. Finding a *smooth* ansatz for duality, consistent with all constraints, is equivalent to finding the infinite-dimensional matrix that mixes the irreducible chiral representations filled out by the vector and axial-vector mesons. We find no smooth solution consistent with the duality and chiral symmetry constraints when the vector and axial-vector squared masses are linear in n and nondegenerate. To avoid this degeneracy it would appear necessary to go beyond the Regge-type linear-spacing ansatz for the squared masses. In the large-N_c limit, the basic constraints of duality and chiral symmetry [see Eq. (6b)] require vector–axial-vector degeneracy in the meson spectrum at sufficiently high excitation energy.[7] The characteristic energy at which degeneracy should set in is unknown. A simple realistic model, which predicts a tower of degenerate vector and axial-vector mesons, is roughly consistent with existing data.

Although hadronic string models provide important insight into how correlators determined by sums of infinite numbers of simple poles can be consistent with chiral symmetry, they do not provide an easy analogue that satisfies the constraints of duality as well. Fundamentally this is because string models exhibit Regge asymptotic behavior for four-point functions, which is governed by fractional powers of the momentum transfer variable Q^2, while duality for two-point functions involves the OPE, which does not see fractional powers of Q^2. Hadronic string models do suggest simple generalizations that give smooth solutions to the joint duality and chiral constraints in the degenerate limit. However, the relation, if any, between these models and large-N_c QCD remains unclear.

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